

## 3-3 REVIEW: FACTORING TRINOMIALS

A method of factoring a trinomial of the form  $ax^2 + bx + c$  is suggested by the way the product of two binomials is obtained.

$$(2x + 7)(x - 5) = 2x(x - 5) + 7(x - 5)$$

$$= 2x^2 - 10x + 7x - 35$$

$$= 2x^2 - 3x - 35$$

The integers  $-10$  and  $7$  have a sum  $-3$  and a product  $-70$ , the same as the product of  $2$  and  $-35$ .

This indicates that a trinomial of the form  $ax^2 + bx + c$  can be factored if two integers can be found with a sum  $b$  and a product  $ac$ .

**Example 1.** Factor.  $2x^2 + 7x + 5$

**Solution.**

$$\underline{2x^2 + 7x + 5} \quad \text{What two integers have a sum 7 and a product 10?}$$

The integers are  $5$  and  $2$ . The trinomial can be factored by writing the second term as  $5x + 2x$  and then grouping the terms to find a common factor.

$$\begin{aligned} 2x^2 + 7x + 5 &= 2x^2 + 5x + 2x + 5 \\ &= x(2x + 5) + 1(2x + 5) \\ &= (2x + 5)(x + 1) \end{aligned}$$

**Example 2.** Factor. a)  $6x^2 - 11xy + 3y^2$                       b)  $12a^3 - 94a^2 - 16a$

**Solution.** a)  $6x^2 - 11xy + 3y^2 = 6x^2 - 9xy - 2xy + 3y^2$   
 $= 3x(2x - 3y) - y(2x - 3y)$   
 $= (2x - 3y)(3x - y)$

b)  $12a^3 - 94a^2 - 16a = 2a(6a^2 - 47a - 8)$                       Extract the  
 $= 2a(6a^2 + a - 48a - 8)$                       common factor  
 $= 2a[a(6a + 1) - 8(6a + 1)]$  first.  
 $= 2a(6a + 1)(a - 8)$

## EXERCISES 3-3

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1. Factor.

a)  $x^2 + 5x + 6$

b)  $m^2 - 9m + 20$

c)  $a^2 + 5a - 14$

d)  $m^2 - 5m - 24$

e)  $x^2 - 15x + 54$

f)  $x^2 + 5x - 84$

2. Factor.

a)  $2m^2 + 7m + 3$

b)  $5x^2 - 7x + 2$

c)  $3a^2 - 10a + 3$

d)  $4x^2 + x - 3$

e)  $3s^2 - s - 2$

f)  $2m^2 + 5m - 7$

3. Factor.

a)  $3x^2 + 7xy + 2y^2$

d)  $5m^2 - 3mn - 2n^2$

b)  $2m^2 - 5mn + 3n^2$

e)  $21x^2 - 10xy + y^2$

c)  $3a^2 + 5ab - 2b^2$

f)  $3x^2 - 16xy + 5y^2$

4. Factor.

a)  $6s^2 + 11s + 5$

d)  $3x^2 - 17x - 6$

b)  $6m^2 - m - 2$

e)  $4m^2 + 8m + 3$

c)  $2a^2 - 11a + 12$

f)  $6m^2 - 17m + 12$

5. Factor.

a)  $2x^2 + 9xy + 4y^2$

d)  $4x^2 + 11xy + 6y^2$

b)  $3a^2 - 8ab + 5b^2$

e)  $2p^2 - pq - 10q^2$

c)  $6m^2 - 7mn + 2n^2$

f)  $6s^2 - st - 15t^2$

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6. Factor.

a)  $6s^2 + 11s - 10$

d)  $8x^2 + 38x + 45$

b)  $6m^2 - m - 40$

e)  $24m^2 - 38m + 15$

c)  $10a^2 + 51a + 27$

f)  $21x^2 + 10x - 16$

7. Factor.

a)  $36x^2 - 48x - 20$

c)  $48x^2 - 200x + 200$

e)  $10a^3b - 55a^2b + 60ab$

b)  $14x^2 + 49x + 42$

d)  $24m^3 + 68m^2 + 48m$

f)  $24m^3n - 66m^2n + 45mn$

8. Factor.

a)  $m^4 + 6m^2 - 16$

d)  $12x^4 - 5x^2y^2 - 2y^4$

b)  $3x^4 - 16x^2y^2 + 5y^4$

e)  $2x^5 + 14x^3 + 20x$

c)  $2a^4 - a^2 - 15$

f)  $16s^5 - 64s^3t^2 + 60st^4$

9. Factor.

a)  $5p^2 + pq - 18q^2$

d)  $32s^2 - 92st + 45t^2$

b)  $8m^2 - 2mn - 21n^2$

e)  $24p^2 + 2pq - 15q^2$

c)  $15x^2 - 34xy + 15y^2$

f)  $-6x^2 - 17xy + 14y^2$

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10. Factor.

