

Solving Exp and Log Eq WS

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9:07 AM

Solving Exponential and Logarithmic Equations

NO CALCULATORS ALLOWED!

1. Solve for x : $\log_2[\log_x(\log_3 9)] = -1$

$$\begin{aligned}x &> 0 \\x &\neq 1\end{aligned}$$

$$\begin{aligned}\log_2[\log_x 2] &= -1 \\2^{-1} &= \log_x 2 \\-\frac{1}{2} &= \log_x 2\end{aligned}$$

$$x^{\frac{1}{2}} = 2$$

$$x = 4$$

2. Solve for x : $\log_2 8 - \log_3 3 = x$.

$$\begin{aligned}3-1 &= x \\2 &= x\end{aligned}$$

3. Solve for x : $\log_5 16 - 2\log_5 x = \log_5 4$.

$$\begin{aligned}x &> 0 \\ \log_5 \frac{16}{x^2} &= \log_5 4\end{aligned}$$

$$\begin{aligned}\frac{16}{x^2} &= 4 \\4 &= x^2\end{aligned}$$

$$x = +2 \text{ or } x = -2$$

\uparrow
reject
 $x > 0$

4. Solve for x : $3(2^x) = 6^{x-2}$

$$\begin{aligned}\log 3 + x \log 2 &= (x-2) \log 6 \\ \log 3 + x \log 2 &= x \log 6 - 2 \log 6 \\ \log 3 + 2 \log 6 &= x (\log 6 - \log 2)\end{aligned}$$

$$\begin{aligned}x &= \frac{\log 3 + 2 \log 6}{\log 6 - \log 2} \\x &= \frac{\log 108}{\log 3} = \log_3 108\end{aligned}$$

5.. Solve for x : $2\log(3-x) = \log 4 + \log(6-x)$

$(x < 3)$

$$\log(3-x)^2 - \log(6-x) = \log 4$$

$$\log \frac{(3-x)^2}{6-x} = \log 4$$

$$9-6x+x^2 = 4(6-x)$$

$$x^2 - 6x + 9 = 24 - 4x$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$$x = 5 \quad \boxed{x = -3}$$

\uparrow
reject $x < 3$

$\boxed{x > 0}$

6. Solve for x : $\log(x+4) = 1 - \log 2x$

$$\begin{aligned}\log(x+4) + \log 2x &= 1 \\ \log(x+4)(2x) &= 1\end{aligned}$$

$$\begin{aligned}0 &= 2(x+5)(x-1) \\x &= -5 \quad \boxed{x = 1}\end{aligned}$$

$$10' = 2x^2 + 8x$$

$$0 = 2(x^2 + 4x - 5)$$

reject
 $x > 0$

7. Solve for x : $(\log_4 a)(\log_a 2a)(\log_{2a} x) = \log_a a^3$

$$x > 0 \quad \frac{\log a}{\log 4} \cdot \frac{\log 2a}{\log a} \cdot \frac{\log x}{\log 2a} = 3$$

$$\log_4 x = 3$$

$$4^3 = x$$

$$64 = x$$

8. Solve for x : $8^{x+2} = (4^{4x-3}) \left(\frac{1}{2}\right)^5$

$$(2^3)^{x+2} = (2^2)^{4x-3} (2^{-5})$$

$$2^{3x+6} = 2^{8x-6} \cdot 2^{-5}$$

$$2^{3x+6} = 2^{8x-11}$$

$$3x+6 = 8x-11$$

$$17 = 5x$$

$$\frac{17}{5} = x$$

9. Solve for x : $a^3 b^{x-2} = c$

$$\log_b \left(\frac{c}{a^3} \right) = x-2$$

$$b^{x-2} = \frac{c}{a^3}$$

$$\log_b \left(\frac{c}{a^3} \right) + 2 = x$$

or $\log_b c - 3 \log_b a + 2 = x$

10. Solve for x : $9^{x+2} + 3^{2x+4} + 81^{0.5x+1} = \left(\frac{1}{27}\right)^{2-x}$

$$(3^2)^{x+2} + 3^{2x+4} + (3^4)^{0.5x+1} = (3^{-3})^{2-x}$$

$$3^{2x+4} + 3^{2x+4} + 3^{2x+4} = 3^{3x-6}$$

$$3 \cdot 3^{2x+4} = 3^{3x-6}$$

$$3^{2x+5} = 3^{3x-6}$$

$$2x+5 = 3x-6$$

$$11 = x$$

Let $y = 3^{2x+4}$

$$y + y + y = 3^{3x-6}$$

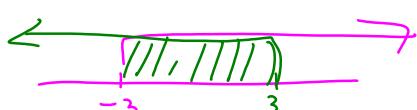
$$3y = 3^{3x-6}$$

$$3'(3^{2x+4}) = 3^{3x-6}$$

$$3^{2x+5} = 3^{3x-6}$$

11. Solve for x : $\log_2(\log_4(\log_5 x)) = -1$

$x > 0$



$$2^{-1} = \log_4 (\log_5 x)$$

$$\frac{1}{2} = \log_4 (\log_5 x)$$

$$4^{\frac{1}{2}} = \log_5 x$$

$$2 = \log_5 x$$

$$5^2 = x$$

$$\begin{array}{c} -3 \quad 3 \\ \hline x < 3 \quad x > -3 \end{array} \quad \begin{array}{l} 5^x = x \\ 25 = x \end{array}$$

12. Solve for x : $\log(3-x) + \log(3+x) = \log 5$

$$-3 < x < 3$$

$$\log(3-x)(3+x) = \log 5$$

$$9 - x^2 = 5$$

$$4 = x^2$$

$$x = \pm 2$$

13. Solve for x : $a^{5x} = (\log_c c^a)^{3x+8}$

$$a^{5x} = a^{3x+8} \quad x = 4$$

$$5x = 3x + 8$$

$$2x = 8$$

14. Solve for x in terms of b , given $\log_{27}(81x) = b$.

$$27^b = 81x \quad \frac{3^{3b}}{3^4} = x$$

$$3^{3b-4} = x$$

15. Solve for x in terms of b , given $x^{1.25} = 2b$.

$$x^{\frac{5}{4}} = 2b \quad x = (2b)^{\frac{4}{5}}$$

16. If $\log_x 3 = a$ and $\log_x 25 = b$, determine an expression for $\log_x \frac{9}{5}$.

$$\log_x 9 - \log_x 5$$

$$\log_x 3^2 - \log_x 25^{\frac{1}{2}}$$

$$2\log_x 3 - \frac{1}{2}\log_x 25$$

$$2a - \frac{1}{2}b$$

An expression equivalent to $(a^{\log_b c})(a^{\log_b c})$ is

A. $a^2 + a^{\log_b c}$

B. $(a^2)^{\log_b c}$

C. $2a^{\log_b c}$

D. $2^{\log_c a^b}$

$$a^{\log_b c + \log_b c}$$

$$a^{\log_b c^2}$$

$$a^{2 \log_b c}$$

$$(a^2)^{\log_b c}$$