# Forces and Equilibrium

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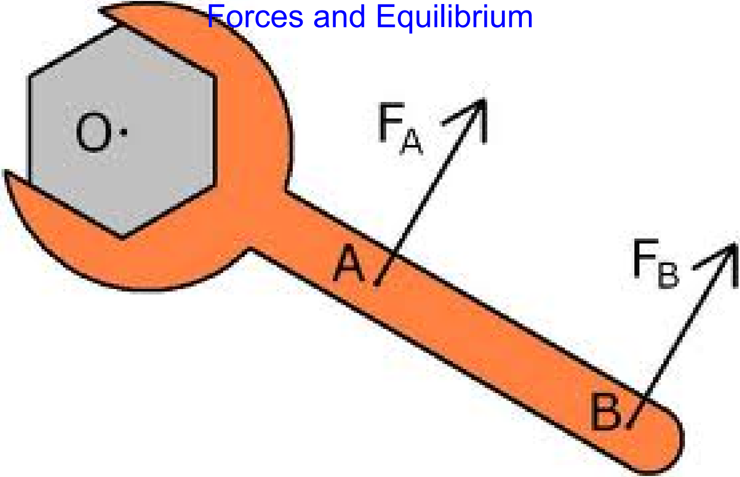


Lesson 1

**Physics 12 – Introduction to <u>Translational Equilibrium</u>** Name:

Objects on Earth have at least one force acting on them at all times (gravity). However, since many objects are at rest, there must be other forces acting on them in order for $\sum F = 0N$ (so that $a = 0m/s^2$).
For a body to be at rest, the sum of the forces acting on the body must add up to zero.

**Equilibrium** –

Since we deal with an x-component and y-component for most forces and motion, both components must add up to a net force of zero as well.

**<u>First Condition of Equilibrium</u>**

$$\sum F_X = 0 \qquad \text{AND} \qquad \sum F_Y = 0$$

A simplest case of equilibrium is the case in which two equal forces act in opposite directions on an object. The resultant force is zero, and the object does not move.

<u>Example 1</u> – Two dogs are fighting over a steak. One dog (a pug) is pulling north with a force of 3.0 N, and another dog (a schnauzer) is pulling west with a force of 4.0 N. How much force and in what direction would I have to pull on the steak so that the steak does not move?

<u>Vector Component Analysis:</u>

With what TOTAL force must you pull on the steak to hold it still (magnitude and direction)?

# Forces and Equilibrium

In order to solve equilibrium problems, you must have a good grasp of *Free Body diagrams* and of *vector components*.

$$\sum F_{net} = 0$$

$$\downarrow$$

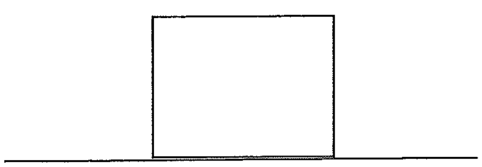
$$\sum F_x = 0 \qquad and \qquad \sum F_y = 0$$

If we find all the vector components of all the forces in the x direction and they sum to zero, can there be any acceleration in the x direction?

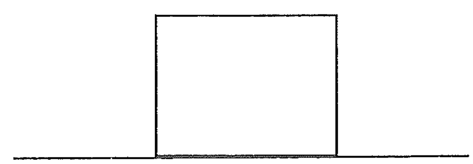
If we find all the vector components of all the forces in the y direction and they sum to zero, can there be any acceleration in the y direction?

If there is no acceleration in the x and y directions, is there any acceleration at all?

<u>**Example 1**</u> - A 2 kg book is sitting motionless on a table. Draw a diagram of the book on the table, and then draw a F.B.D. of the book to find the force applied by the table.



<u>**Example 2**</u> - A 2 kg book is sitting motionless on a table. A helium balloon is attached to the book and the tension in the balloon string is 2 N. Draw a diagram of the book on the table, and then draw a F.B.D. of the book to find the force applied by the table.



**Example 3** - A 54 N force acts at 18° and a second 45 N force acts at 50°
a. What is the magnitude and direction of the force that produces equilibrium?

**Example 4** - A 2 kg book is sitting motionless on a ramp that has an angle of 80° with respect to the vertical. For part 'C' assume that if angle decreases to 79° to the vertical, the book will begin to slide.

A) Why does the book not slide down the ramp?
B) How big is this force
C) Calculate $\mu_s$

**Introduction to Equilibrium Problems**

1. A force of 55 N acts due west on an object. What single force is required to produce equilibrium? (55 N [E])

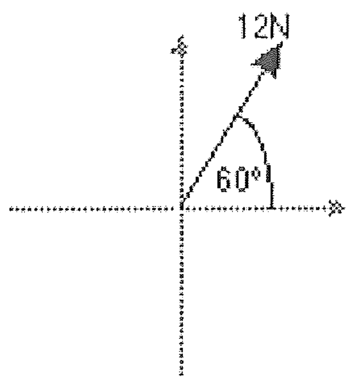
2. Two forces act on an object. One force is 6.0 N east; the second force is 8.0 N north. If the object is in equilibrium, find the magnitude and direction of the force that produces equilibrium. (10 N @ 53° S of W)

3. A 62 N force acts at 30° and a second 62 N force acts at 60°. What is the magnitude and direction of the force that produces equilibrium? (120 N @ 45° S of W)

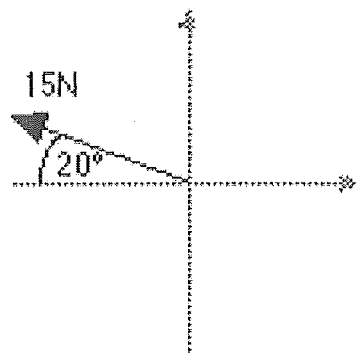
4. What force is required to satisfy the $\Sigma F_x = 0N$ condition for diagram A below? (-6.0N)

5. What force is required to satisfy the $\Sigma F_y = 0N$ condition for diagram B below?(-5.1N)

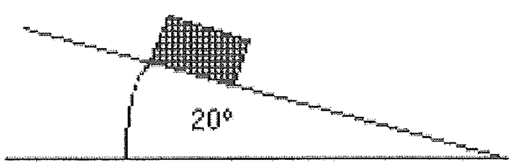
A.



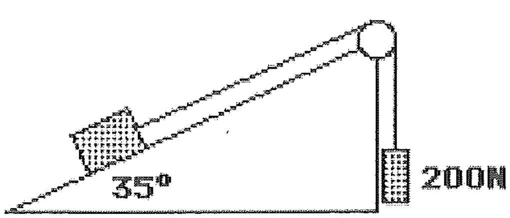

B.




6. An 8.0 kg box rests motionless on an incline. Draw an F.B.D. for the box and determine the values of each force keeping the box in an equilibrium state. (-26.8N, +73.7N)




7. In the system below the pulley and ramp are frictionless and the block is in static equilibrium. What is the mass of the block? (35.6 kg)



**Physics 12 -Translational Equilibrium** Name:

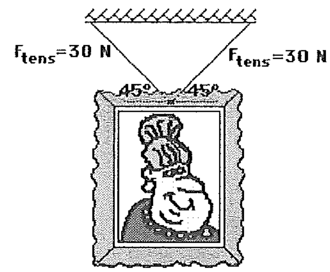
**First Condition of Equilibrium**

$$\sum F_X = 0 \qquad \text{AND} \qquad \sum F_Y = 0$$

**A body in translational equilibrium will have no acceleration in the x or y directions.**

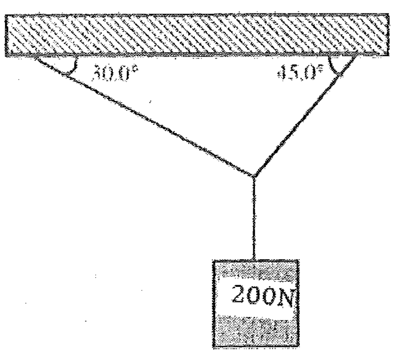
**Equilibrium is a very important concept in the Engineering and Design of any structure as this is a good thing for stationary objects like bridges and buildings.**

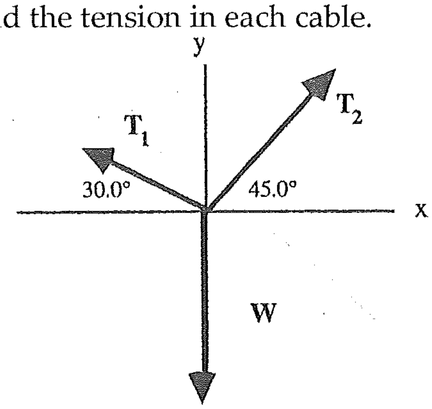
Yesterday we discussed the simplest cases of equilibrium in which forces were acting on an object and we determined what would "balance" those forces to create a state of equilibrium for the object.



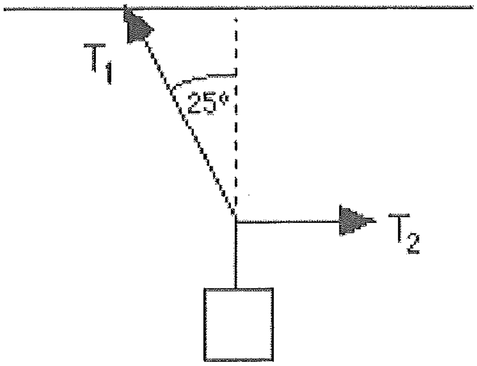

**Another example type of equilibrium is when objects are hung and tensional forces must be in equilibrium with the weight of the object** (such as a sign or picture).

**Example 1:** A weight suspended by cables and the system is in equilibrium. A 200 N block is suspended by two cables as shown in the diagram. Find the tension in each cable.

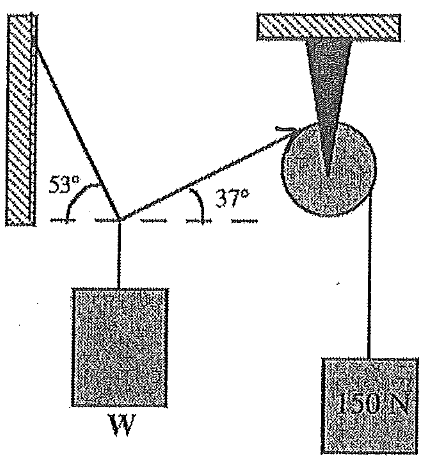


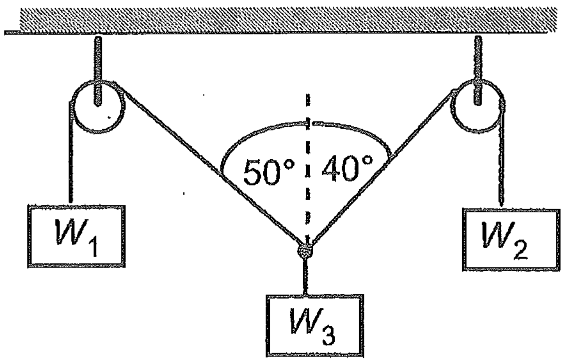
**Example 2:** For the force diagram shown below, let $T_1 = 85$ N. Determine the magnitude of $T_2$ and the mass of the box.




**Example 3:** If the system is in equilibrium, determine the value of the weight, W and the tension in the left wire.



**Example 4:** $W_1$, $W_2$, and $W_3$ are the weights of three objects suspended by pulleys. Assuming that the pulleys in this system are frictionless and weightless and that $W_3 = 12$ N, what are the values of $W_1$ and $W_2$?

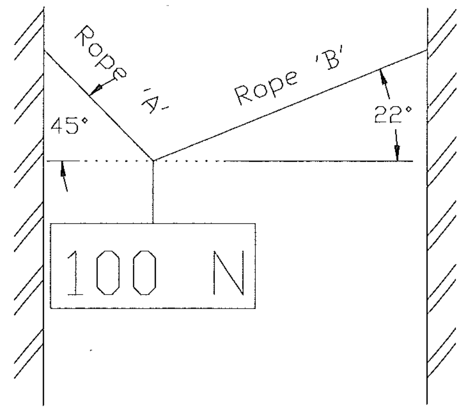


Do 1-11, 13

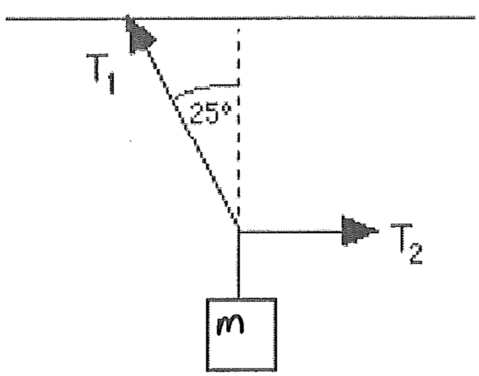
Lesson 1

**Equilibrium – First Condition Part One Problems**

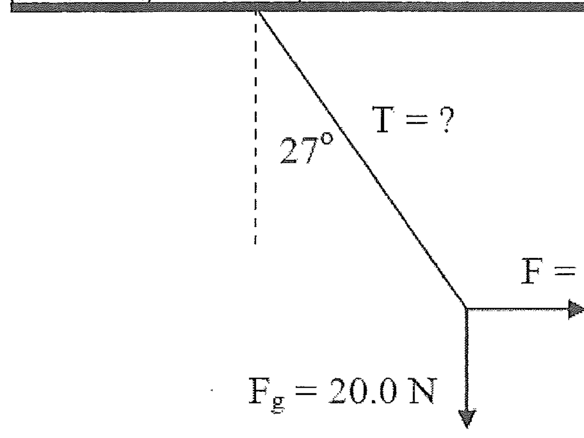
1. A 100 N sign hangs in between two buildings as shown. Which rope will have the higher tension force, rope 'A' or Rope 'B'? (Rope A with +101 N versus +77 N for Rope B)



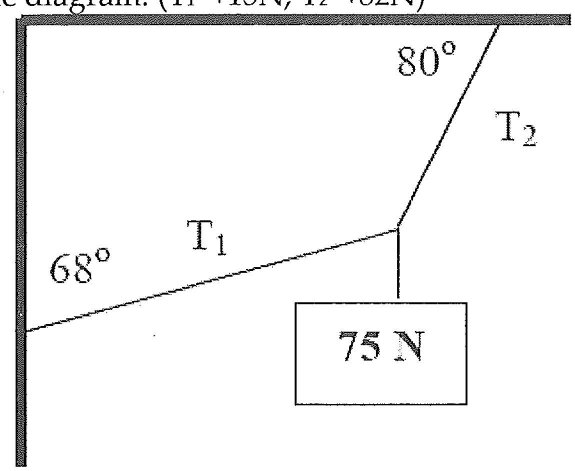

2. For the force diagram shown below, let $T_1 = 50$ N. Determine the magnitude of $T_2$ and the mass of the box. (+21N, 4.6 kg)



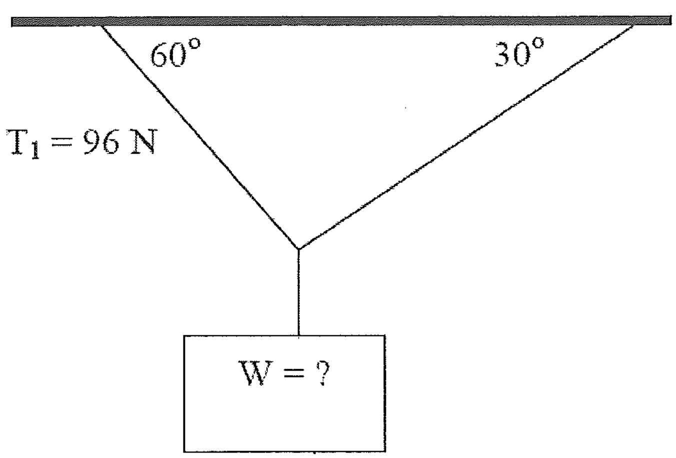
3. A 20.0 N child sitting on a playground swing is being pushed by her father. When the rope makes an angle of 27° to the vertical, what is the force exerted by her father when the swing is pulled back? What is the tension in the rope, T? (F=+10.2N, T=+22.4N)



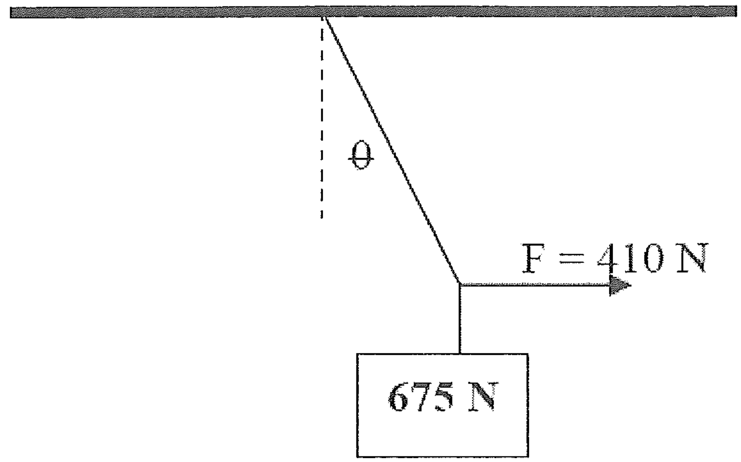

4. Find the tensions $T_1$ and $T_2$ in the ropes shown in the diagram. ($T_1$=+15N, $T_2$=+82N)



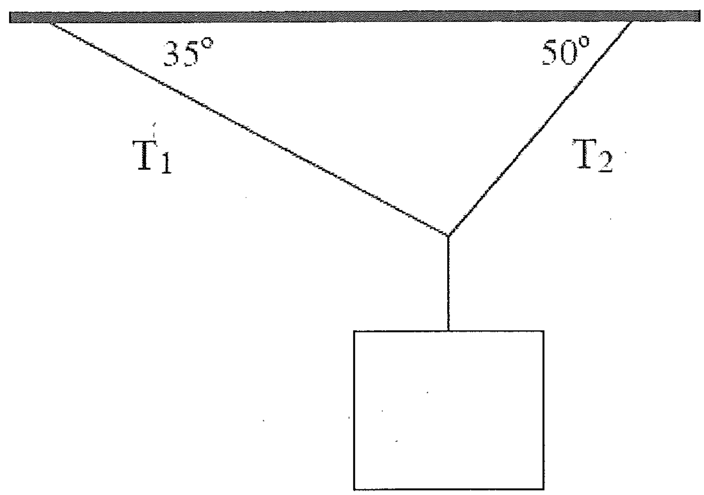

5. Given the following diagram, find W and $T_2$. ($T_2$=+55N, W=-111N)



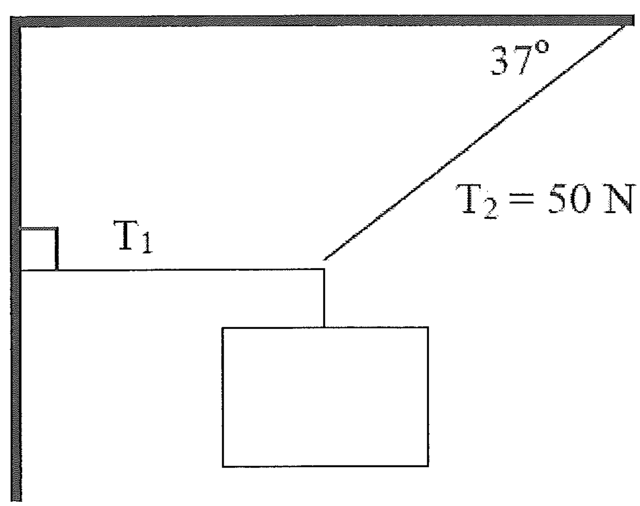
6. A 675 N object is pulled horizontally by a force of 410 N as shown. What is the angle, θ, between the rope and the vertical? (31°)



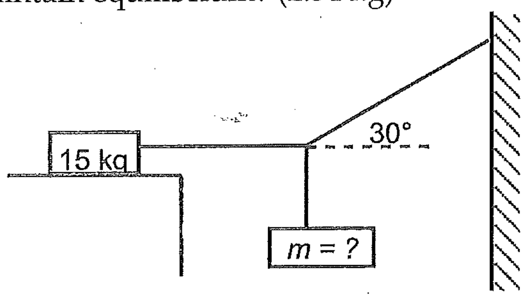

7. A 64 N object is suspended using ropes as shown in the diagram. Calculate tensions $T_1$ and $T_2$ in the ropes. (+41N, +53N)



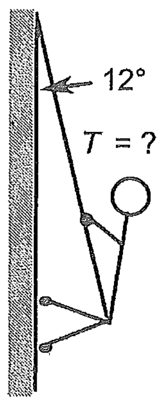

8. An object is suspended as shown. If the tension in one of the ropes is 50 N as shown, what is the weight of the object? (-30N)



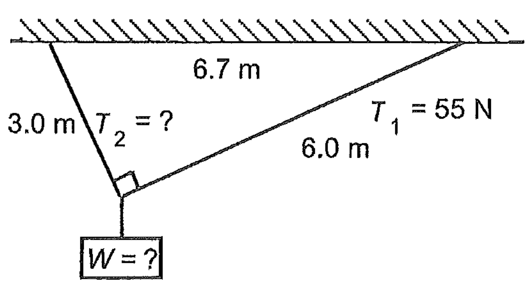
9. A 15 kg object rests on a table. A cord is attached to this object and also to a wall. Another object is hung from this cord as shown in the diagram. Is the coefficient of friction between the 15 kg object and the table is 0.27, what is the maximum mass that can be hung and still maintain equilibrium? (2.34 kg)



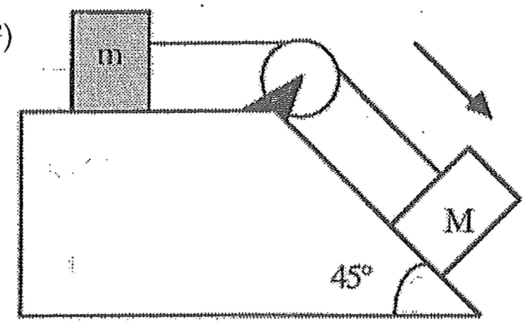

10. A 735 N mountain climber is rappelling down the face of a vertical cliff. If the rope makes an angle of 12.0º with the vertical face, what is the tension in the rope? (+751 N)



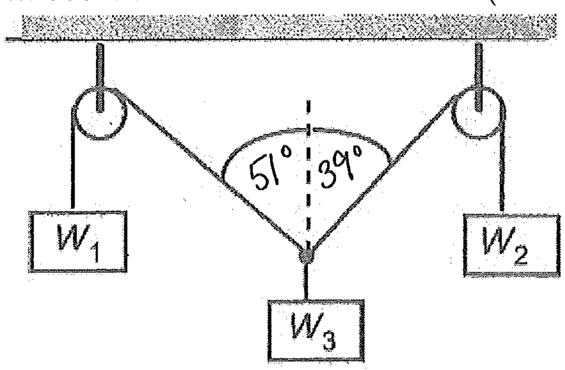

11. In the equilibrium state as shown in the diagram, find $W$ and $T_2$.



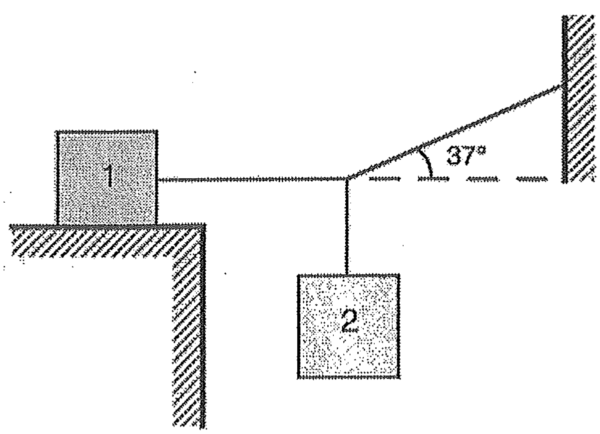
12. Find the acceleration of the system (M = 7.5 kg, m = 2.5 kg) if,
a) There is no friction. (5.2 m/s²)
b) The coefficient of kinetic friction is 0.33 (2.7 m/s²)