a)  $(3x)^2 + 2(3x) + 1$

c)  $(6m)^2 + 8(6m) + 7$

e)  $(4p)^2 + 2(4p) - 15$

b)  $(5x)^2 - 3(5x) + 2$

d)  $(2a)^2 + 8(2a) + 15$

f)  $(3x)^2 - 2(3x) - 8$

11. Factor.

a)  $(a + b)^2 + 7(a + b) + 12$

c)  $(2m + n)^2 + 3(2m + n) - 10$

e)  $(x - 2y)^2 - 10(x - 2y) + 21$

b)  $(p - q)^2 - 5(p - q) + 6$

d)  $(4x + y)^2 - 8(4x + y) + 15$

f)  $(3x + 5y)^2 - 3(3x + 5y) - 18$

12. Factor.

a)  $2(5x)^2 + 7(5x) + 3$

c)  $4(3a - b)^2 + 21(3a - b) + 5$

e)  $8(2p + q)^2 - 10(2p + q) + 3$

b)  $3(2m)^2 + 8(2m) - 3$

d)  $6(x + 5y)^2 - 7(x + 5y) + 2$

f)  $10(3x^2 + 2y)^2 - 29(3x^2 + 2y) - 21$

13. In how many ways can each trinomial be factored?

a)  $5x^2 + 20x + 20$

d)  $4x^2 + 4x - 8$

b)  $5x^2 + 5x - 10$

e)  $6x^2 + 24x + 24$

c)  $4x^2 + 16x + 16$

f)  $6x^2 + 6x - 12$

## EXERCISES 3-4

A

1. Factor.

a)  $4x^2 - 25$

d)  $9s^2 - 49t^2$

b)  $16m^2 - 81n^2$

e)  $64x^2 - 169y^2$

c)  $36a^2 - 121$

f)  $400a^2 - 81b^2$

2. Factor.

a)  $48a^2 - 147b^2$

d)  $63a^3 - 112ab^2$

b)  $50m^3 - 18m$

e)  $100x^3y^2 - 324xy^4$

c)  $20x^3 - 405xy^2$

f)  $54y - 384x^2y$

3. Factor.

a)  $x^4 - 81$

d)  $9s^4t - \frac{1}{4}t^3$

b)  $12m^4 - 75n^4$

e)  $\frac{16}{25}x^2y^2 - \frac{36}{49}y^4$

c)  $32a^4 - 1250b^4$

f)  $256x^8 - y^8$

4. Factor.

a)  $(5x - 2)^2 - 49$

c)  $16(3x - y)^2 - 81y^2$

e)  $(5m + 2)^2 - (3m - 8)^2$

b)  $4m^2 - (6m - 7)^2$

d)  $(2x - 7y)^2 - (3x + 2y)^2$

f)  $9(2a + 5b)^2 - 4(7a - 3b)^2$

5. Factor.

a)  $(a^2 + 2a)^2 - 64$

c)  $(2x^2 + 3xy)^2 - 4y^4$

e)  $(a^2 - 13a)^2 - 900$

b)  $(2x^2 - 6x)^2 - 1296$

d)  $36n^4 - (3m^2 + 7mn)^2$

f)  $2(x^2 - 10xy)^2 - 1152y^4$

6. Factor.

a)  $x^2 + 10x + 25$

d)  $x^2 - 18xy + 81y^2$

b)  $m^2 - 14m + 49$

e)  $36x^2 + 132xy + 121y^2$

c)  $4a^2 + 12a + 9$

f)  $9x^2 - 42xy + 49y^2$

**B**

7. Factor.

a)  $49m^2 + 70m + 25$

c)  $16s^2 + 88s + 121$

e)  $9m^2 - 60mn + 100n^2$

b)  $12a^2 - 108a + 243$

d)  $-32x^2 + 48xy - 18y^2$

f)  $45x^2 - 210xy + 245y^2$

8. Factor.

a)  $m^2 + 6m + 9 - n^2$

c)  $9a^2 - 12a + 4 - 49b^2$

e)  $4s^2 - 20st + 25t^2 - 9$

b)  $4x^2 - 20x + 25 - 16y^2$

d)  $x^2 + 8xy + 16y^2 - 81$

f)  $25x^2 - 80x + 64 - 64y^2$

9. Factor.

a)  $a^2 - b^2 + 8bc - 16c^2$

c)  $25 - m^2 - 12mn - 36n^2$

e)  $x^2 - a^2 - y^2 - 2ay$

b)  $x^2 - y^2 - 14yz - 49z^2$

d)  $4s^2 - 9t^2 - 12t - 4$

f)  $a^2 - 2a + 1 - b^2 + 2bc - c^2$

10. Factor.

a)  $x^2 + 9y^2 - 25z^2 - 6xy$

c)  $x^3 + x^2 - x - 1$

e)  $a^{2n} - b^{2n}$

b)  $9m^2 - 49p^2 - 4n^2 - 28np$

d)  $a^2 + 2a + 1 - b^2 + 6b - 9$

f)  $2x^4 - 20x^2 + 18$