# TRIGONOMETRIC EQUATIONS & IDENTITIES UNIT REVIEW

Name: Key

Block: ________________

Date: ________________

**Total** $\frac{\quad\quad}{95}$ = ________%

**All answers rounded to 2 decimal places unless otherwise stated.**
**Show all work where possible.**

1. Within the domain $0 \le x < 2\pi$, how many solutions are there for the following equations.

a) $\cos 5x = 0.78$

HC by $\frac{1}{5}$

2 answers on each cycle

$y = \cos\theta$

$P = \frac{2\pi}{5}$ ∴ 5 cylcles between $0 \le x < 2\pi$

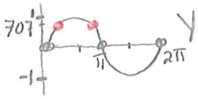
a) $2 \times 5 = 10$ solutions

*1 mark*

b) $\sin 3x = \frac{1}{\sqrt{2}}$

HC by $\frac{1}{3}$

$\approx 0.707$



2 answers on each cycle

$P = \frac{2\pi}{3}$ ∴ 3 cycles between $0 \le x < 2\pi$

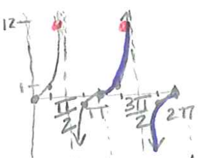
b) $2 \times 3 = 6$ solutions

*1 mark*

c) $\tan 4x = 12$

HC by $\frac{1}{4}$

or



1cycle 1cycle

one answer on each cycle

$P = \frac{\pi}{4}$ ∴ 8 cycles between $0 \le x < 2\pi$

$1 \times 8$

c) ~~8x1~~ = 8 solutions

*1 mark*

d) $\tan bx = 5$

HC by b

$P = \frac{\pi}{b}$ ∴ 2b cycles between $0 \le x < 2\pi$

1 answer on each cycle

d) $1 \times 2b = 2b$ solutions

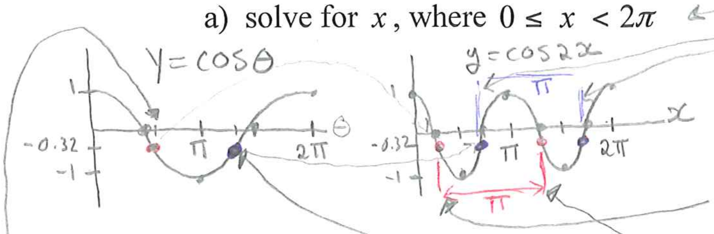
*1 mark*

2. For the equation $\cos 2x = -0.32$

HC by 2

$\boxed{P = \frac{2\pi}{2} = \pi}$ ∴ 2cycles

a) solve for $x$, where $0 \le x < 2\pi$



$\theta_2 = 2x$
$1.8965 = 2x$
$0.9482 = x_1$
$0.9482 + \pi = x_3$
$4.0898 = x_3$

$\theta_3 = 2x$
$4.3866 = 2x$
$2.1933 = x_2$
$2.1933 + \pi = x_4$
$5.3349 = x_4$

a) $x = 0.95, 2.19, 4.09, 5.33$

*2 marks*

b) determine the general solution.

$\cos\theta = -0.32$

$\theta_R = \cos^{-1}(0.32)$
$\approx 1.245$

$\theta_2 = \pi - \theta_R$
$\approx 1.8965$

$\theta_3 = \pi + \theta_R$
$= 4.38666$

$$x = \left.\begin{array}{l} 1.90 + \pi n \\ 2.19 + \pi n \\ 4.09 + \pi n \\ 5.33 + \pi n \end{array}\right\} n \in I$$

b) ________

*2 marks*

omit

~~3.~~ For the equation $3\sin x = x$, solve for $x$ and determine all solutions.

3) ____________________

*2 marks*

4. For the equation $\cos^2 x + 4\cos x - 2 = 0$

$\cos x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-2)}}{2(1)} = \frac{-4 \pm \sqrt{24}}{2}$

$\approx 0.449$ or $-4.449$

a) solve for $x$, where $0^\circ \le x < 360^\circ$

$\cos x = 0.4494897$

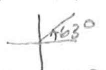
$\cos x = -4.449$

no solution

$-1 \le \cos\theta \le 1$

$x_R = \cos^{-1}(0.4494897)$

$\approx 63^\circ$



$x_1 = 63^\circ$ $x_4 = 360 - 63^\circ = 297^\circ$

a) $X = 63^\circ, 297^\circ$

*2 marks*

b) determine the general solution.