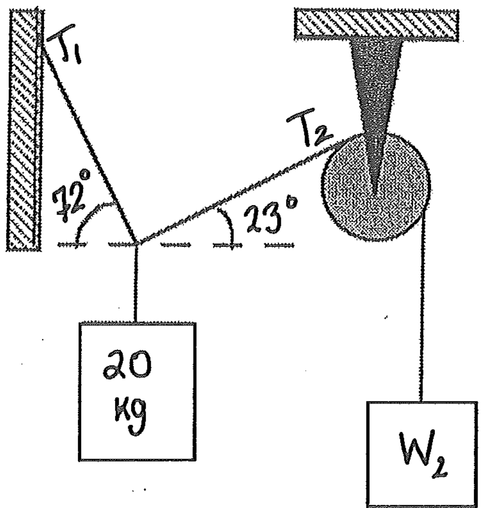

13. Assuming that the pulleys in this system are frictionless and weightless and that $W_2 = 25$ N, what are the values of $W_1$ and $W_3$? (-32 N, -20 N)



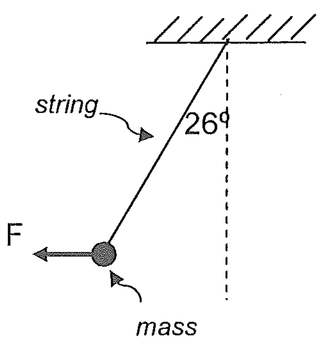

14. Block 1 weighs 96 N. The coefficient of static friction between block 1 and the table is 0.50. Find the maximum weight of block 2 for which the system will remain in equilibrium. (-36N)



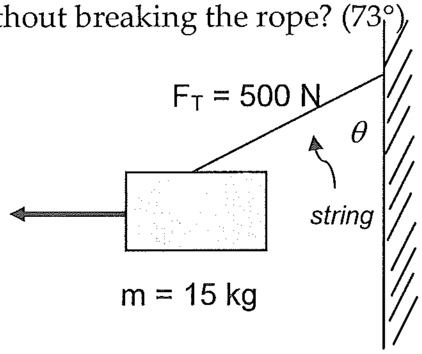
15. Determine the weight and tension in the left wire. ($W_2$ = -61N, $T_1$ = +181N)



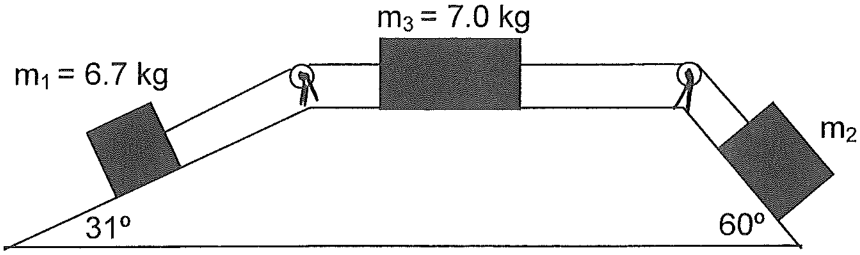

16. A mass of 3.0 kg is suspended from a cord as shown in the diagram below. What horizontal force *F* is necessary to hold the mass in the position shown? (14.3 N)




17. A 15 kg block is pulled by a horizontal force. The supporting rope can withstand a maximum tension force of 500 N. To what maximum angle, $\theta$, can the block be pulled without breaking the rope? (73°)



18. Three masses connected by a string are arranged on frictionless surfaces, as shown in the diagram below. If this system is in equilibrium, what is the mass of $m_2$? (3.98 kg)



Lesson 2

# Physics 12 – Torque and Rotational Equilibrium

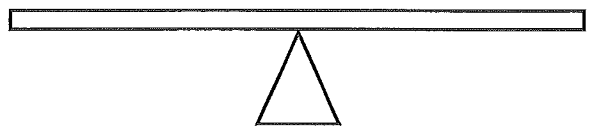
A body **in translational equilibrium** will have **no acceleration** in the *x or y* directions. However, it could still be ______________________.

Consider a teeter totter, with a 100 kg student on one end and a 50 kg student on the other.

What are the net translational forces in:

The x-direction? ______________

The y-direction? ______________



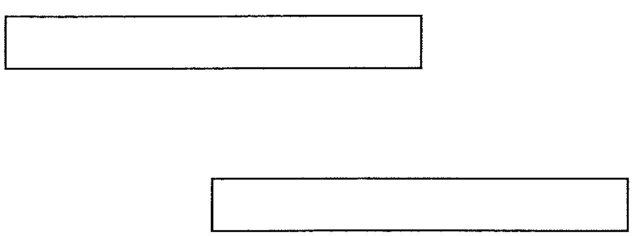
Although the net translational forces are zero, the system has a **net torque** – so it is not in equilibrium.

An object in equilibrium must have both translational and **rotational** equilibrium.

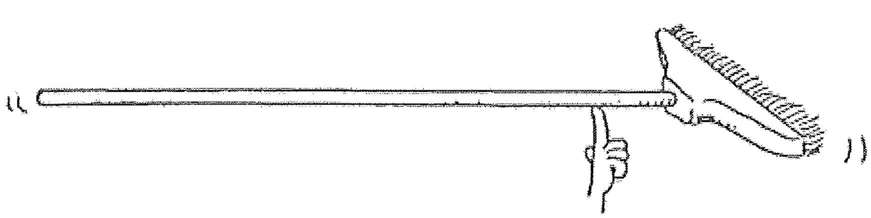
## Center of Gravity -

The **center of gravity** is the point at which the system's whole mass can be considered to be concentrated for the purpose of calculations.

To this point in your physics experience, it has always been assumed that any forces applied to an object are applied to its center of mass. This is illustrated when the object accelerates in the same direction as the force.



The center of mass of a body does not always coincide with its intuitive geometric center. **If the object's mass is not of a uniform density or shape, the center of mass will fall in the area of the greater density or mass distribution.**



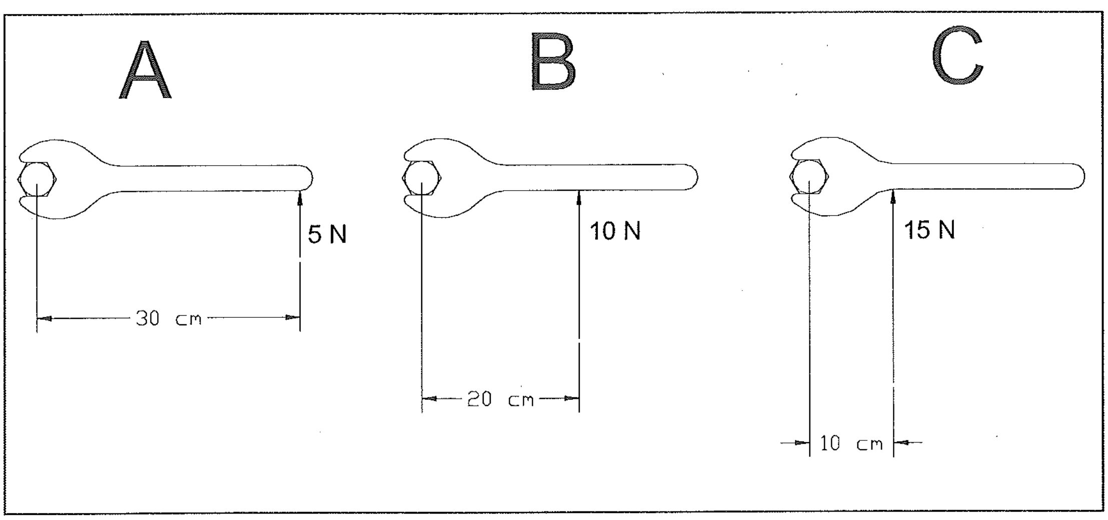
However, when dealing with objects in every day life, **<u>forces are not always applied to the center of mass.</u>** What will happen in these cases?

## <u>Torque</u> –

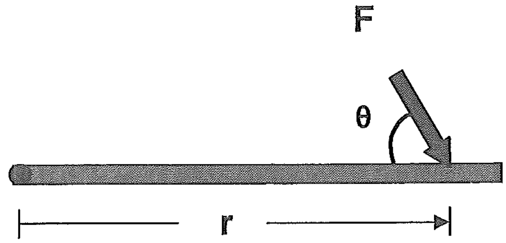
When the force is not applied directly to the center of mass, we just learned that the object rotates.

***Torque* is a measure of how much the force acting on an object causes that object to rotate.** The object rotates about an axis, which is called the **pivot point** or **point of rotation**. We will call the force '**F**'.

**Torque is also directly proportional to the perpendicular distance of the force. Which example will 'twist' the bolt with the most *torque*? Why?**



The **distance from the pivot point to the point where the force acts** is called the **radius of rotation**, and is represented by '**r**'. Note that this distance, '**r**', is also a vector, and points from the axis of rotation to the point where the force acts.

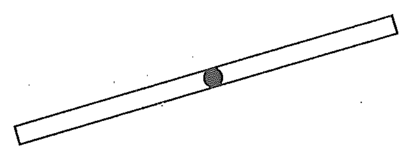



## <u>Lever Arm –</u>

The lever arm of a force is the **<u>perpendicular</u>** distance from the axis of rotation to the line along which the force acts.

**When the force (F) is perpendicular to the radius of rotation (r), the lever arm and the radius of rotation are the <u>same</u>.**

**When the force (F) is applied at an angle, the lever arm is found through the formula:**



The symbol for torque is the Greek letter tau: $\tau$

Torque is defined as:

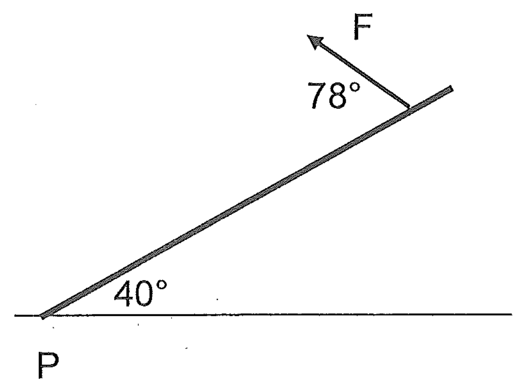
**Torque is a vector quantity.** A torque that will cause an object to rotate **counter-clockwise** is assigned a **positive sign**, and a torque that will cause an object to rotate **clockwise** is assigned a **negative sign**.

## Net Torque:

$\sum \tau = \tau_{\text{clockwise}} + \tau_{\text{counterclockwise}}$

Remember – Counterclockwise rotation is assigned (+) and clockwise rotation is assigned (-)

**Example Two:** A uniform 8.0 m long beam of mass 20 kg has a force of 210 N applied 6.5 m from point P as shown in the diagram. Find the magnitude and direction of the net torque acting on the beam about the pivot point P.

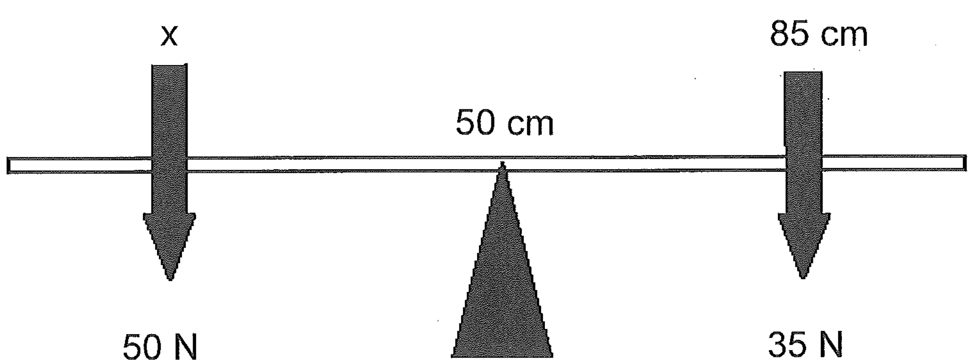



also
• Show 8 & 9 at end of Lesson 2 (extra practice lesson 2)

**Example Three**: A 100 cm meter rule is pivoted at its middle point (that is, at the 50 cm point). If a weight of 10 N is hanged from the 30 cm mark (from the left) and a weight of 20 N is hanged from its 60 cm mark, find out whether the meter rule will remain balanced about its pivot or not?

**Example Four:**

Determine the location of the 50 N force if this system is in rotational equilibrium ($\Sigma \tau$ = 0 N•m)




* ADD a Bridge problem

Physics 12 – **Rotational Equilibrium**

An object is in **translational equilibrium** when -

An object is in **rotational equilibrium** when -

An object is rotational equilibrium will rotate with constant angular velocity, which could be zero.

> The second condition of equilibrium is that in order to have no rotation, there must be no net torque.
>
> Torque is defined as: **force x distance to pivot**

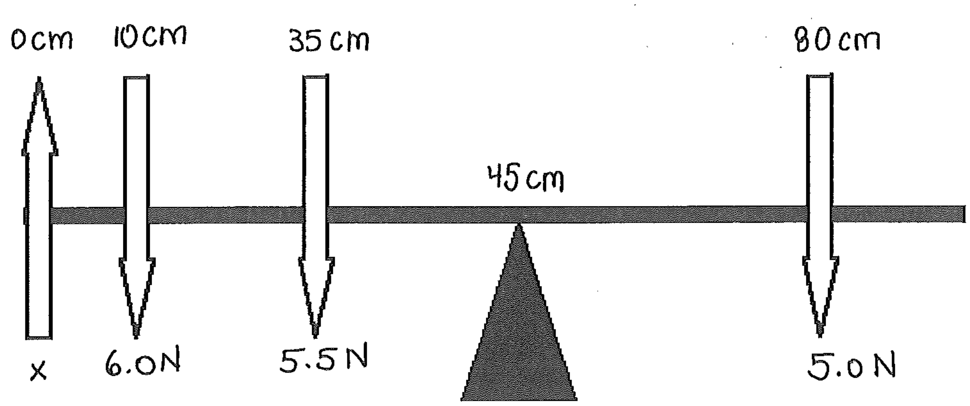
Terms to know:

**Center of Gravity:**

**Uniform Beam:**

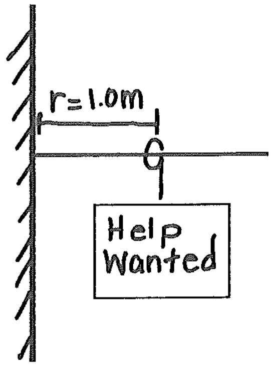
**Arbitrary Position of Rotation:**

With this in mind we can determine unknown measurements for a system in rotational equilibrium. Determine the force applied at x.

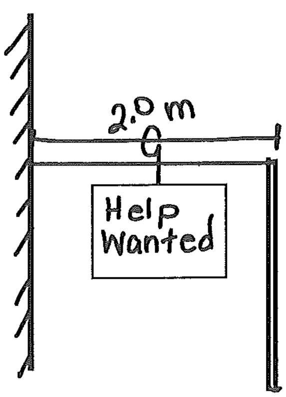


We can also **combine this idea with translational equilibrium** and this allows us to determine the tension is slightly more complicated systems.

Suppose you need to hang a sign that is 10 kg 1.0 m from a rod that is not very strong. The force of gravity on the sign produces a torque around the rod's base.

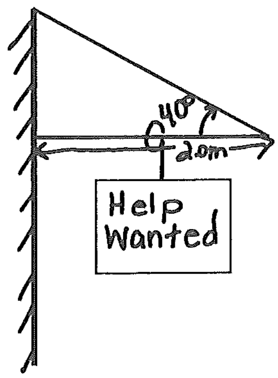



**Option One:** Place a pole under the end of the rod in order to prevent the rod from rotating. How much normal force must the pole provide to keep the system in equilibrium?



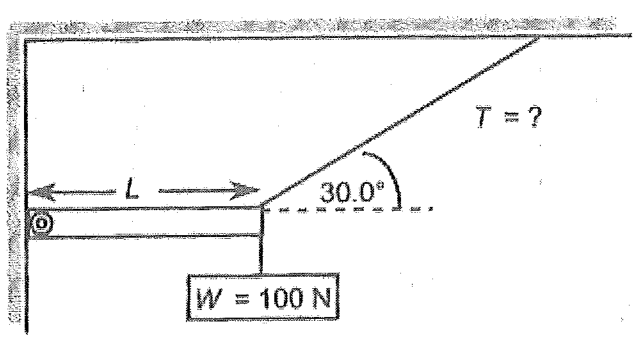
**Option Two: Support the rod with a wire.**

If the wire attaches to the end of the rod at 40°, how much tension must the wire provide to keep the system in equilibrium?

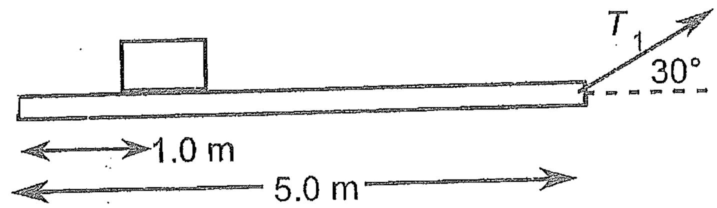



Examples:

A 25.0 N uniform beam is attached to a wall by means of a hinge. Attached to the other end of this beam is a 100 N weight. A rope also helps to support the beam as shown in the diagram. What is the tension in the rope?



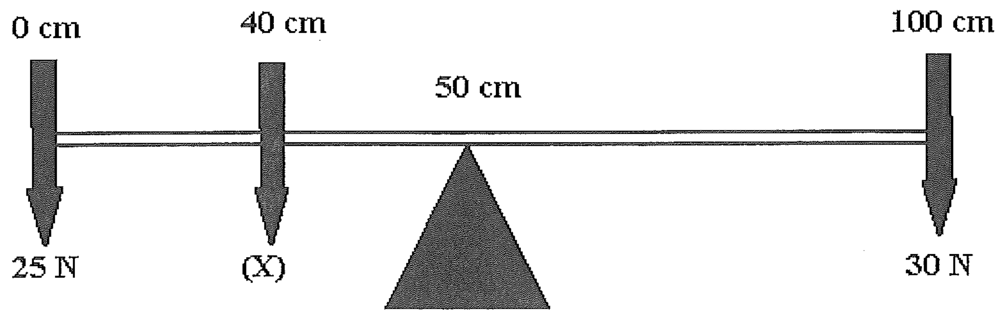
A uniform beam 5.0 m long has a weight of 200 N and is suspended by three ropes as shown in the diagram. If an 800 N object is placed as shown in the diagram, what is the tension of the ropes?



Lesson 2

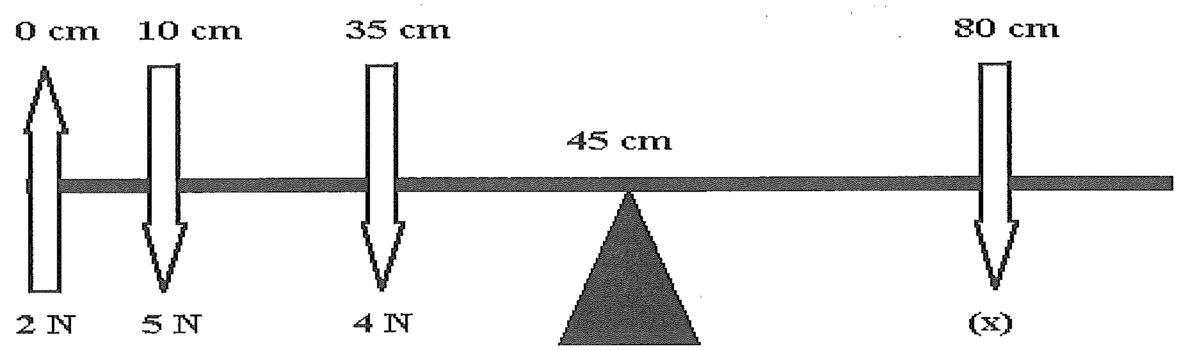
**Rotational Equilibrium Problems:**

1. Determine the force at 40 cm. (25 N)



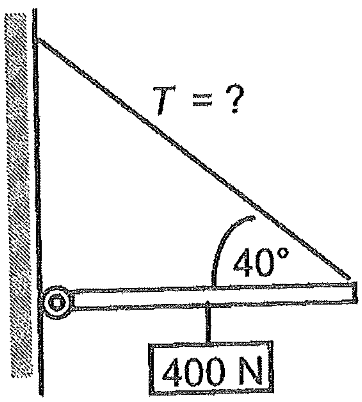

2. Determine the force at 80 cm. (3.64 N)

mass meter stick = 0.5 N

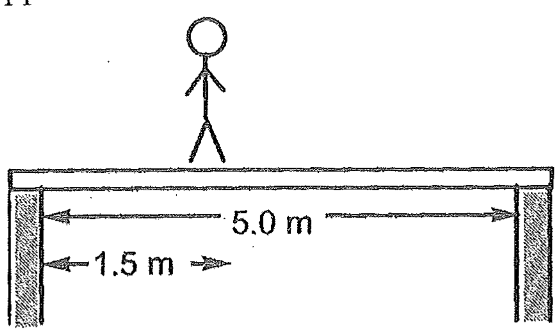


3. A beam of negligible mass is attached to a wall by means of a hinge. Attached to the center of the beam is a 400 N weight. A rope also helps to support this beam as shown in the diagram.

