

Exponents & Logarithms

Unit Review

Name: _____

Solutions

Block: _____

Date: _____

Total _____ = _____ %
73

All answers rounded to 2 decimal places unless otherwise stated.

1. Change the following to exponential form: $\log_m a = p$

$$1. \quad \underline{a = m^p} \quad 1 \text{ mark}$$

2. Change the following to logarithmic form: $c^d = e$

$$2. \quad \underline{\log_c e = d} \quad 1 \text{ mark}$$

3. Evaluate: $\log_5 70 = \frac{\log 70}{\log 5} =$

$$70 = 5^{\square}$$

$$3. \quad \underline{\approx 2.64} \quad 1 \text{ mark}$$

4. Solve the following for x .

a) $9^{x+1} = 81^{x-4}$

$$9^{x+1} = (9^2)^{x-4}$$

$$x+1 = 2x-8$$

$$9 = x$$

$$\boxed{9}^{x+1} = \boxed{9}^{2x-8}$$

$$x = 9$$

a) $\underline{\hspace{10em}}$ 2 marks

b) $125^{x+2} = \left(\frac{1}{5}\right)^{1-5x}$

$$(5^3)^{x+2} = (5^{-1})^{1-5x}$$

$$\boxed{5}^{3x+6} = \boxed{5}^{-1+5x}$$

$$3x+6 = -1+5x$$

$$-2x = -7$$

$$x = \frac{7}{2}$$

b) $\underline{x = \frac{7}{2}}$ 2 marks

c) $7^{3x+1} = 5^x$

$$\log 7^{3x+1} = \log 5^x$$

$$(3x+1) \log 7 = x \log 5$$

$$3x \log 7 + \log 7 = x \log 5$$

$$3x \log 7 - x \log 5 = -\log 7$$

d) $2(5)^x = 3^{x+1}$

$$\log 2 + x \log 5 = (x+1) \log 3$$

$$\log 2 + x \log 5 = x \log 3 + \log 3$$

$$x \log 5 - x \log 3 = \log 3 - \log 2$$

$$x(\log 5 - \log 3) = \log 3 - \log 2$$

$$x = \frac{\log 3 - \log 2}{\log 5 - \log 3}$$

e) $\log_5 x = 4$

$$x = 5^4$$

$$x = 625$$

f) $\log_x 32 = \frac{5}{3}$

$$32 = x^{\frac{5}{3}}$$

$$\sqrt[3/5]{32} = \left(x^{\frac{5}{3}}\right)^{3/5}$$

$$\left(\sqrt[3/5]{32}\right)^3 = x$$

$$2^3 = x$$

$$8 = x$$

$$x(3 \log 7 - \log 5) = -\log 7$$

$$x = \frac{-\log 7}{3 \log 7 - \log 5}$$

$$x = \frac{-\log 7}{\log \left(\frac{3^3 7}{5}\right)}$$

c)

2 marks

$$x = 0.46$$

$$x = \frac{\log \left(\frac{3}{2}\right)}{\log \left(\frac{5}{3}\right)}$$

$$x = 0.79$$

d)

2 marks

e)

1 mark

f)

2 marks

$x > 0$

g) $\log_{3x} 81 = 2$

$$\sqrt{81} = \sqrt{(3x)^2}$$

$$9 = 3x$$

$$3 = x$$

g) $x = 3$
2 marks

h) $\log_3(\log_x(\log_4 16)) = -1$

$$\log_3(\log_x 2) = -1$$

$$\log_x 2 = 3^{-1}$$

$$x = 8$$

$$\log_x 2 = \frac{1}{3}$$

$$(2)^3 = \left(x^{\frac{1}{3}}\right)^3$$

h) $x = 8$
2 marks

i) $6^{2\log_6 x + \log_6 x} = 125$

$$6^{\log_6 x^2 + \log_6 x} = 125$$

$$6^{\log_6(x^3)} = 125$$

$$\therefore x^3 = 125$$

$$x = 5$$

i) $x = 5$
2 marks

5. Use the laws of logarithms for the following.

a) Write the expression in terms of $\log x$ and $\log y$. $\log \frac{x^3}{10y}$