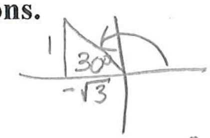
$x = \left. \begin{matrix} 63^\circ + 360^\circ n \\ 297^\circ + 360^\circ n \end{matrix} \right\} n \in I$

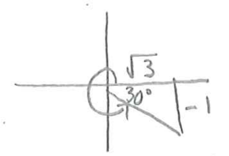
b) ____________________

*2 marks*

5. Solve the following equations **algebraically**, where $0^\circ \le x < 360^\circ$.
**Give answers as exact solutions.**

a) $\sqrt{3}\tan x = -1$





$\tan x = \frac{-1}{\sqrt{3}} \frac{y}{x}$

$x_2 = 180^\circ - 30^\circ = 150^\circ$

$x_4 = 360^\circ - 30^\circ = 330^\circ$

$x_R = 30^\circ$

a) $X = 150^\circ, 330^\circ$

*2 marks*

b) $\sec x = -2$ $\quad \frac{-2}{1} \quad \frac{r}{x}$


$x_R = 60^\circ$

$x_2 = 180^\circ - 60^\circ = 120^\circ$

$x_3 = 180^\circ + 60^\circ$
$= 240^\circ$

b) $x = 120^\circ, 240^\circ$

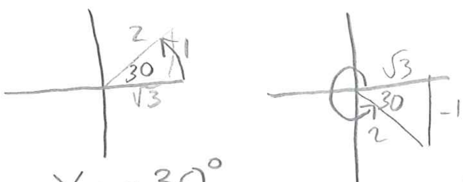
*2 marks*

c) $\cos x = \sqrt{3} - \cos x$

$+\cos x \qquad + \cos x$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2} \quad \frac{x}{r}$




$x_R = 30^\circ$

$x_1 = 30^\circ$

$x_4 = 360^\circ - 30^\circ = 330^\circ$

c) $x = 30^\circ, 330^\circ$

*2 marks*

d) $3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm \frac{1}{\sqrt{3}} \quad \frac{y}{x}$


$x_R = 30^\circ$

$x_1 = 30^\circ$

$x_3 = 30^\circ + 180^\circ$
$= 210^\circ$

$x_2 = 180^\circ - 30^\circ$
$= 150^\circ$

$x_4 = 150^\circ + 180^\circ$
$= 330^\circ$

d) $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

*3 marks*

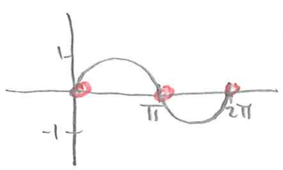
6. Solve the following equations **algebraically**, where $0 \le x < 2\pi$.
**Give answers as exact solutions.**

$2\pi$ is not part of domain

a) $2\sin^2 x + \sin x = 0$

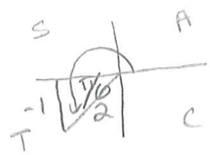
$\sin x(2\sin x + 1) = 0$

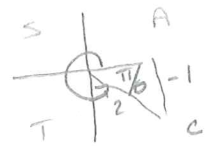
$\sin x = 0$ $\quad$ $\sin x = \frac{-1}{2}$




$X = 0$

$X = \pi$




$X_R = \frac{\pi}{6}$

$X_3 = \pi + \frac{\pi}{6}$

$= \frac{7\pi}{6}$

$X_4 = 2\pi - \frac{\pi}{6}$

$= \frac{11\pi}{6}$

a) $X = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

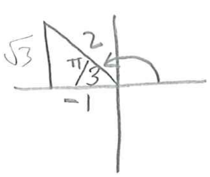
*3 marks*

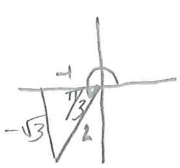
b) $2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$\cos x = \frac{-1}{2}$ $\quad$ $\cos x = -1$