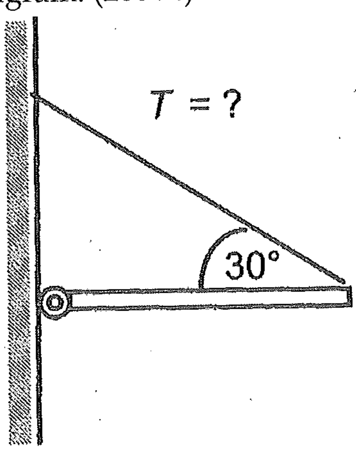
What is the tension in the rope? (311 N)



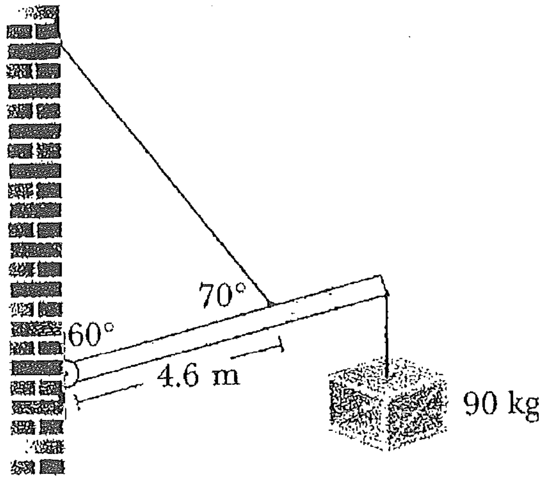

4. A 650 N student stands on a 250 N uniform beam that is supported by two supports as shown in the diagram. If the supports are 5.0 m apart and the student stands 1.5 m from the left support, what is the force that the right support exerts on the beam? (+320N)



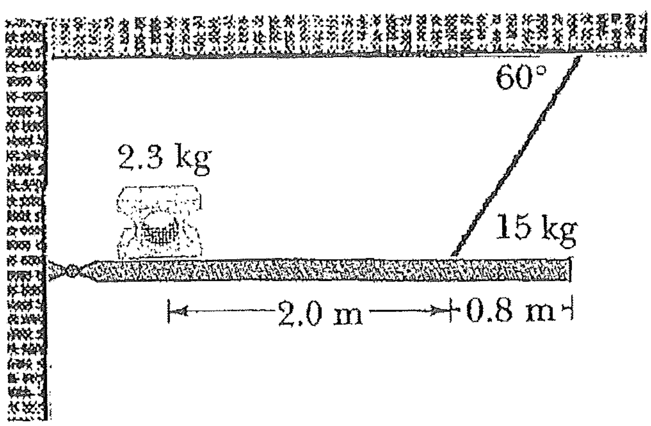

5. Find the tension in the rope supporting the 200 N hinged uniform beam shown in the diagram. (200 N)



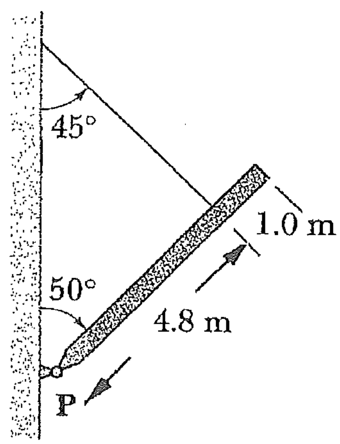
6. A 7.0 m uniform beam of mass 30 kg is attached to a vertical wall by a cable as shown in the diagram. A 90 kg crate hands from the far end of the beam. Find the tension in the cable connected to the wall. (1450 N)



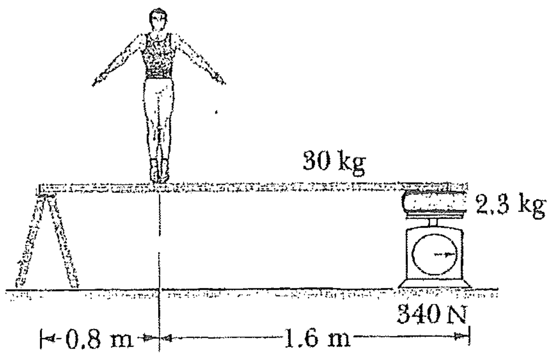

7. A uniform 15 kg plank of length 4.0 m holding a 2.3 kg block is attached by a rope to a ceiling as shown in the diagram. What is the tension in the rope? (116 N)



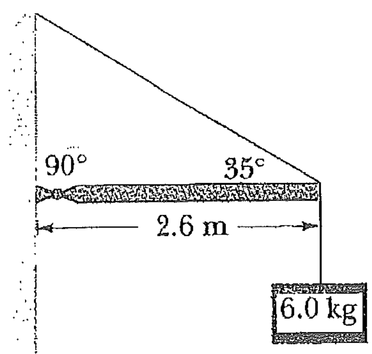

8. A 5.8 m uniform beam is supported by a cable having a tension of 1300 N. What is the mass of this beam? (286 kg)



9. A gymnast stands on a uniform 2.4 m beam with a mass of 30 kg. The beam is held level by a 2.3 kg book resting on a scale at the other end. The scale on the right end reads 340 N. What is the mass of the gymnast? (52 kg)




10. A 6.0 kg box is supported by a uniform 2.6 m beam as shown in the diagram. The beam has a mass of 1.8 kg. Find the tension in the cable connected to the wall. (118 N)

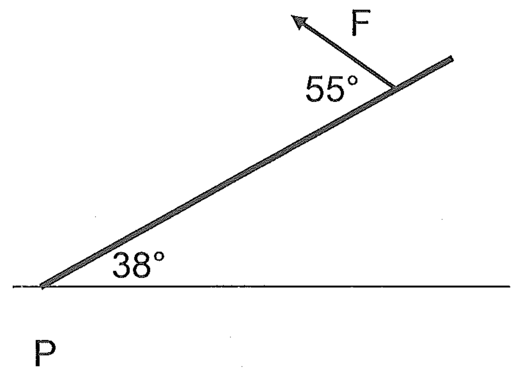


Lesson 2 - extra practice

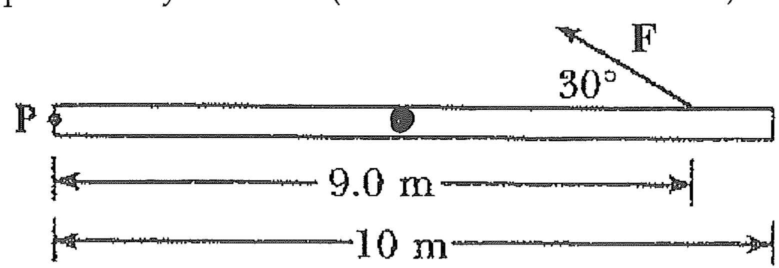
**Torque and Net Torque Problems:**

1. If the torque needed to loosen a lug nut holding the wheel of a car is 45 N•m and you are using a wheel wrench that is 35 cm long, what force must you exert perpendicular to the end of the wrench? (129 N)

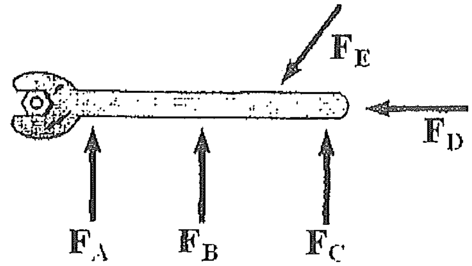
2. A uniform 5.0 m long beam of mass 15 kg has a force of 165 N applied 3.8 m from the pivot point. Find the magnitude and direction of **net torque** acting on the beam about the pivot point. (224 N•m counter-clockwise)




3. A uniform horizontal beam of length 10 m and mass 2.0 kg, has a force 55 N applied 9.0 m from the pivot point (P). Find the magnitude and direction of the net torque about the pivot point produced by this force. (152 N•m counter-clockwise)

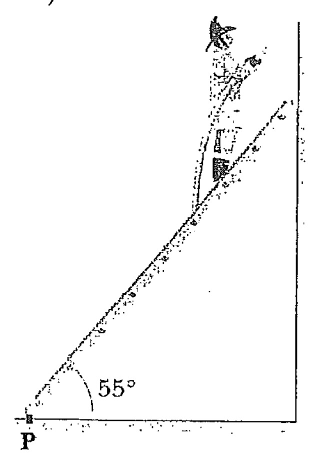


4. The diagram shows situations in which a force acts along different directions and at different points.

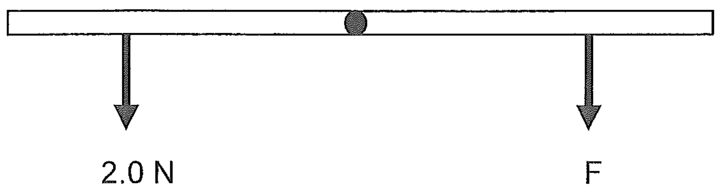



a) If the forces have the same magnitude, in which situation will the force produce the greatest torque around the bolt?
b) In which situation is the greatest force required to loosen the bolt?
c) In which situation does the force produce a clockwise torque around the bolt?

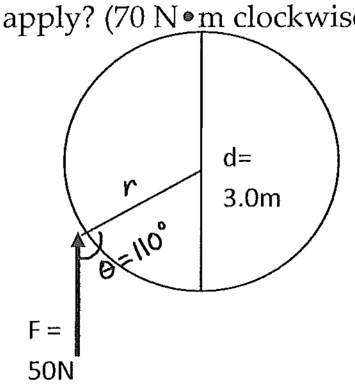
5. A 70 kg firefighter stands 4.8 m from the bottom of the 6.0 m ladder as shown in the diagram. What are the magnitude and direction of the torque about the base of the ladder at P produced by the firefighter? ($1.89 \times 10^3$ N•m clockwise)



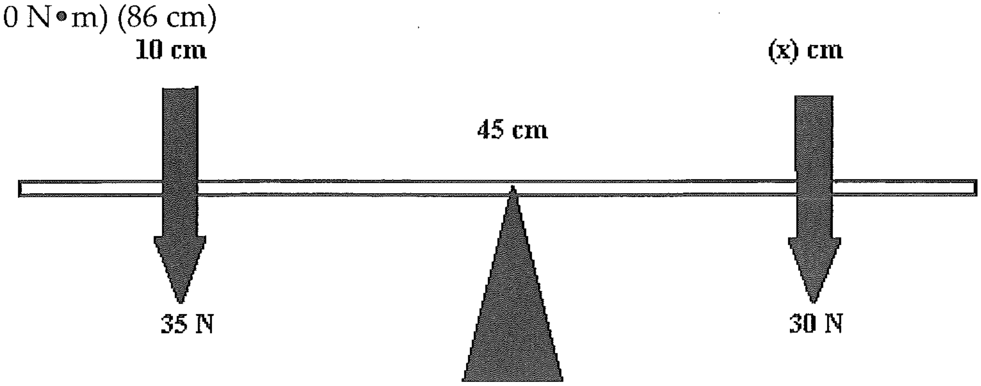
6. A 100 cm meter rule is pivoted at its middle point (that is, at the 50 cm point). If a weight of 2.0 N is hanged from the 20 cm point, prove that amount of weight needed to be applied at the 80 cm mark so as to keep it in a balanced position is 2.0 N as well.



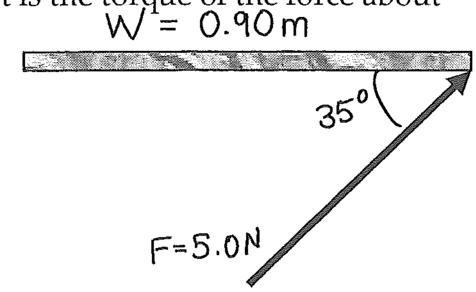

7. Several children are playing in the park. One child pushes the merry-go-round with a force of 50 N. The diameter of the merry-go-round is 3.0 m. What torque does the child apply? (70 N•m clockwise)




8. Determine the location of the 30N Force if this system is in rotational equilibrium ($\Sigma \tau = 0$ N•m) (86 cm)



9. A person pushes on the edge of a 0.90 m wide door with a horizontal force of 5.0 N acting at an angle of 35° to the plane of the door. What is the torque of the force about the door hinge? (2.6 N•m counter-clockwise)



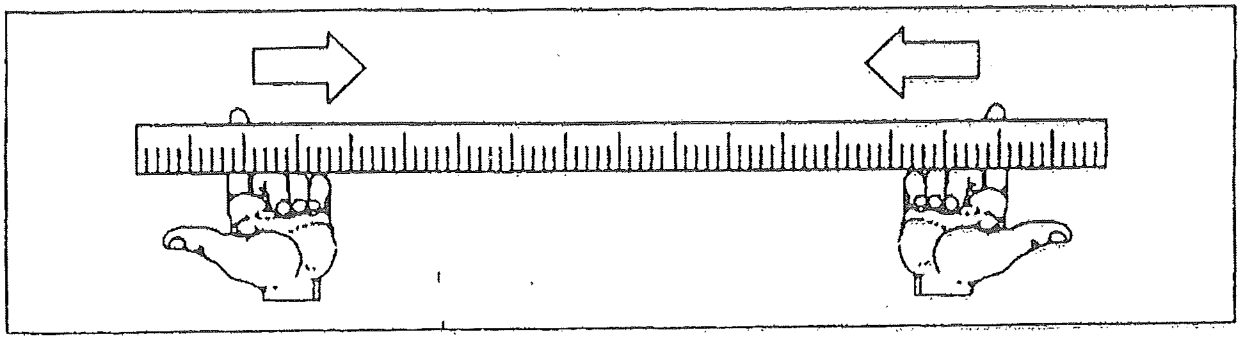

10. The crank handle on an ice cream freezer is 0.20 m from the shaft. At some point in making ice cream, a boy must exert 50 N to turn the crank. How much torque does the boy apply about an axis through the shaft? (10 N•m clockwise)

Lesson 3 (+ Lesson 2 extra practice 1-8)

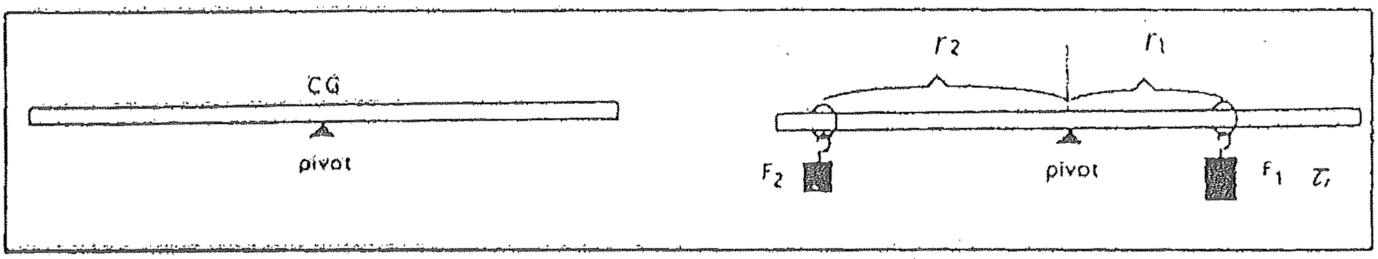
Physics 12 – **Lab: Rotational Equilibrium Investigation** Name:

**Purpose**: To investigate the conditions necessary to prevent the rotation of a loaded beam.

**Procedure:**



1. The diagram above shows you how you can find the **center of gravity** of a meter stick very quickly. Hold the meter stick on your fingers, as in the diagram. Slowly slide your fingers toward each other. When they meet, they will have the center of gravity's "surrounded". Try this several times.




2. Mount your meter stick on a stand (as shown above) with the pivot exactly at the center of gravity. (Few meter sticks are perfectly uniform, so do not assume that the center of gravity (CG) is at the 50.0 cm mark). Adjust the pivot point precisely until the meter stick is in equilibrium. Record the position of the CG to the nearest millimeter.

3. Use a very thin, light piece of wire to attach a 1.00 kg mass at a distance of 20.0 cm from the pivot. The force of gravity on this mass will be 9.80 N. This force is labeled $F_1$ on the diagram. The distance from the pivot to the point where $F_1$ acts is called the **lever arm** and is labeled $r_1$.The torque due to $F_1$ produces a clockwise turning effect, and is called a **clockwise torque.** It is labeled as $\tau_1$. ($\tau_1 = F_1 r_1$)

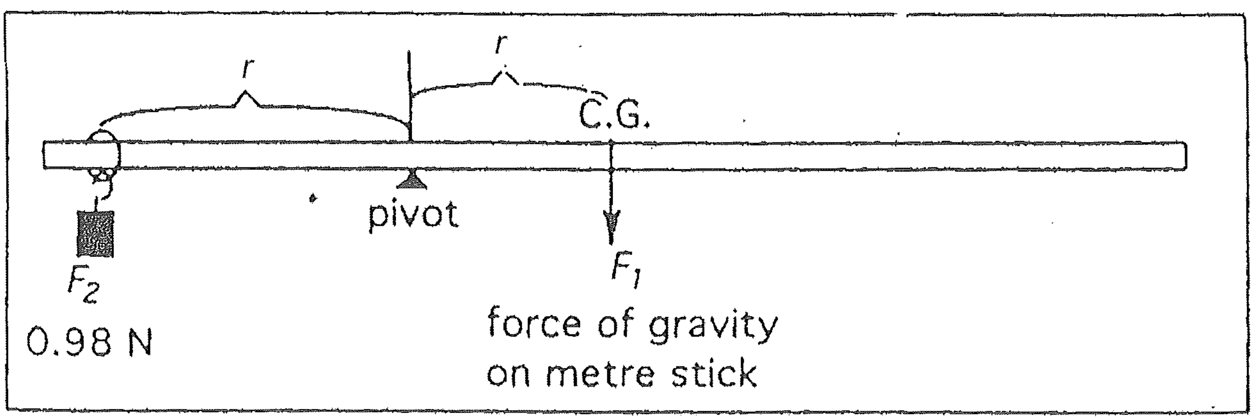
4. Suspend a 500 g mass by a light piece of wire on the other side of the meter stick. Adjust its lever arm until the meter stick is in a state of equilibrium. The force of gravity on the 500 g mass is 4.90 N. Call this force $F_2$ and its lever arm, $r_2$ . The torque, $\tau_2$ is a **counterclockwise torque.** $(\tau_2 = F_2 r_2)$ Record the forces and lever arms in the data table. Calculate the torques and enter them in the **data table** for **Trial 1.** (Torque is expressed in N•m)

Data Table – Observations for Torque

| Trial | $F_1$ | $r_1$ | $\tau_1$ | $F_2$ | $r_2$ | $\tau_2$ | $F_3$ | $r_3$ | $\tau_3$ | $\tau_2 + \tau_3$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | (N) | (m) | (N•m) | (N) | (m) | (N•m) | (N) | (m) | (N•m) | (N•m) |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |

5. Repeat the experiment, but place the 1.0 kg mass 25.0 cm from the pivot. Use two masses to produce **counterclockwise torques** and adjust their positions until the meter stick is in equilibrium. Measure and record all the forces and lever arms in the **data table**. Calculate torques $\tau_1$, $\tau_2$, and $\tau_3$ and record them in the data table. **(Trial 2)**

6. Try another combination of at least three force of your own choosing. Record all the forces and lever arms and calculate all the torques needed to produce rotational equilibrium. (**Trial 3)**.



7. Finally, set up your meter stick as in the diagram above so that the force of gravity on the meter stick produces a clockwise torque. Place the CG 20.0 cm to the right of the pivot. Use a 100 g mass (force of gravity 0.980 N) to balance the meter stick. Move the mass to a position where the meter stick stays balanced. Record all of your data in the **data table as Trial 4**. Note that in this case $F_1$ is the unknown force of gravity on the meter stick.

8. Hang the meter stick from a spring balance and measure the force of gravity on the meter stick, according to the spring balance. Record this value.

| Fg on meter stick = |
|---|
| |

**Concluding Questions:**

1. Examine your calculated torques for each trial. When rotational equilibrium is achieved, what can you conclude regarding the **clockwise torque(s)** when compared with the **counterclockwise torque(s)**? State a general rule describing the condition(s) required for rotational equilibrium.

2. What are some likely sources of error which might cause discrepancies in your results? What is the maximum percent difference you observed when comparing the sum of clockwise torques with the sum of counterclockwise torques?

3. Use the general rule that you described in question #1 to calculate the force of gravity on your meter stick using only the torques involved (**Procedure** 7). Calculate the percent difference between your calculated force of gravity and the force of gravity according to the spring balance. Explain any discrepancy you observe.

Lesson 4

# Physics 12 – Static Equilibrium

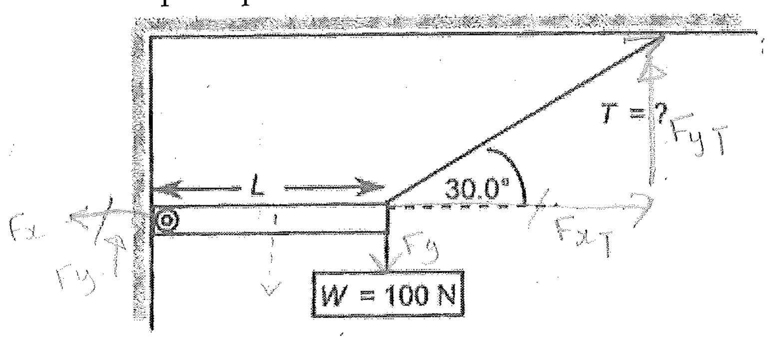
**An object is in static equilibrium if it is at rest.** In static equilibrium, the sum of the external forces is zero, and the sum of the external torques about any pivot point is zero.

**Both conditions of equilibrium must be met** to be in static equilibrium.

By using **both** conditions together we can determine even more about the forces acting on a structure (very helpful when designing a structure!) or a system.

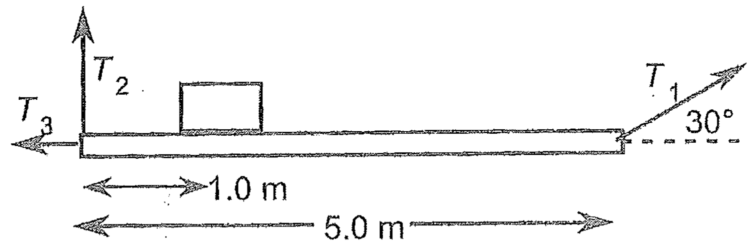
Last lesson we determined the tension for the following situation using the second condition of equilibrium - $\Sigma \tau = 0$.

For the same situation, what are the horizontal and vertical forces that the wall exerts on the beam? We can no longer use torque because no torque is produced when applied to the pivot point.



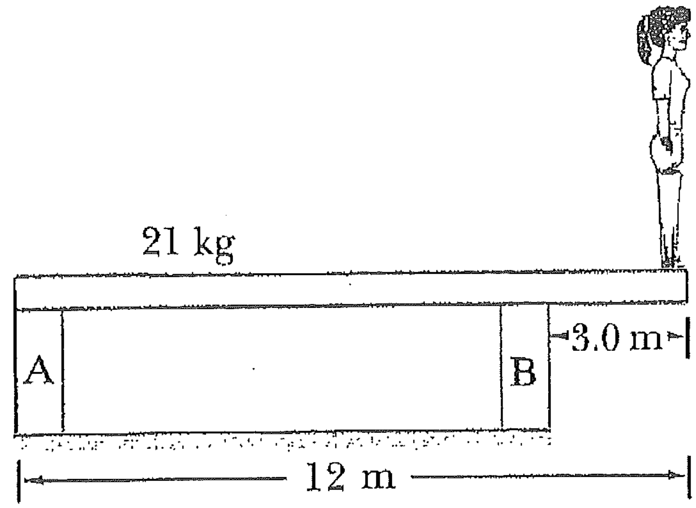
We determined $T_1$ last lesson by using the first condition of equilibrium. However, no torque is produced by $T_2$ or $T_3$ (they are applied to the pivot point). In order to determine these tensions we will now need to use the first condition of equilibrium.

A uniform beam 5.0 m long has a weight of 200 N and is suspended by three ropes as shown in the diagram. If an 800 N object is placed as shown in the diagram, what is the tension of the ropes?