$$\begin{aligned} & \log \left(\frac{x^3}{10y} \right) \\ &= \log x^3 - \log 10y \\ &= 3 \log x - [\log 10 + \log y] \\ &= 3 \log x - \log 10 - \log y \\ &= 3 \log x - 1 - \log y \end{aligned}$$

a) _____
3 marks

b) Write the expression as a single logarithm. $3 \log a - \log b - \frac{1}{2} \log c$

$$\begin{aligned} &= 3 \log a - \log b - \frac{1}{2} \log c \\ &= \log a^3 - \log b - \log c^{\frac{1}{2}} \\ &= \log a^3 - [\log b + \log c^{\frac{1}{2}}] \\ &= \log a^3 - \log(bc^{\frac{1}{2}}) \\ &= \log \left(\frac{a^3}{bc^{\frac{1}{2}}} \right) \end{aligned}$$

b) _____
3 marks

- c) If $\log 5 = a$ and $\log 36 = b$, determine an expression for $\log \frac{6}{25}$ in terms of a and b .

$$\begin{aligned} \log 5 &= a & \log 6^2 &= b & \therefore &= \log \left(\frac{6}{25} \right) \\ & & 2 \log 6 &= b & &= \log 6 - \log 25 \\ & & \log 6 &= \frac{b}{2} & &= \frac{b}{2} - \log 5^2 \\ & & & & &= \frac{b}{2} - 2 \log 5 \\ & & & & &= \frac{b}{2} - 2a \end{aligned}$$

3 marks

- d) If $\log x = a$ and $\log y = b$ what is $\log \left(\frac{100x^2}{y^3} \right)$ in terms of a and b .

$$\begin{aligned} &= \log \left(\frac{100x^2}{y^3} \right) \\ &= \log 100x^2 - \log y^3 \\ &= \log 100 + \log x^2 - 3 \log y \\ &= 2 + 2 \log x - 3 \log y \end{aligned}$$

d) $2 + 2a - 3b$

3 marks

6. Solve for x algebraically.

a) $\log_3(x-4) + \log_3(x-2) = 1$

$x > 4$ Domain

$$\log_3(x-4)(x-2) = 1$$

$$\therefore (x-4)(x-2) = 3^1$$

$$x^2 - 6x + 8 = 3$$

$$x^2 - 6x + 5 = 0$$

$$(x-5)(x-1) = 0$$

$$x = 5 \quad \text{or} \quad x = 1$$

$x = 1$ reject

a)

$$x = 5$$

3 marks

$$b) \log_7 x + \log_7(x-1) = \log_7 12$$

$$x > 1$$

$$\log_7 x(x-1) = \log_7 12$$

$$\therefore x(x-1) = 12$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x = 4 \text{ or } x = -3$$

$$x = -3$$

reject

$$x = 4$$

b) _____
3 marks

$$c) 2\log_2(x-6) - \log_2 x = 3$$

$$x > 6$$

$$\log_2(x-6)^2 - \log_2 x = 3$$

$$\log_2 \left[\frac{(x-6)^2}{x} \right] = 3$$

$$x = 18 \text{ or } x = 2$$

reject

$$\therefore \frac{(x-6)^2}{x} = 2^3$$

$$\frac{x^2 - 12x + 36}{x} = 8$$

$$x^2 - 12x + 36 = 8x$$

$$x^2 - 20x + 36 = 0$$

$$(x-18)(x-2) = 0$$

c) _____
3 marks

$$x = 18$$

$$x = -1 \quad y = 2^1 - 3 = -1$$

$$x = 1 \quad y = 2^3 - 3 = 5$$

$$x = -2 \quad y = 2^0 - 3 = -2$$

7. a) Graph $y = 2^{x+2} - 3$. Indicate the asymptote with a dotted line. Clearly show at least four points on the graph.