$X = \pi$




$X_R = \frac{\pi}{3}$

$X_2 = \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$X_3 = \pi + \frac{\pi}{3}$

$= \frac{4\pi}{3}$

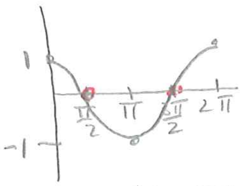
b) $X = \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

*3 marks*

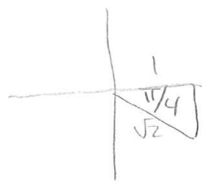
c) $\sqrt{2}\cos^2 x - \cos x = 0$

$\cos x(\sqrt{2}\cos x - 1) = 0$

$\cos x = 0$ $\cos x = \frac{1}{\sqrt{2}} \frac{x}{r}$








$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $x_1 = \frac{\pi}{4}$ $x_2 = 2\pi - \frac{\pi}{4}$

$= \frac{7\pi}{4}$

c) $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{4}$

*3 marks*

HC by $\frac{1}{2}$

d) $\sin 2x = -\frac{1}{2}$

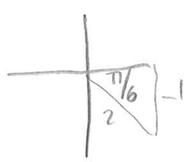
$P = \frac{2\pi}{2} = \pi$ $\therefore$ 2 cyles for $0 \le x < 2\pi$

$\theta = 2x$

$\sin\theta = -\frac{1}{2}$

2 answers on each cylce x 2 cycles

= 4 solutions


$\frac{7\pi}{12} + \pi = \frac{19\pi}{12}$

$\theta_3 = \pi + \frac{\pi}{6}$ $\theta_4 = 2\pi - \frac{\pi}{6}$

$= \frac{7\pi}{6}$ $= \frac{11\pi}{6}$

$\frac{11\pi}{12} + \pi = \frac{23\pi}{12}$

$\frac{7\pi}{6} = 2x$ $\frac{11\pi}{6} = 2x$

$x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$\frac{7\pi}{6} \times \frac{1}{2} = x$ $\frac{11\pi}{6} \times \frac{1}{2} = x$

d) ____________

*3 marks*

$\frac{7\pi}{12} = x$ $\frac{11\pi}{12} = x$

Add Period $= \pi$ to get the next 2 solutions

H & by 2

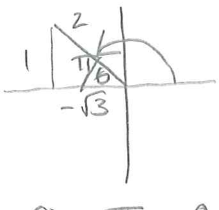
e) $\cos\frac{1}{2}x = -\frac{\sqrt{3}}{2}$

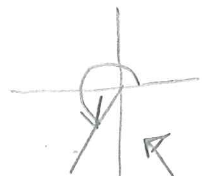
$P = \frac{2\pi}{\frac{1}{2}} = 4\pi$ ∴ half a cycle for $0 \le x < 2\pi$


only 1 solution in domain

$\theta = \frac{1}{2}x$

$\cos\theta = -\frac{\sqrt{3}}{2}$



on second half of cycle so will not be in domain

$\theta_R = \frac{\pi}{6}$ $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\frac{1}{2}x = \frac{5\pi}{6}$

$x = 2 \cdot \frac{5\pi}{6} = \frac{5\pi}{3}$

e) $x = \frac{5\pi}{3}$

*3 marks*

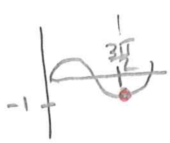
7. Simplify using the sum and difference identities.

a) $\cos\left(\frac{3\pi}{2} - x\right) = \cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x$

$0(\cos x) + -1(\sin x)$

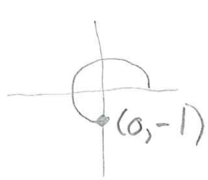
$0 + -\sin x$

$= -\sin x$




sinθ

cosθ.




$\cos\frac{3\pi}{2} = \frac{x}{r} = \frac{0}{r} = 0$

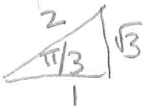
$\sin\frac{3\pi}{2} = \frac{y}{r} = \frac{-1}{1} = -1$

a) $-\sin x$

*2 marks*

b) If $\tan x$ is $-1$, simplify $\tan\left(\frac{\pi}{3} + x\right)$. Answer exact value and rationalize denominator.