**Example:**

A uniform 12 m diving board with a mass of 21 kg rests on two supports as shown in the diagram. A diver is standing at the right end of the board. If support B exerts a reaction force of 960 N on the board, what reaction force does support A exert?



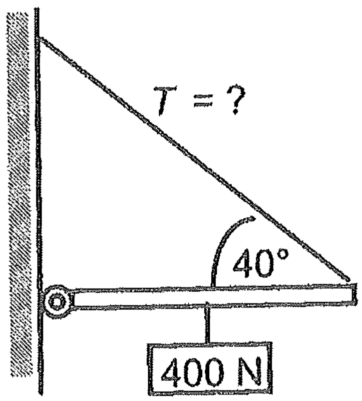
Lesson 4

**Static Equilibrium Problems:**

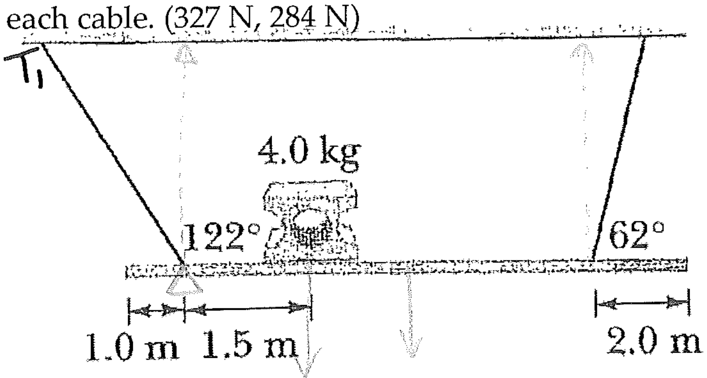
1. A beam of negligible mass is attached to a wall by means of a hinge. Attached to the center of the beam is a 400 N weight. A rope also helps to support this beam as shown in the diagram.

a) What is the tension in the rope? (311 N)

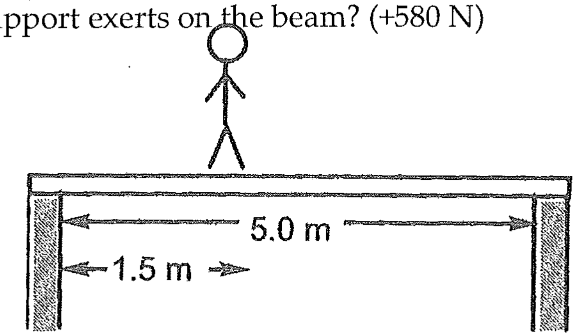
b) What are the vertical and horizontal forces that the wall exerts on the beam? ($F_x$=+238N, $F_y$=+200N)



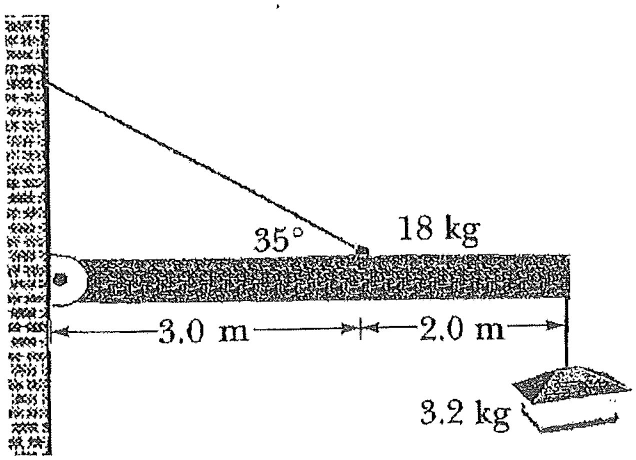

2. A uniform 50 kg plank of length 10 m is suspended horizontally by two cables. A 4.0 kg block 2.5 m from the left end of the plank as shown in the diagram. Find the tension in each cable. (327 N, 284 N)



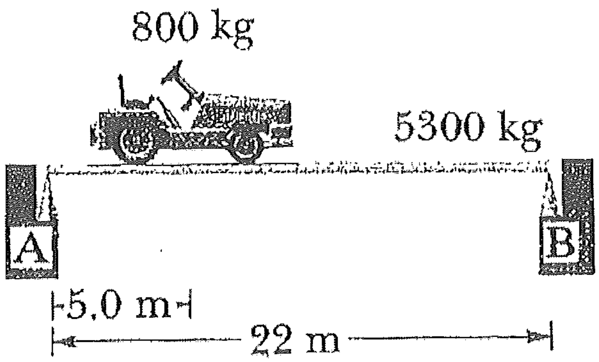
3. A 650 N student stands on a 250 N uniform beam that is supported by two supports as shown in the diagram. If the supports are 5.0 m apart and the student stands 1.5 m from the left support, what is the force that the left support exerts on the beam? (+580 N)



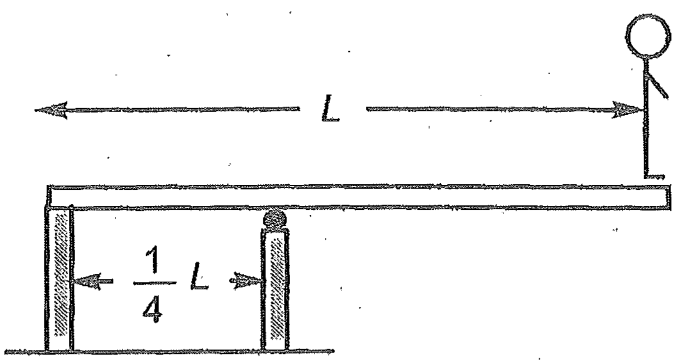

4. A uniform 18 kg beam with a length of 5.0 m is mounted by a hinge on a wall. The beam is held in a horizontal position by a cable with an angle of 35° and supports a cage with a mass of 3.2 kg as shown in the diagram. Find the force that the hinge exerts on the beam. (+285 N)

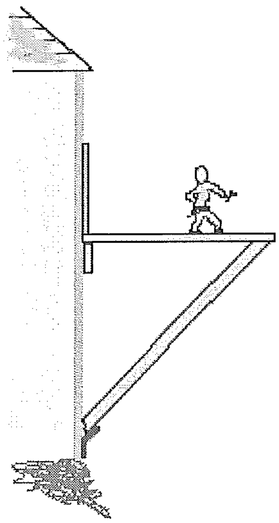


5. An 800 kg automobile moves toward the right end of the bridge which has a mass of 5300 kg and a length of 22 m. Find the reaction forces exerted by supports A and B on the bridge at the position shown in the diagram. (+2.78x10⁴ N, +3.20x10⁴ N)



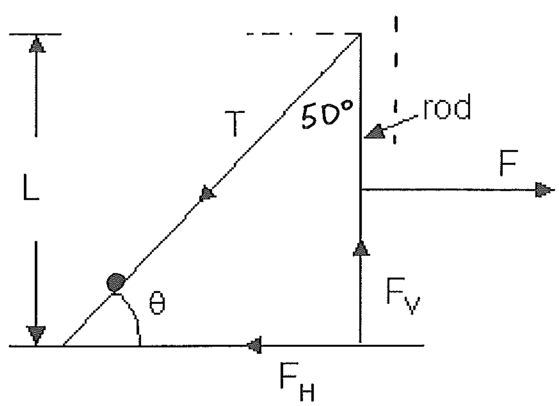

6. A uniform 400 N diving board is supported at two points as shown below. If a 75 kg diver stands at the end of the board, what are the forces acting on each support? (+3.74x10³N, -2.61x10³N)



7. 

You've decided to build a new deck. It is to be 10 m wide and run the 30 m length of the house. You must estimate the weight of the deck, as well as, the maximum load you expect it to support to be 6000 N. The deck is to be supported where it is attached to the house and at the outside edge by support beams that make a 45° angle as shown. What will the compressive tension in the support beam be? (You can ignore the weight of the man and the beam as they are included in the max 6000N load) ($-4.24 \times 10^3$ N)

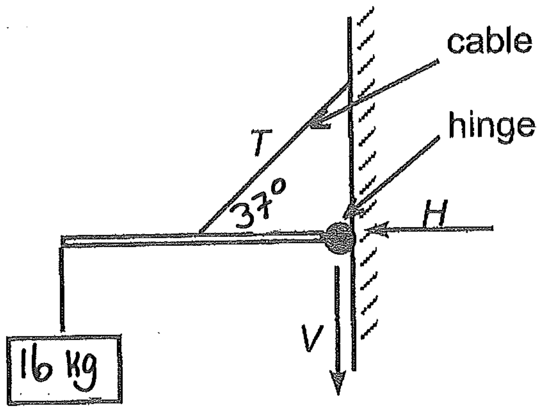
8. A rigid, vertical rod (2.0 m) of negligible mass is connected to the floor by an axle through its lower end. The rod also has a wire connected between its top end and the floor. If a horizontal force of 200 N (F) is applied at the midpoint of the rod, find a) the tension in the wire, and b) the horizontal and vertical components of force exerted by the bolt on the rod. ($F_H$ = -100N, $F_V$ = +84.2N)



9. A uniform beam (mass = 40 kg) is supported by a cable that is attached to the center of the beam as shown in the diagram.

a) Find the tension (T) in the cable. ($1.17 \times 10^3$ N)

b) Find H and V (the horizontal and vertical forces acting on the hinge) (H = -936N, V = -157N)



Physics 12 – **Lab: Equilibrium** Names:

**Purpose:** To determine the supportive X-component and Y- component forces provided by the plasticine bases required to keep your structure in static equilibrium.

**Task:**

Working with a partner your task is to build a simple structure to support a 200 g mass between your two desks spread 13.0 cm apart. The mass must be suspended at least 3.0 cm above the desk top and must be attached to a connection point. No more than two tongue depressors can be used to connect two tongue depressors. The structure can be anchored/supported laterally with **small** pieces of plasticine.

**You have the following materials available:**

- Brass connectors
- 3 tongue depressors
- One piece of string
- One 200 g mass
- Two small pieces of plasticine
- Use of an electric drill to drill holes in your tongue depressors (your teacher will do this, but you will need to indicate exactly where you want the holes drilled).

**Design:** Before you begin building your structure, draw the design. Draw the FBD's for each pivot point in your design. When you feel you have a complete design that will hold the 200g weight, you may begin building your structure.

**Testing:** Once you have built your structure, you must have your teacher come over and observe the structure supporting the 200 g mass.

**Teacher's initial:**

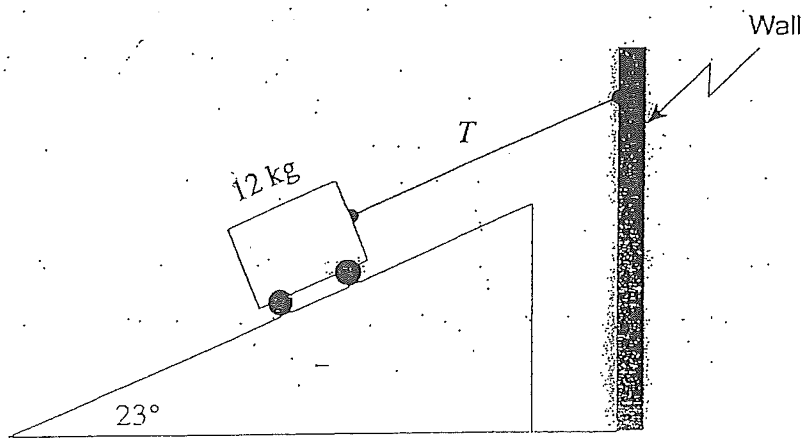
____________

**Write-up Instructions:**

1. On your FBD (**<u>you must use a ruler</u>**), label all the forces, pivot points, torques, and distances.
2. Using what you have learned about rotational equilibrium, calculate the reaction forces produced by your structure.
3. Using what you have learned about translational equilibrium, calculate the X and Y component forces provided by the plasticine supports.
4. Label your calculations (so I can easily tell what you are working out!)
5. Attach the calculations to your free-body diagrams.

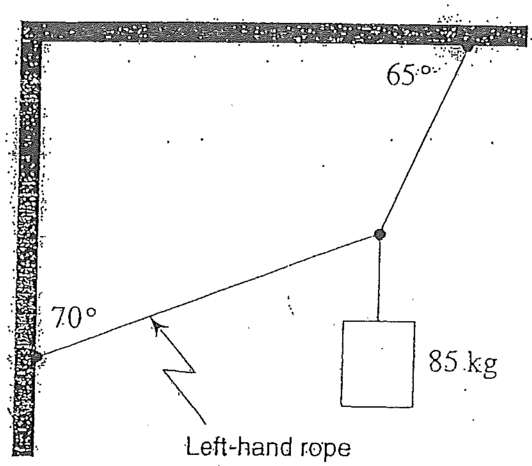
# Physics 12 Lesson 5 (6)
## Forces and Equilibrium
## Provincial Exam Questions

1. A 12 kg cart on a 23° frictionless incline is connected to a wall as shown.



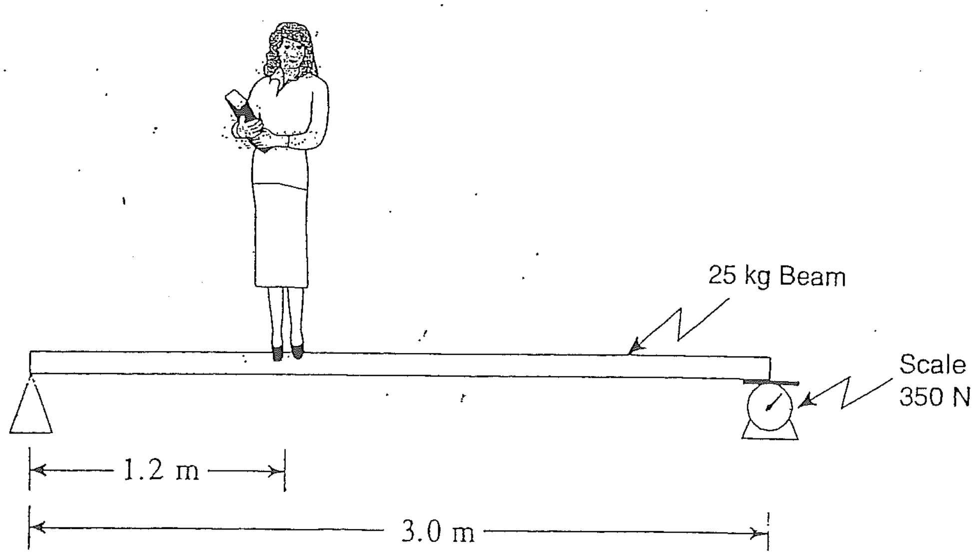

What is the tension $T$ in the cord?

2. An 85 kg object is suspended from a ceiling and attached to a wall.

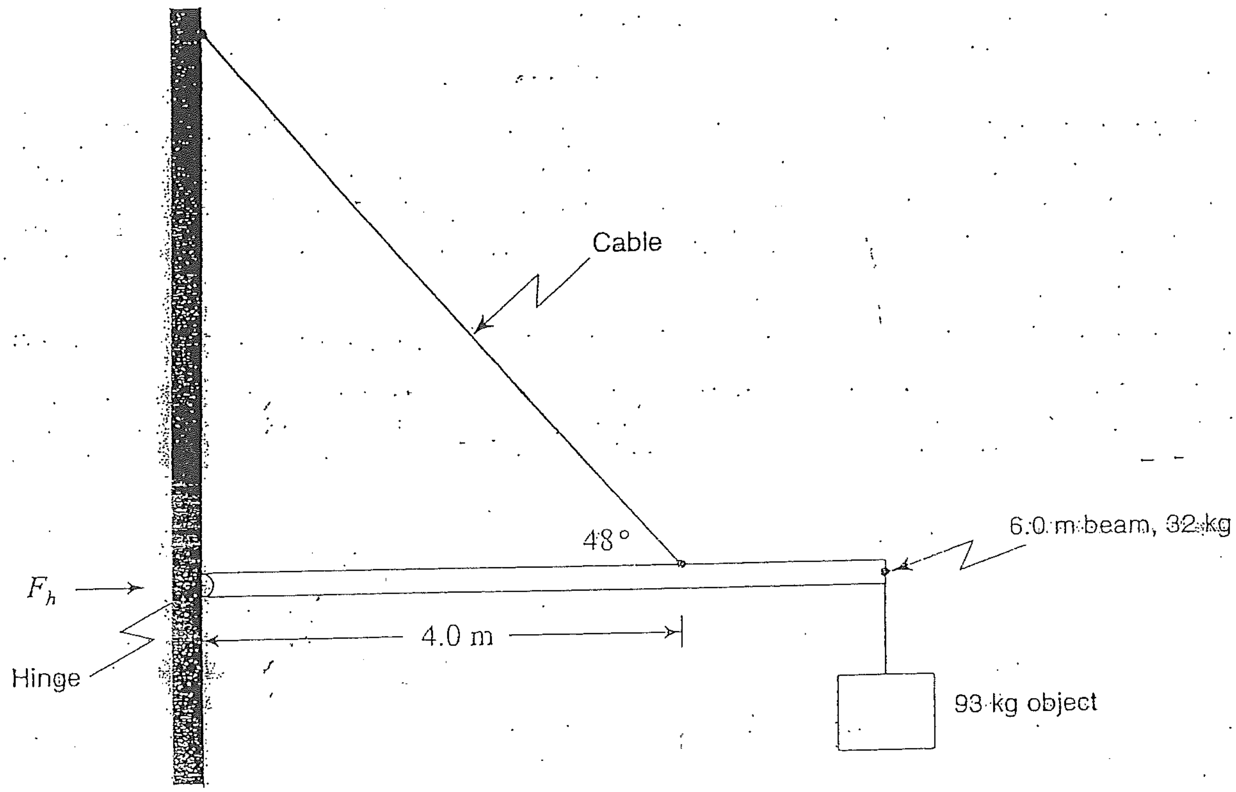



What is the tension in the left-hand rope?

3. A student stands on a uniform 25 kg beam. The scale on the right end reads 350 N.



4. A 6.0 m uniform beam of mass 32 kg is suspended horizontally by a hinged end and a cable. A 93 kg object is connected to one end of the beam.

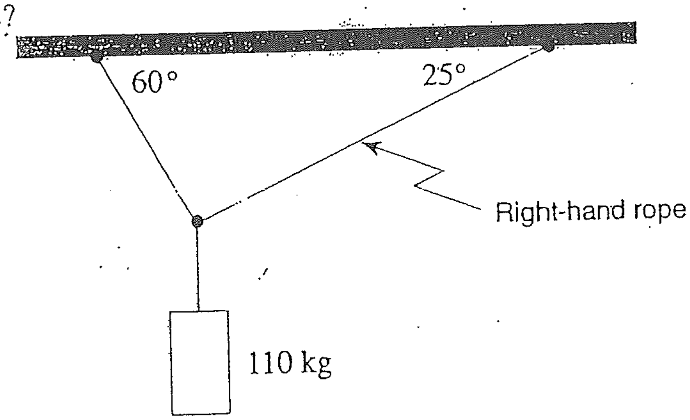



What is the magnitude of the horizontal force $F_h$ that the hinge exerts on the beam? (7 marks)

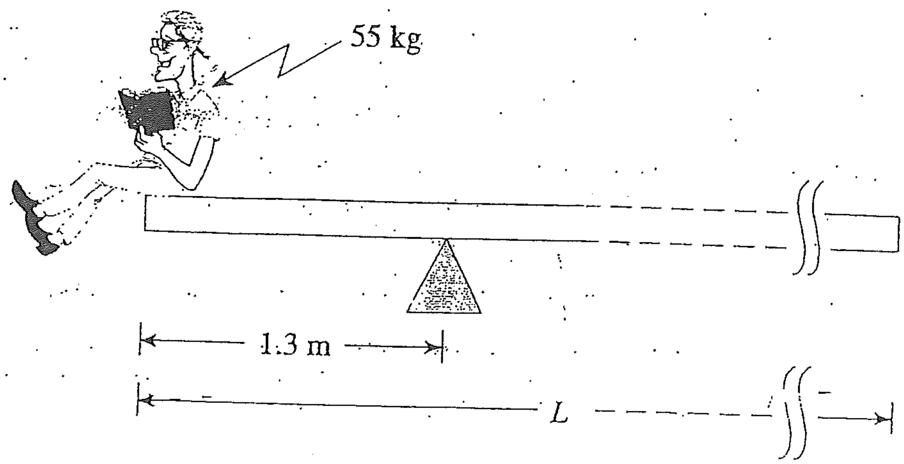
5. A body is in rotational equilibrium when

A. $\Sigma\tau = 0$
B. $\Sigma F = 0$
C. $\Sigma p = 0$
D. $\Sigma E_k = 0$

6. A 110 kg object is supported by two ropes attached to the ceiling. What is the tension $T$ in the right-hand rope?

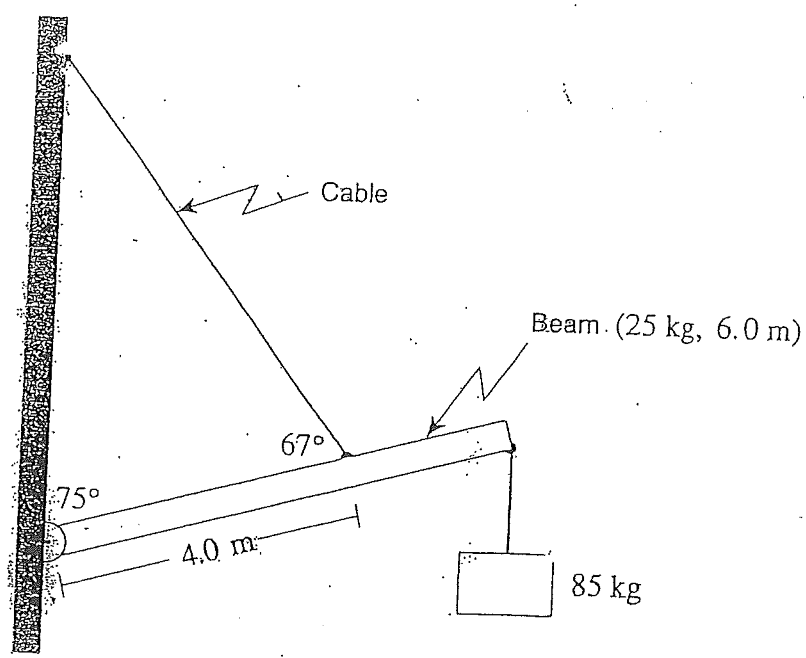


7. A 35 kg uniform plank is balanced at one end by a 55 kg student as shown.




What is the overall length of this plank?

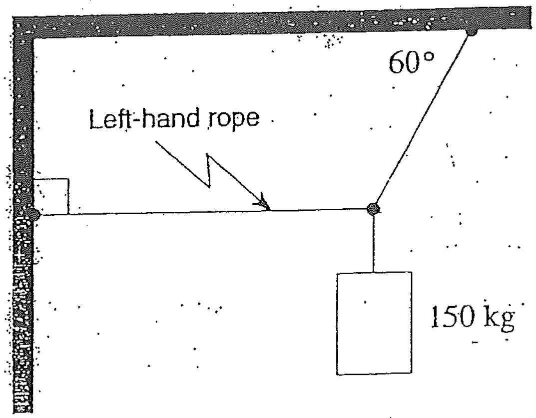
8. A 6.0 m uniform beam of mass 25 kg is suspended by a cable as shown. An 85 kg object hangs from one end.



