

1. Suppose b is a positive constant greater than 1, and let A , B , and C be defined as follows:

$$\log_b 2 = A \quad \log_b 3 = B \quad \log_b 5 = C$$

In each case, use the properties of logarithms to evaluate the given expression in terms of A , B , and or C

a) $\log_b 6$
 $= \log_b 2 + \log_b 3$
 $= A + B$

b) $\log_b \frac{1}{6}$
 $= \log_b 1 - \log_b 6$
 $= \log_b 1 - \log_b 2 - \log_b 3$
 $= 0 - A - B = -A - B$
 or $\log_b 6^{-1}$
 $= 1 \cdot \log_b 6$
 $= (\log_b 2 + \log_b 3)$
 $= -A - B$

c) $\log_b 27$
 $= \log_b 3^3$
 $= 3 \log_b 3$
 $= 3B$

d) $\log_b \frac{1}{27}$
 $= \log_b 27^{-1}$
 $= -\log_b 27$
 $= -\log_b 3^3$
 $= -3 \log_b 3$
 $= -3B$

e) $\log_b 10$
 $= \log_b 2 + \log_b 5$
 $= A + C$

f) $\log_b 0.01$
 $= \log_b \frac{1}{100}$
 $= \log_b 100^{-1}$
 $= -\log_b 100$
 $= -\log_b 10^2$
 $= -2 \log_b 10$
 $= -2(\log_b 2 + \log_b 5)$
 $= -2(A + C)$
 $= -2A - 2C$

g) $\log_b 0.3$
 $= \log_b \frac{3}{10}$
 $= \log_b 3 - \log_b 10$
 $= \log_b 3 - (\log_b 2 + \log_b 5)$
 $= B - A - C$

h) $\log_b \left(\frac{5}{3}\right)$
 $= \log_b 5 - \log_b 3$
 $= C - B$

i) $\log_b 0.6$

$\log_b \left(\frac{3}{5}\right)$
 $\log_b 3 - \log_b 5$
 $B - C$

j) $\log_b \frac{5}{9}$
 $= \log_b 5 - \log_b 9$
 $= \log_b 5 - \log_b 3^2$
 $= \log_b 5 - 2 \log_b 3$
 $= C - 2B$

k) $\log_b \frac{5}{16}$
 $= \log_b 5 - \log_b 16$
 $= \log_b 5 - \log_b 2^4$
 $= \log_b 5 - 4 \log_b 2$
 $= C - 4A$

l) $\log_b \sqrt{5}$
 $= \frac{1}{2} \log_b 5$
 $= \frac{1}{2} C$
 $= \frac{C}{2}$

m) $\log_b \sqrt{15}$

$\frac{1}{2} \log_b 15$
 $\frac{1}{2} (\log_b 3 + \log_b 5)$
 $\frac{1}{2} (B + C)$
 $\frac{B+C}{2}$

n) $\log_b \sqrt[3]{0.4}$
 $= \frac{1}{3} \log_b 0.4$
 $= \frac{1}{3} \log_b \frac{2}{5}$
 $= \frac{1}{3} (\log_b 2 - \log_b 5)$
 $= \frac{1}{3} (A - C)$
 $= \frac{A - C}{3}$

o) $\log_b \sqrt[4]{60}$
 $= \frac{1}{4} \log_b 60$
 $= \frac{1}{4} \log_b (2^2 \cdot 3 \cdot 5)$
 $= \frac{1}{4} (\log_b 2^2 + \log_b 3 + \log_b 5)$
 $= \frac{1}{4} (2 \log_b 2 + \log_b 3 + \log_b 5)$
 $= \frac{1}{2} \log_b 2 + \frac{1}{4} \log_b 3 + \frac{1}{4} \log_b 5$
 $= \frac{1}{2} A + \frac{1}{4} B + \frac{1}{4} C$

p) $\log_b b = \frac{\log_b b}{\log_b 3}$
 $= \frac{1}{B}$

q) $\log_3(10b)$

$\frac{\log_b 10b}{\log_b 3}$
 $\frac{\log_b 10 + \log_b b}{\log_b 3}$
 $\frac{\log_b 2 + \log_b 5 + 1}{\log_b 3}$

r) $\log_{b^2} 5$

$\frac{\log_b 5}{\log_b b^2}$
 $\frac{\log_b 5}{2 \log_b b}$
 $\frac{C}{2(I)}$

s) $\log_{\sqrt{b}} 2$

$\frac{\log_b 2}{\log_b b^{1/2}}$
 $\frac{\log_b 2}{\frac{1}{2} \log_b b}$
 $\frac{A}{\frac{1}{2}(I)}$

t) $\log_{3b} 2$

$\frac{\log_b 2}{\log_b 3b}$
 $\frac{\log_b 2}{\log_b 3 + \log_b b}$
 $\frac{A}{B+1}$