horizontal asymptote at $y = -3$

y-intercept $x = 0$

$$y = 2^{0+2} - 3$$

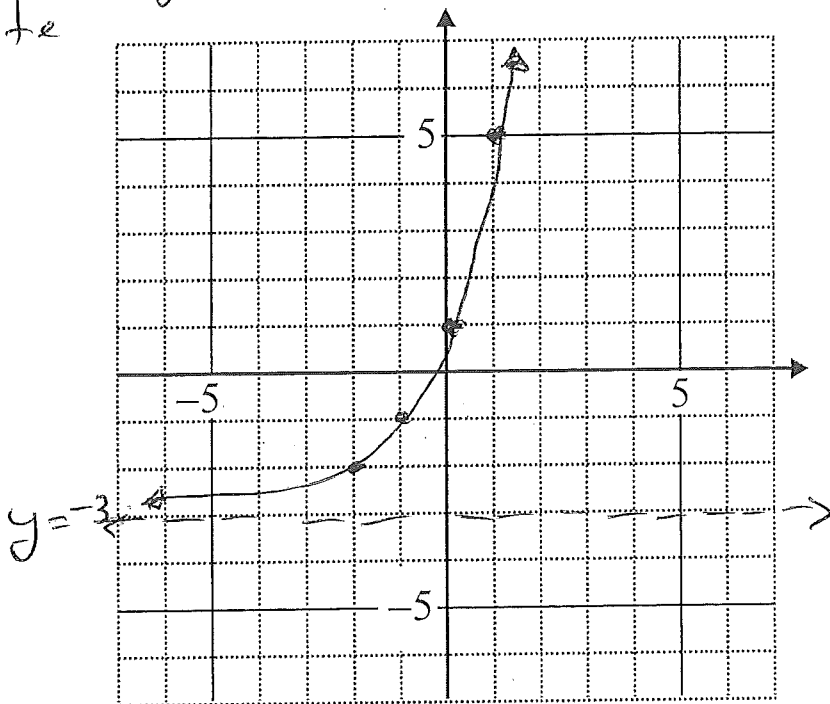
$$y = 2^2 - 3$$

$$y = 1$$

x-intercept

$$0 = 2^{x+3} - 3$$

$$2^{x+3} = 3$$



2 marks

b) Determine the range of $y = 2^{x+2} - 3$.

$$\{y \mid y > -3, y \in \mathbb{R}\}$$

b) _____
1 mark

c) Algebraically determine the x-intercept of $y = 2^{x+2} - 3$.

$$0 = 2^{x+2} - 3$$

$$2^{x+2} = 3$$

$$(x+2) \log 2 = \log 3$$

$$x+2 = \frac{\log 3}{\log 2}$$

$$x = \frac{\log 3}{\log 2} - 2$$

c) $x = -0.42$

1 mark

8. If you invest \$500 at 6% compounded annually, how many years (to the nearest tenth) would it take for your investment to grow to \$1300?

$$A = P(1+i)^n$$

$$1300 = 500(1+0.06)^n$$

$$1300 = 500(1.06)^n$$

$$\frac{13}{5} = (1.06)^n$$

$$\log\left(\frac{13}{5}\right) = n \log(1.06)$$

$$n = \frac{\log\left(\frac{13}{5}\right)}{\log(1.06)}$$

$$n = 16.4 \text{ years}$$

8. _____
3 marks

9. The population of a certain bacteria can multiply threefold in 24 hours. If there are 500 bacteria now, how many will there be in 96 hours?

$$A = A_0(1+r)^{\frac{t}{P}}$$

$$A = 500(3)^{\frac{96}{24}}$$

$$= 500(3)^4$$

$$= 40500$$

9. 40500 bacteria

3 marks

10. A radioactive isotope has a half-life of 173 days. How much of a sample of 100 grams of the isotope would remain after 732 days?