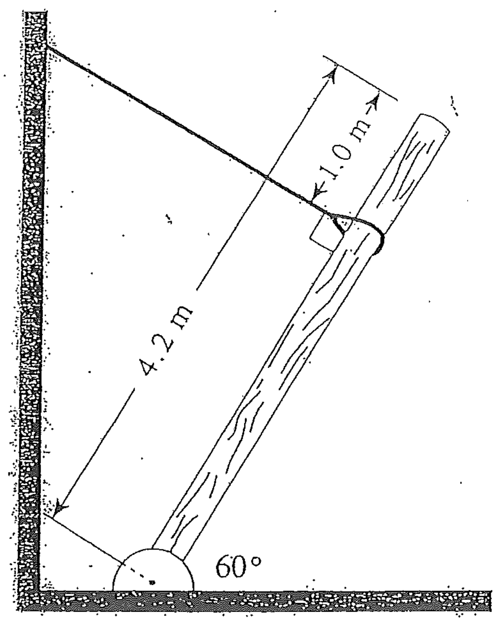

What is the tension in the cable?

**(7 marks)**

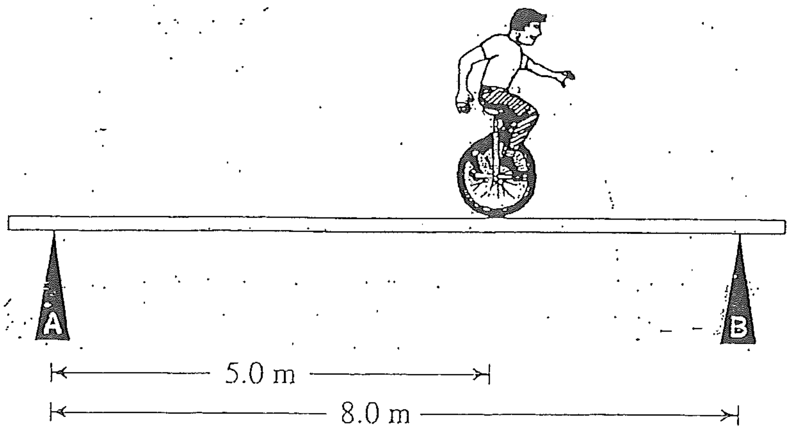
9. A 150 kg object is suspended from a ceiling and attached to a wall. What is the tension in the left-hand rope?




10. A 4.2 m long uniform post is supported by a cable having a tension of 1 700 N. What is the mass of this post?



11. A circus performer on a unicycle of total mass 55 kg rides across a uniform 30 kg beam. The supports are placed equal distances from the ends of the beam.




a) When he is at the position shown, determine the forces exerted by the supports on the beam. (5 marks)

b) As the performer moves toward the right the force exerted by support B will

☐ remain the same.

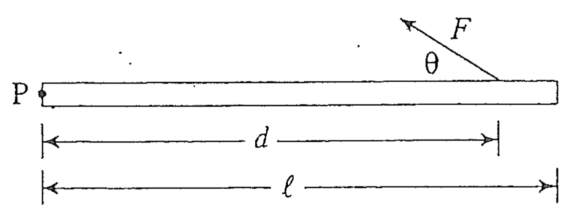
☐ increase.

☐ decrease.

(Check one response.)

c) Using principles of physics, explain your answer to b). (3 marks)

12. A force F is applied to a uniform horizontal beam as shown in the diagram below.



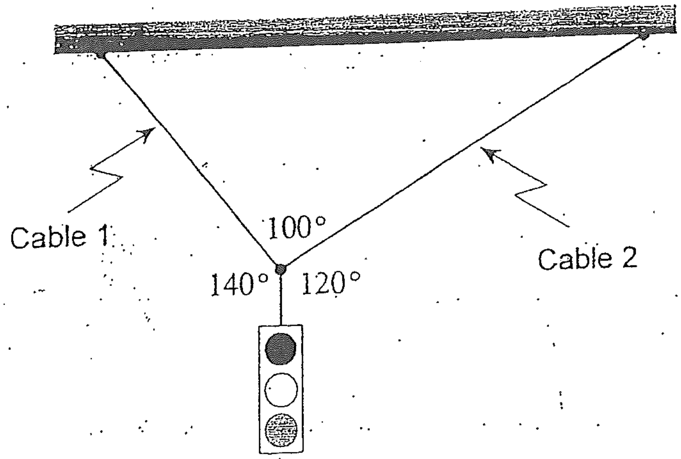

Which of the following is a correct expression for the torque on the beam about pivot point P due to this force?

A. $F \sin\theta \cdot d$

B. $F \sin\theta \cdot d/\ell$

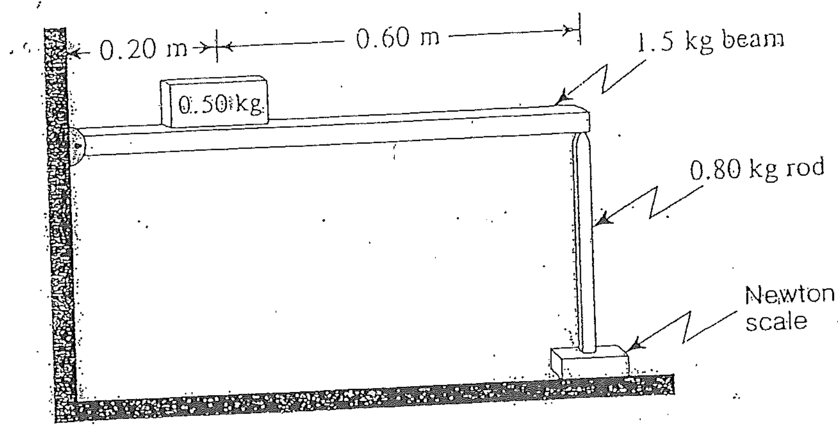
C. $F \cos\theta \cdot d$

D. $F \cos\theta \cdot d/\ell$




What is the tension in each of these cables? (7 marks)

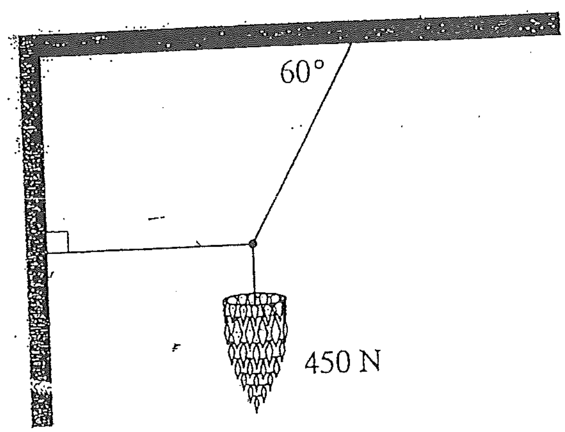
14. A uniform 1.5 kg beam hinged at one end supports a 0.50 kg block. The beam is held level by a vertical 0.80 kg rod resting on a Newton scale at the other end.




What is the reading on the scale?

A. 8.6 N
B. 9.1 N
C. 16 N
D. 27 N

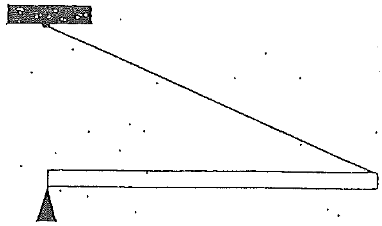
15. A 450 N chandelier is supported by three cables as shown in the diagram.



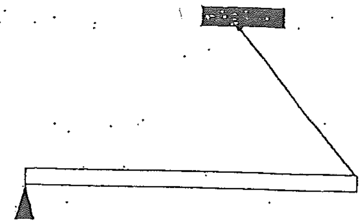

What is the tension in the horizontal cable?

16. A beam is to be kept horizontal by a cord. In which of the four situations shown below will the tension in the cord be least?

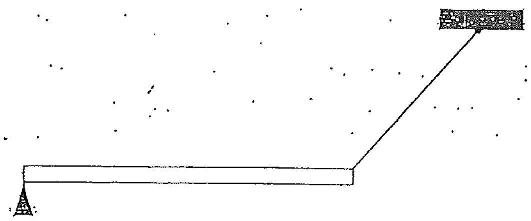
A.



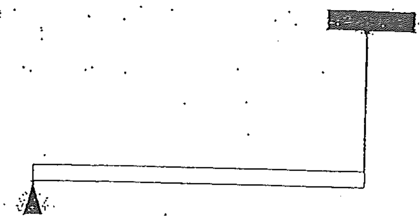
B.



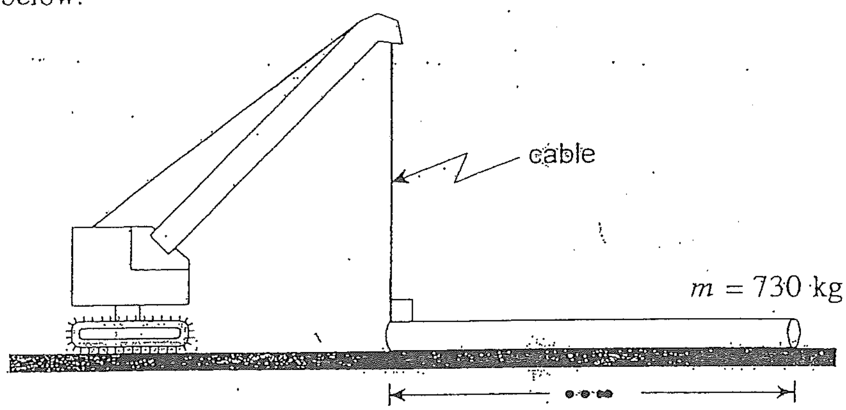
C.



D.

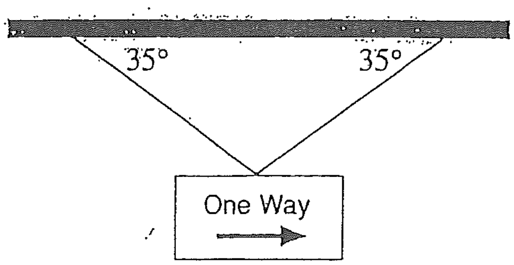


17. A crane is used to lift one end of a uniform 15 m long pipe with a mass of 730 kg as shown in the diagram below.




What is the minimum force of tension in the crane cable to just lift the end of the pipe off the ground?

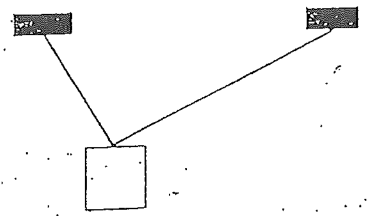
18. A traffic sign hangs from two cables as shown.




If the tension in each cable is 220 N, what is the weight of the sign?

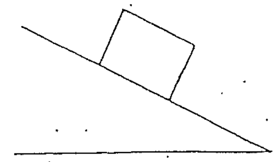
19. Which of the four problems shown requires the application of torque?

A.



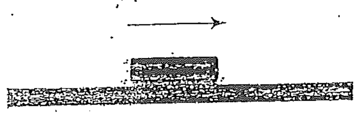
What is the tension in the supporting cables?

B.



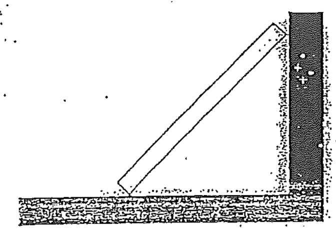
What is the friction force acting on the block?

C.



What is the acceleration of the puck?

D.



What force does the wall exert on the board?

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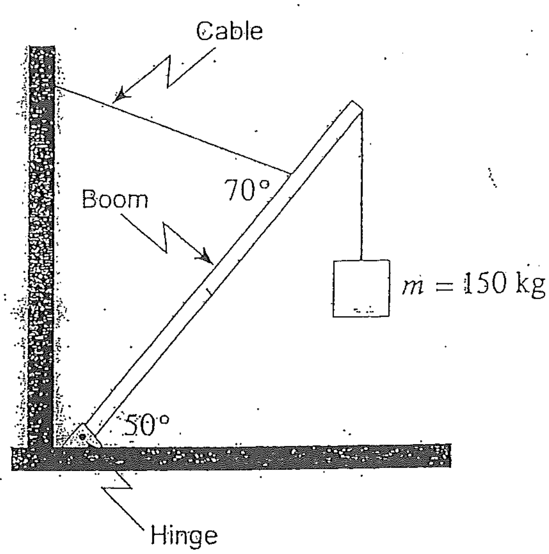
20. A uniform 6.0 m-long boom has a mass of 55 kg. It is kept in position by a restraining cable attached three-quarters of the way along the boom.




What is the tension in this cable when the boom supports a 150 kg mass as shown? **(7 marks)**

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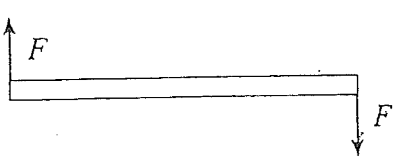
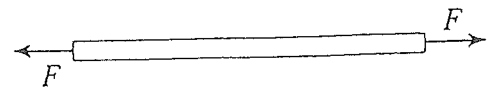
21. A metre stick, as seen from above, is sitting on a table and is then subjected to two forces of equal magnitude as shown. In which case would the metre stick be in rotational equilibrium?

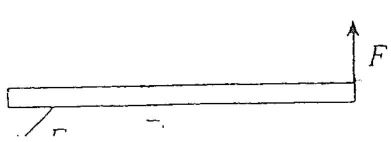
A.



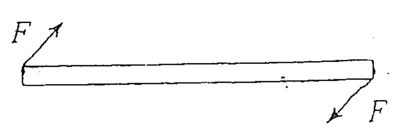

B.

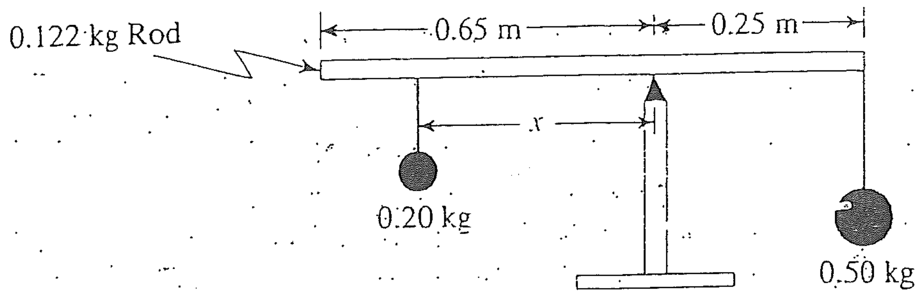



C.




D.

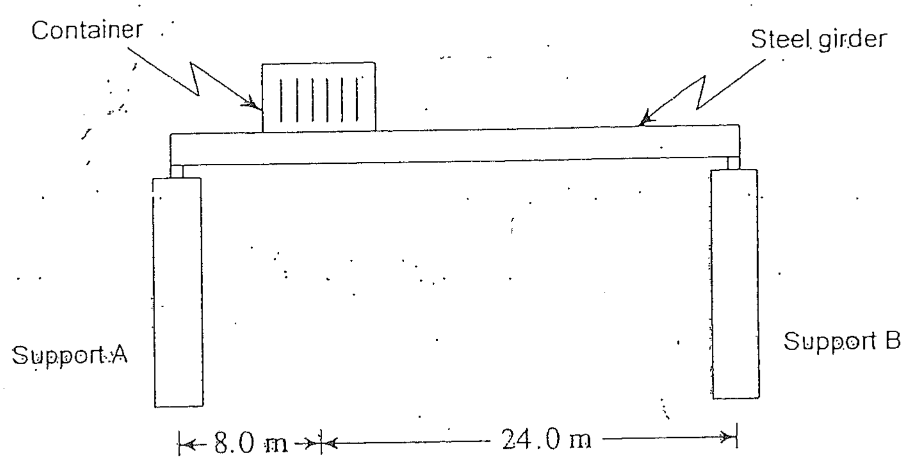





At what distance $x$ should the 0.20 kg mass be placed to achieve static equilibrium?

---

a3 A uniform 1 200 kg steel girder is supported horizontally at its endpoints as shown in the diagram.




What are the upward forces at the girder end points when it is bearing a 3 700 kg shipping container 8.0 m from support A? (7 marks)

---

# Forces and Equilibrium
## Answers

1. 46 N
2. 500N
3. 58 kg
4. $1.4 \times 10^3$ N
5. A
6. 540 N
7. 6.7 m
8. $1.5 \times 10^3$ N
9. $8.5 \times 10^2$ N
10. $5.3 \times 10^2$ kg
11. a) $F_B = 4.8 \times 10^2$ N, $F_A = 3.5 \times 10^2$ N
    b) increase
    c) As the cyclist moves toward B the lever arm increases creating a larger clockwise torque. To maintain static equilibrium the counterclockwise torque must also increase and since d is fixed (d = 8.0m), the force must increase.
12. A
13. $T_1 = 302$ N, $T_2 = 224$ N
14. C
15. 260 N
16. D
17. $3.58 \times 10^3$ N
18. $2.5 \times 10^2$ N
19. D
20. $1.6 \times 10^3$ N
21. B
22. 0.50 m
23. $F_A = 3.31 \times 10^4$ N, $F_B = 1.5 \times 10^4$ N

Torque

Key

**Physics 12 – Introduction to <u>Translational Equilibrium</u>** Name:

Objects on Earth have at least one force acting on them at all times (gravity). However, since many objects are at rest, there must be other forces acting on them in order for $\sum F = 0N$ (so that $a = 0 m/s^2$).

For a body to be at rest, the sum of the forces acting on the body must add up to zero.

**Equilibrium** – <u>When the net force (the sum of all the forces) on an object is zero, the object is said to be in equilibrium.</u>

Since we deal with an x-component and y-component for most forces and motion, both components must add up to a net force of zero as well.

**<u>First Condition of Equilibrium</u>**

$$\sum F_X = 0 \qquad \text{AND} \qquad \sum F_Y = 0$$

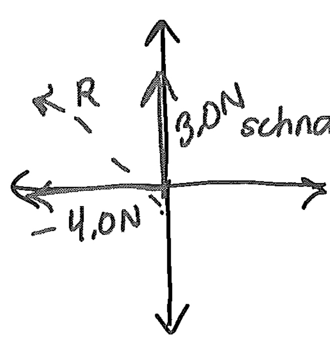
A simplest case of equilibrium is the case in which two equal forces act in opposite directions on an object. The resultant force is zero, and the object does not move.

<u>Example 1</u> – Two dogs are fighting over a steak. One dog (a pug) is pulling north with a force of 3.0 N, and another dog (a schnauzer) is pulling west with a force of 4.0 N. How much force and in what direction would I have to pull on the steak so that the steak does not move?

<u>Vector Component Analysis:</u>

| | X | Y |
|---|---|---|
| pug: | 0N | +3.0N |
| schnauzer: | -4.0N | 0N |

$$\boxed{\begin{matrix} \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{matrix}}$$

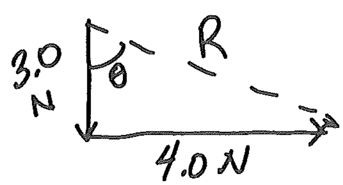



$\Sigma F_x = 0 + (-4.0) + F_x = 0$

$\Sigma F_x = 3.0 + 0 + F_y = 0$

$F_x = +4.0\,N \qquad F_y = -3.0\,N$

With what TOTAL force must you pull on the steak to hold it still (magnitude and direction)?




$R = \sqrt{3^2 + 4^2} = 5.0N$

$\tan^{-1}\left(\frac{4.0}{3.0}\right) = 53°$ E of S

Total $F = 5.0N$ @ $53°$ E of S

In order to solve equilibrium problems, you must have a good grasp of *Free Body diagrams* and of *vector components.*

$$\sum F_{net} = 0$$

$$\downarrow$$

$$\sum F_x = 0 \quad \textit{and} \quad \sum F_y = 0$$

If we find all the vector components of all the forces in the x direction and they sum to zero, can there be any acceleration in the x direction? no $\quad \Sigma F_x > 0$ for $\vec{a}$ in that direction

If we find all the vector components of all the forces in the y direction and they sum to zero, can there be any acceleration in the y direction? no $\quad \Sigma F_y > 0$

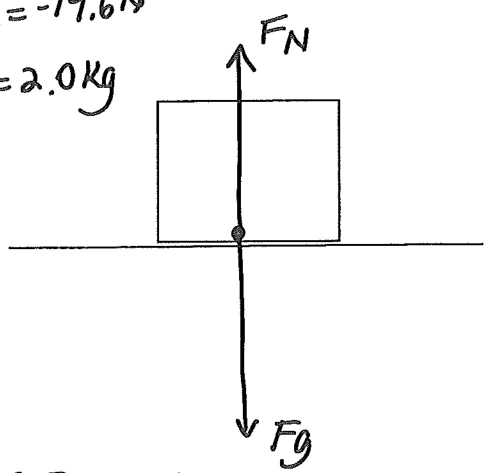
If there is no acceleration in the x and y directions, is there any acceleration at all? NO =equilibrium state

**Example 1** - A 2 kg book is sitting motionless on a table. Draw a diagram of the book on the table, and then draw a F.B.D. of the book to find the force applied by the table.

$F_N = +19.6N$

$F_g = -19.6N$

$m = 2.0kg$




$\Sigma F = 0N$

✓$\Sigma F_y = -19.6 + 19.6 = 0$

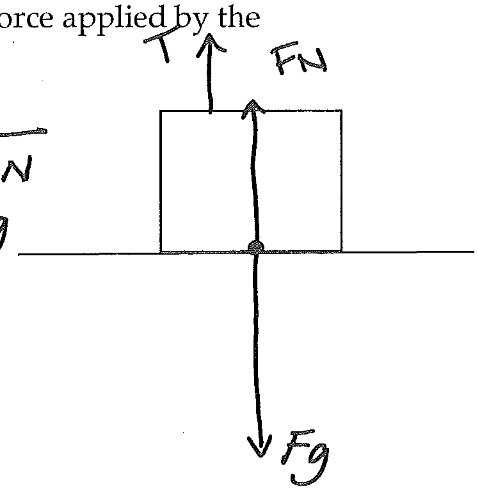
✓$\Sigma F_x = 0 + 0 = 0$

**Example 2** - A 2 kg book is sitting motionless on a table. A helium balloon is attached to the book and the tension in the balloon string is 2 N. Draw a diagram of the book on the table, and then draw a F.B.D. of the book to find the force applied by the table. $T = +2N$

$F_N =$ ______

$F_g = -19.6N$

$m = 2.0kg$




$\Sigma F = 0N$

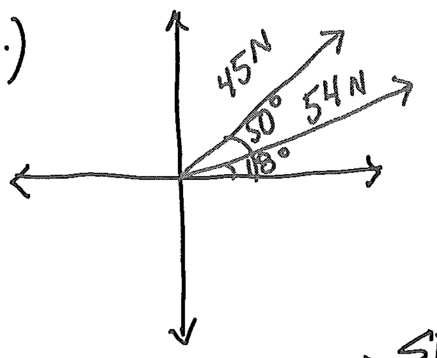
✓$\Sigma F_y = 2 + F_N + (-19.6) = 0$

$F_N = +17.6\ N$

✓$\Sigma F_x = 0 + 0 = 0$

**Example 3** - A 54 N force acts at 18° and a second 45 N force acts at 50°
a. Determine the resultant force.
b. What is the magnitude and direction of the force that produces equilibrium?