$\tan x = -1$




$\tan\frac{\pi}{3} = \frac{y}{x} = \frac{\sqrt{3}}{1}$

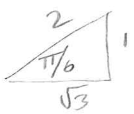
$\tan\frac{\pi}{3} = \sqrt{3}$

$= \frac{\tan\frac{\pi}{3} + \tan x}{1 - \tan\frac{\pi}{3}\tan x} = \frac{\sqrt{3} + -1}{1 - \sqrt{3}(-1)}$

$= \frac{(\sqrt{3} - 1)}{(1 + \sqrt{3})}\frac{(1 - \sqrt{3})}{(1 - \sqrt{3})}$

$= \frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - 3}$

$= \frac{2\sqrt{3} - 4}{-2}$




$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\sin\frac{\pi}{6} = \frac{1}{2}$

$\cos\frac{\pi}{3} = \frac{1}{2}$ $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

b) $-\sqrt{3} + 2$

*3 marks*

c) $\sin\left(\frac{\pi}{3} - x\right) - \left[\cos\left(\frac{\pi}{6} + x\right)\right]$

$= \sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\sin x - \left(\cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x\right)$

$= \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$

$= 0$

c) $0$

*3 marks*

$\cos\alpha\cos\beta - \sin\alpha\sin\beta = \cos(\alpha+\beta)$

d) $\cos 3x\cos 5x - \sin 3x\sin 5x$ $= \cos(3x+5x)$

$= \cos 8x$

d) $\cos 8x$

*2 marks*

$\sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin(\alpha+\beta)$

e) $\sin 6.7\cos 2.3 + \cos 6.7\sin 2.3$

$= \sin(6.7+2.3)$

$\sin 9$

e) $\sin 9$

*2 marks*

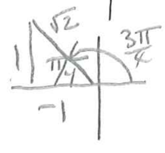
f) Find the exact value of $\sin\left(\frac{17\pi}{12}\right)$ Hint: use sum identity

$\frac{\pi}{3} = \frac{4\pi}{12}$

$\frac{\pi}{4} = \frac{3\pi}{12}$

$\frac{2\pi}{3} = \frac{8\pi}{12}$

$\frac{3\pi}{4} = \frac{9\pi}{12}$


$\frac{8\pi}{12} + \frac{9\pi}{12} = \frac{17\pi}{12}$

$\sin\left(\frac{2\pi}{3} + \frac{3\pi}{4}\right) = \sin\frac{2\pi}{3}\cos\frac{3\pi}{4} + \cos\frac{2\pi}{3}\sin\frac{3\pi}{4}$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{-1}{\sqrt{2}}\right) + \left(\frac{-1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$\frac{-\sqrt{3}}{2\sqrt{2}} + \frac{-1}{2\sqrt{2}}$

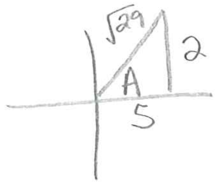
$\frac{-\sqrt{3}-1}{2\sqrt{2}} \frac{\times\sqrt{2}}{\times\sqrt{2}} = \frac{-\sqrt{6}-\sqrt{2}}{4}$

f) $\frac{-\sqrt{6}-\sqrt{2}}{4}$

*3 marks*

g) Find the exact value of $\cos(A - B)$ given:

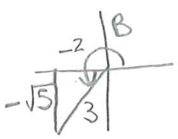
$\tan A = \frac{2}{5}$ if angle $A$ is in quadrant 1 and $\cos B = -\frac{2}{3}$ if angle $B$ is in quadrant 3.