$$A = A_0(1-d)^{\frac{t}{P}}$$

$$A = 100(0.5)^{\frac{732}{173}}$$

$$A = 5.32 \text{ grams}$$

10. A = 5.32 grams

3 marks

11. A radioactive substance decays to 30% of its original mass in 15 months. Determine the half-life of this radioactive substance to the nearest month.

$$A = A_0 (1-d)^{\frac{t}{P}}$$

$$0.3 = 1 (0.5)^{\frac{15}{P}}$$

$$\log 0.3 = \frac{15}{P} \log 0.5$$

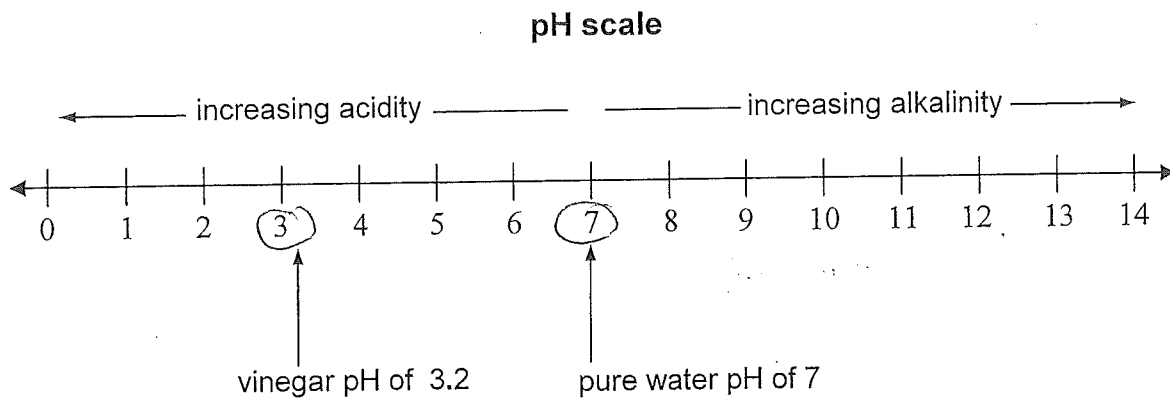
$$P \log 0.3 = 15 \log 0.5$$

$$P = \frac{15 \log 0.5}{\log 0.3}$$

$$P = 8.64 \text{ months}$$

11. _____
3 marks

12. On the pH scale, each unit change in pH represents a tenfold increase in acidity or alkalinity. According to the diagram, vinegar is how many times as acidic as pure water?



$$\frac{10^7}{10^3} = 10^4 = 10,000 \text{ times more acidic than pure water.}$$

12. _____
2 marks

13. An earthquake off the coast of Vancouver Island was measured at 8.9 on the Richter Scale and an earthquake off the coast of Alaska was measured at 6.5. How many times more intense, to the nearest whole number, was the earthquake off the coast of Vancouver Island than the one off the coast of Alaska?

$$\frac{I_V}{I_A} = \frac{10^{8.9}}{10^{6.5}} = 10^{2.4} \approx 251 \text{ times as strong.}$$

13.

2 marks

14. A major earthquake of magnitude 8.3 is 120 times as intense as a minor earthquake. Determine the magnitude, to the nearest tenth, of the minor earthquake.

Let 10^x be Intensity of minor quake

Then

$$\frac{10^{8.3}}{10^x} = 120$$

$$\therefore x = 8.3 - \log 120$$

$$x = 6.2$$

$$10^{8.3-x} = 120$$

$$(8.3-x) \log 10 = \log 120$$

$$(8.3-x)(1) = \log 120$$

2 marks

15. Determine the inverse function of $f(x) = 3^{x-1} - 2$.

$$x = 3^{y-1} - 2$$

$$x+2 = 3^{y-1}$$

$$\log(x+2) = (y-1) \log 3$$

$$\frac{\log(x+2)}{\log 3} = y-1$$

$$y = \log_3(x+2) + 1$$

$$f^{-1}(x) = \log_3(x+2) + 1$$

15.