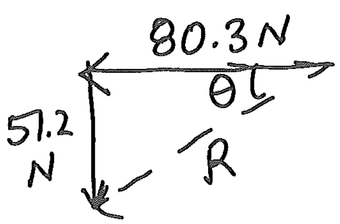
a.)




| X | Y |
|---|---|
| $\cos 18(54) = +51.4N$ | $\sin 18(54) = +16.7N$ |
| $\cos 50(45) = +28.9N$ | $\sin 50(45) = +34.5N$ |

$\Sigma F = 0N \rightarrow \Sigma F_x = 51.4 + 28.9 + F_x = 0 \quad F_x = -80.3N$

$\rightarrow \Sigma F_y = 16.7 + 34.5 + F_y = 0 \quad F_y = -51.2N$




$R = \sqrt{80.3^2 + 51.2^2} = 95.2N$

$\tan^{-1}\left(\frac{51.2}{80.3}\right) = 32.5° \text{ S of W}$

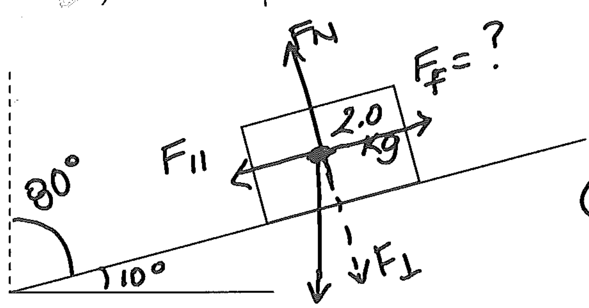
Equilibrium Force = 95.2N @ 32.5° S of W

**Example 4** - A 2 kg book is sitting motionless on a ramp that has an angle of 80° with respect to the vertical. For part 'C' assume that if angle decreases to 79° to the vertical, the book will begin to slide.

A) Why does the book not slide down the ramp?
B) How big is this force
C) Calculate $\mu_s$

Ⓐ $F_f = F_{\parallel}$ so $\Sigma F = 0N$

Ⓑ $F_{\parallel} = mg\sin 10°$
$= (2.0)(9.8)\sin 10° = 3.4N$




Ⓒ $F_f = \mu_s F_N$ since the ∠ is now 11° need to calc.:

* $F_N$ for 11°
$= (2.0 \cdot 9.8)\cos 11 = \underline{19.2N}$

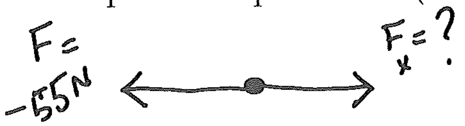
*assume $F_{\parallel} = F_f$ for 11° as this will be what overcomes $\mu_s$
$= (2.0 \cdot 9.8)\sin 11 = \underline{3.74N}$

$F_f = \mu_s F_N$
$3.74 = \mu_s 19.2$

$\underline{\mu_s = 0.195}$

**Introduction to Equilibrium Problems**

1. A force of 55 N acts due west on an object. What single force is required to produce equilibrium? (55 N [E])



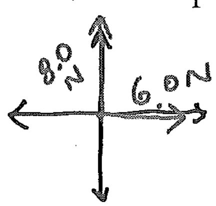

$\Sigma F_y = 0\,N$

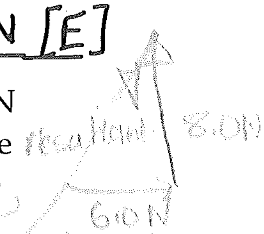
$\Sigma F_x = -55 + F_x = 0$

$\Sigma F = 0\,N$

$F_x = +55\,N$ or 55 N [E]

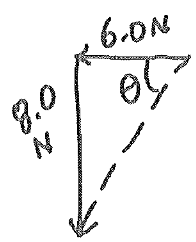
2. Two forces act on an object. One force is 6.0 N east; the second force is 8.0 N north. If the object is in equilibrium, find the magnitude and direction of the force that produces equilibrium. (10 N @ 53° S of W)




$\Sigma F_x = 0 + 6.0 + F_x = 0 \quad F_x = -6.0\,N$

$\Sigma F_y = 8.0 + 0 + F_y = 0 \quad F_y = -8.0\,N$



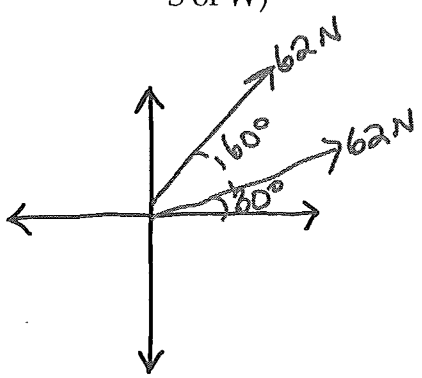

$R = \sqrt{6.0^2 + 8.0^2} = 10\,N$

$\tan^{-1}\left(\frac{8.0}{6.0}\right) = 53°$ S of W

Equilibrium Force = 10 N @ 53° S of W

3. A 62 N force acts at 30° and a second 62 N force acts at 60°. What is the magnitude and direction of the force that produces equilibrium? (120 N @ 45° S of W)

X | Y

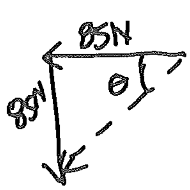



$\cos 30(62) = +54\,N$ | $\sin 30(62) = +31\,N$

$\cos 60(62) = +31\,N$ | $\sin 60(62) = +54\,N$

$\Sigma F_x = 54 + 31 + F_x = 0 \quad F_x = -85\,N$

$\Sigma F_y = 31 + 54 + F_y = 0 \quad F_y = -85\,N$




$R = \sqrt{85^2 + 85^2} = 120\,N$ @ 45° S of W

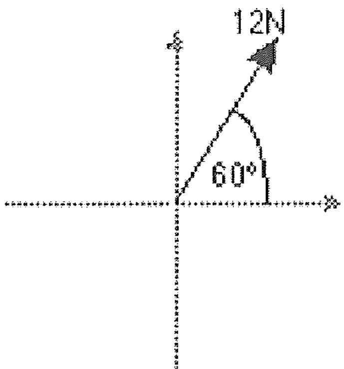
4. What force is required to satisfy the $\Sigma F_x = 0N$ condition for diagram A below? (-6.0N)

$$\Sigma F_x = \cos 60(12) + F_x = 0 \qquad F_x = -6.0\ N$$

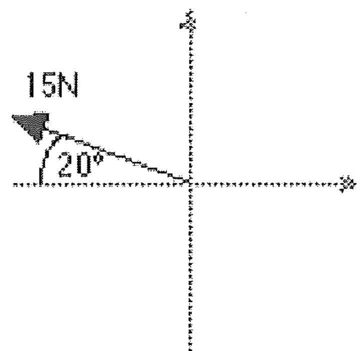
5. What force is required to satisfy the $\Sigma F_y = 0N$ condition for diagram B below?(-5.1N)

$$\Sigma F_y = \sin 20(15) + F_y = 0 \qquad F_y = -5.1\ N$$

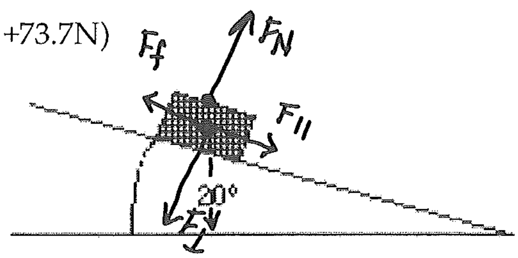
A.




B.




6. An 8.0 kg box rests motionless on an incline. Draw an F.B.D. for the box and determine the values of each force keeping the box in an equilibrium state. (-26.8N, +73.7N)




$$\Sigma F = 0\ N$$

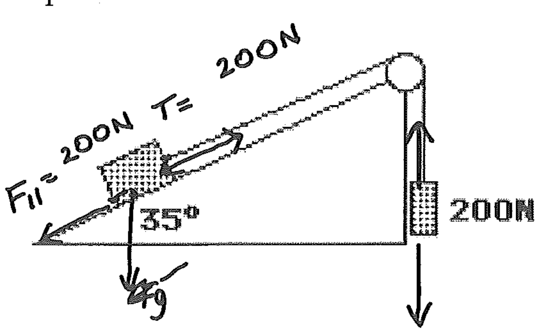
$$\Sigma F_x = \sin 20(8.0 \cdot 9.8) + F_f = 0$$

$$F_f = -26.8\ N$$

$$\Sigma F_y = -\cos 20(8.0 \cdot 9.8) + F_N = 0$$

$$F_N = +73.7\ N$$

7. In the system below the pulley and ramp are frictionless and the block is in static equilibrium. What is the mass of the block? (35.6 kg)




$$F_{\parallel} = F_g \sin 35 \qquad F_g = 348.7\ N$$

$$200 = F_g \sin 35$$

$$F_g = mg$$

$$348.7 = m(9.8) \qquad m = \underline{35.6\ kg}$$

**Physics 12 -Translational Equilibrium** Name:

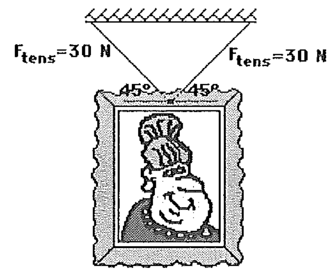
**First Condition of Equilibrium**

$\sum F_X = 0$ AND $\sum F_Y = 0$

**A body in translational equilibrium will have no acceleration in the x or y directions.**

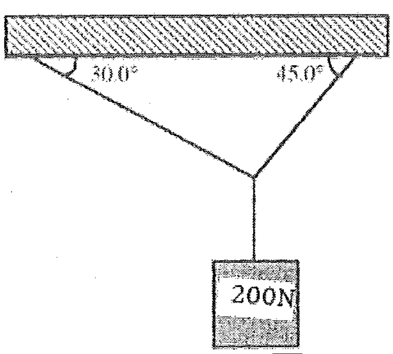
**Equilibrium is a very important concept in the Engineering and Design of any structure as this is a good thing for stationary objects like bridges and buildings.**

Yesterday we discussed the simplest cases of equilibrium in which forces were acting on an object and we determined what would "balance" those forces to create a state of equilibrium for the object.

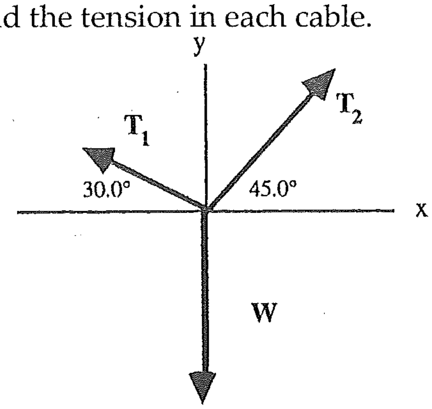



**Another example type of equilibrium is when objects are hung and tensional forces must be in equilibrium with the weight of the object** (such as a sign or picture).

**Example 1:** A weight suspended by cables and the system is in equilibrium. A 200 N block is suspended by two cables as shown in the diagram. Find the tension in each cable.




translate into vectors →




| | x | y |
|---|---|---|
| $T_1$: | $-\cos 30 T_1$ | $+\sin 30 T_1$ |
| $T_2$: | $+\cos 45 T_2$ | $+\sin 45 T_2$ |
| W: | 0 N | $-200$ N |

$\Sigma F_x = -\cos 30 T_1 + \cos 45 T_2 + 0 = 0$

*solve for $T_1$: $T_1 = \dfrac{\cos 45 T_2}{\cos 30}$

$T_1 = 0.816(T_2)$

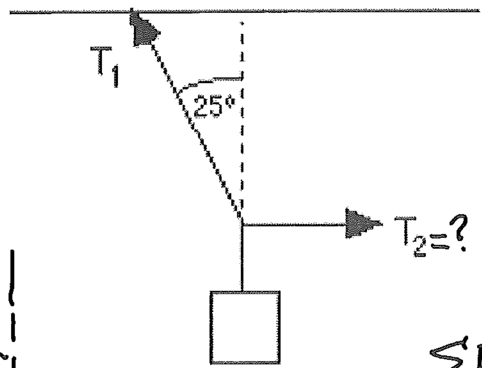
$\Sigma F_y = \sin 30(0.816 T_2) + \sin 45 T_2 - 200 = 0$

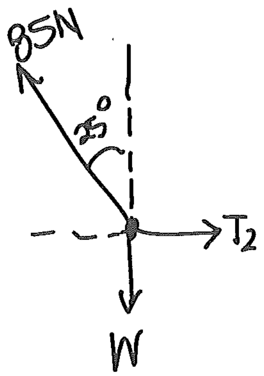
$200 = 1.115 T_2$ $\quad T_2 = \underline{+179\text{ N}}$

$T_1 = 0.816(179)$

$= \underline{+146\text{ N}}$

**Example 2:** For the force diagram shown below, let $T_1 = 85$ N. Determine the magnitude of $T_2$ and the mass of the box.



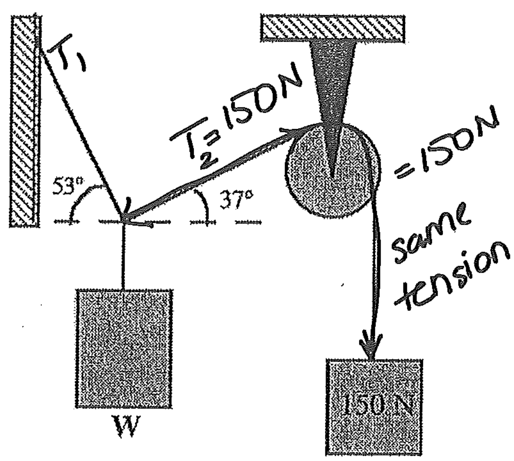

| | X | Y |
|---|---|---|
| $T_1 =$ | $-\sin 25(85) = -36$ N | $+\cos 25(85) = +77$ N |
| $T_2 =$ | $T_2$ | 0 N |
| $W =$ | 0 | $-W$ |

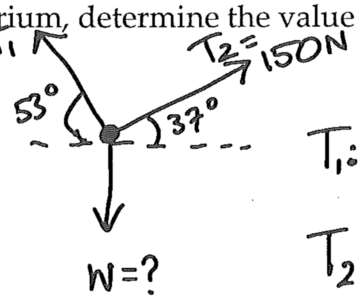
$\Sigma F_x = -36 + T_2 + 0 = 0 \quad T_2 = +36 \text{ N}$

$\Sigma F_y = 77 + 0 + (-W) = 0 \quad W = -77 \text{ N}$

$$\text{mass} = \frac{W}{9.8} = \frac{77}{9.8} = 7.9 \text{ kg}$$

**Example 3:** If the system is in equilibrium, determine the value of the weight, W and the tension in the left wire.




| | X | Y |
|---|---|---|
| $T_1$: | $-\cos 53 T_1$ | $\sin 53 T_1$ |
| $T_2$: | $\cos 37(150)$ | $\sin 37(150)$ |
| W: | 0 | $-W$ |

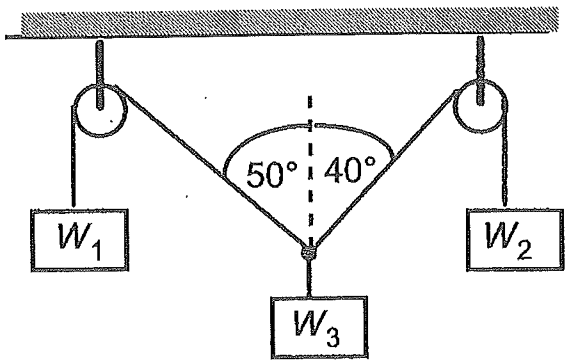
$\Sigma F_x = -\cos 53 T_1 + \cos 37(150) + 0 = 0$

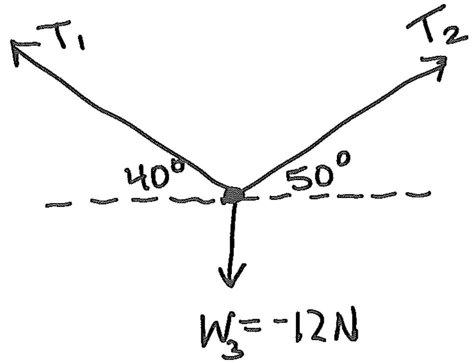
$T_1 = 199$ N

$\Sigma F_y = \sin 53(199) + \sin 37(150) - W = 0$

$W = 249$ N

**Example 4:** $W_1$, $W_2$, and $W_3$ are the weights of three objects suspended by pulleys. Assuming that the pulleys in this system are frictionless and weightless and that $W_3 = 12$ N, what are the values of $W_1$ and $W_2$?




$$\underbrace{T_1 = W_1 \qquad T_2 = W_2}_{\text{if in equilibrium } (\Sigma F = 0N) \;=\text{ balanced}}$$

| | $x$ | $y$ |
|---|---|---|
| $T_1$: | $-\cos 40 T_1$ | $\sin 40 T_1$ |
| $T_2$: | $\cos 50 T_2$ | $\sin 50 T_2$ |
| $W_3$: | $0$ | $-12$ |

$\Sigma F_x = -\cos 40 T_1 + \cos 50 T_2 + 0 = 0$

$T_1 = 0.839 T_2$

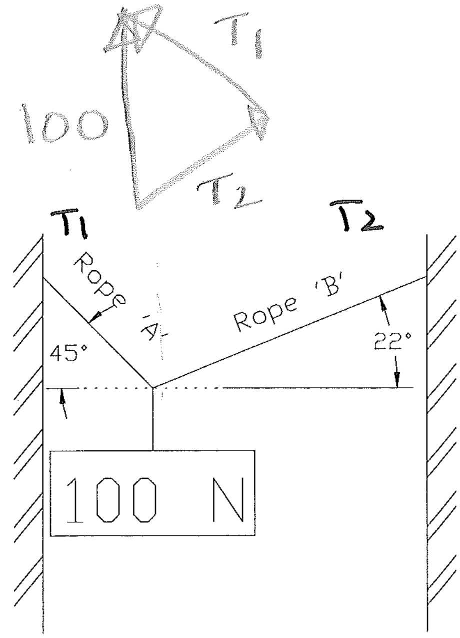
$\Sigma F_y = \sin 40(0.839 T_2) + \sin 50 T_2 - 12 = 0$

$1.31 T_2 = 12 \qquad T_2 = +9.2$ N

$T_1 = 0.839(9.2) = +7.7$ N

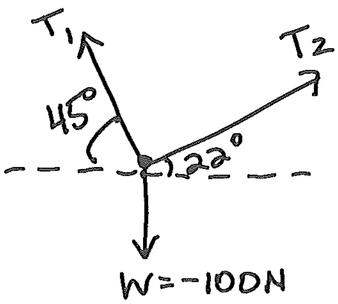
$$\boxed{\begin{aligned} W_1 &= -7.7 \text{ N} \\ W_2 &= -9.2 \text{ N} \end{aligned}}$$

## Equilibrium – First Condition Part One Problems




1. A 100 N sign hangs in between two buildings as shown. Which rope will have the higher tension force, rope 'A' or Rope 'B'? (Rope A with +101 N versus +77 N for Rope B)

41 N



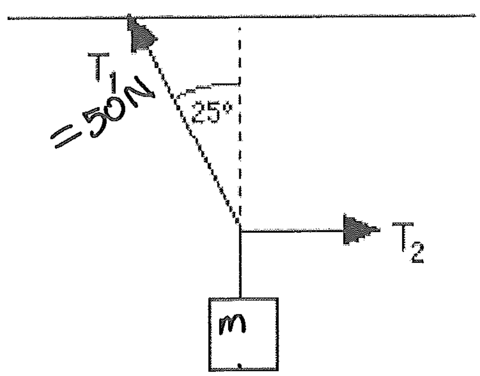

| | X | Y |
|---|---|---|
| $T_1$: | $-\cos 45 T_1$ | $\sin 45 T_1$ |
| $T_2$: | $\cos 22 T_2$ | $\sin 22 T_2$ |
| W: | 0 | $-100$ |

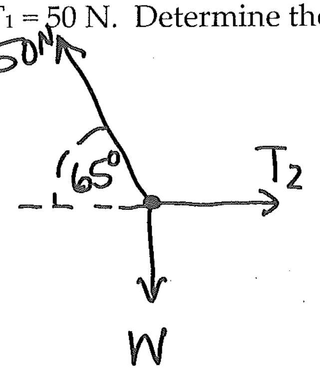
$$\Sigma F_x = -\cos 45 T_1 + \cos 22 T_2 + 0 = 0 \qquad T_1 = 1.31 T_2$$

$$\Sigma F_y = \sin 45(1.31 T_2) + \sin 22 T_2 - 100 = 0$$

$$100 = 1.30 T_2 \qquad \underline{T_2 = +77\ N} \qquad T_1 = 1.31(77) = \underline{+101\ N}$$

2. For the force diagram shown below, let $T_1 = 50$ N. Determine the magnitude of $T_2$ and the mass of the box. (+21N, 4.6 kg)




| | X | Y |
|---|---|---|
| $T_1$: | $-\cos 65(50) = -21.1$ N | $\sin 65(50) = 45.3$ N |
| $T_2$: | $T_2$ | 0 |
| W: | 0 | $-W$ |

$$\Sigma F_x = -21.1 + T_2 + 0 = 0$$

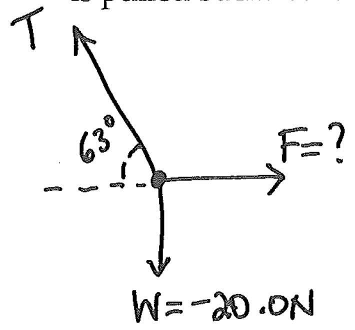
$$\underline{T_2 = +21\ N}$$

$$\Sigma F_y = 45.3 + 0 - W = 0$$

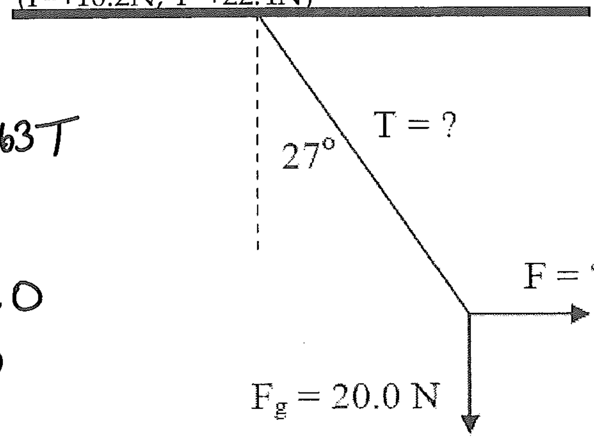
$$W = 45.3\ N$$

$$mass = \frac{45.3}{9.8} = \underline{4.6\ Kg}$$

3. A 20.0 N child sitting on a playground swing is being pushed by her father. When the rope makes an angle of 27° to the vertical, what is the force exerted by her father when the swing is pulled back? What is the tension in the rope, T? (F=+10.2N, T=+22.4N)




| | X | Y |
|---|---|---|
| T: | $-\cos 63 T$ | $\sin 63 T$ |
| F: | F | 0 |
| W: | 0 | -20.0 |