$2^2 + 5^2 = r^2$

$29 = r^2$

$\pm\sqrt{29} = r$




$3^2 - (-2)^2 = y^2$

$9 - 4 = y^2$

$\pm\sqrt{5} = y$

$$\cos(A-B) = \cos A\cos B + \sin A \sin B$$

$$= \left(\frac{5}{\sqrt{29}}\right)\left(\frac{-2}{3}\right) + \left(\frac{2}{\sqrt{29}}\right)\left(\frac{-\sqrt{5}}{3}\right)$$

$$= \frac{-10}{3\sqrt{29}} + \frac{-2\sqrt{5}}{3\sqrt{29}}$$

$$= \frac{(-10 - 2\sqrt{5}) \times \sqrt{29}}{3\sqrt{29} \times \sqrt{29}}$$

$$= \frac{-10\sqrt{29} - 2\sqrt{145}}{87}$$

g) $\frac{-10\sqrt{29} - 2\sqrt{145}}{87}$

*3 marks*

8. Write the following as a single trigonometric function.

a) $\cos 24 \sin 24$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\frac{1}{2}\sin 2\theta = \sin\theta\cos\theta$

$= \frac{1}{2}\sin(2 \cdot 24)$

$= \frac{1}{2}\sin 48$

a) $\frac{\sin 48}{2}$

*2 marks*

b) $1 - 2\sin^2\frac{\pi}{6}$

$\cos 2\theta = 1 - 2\sin^2\theta$

$= \cos\left[2\left(\frac{\pi}{6}\right)\right]$

$= \cos\frac{\pi}{3}$

b) $\cos\frac{\pi}{3}$

*2 marks*

c) $\frac{1}{2}\sin 12\theta \cos 12\theta$

$\frac{2\sin\theta\cos\theta}{4} = \frac{\sin 2\theta}{4}$

$\frac{1}{2}\sin\theta\cos\theta = \frac{\sin 2\theta}{4}$

$= \frac{\sin(2(12\theta))}{4}$

$= \frac{\sin 24\theta}{4}$

c) $\frac{\sin 24\theta}{4}$

*2 marks*

d) $\frac{2\tan 4\theta}{5 - 5\tan^2 4\theta}$

$\frac{2\tan 4\theta}{5(1-\tan^2 4\theta)}$

$= \frac{1}{5}\left(\frac{2\tan 4\theta}{1-\tan^2 4\theta}\right)$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

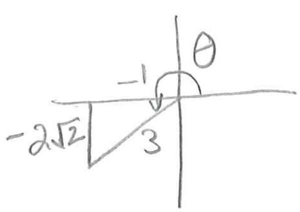
$= \frac{1}{5}\tan(2(4\theta))$

$= \frac{1}{5}\tan 8\theta$

d) $\frac{\tan 8\theta}{5}$

*2 marks*

9. If $\cos\theta = -\frac{1}{3}$ and $\theta$ is in Quadrant III. Evaluate $\sin 2\theta$.




$y^2 = 3^2 - (-1)^2$

$= 9 - 1$

$y^2 = 8$

$y = \pm 2\sqrt{2}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-2\sqrt{2}}{3}\right)\left(\frac{-1}{3}\right)$

$= \frac{4\sqrt{2}}{9}$

9. $\frac{4\sqrt{2}}{9}$

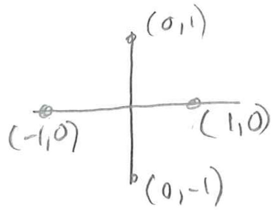
*3 marks*

10. Determine all restrictions for $\dfrac{\csc\theta}{1-\cos\theta}$.

$\csc\theta \neq \phi \qquad \theta = 0, \pi, 2\pi$

$1-\cos\theta \neq 0$

$\cos\theta \neq 1 \qquad \theta = 0, 2\pi$




$\cos\theta = \frac{x}{r}$

$\csc\theta = \frac{r}{y}$ 0

10. $\theta \neq \pi n,\ n \in I$

*2 marks*

11. Prove the following identities.

a)

$$\cot\theta\sec\theta\sin\theta = 1$$

| | |
|---|---|
| $\frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}\ \frac{1}{\cancel{\cos\theta}}\ \cancel{\sin\theta}$ | |
| 1 | 1 |
| LS = | RS |