3 marks

16. Graph $y = \log_2(x + 3)$. Indicate the asymptote with a dotted line. Clearly show at least four points on the graph.

Vertical asymptote
at $x = -3$

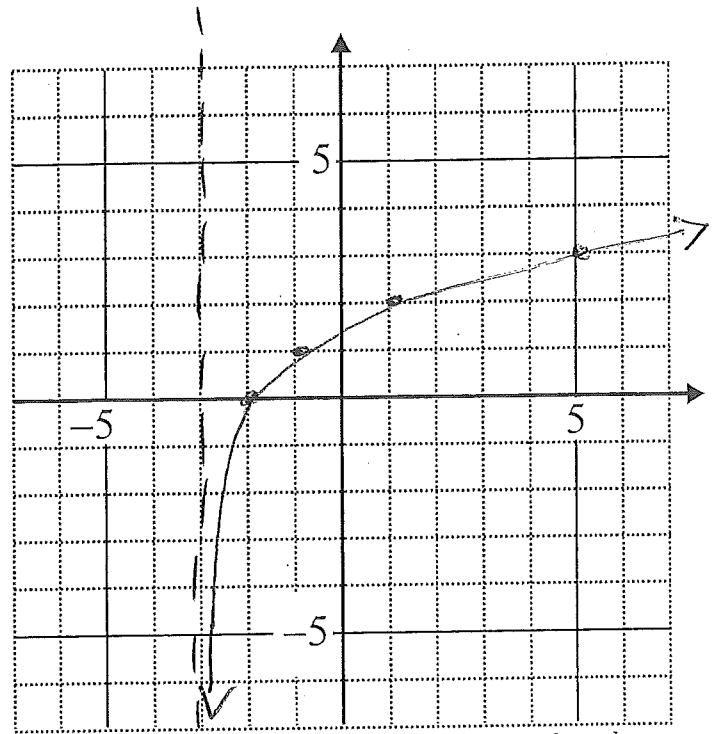
Domain: $x > -3$

$$x = 1 \text{ then } y = \log_2(4) \\ y = 2$$

$$x = -1 \text{ then } y = \log_2(2) \\ y = 1$$

$$x = -2, \text{ then } y = \log_2(1) \\ y = 0$$

$$x = 5, \text{ then } y = \log_2(8) \\ = 3$$



2 marks

$$x = -3$$

17. Algebraically determine the domain and the y -intercept of the function $y = \log_4(2x + 1) - 3$.

$$\text{Domain: } 2x + 1 > 0 \\ 2x > -1 \\ x > -\frac{1}{2}$$

$$y\text{-intercept } x = 0$$

$$y = \log_4(2 \cdot 0 + 1) - 3$$

$$y = \log_4(1) - 3$$

$$y = 0 - 3$$

$$y = -3$$

18. domain $\left\{x \mid x > -\frac{1}{2}, x \in \mathbb{R}\right\}$

1 mark

18. y -intercept $y = -3$

1 mark

19. The graph of the function $y = \log_b x$ passes through the point $(729, 6)$, determine b .

$$6 = \log_b 729$$

$$\therefore (729)^{\frac{1}{6}} = (b^6)^{\frac{1}{6}}$$

$$3 = b$$

$$b = 3$$

19.

2 marks

20. If the point $(3, 10)$ is on the graph of $y = a^x$, then what is this point on the graph of $y = \log_a x$?

Inverse of $y = a^x$

$$x = a^y$$

$$\log_a x = y$$

$$\therefore (3, 10) \rightarrow (10, 3)$$

20.

1 mark