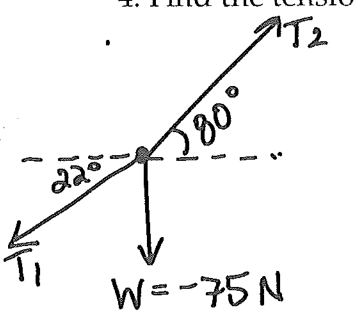

$\Sigma F_y = \sin 63 T + 0 - 20.0 = 0$

$T = +22.4\,N$

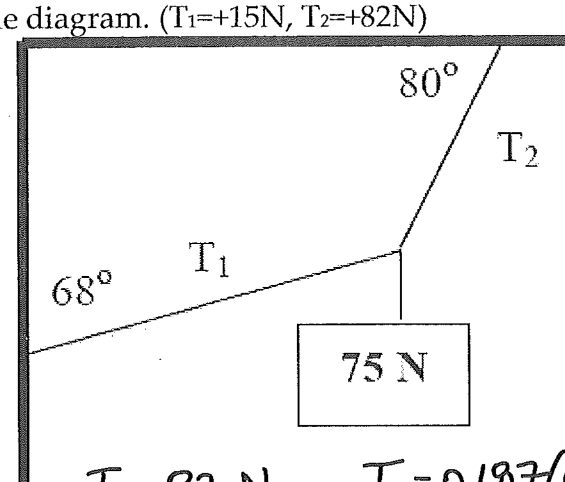
$\Sigma F_x = -\cos 63(22.4) + F + 0 = 0$

$F = +10.2\,N$

4. Find the tensions $T_1$ and $T_2$ in the ropes shown in the diagram. ($T_1$=+15N, $T_2$=+82N)




| | X | Y |
|---|---|---|
| $T_1$: | $-\cos 22 T_1$ | $-\sin 22 T_1$ |
| $T_2$: | $\cos 80 T_2$ | $\sin 80 T_2$ |
| W: | 0 | -75 |




$\Sigma F_x = -\cos 22 T_1 + \cos 80 T_2 + 0 = 0$

$T_1 = 0.187 T_2$

$\Sigma F_y = -\sin 22(0.187 T_2) + \sin 80 T_2 - 75 = 0$

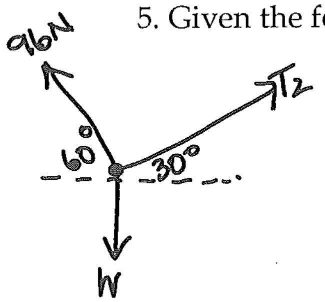
$0.915 T_2 = 75$

$T_2 = +82\,N$

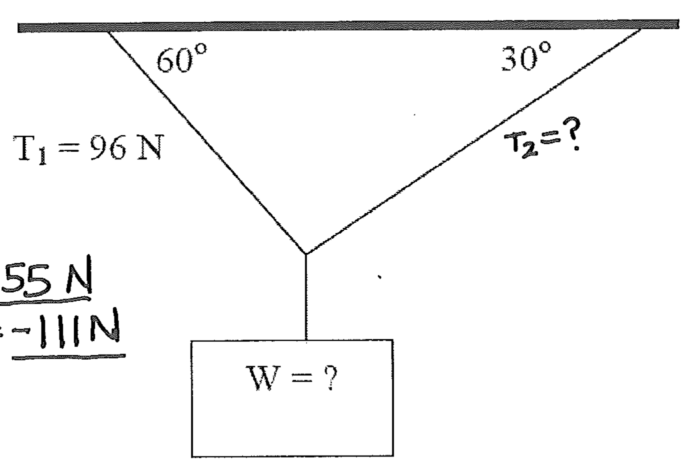
$T_1 = 0.187(82)$

$= +15\,N$

5. Given the following diagram, find W and $T_2$. ($T_2$=+55N, W=-111N)



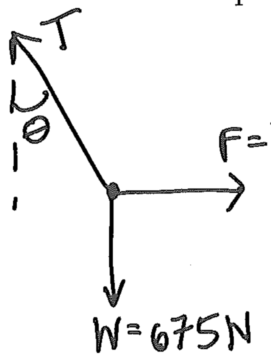

| | X | Y |
|---|---|---|
| $T_1$: | $-\cos 60(96)$ | $\sin 60(96)$ |
| $T_2$: | $\cos 30 T_2$ | $\sin 30 T_2$ |
| W: | 0 | -W |

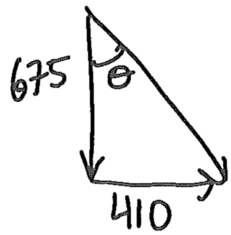



$\Sigma F_x = -48 + \cos 30 T_2 + 0 = 0$ $\quad T_2 = +55\,N$

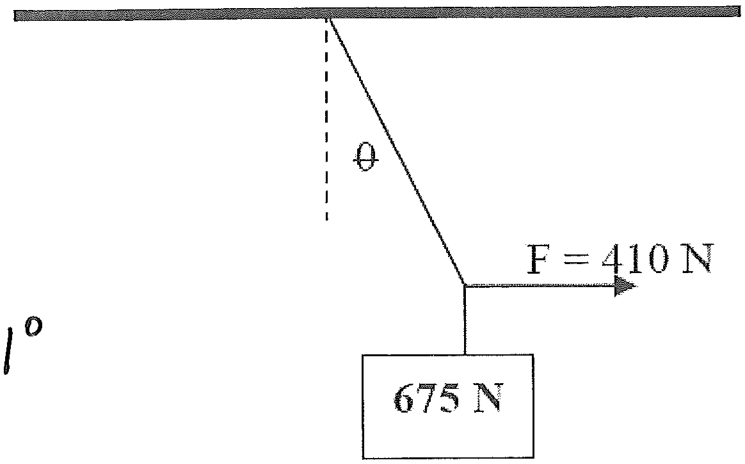
$\Sigma F_y = 83.1 + \sin 30(55) - W = 0$ $\quad W = -111\,N$

6. A 675 N object is pulled horizontally by a force of 410 N as shown. What is the angle, θ, between the rope and the vertical? (31°)

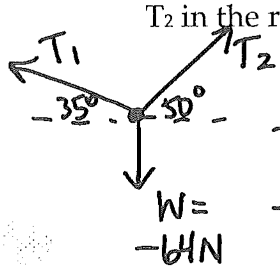



$\tan^{-1}\left(\frac{410}{675}\right) = 31°$




7. A 64 N object is suspended using ropes as shown in the diagram. Calculate tensions $T_1$ and $T_2$ in the ropes. (+41N, +53N)

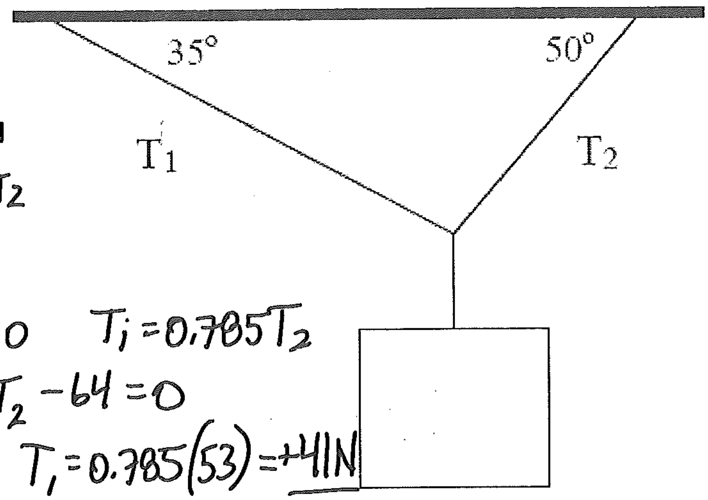



| | X | Y |
|---|---|---|
| $T_1$: | $-\cos 35 T_1$ | $\sin 35 T_1$ |
| $T_2$: | $\cos 50 T_2$ | $\sin 50 T_2$ |
| W: | 0 | $-64$ |

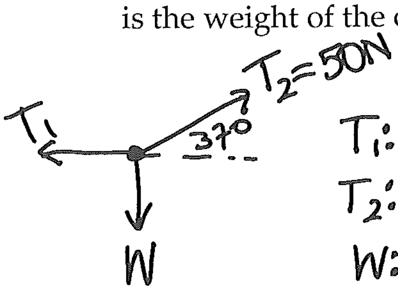
$\Sigma F_x = -\cos 35 T_1 + \cos 50 T_2 + 0 = 0 \quad T_1 = 0.785 T_2$

$\Sigma F_y = \sin 35(0.785 T_2) + \sin 50 T_2 - 64 = 0$

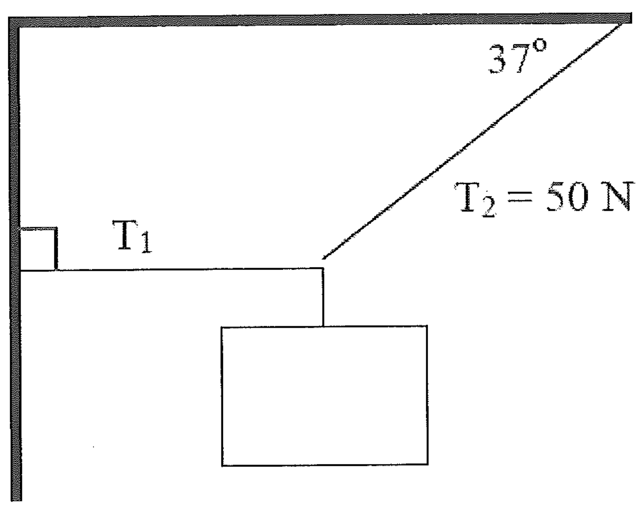
$1.22 T_2 = 64 \quad T_2 = +53\,N \quad T_1 = 0.785(53) = +41\,N$




8. An object is suspended as shown. If the tension in one of the ropes is 50 N as shown, what is the weight of the object? (-30N)




| | X | Y |
|---|---|---|
| $T_1$: | $-T_1$ | 0 |
| $T_2$: | $\cos 37(50)$ | $\sin 37(50)$ |
| W: | 0 | $-W$ |



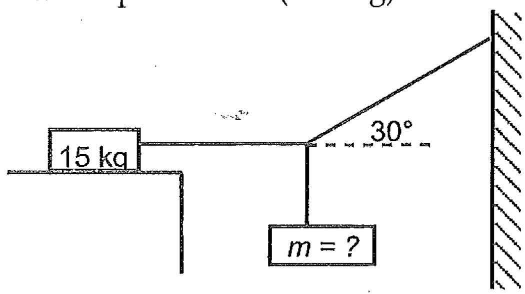

$\Sigma F_x = -T_1 + \cos 37(50) + 0 = 0$

$T_1 = +40\,N$

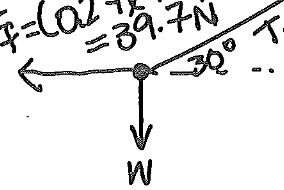
$\Sigma F_y = 0 + \sin 37(50) - W = 0$

$W = -30\,N$

9. A 15 kg object rests on a table. A cord is attached to this object and also to a wall. Another object is hung from this cord as shown in the diagram. Is the coefficient of friction between the 15 kg object and the table is 0.27, what is the maximum mass that can be hung and still maintain equilibrium? (2.34 kg)




$F_f = (0.27)(9.8)(15)$
$= 39.7\text{N}$




| | X | Y |
|---|---|---|
| $T_1$: | $\cos 30 T_1$ | $\sin 30 T_1$ |
| $F_f$: | $-39.7$ | $0$ |
| W: | $0$ | $-W$ |

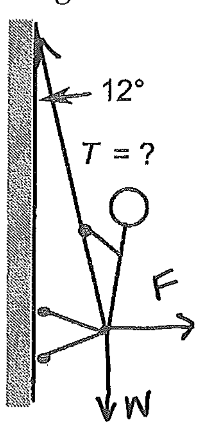
$\Sigma F_x = \cos 30 T_1 + (-39.7) + 0 = 0$

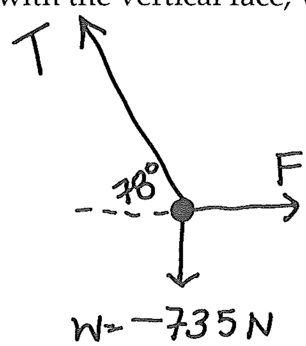
$T_1 = +46\text{N}$

$\Sigma F_y = \sin 30(46) - W + 0 = 0$

$W = 22.9\text{N} \div 9.8 = \underline{2.34\text{ kg}}$

10. A 735 N mountain climber is rappelling down the face of a vertical cliff. If the rope makes an angle of 12.0º with the vertical face, what is the tension in the rope? (+751 N)



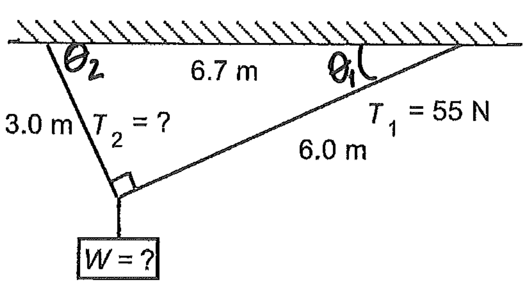

| | X | Y |
|---|---|---|
| T: | $-\cos 78(T)$ | $\sin 78 T$ |
| F: | $F$ | $0$ |
| W: | $0$ | $-735\text{ N}$ |

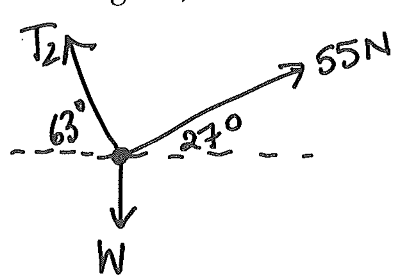
$\Sigma F_y = \sin 78 T + 0 + (-735) = 0$

$T = +\underline{751}\text{N}$

$\Sigma F_x = -\cos 78(751) + F + 0 = 0 \quad F = \underline{+156\text{N}}$

11. In the equilibrium state as shown in the diagram, find W and $T_2$.




| | X | Y |
|---|---|---|
| $T_1$: | $\cos 27(55)$ | $\sin 27(55)$ |
| $T_2$: | $-\cos 63 T_2$ | $\sin 63 T_2$ |
| W: | $0$ | $-W$ |

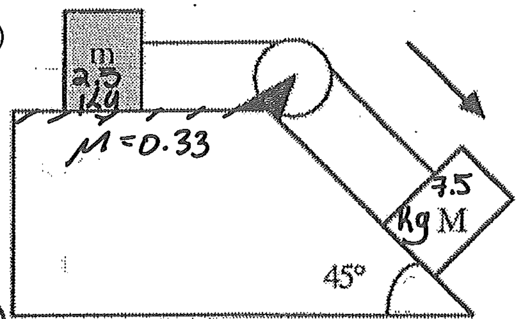
$\theta_1 = \tan^{-1}\left(\frac{3.0}{6.0}\right) = 27°$

$\theta_2 = \tan^{-1}\left(\frac{6.0}{3.0}\right) = 63°$

$\Sigma F_x = \cos 27(55) - \cos 63 T_2 + 0 = 0 \quad T_2 = \underline{+108\text{N}}$

$\Sigma F_y = \sin 27(55) + \sin 63(108) - W = 0 \quad W = \underline{+121}\text{N}$

12. Find the acceleration of the system (M = 7.5 kg, m = 2.5 kg) if,
a) There is no friction. (5.2 m/s²)
b) The coefficient of kinetic friction is 0.33 (2.7 m/s²)




a) $\sin 45(7.5 \cdot 9.8) = 52\text{ N} = F_{\parallel}$

$F_f = 0$ $F_{\parallel}$ $\quad \Sigma F = 52 \quad \vec{a} = \frac{52}{10}$

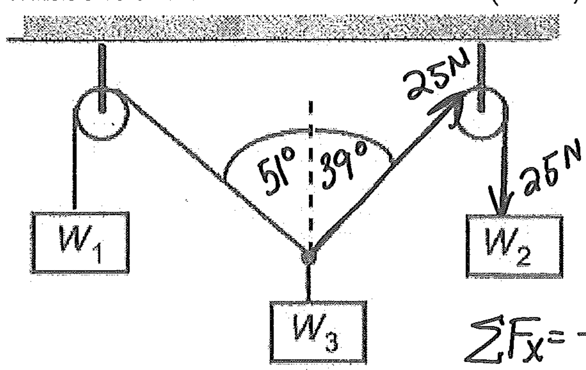
$\vec{a} = 5.2\text{ m/s}^2$

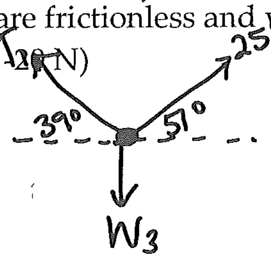
b) $F_{\parallel} = 52\text{N}$

$F_f = (0.33)(2.5 \cdot 9.8) + \cos 45(7.5 \cdot 9.8)(0.33) = 25.2\text{ N}$

$\Sigma F = 52 + (-25.2) = 26.8\text{N} \quad \vec{a} = \frac{26.8}{10} = 2.7\text{ m/s}^2$

13. Assuming that the pulleys in this system are frictionless and weightless and that $W_2$ = 25 N, what are the values of $W_1$ and $W_3$? (-32 N, -20 N)



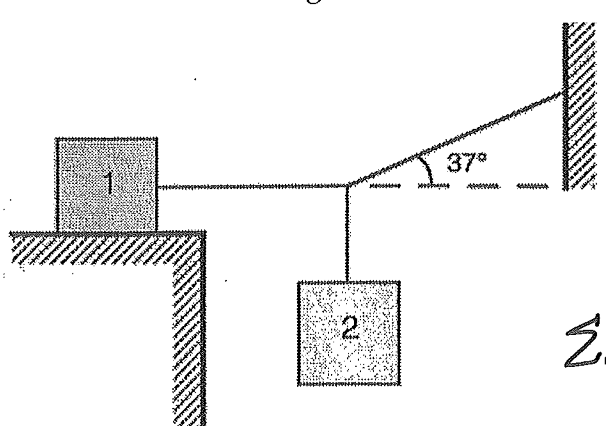

| | X | Y |
|---|---|---|
| $T_1$: | $-\cos 39 T_1$ | $\sin 39 T_1$ |
| $T_2$: | $\cos 51(25)$ | $\sin 51(25)$ |
| $W_3$: | 0 | $-W_3$ |

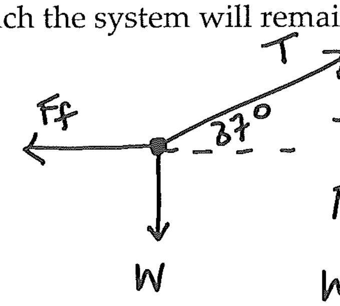
$\Sigma F_x = -\cos 39 T_1 + \cos 51(25) + 0 = 0 \quad T_1 = 20.2\text{N}$

$\Sigma F_y = \sin 39(20.2) + \sin 51(25) - W_3 = 0$

$W_3 = 32\text{ N} \qquad W_1 = 20.2\text{N}$

14. Block 1 weighs 96 N. The coefficient of static friction between block 1 and the table is 0.50. Find the maximum weight of block 2 for which the system will remain in equilibrium. (-36N)




| | X | Y |
|---|---|---|
| T: | $\cos 37 T$ | $\sin 37 T$ |
| $F_f$: | $-48$ | 0 |
| W: | 0 | $-W$ |

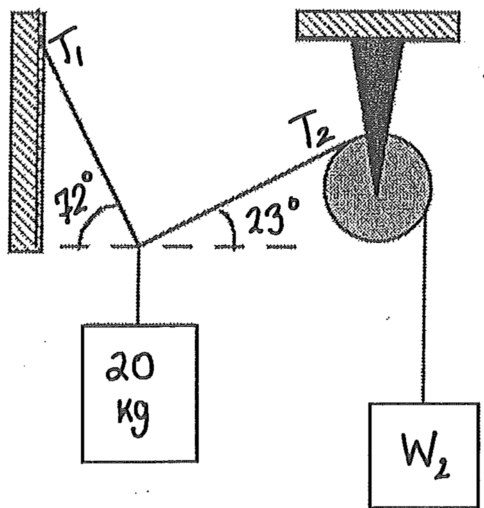
$\Sigma F_x = \cos 37 T - 48 + 0 = 0$

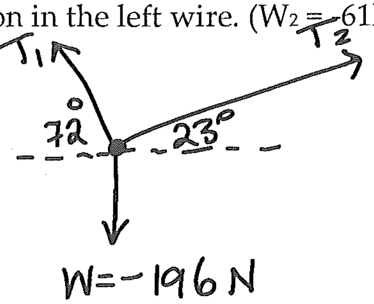
$T = +60\text{N}$

$F_f = (0.5)(96) = -48\text{N}$

$\Sigma F_y = \sin 37(60) + 0 - W = 0 \quad W = -36\text{N}$

15. Determine the weight and tension in the left wire. ($W_2$ = -61N, $T_1$ = +181N)




| | X | Y |
|---|---|---|
| $T_1$: | $-\cos 72 T_1$ | $\sin 72 T_1$ |
| $T_2$: | $\cos 23 T_2$ | $\sin 23 T_2$ |
| W: | 0 | -196 |

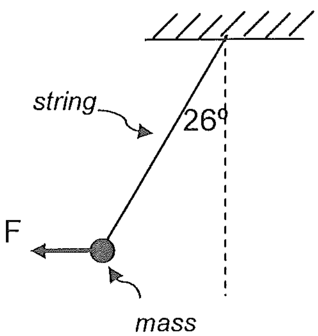
$$\Sigma F_x = -\cos 72 T_1 + \cos 23 T_2 + 0 = 0$$

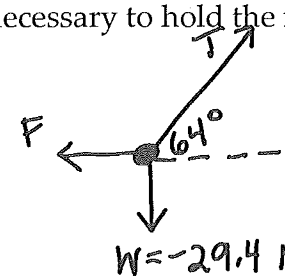
$$T_1 = 2.98 T_2 \quad T_1 = 2.98(61) = \underline{+181\ N}$$

$$\Sigma F_y = \sin 72(2.98 T_2) + \sin 23 T_2 - 196 = 0$$

$$3.22 T_2 = 196 \quad T_2 = +61 N \quad W_2 = \underline{-61 N}$$

16. A mass of 3.0 kg is suspended from a cord as shown in the diagram below. What horizontal force $F$ is necessary to hold the mass in the position shown? (14.3 N)