$$\begin{array}{c|c|c|c}
\frac{\log_b 2 + \log_b 5 + 1}{\log_b 3} & \frac{C}{2(C)} & \frac{A}{2(A)} & \frac{-A}{B+1} \\
\hline
\frac{A+C+1}{B} & \frac{C}{2} & 2A &
\end{array}$$

u) $\log_{3b} 15$

$$\frac{\log_b 15}{\log_b 3b}$$

$$\frac{\log_b 3 + \log_b 5}{\log_b 3 + \log_b b}$$

$$\frac{B+C}{B+1}$$

v) $\log_{5b} 1.2$

$$\frac{\log_{5b} 1.2}{\log_{5b} \frac{6}{5}}$$

$$\frac{\log_b \frac{6}{5}}{\log_b 5b}$$

$$\frac{\log_b 2 + \log_b 3 - \log_b 5}{\log_b 5 + \log_b b}$$

$$\frac{A+B-C}{C+1}$$

w) $\log_{5b} 2.5$

$$\frac{\log_{5b} \frac{5}{2}}{\log_{5b} 5b}$$

$$\frac{\log_b 5 - \log_b 2}{\log_b 5 + \log_b b}$$

x) $(\log_b 5)(\log_5 b)$

~~$$\frac{\log 5}{\log b} \cdot \frac{\log b}{\log 5}$$~~

$$= 1$$

y) $(\log_b 6)(\log_6 b)$

~~$$\frac{\log 6}{\log b} \cdot \frac{\log b}{\log 6}$$~~

$$= 1$$

z) $\log_{2b} 6 + \log_{2b} \left(\frac{1}{6}\right)$

$$\frac{\log_b 6}{\log_b 2b} + \frac{\log_b \frac{1}{6}}{\log_b 2b}$$

$$\frac{\log_b 2 + \log_b 3}{\log_b 2 + \log_b b} + \frac{\log_b 1 - \log_b 6}{\log_b 2 + \log_b b}$$

$$\frac{A+B}{A+1} + \frac{0 - (\log_b 2 + \log_b 3)}{A+1}$$

$$\frac{A+B}{A+1} + \frac{-A-B}{A+1} = 0$$

or $\log_{2b} 6 + \log_{2b} 6^{-1}$

$$\log_{2b} 6 - \log_{2b} 6 = 0$$

$$\frac{\log_b \frac{1}{b}}{\log_b 18}$$

$$= \frac{\log_b 1 - \log_b b}{\log_b 3^2 + \log_b 2}$$

$$= \frac{\log_b 1 - \log_b b}{2 \log_b 3 + \log_b 2}$$

$$= \frac{0 - 1}{2B+A}$$

$$= \frac{-1}{A+2B}$$

2. Suppose that $\log A = a$, $\log B = b$, and $\log C = c$. Express the following logarithms in terms of a , b , and c .

a) $\log AB^2C$

b) $\log 10\sqrt{A}$

c) $\log \sqrt{10ABC}$

$$\log A + \log B^2 + \log C$$

$$\log 10 + \log \sqrt{A}$$

$$\frac{1}{2} \log 10ABC$$

$$\log A + 2\log B + \log C$$

$$\log 10 + \frac{1}{2} \log A$$

$$\frac{1}{2} (\log 10 + \log A + \log B + \log C)$$

$$a + 2b + c$$

$$1 + \frac{1}{2}a$$

$$\frac{1}{2}(1+a+b+c)$$

$$1 + \frac{a}{2}$$

$$\frac{1+a+b+c}{2}$$

d) $\log \frac{10A}{\sqrt{BC}}$

e) $\log A + 2 \log \frac{1}{A}$

f) $\log \frac{A}{10}$

$$\log 10 + \log A - \frac{1}{2} \log B - \frac{1}{2} \log C$$

$$\log A + 2(\log 1 - \log A)$$

$$\frac{\log A - \log 10}{2}$$

\sqrt{BC}

$$\log 10 + \log A - \frac{1}{2} \log B - \frac{1}{2} \log C$$

$$1 + a - \frac{b}{2} - \frac{c}{2}$$

 A 10

$$\left. \begin{array}{l} \log A + 2(\log 1 - \log A) \\ \log A + 2\log 1 - 2\log A \\ -\log A + 2\log 1 \\ -a + 2(0) \\ -a \end{array} \right\}$$

$$\log A - \log 10$$

$$a - 1$$

$$g) \log \frac{100A^2}{B^4 \sqrt[3]{C}}$$

$$h) \log \frac{(AB)^5}{C}$$

$$\log 100 + \log A^2 - \log B^4 - \log C^{1/3}$$

$$2 + 2\log A - 4\log B - \frac{1}{3}\log C$$

$$2 + 2a - 4b - \frac{1}{3}c$$

$$\log A^5 B^5 - \log C$$

$$\log A^5 + \log B^5 - \log C$$

$$5\log A + 5\log B - \log C$$

$$5a + 5b - c$$