*3 marks*

b)

$$\cot\theta + \tan\theta = \csc\theta\sec\theta$$

| | |
|---|---|
| $\left(\frac{\cos\theta}{\cos\theta}\right)\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\left(\frac{\sin\theta}{\sin\theta}\right)$ | $\frac{1}{\sin\theta}\ \frac{1}{\cos\theta}$ |
| $\frac{\cos^2\theta + \sin^2\theta}{\cos\theta\sin\theta}$ | $\frac{1}{\cos\theta\sin\theta}$ |
| $\frac{1}{\cos\theta\sin\theta}$ | |
| LS | RS |

*3 marks*

c) $\dfrac{\sin\theta + \tan\theta}{\cos\theta + 1} = \tan\theta$

$$\frac{\frac{(\cos\theta)}{(\cos\theta)}\frac{\sin\theta}{1} + \frac{\sin\theta}{\cos\theta}}{\cos\theta + 1}$$

$$\frac{\cos\theta\sin\theta + \sin\theta}{\cos\theta} \div (\cos\theta + 1)$$

$$\frac{\sin\theta\cancel{(\cos\theta+1)}}{\cos\theta} \times \frac{1}{\cancel{\cos\theta+1}}$$

$$\frac{\sin\theta}{\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta}$$

LS RS

*3 marks*

d) $\cos^2\theta = \dfrac{\cot^2\theta}{1 + \cot^2\theta}$

$$\frac{\frac{\cos^2\theta}{\sin^2\theta}}{\csc^2\theta}$$

$$\frac{\cos^2\theta}{\sin^2\theta} \div \frac{1}{\sin^2\theta}$$

$$\frac{\cos^2\theta}{\cancel{\sin^2\theta}} \times \frac{\cancel{\sin^2\theta}}{1}$$

$$\cos^2\theta$$

*3 marks*

e) $$\frac{(\cot\theta - 1)}{(\cot\theta - 1)}\frac{(1 + \tan\theta)}{(1 + \cot\theta)} = \frac{1 - \tan\theta}{(\cot\theta - 1)}\frac{(\cot\theta + 1)}{(\cot\theta + 1)}$$

$$\frac{\cot\theta + \cot\theta\tan\theta - 1 + \tan\theta}{\cot^2\theta - 1}$$

$$\frac{\cot\theta + 1 - 1 + \tan\theta}{\cot^2\theta - 1}$$

$$\frac{\cot\theta + \tan\theta}{\cot^2\theta - 1}$$

$$\frac{\cot\theta + 1 - \tan\theta\cot\theta + \tan\theta}{\cot^2\theta - 1}$$

$$\frac{\cot\theta + 1 - 1 + \tan\theta}{\cot^2\theta - 1}$$

$$\frac{\cot\theta + \tan\theta}{\cos^2\theta - 1}$$

LS | RS

*3 marks*

f) $$\frac{\tan\theta}{\cos\theta - \sec\theta} = -\csc\theta$$

$$\frac{-1}{\sin\theta}$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{\cos\theta}{\cos\theta}\right)\cos\theta - \frac{1}{\cos\theta}}$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta - 1}{\cos\theta}}$$

$$\frac{\sin\theta}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{\cos^2\theta + 1}$$

*3 marks*

$$\frac{\sin\theta}{-\sin^2\theta}$$

$$\frac{-1}{\sin\theta}$$

g)

$$\frac{\sin\theta}{1+\cos\theta} = \left(\frac{1-\cos\theta}{\sin\theta}\right)\frac{(1+\cos\theta)}{1+\cos\theta}$$

$$\frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$\frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}(1+\cos\theta)}$$

$$\frac{\sin\theta}{1+\cos\theta}$$

LS. | RS

*3 marks*

h)

$$\frac{\sin 2\theta}{1-\cos 2\theta} = \cot\theta$$

$$\frac{2\sin\theta\cos\theta}{1-(1-2\sin^2\theta)} \quad \Bigg| \quad \frac{\cos\theta}{\sin\theta}$$

$$\frac{2\sin\theta\cos\theta}{1-1+2\sin^2\theta}$$

$$\frac{\cancel{2}\cancel{\sin\theta}\cos\theta}{\cancel{2}\sin^{\cancel{2}}\theta}$$

$$\frac{\cos\theta}{\sin\theta}$$

*3 marks*