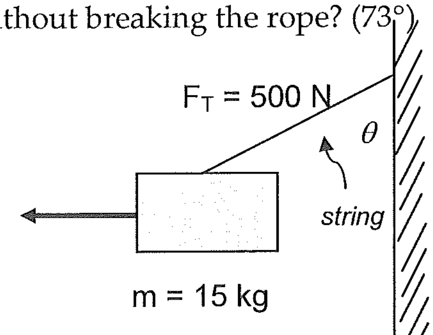



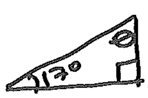

| | X | Y |
|---|---|---|
| T: | cos64T | sin64T |
| F: | -F | 0 |
| W: | 0 | -29.4 |

$$\Sigma F_y = \sin 64 T + 0 - 29.4 = 0 \quad T = +32.7 N$$

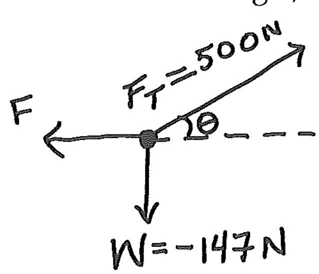
$$\Sigma F_x = \cos 64(32.7) - F + 0 = 0 \quad F = \underline{-14.3 N}$$

17. A 15 kg block is pulled by a horizontal force. The supporting rope can withstand a maximum tension force of 500 N. To what maximum angle, $\theta$, can the block be pulled without breaking the rope? (73°)




$\theta = \underline{73°}$

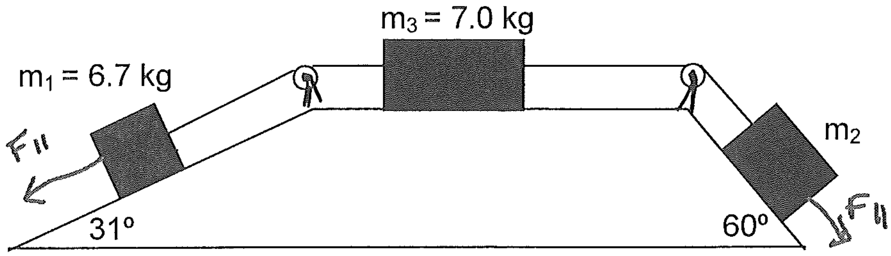



| | X | Y |
|---|---|---|
| $F_T$: | cosθ(500) | sinθ(500) |
| F: | -F | 0 |
| W: | 0 | -147 N |

$$\Sigma F_y = \sin\theta(500) + 0 - 147 = 0$$

$$\sin^{-1}\left(\frac{147}{500}\right) = 17°$$

18. Three masses connected by a string are arranged on frictionless surfaces, as shown in the diagram below. If this system is in equilibrium, what is the mass of $m_2$? (3.98 kg)




$$F_{\parallel} = \sin 31 \cdot 6.7 \cdot 9.8$$
$$= 33.8\,N$$

$$F_{\parallel} = \sin 60 \cdot F_g$$
$$33.8 = \sin 60 \cdot F_g$$

$$F_g = 39.0\,N$$

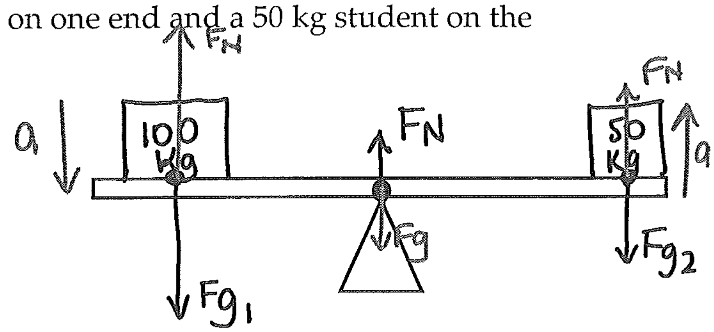
$$m = \frac{F_g}{9.8} = \frac{39.0\,N}{9.8}$$

$$m = \underline{3.98\,Kg}$$

## Physics 12 – Torque and Rotational Equilibrium

A body **in translational equilibrium** will have **no acceleration** in the *x or y* directions. However, it could still be rotating.

Consider a teeter totter, with a 100 kg student on one end and a 50 kg student on the other.




What are the net translational forces in:

The x-direction? $\Sigma F_x = 0\,N$

The y-direction? $\Sigma F_y = 0\,N$

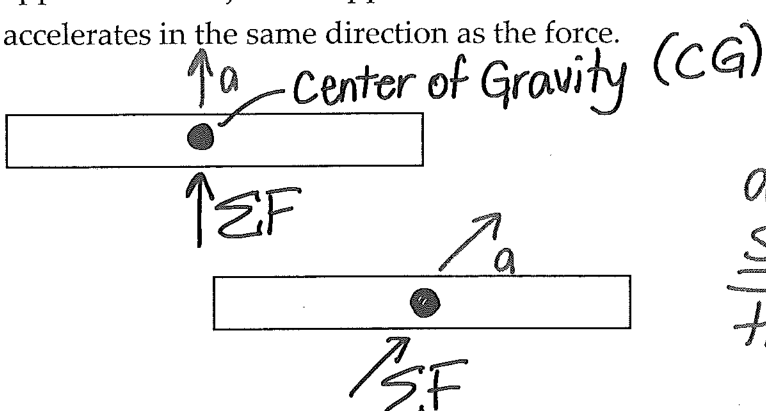
Although the net translational forces are zero, the system has a **net torque** – so it is not in equilibrium. $\Sigma\tau > 0$

An object in equilibrium must have both translational and **<u>rotational</u>** equilibrium.

### <u>Center of Gravity</u> -

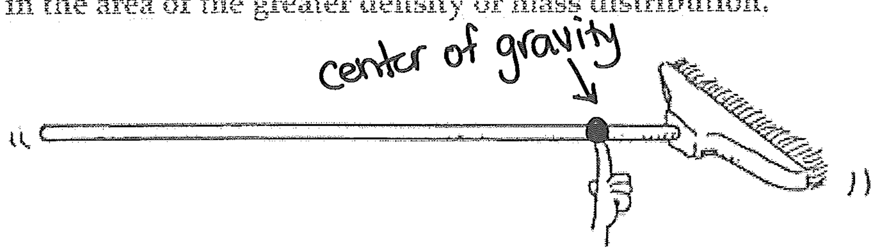
The center of gravity is the point at which the system's whole mass can be considered to be concentrated for the purpose of calculations.

To this point in your physics experience, it has always been assumed that any forces applied to an object are applied to its center of mass. This is illustrated when the object accelerates in the same direction as the force.



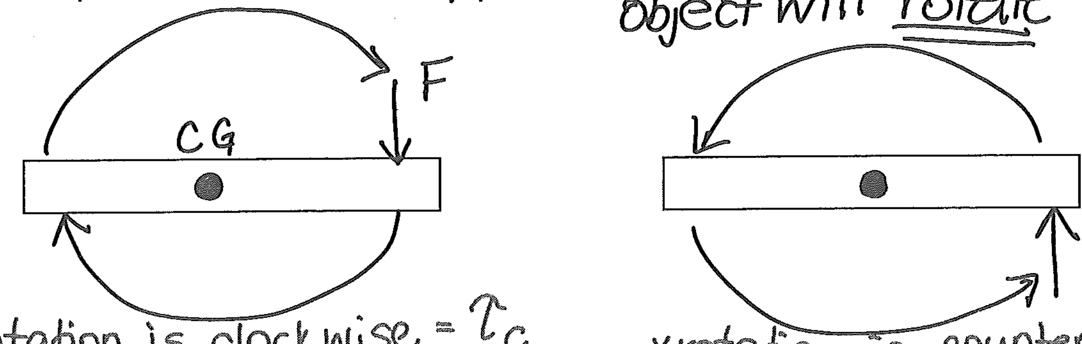

*the object accelerates in the <u>same</u> direction as the $\Sigma F$

The center of mass of a body does not always coincide with its intuitive geometric center. If the object's mass is not of a uniform density or shap the center of mass will fall in the area of the greater density or mass distribution.



However, when dealing with objects in every day life, **forces are not always applied to the center of mass.** What will happen in these cases?

* if the force is applied away from the CG, the object will rotate




*rotation is clockwise = $\tau_c$

*rotation is counter-clockwise = $\tau_{cc}$

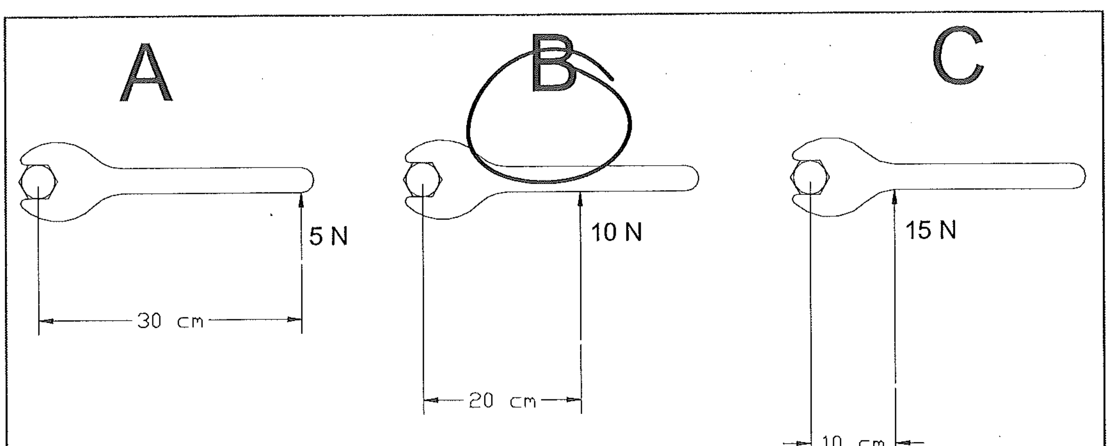
**Torque** –

When the force is not applied directly to the center of mass, we just learned that the object rotates.

*Torque* is a measure of how much the force acting on an object causes that object to rotate. The object rotates about an axis, which is called the pivot point or point of rotation. We will call the force 'F'.

↖ P

**Torque is also directly proportional to the perpendicular distance of the force. Which example will 'twist' the bolt with the most *torque*? Why?**



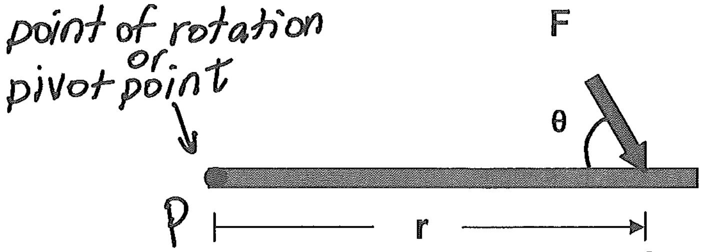

*need to use both the F and lever arm (perpendicular)

A. $\tau = F \cdot d$
$= (5.0)(0.30)$
$= 1.5\ N \cdot m$

B. $\tau = F \cdot d$
$= (10)(0.20)$
$= 2.0\ N \cdot m$

C. $\tau = F \cdot d$
$= (15)(0.10)$
$= 1.5\ N \cdot m$

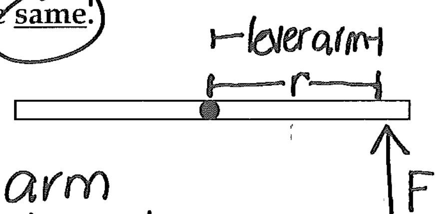
The **distance from the pivot point to the point where the force acts** is called the **radius of rotation**, and is represented by '**r**'. Note that this distance, '**r**', is also a vector, and points from the axis of rotation to the point where the force acts.




**<u>Lever Arm</u> –** (distance from force to P)

The lever arm of a force is the **<u>perpendicular</u>** distance from the axis of rotation to the line along which the force acts.

(1) **When the force (F) is perpendicular to the radius of rotation (r), the lever arm and the radius of rotation are the <u>same</u>.**



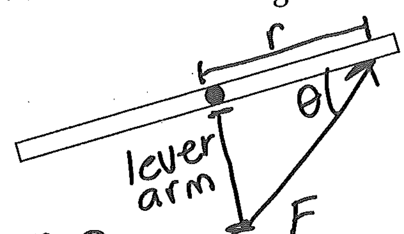

F is ⊥ to r
lever arm = r

* a <u>longer</u> lever arm will produce <u>higher</u> torque

(2) **When the force (F) is applied at an angle, the lever arm is found through the formula:**

* not the same

$$\boxed{\text{lever arm} = F \cdot \sin\theta}$$




so... in the equation for torque
$\tau = r \cdot F \cdot \sin\theta$ (the $r \cdot \sin\theta$ represents the lever arm)

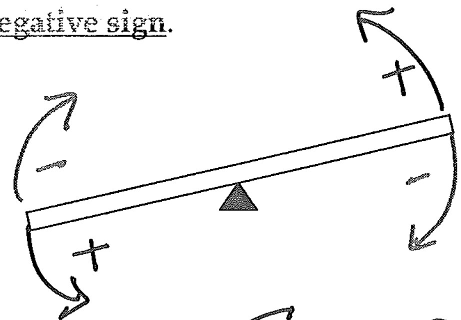
The symbol for torque is the Greek letter tau: $\tau$

Torque is defined as: $\boxed{\tau = r \cdot F = r \cdot F \cdot \sin\theta}$

2 factors affect torque:

1. lever arm
2. Force applied

Torque is a vector quantity. A torque that will cause an object to rotate counter-clockwise is assigned a positive sign, and a torque that will cause an object to rotate clockwise is assigned a negative sign.




$\tau_{cc} = (+)$
$\tau_{c} = (-)$

## Net Torque:

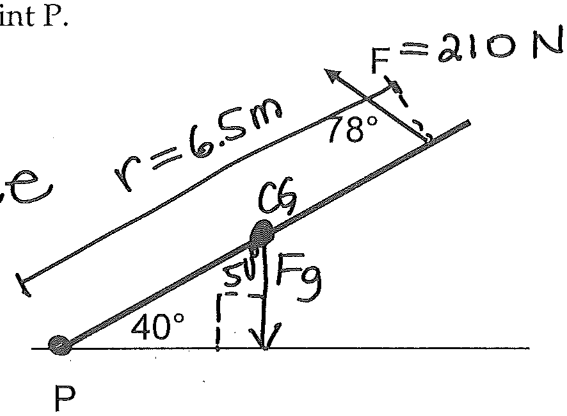
$\sum \tau = \tau_{\text{clockwise}} + \tau_{\text{counterclockwise}}$ → $\Sigma\tau = \tau_c + \tau_{cc}$

Remember – Counterclockwise rotation is assigned (+) and clockwise rotation is assigned (-)

**Example Two:** A uniform 8.0 m long beam of mass 20 kg has a force of 210 N applied 6.5 m from point P as shown in the diagram. Find the magnitude and direction of the net torque acting on the beam about the pivot point P.

uniform beam = CG is at the mid-point

Fg on beam will cause a torque




$\tau_{cc} = F \cdot r \cdot \sin\theta$
$= (210)(6.5)\sin 78$
$= +1335\ N\cdot m$

F is causing $\tau_{cc}$ rotation

$\tau_c = Fg \cdot \underset{\text{to CG}}{\text{distance}} \cdot \sin\theta$

↓

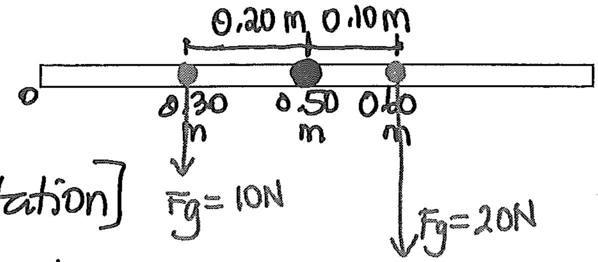
½ length of beam $\left(\frac{8.0\text{m}}{2} = 4.0\text{m}\right)$

$= (20 \cdot 9.8)(4.0\text{m})(\sin 50) = -600\ N\cdot m$

$\Sigma\tau = \tau_{cc} + \tau_c = 1335 + (-600) = +735\ N\cdot m$

↑ indicates counter clockwise net rotation

**Example Three**: A 100 cm meter rule is pivoted at its middle point (that is, at the 50 cm point). If a weight of 10 N is hanged from the 30 cm mark (from the left) and a weight of 20 N is hanged from its 60 cm mark, find out whether the meter rule will remain balanced about its pivot or not?




$\tau_{cc} = r \cdot F \cdot \sin\theta$

[$F_g$=10N will cause the cc rotation]

$\tau_{cc} = (0.20)(10)\sin 90 = +2.0\ N \cdot m$

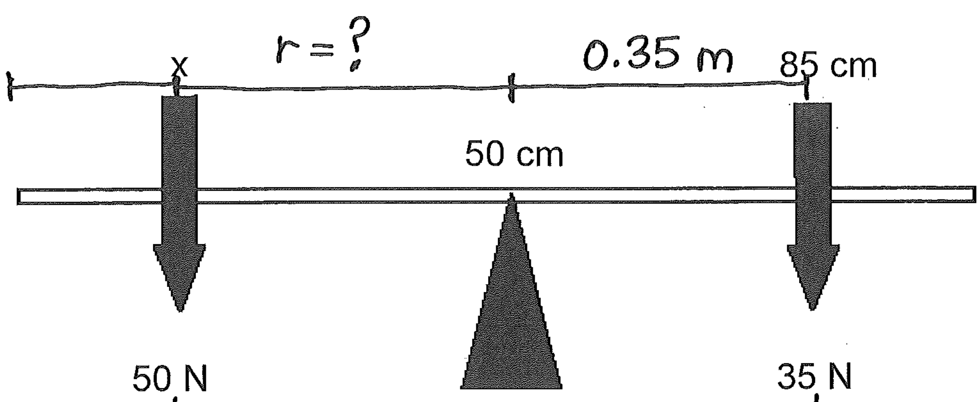
$\tau_c = r \cdot F \cdot \sin\theta$
$= (0.10)(20)\sin 90 = -2.0\ N \cdot m$

$\Sigma\tau = \tau_{cc} + \tau_c = 2.0 + (-2.0) = 0\ N \cdot m$

*since $\Sigma\tau = 0\ N \cdot m$ it is <u>balanced</u> and is in <u>rotational equilibrium</u> (will <u>not</u> rotate)

**Example Four:**

Determine the location of the 50 N force if this system is in rotational equilibrium ($\Sigma\tau$ = 0 N•m)




$\tau_{cc}$ $\qquad$ $\tau_c$

clockwise

$\boxed{\Sigma\tau = 0\ N \cdot m}$ $\qquad$ $\Sigma\tau = (r \cdot 50 \cdot \sin 90) + (-(0.35 \cdot 35 \cdot \sin 90)) = 0$

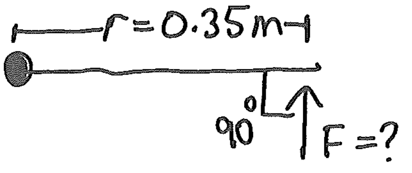
$50r + 12.25 = 0$

$\tau_{cc} + \tau_c = 0\ N \cdot m$ $\qquad$ $r = 0.25m$ or $25$ cm

$(d\ to\ P) - (r)$
$0.50m - 0.25m = \boxed{0.25m}$ or $\boxed{25\ cm}$ mark

**Torque and Net Torque Problems:**

1. If the torque needed to loosen a lug nut holding the wheel of a car is 45 N•m and you are using a wheel wrench that is 35 cm long, what force must you exert perpendicular to the end of the wrench? (129 N)

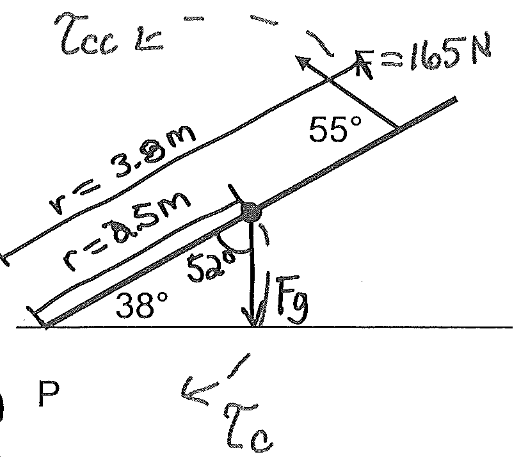



$\tau = F \cdot r \cdot \sin\theta$

$45 = F(0.35)\sin 90$

$F = 129\,N$

2. A uniform 5.0 m long beam of mass 15 kg has a force of 165 N applied 3.8 m from the pivot point. Find the magnitude and direction of **net torque** acting on the beam about the pivot point. (224 N•m counter-clockwise)



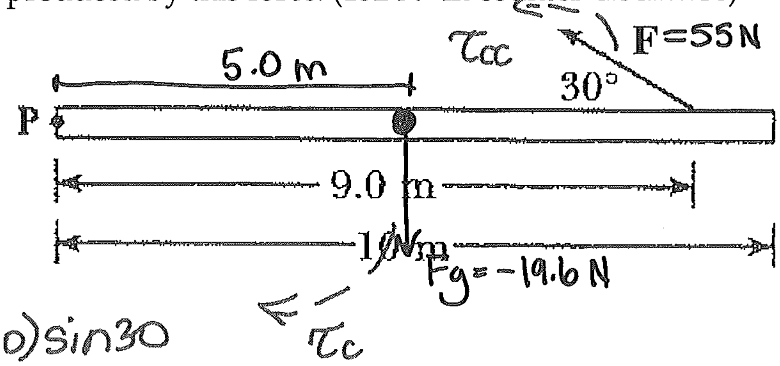

$\Sigma\tau = \tau_c + \tau_{cc}$

$\tau_{cc} = F \cdot r \cdot \sin\theta = 165(3.8)\sin 55$
$= +513.6\ N \cdot m$

$\tau_c = F_g \cdot r \cdot \sin\theta = (15 \cdot 9.8)(2.5)\sin 52$
$= -289.6\ N \cdot m$

$\Sigma\tau = 513.6 + (-289.6) = +224\ N \cdot m$
(counter-clockwise)

3. A uniform horizontal beam of length 10 m and mass 2.0 kg, has a force 55 N applied 9.0 m from the pivot point (P). Find the magnitude and direction of the net torque about the pivot point produced by this force. (152 N•m counter-clockwise)

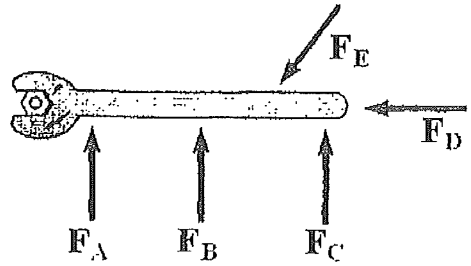



$\tau_{cc} = (55)(9.0)\sin 30$
$= +250\ N \cdot m$

$\tau_c = (19.6)(5.0)\sin 90$
$= -98\ N$

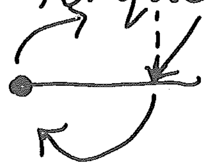
$\Sigma\tau = 250 + (-98)$
$= +152\ N \cdot m$
(counterclockwise)

4. The diagram shows situations in which a force acts along different directions and at different points.




a) If the forces have the same magnitude, in which situation will the force produce the greatest torque around the bolt?
b) In which situation is the greatest force required to loosen the bolt?
c) In which situation does the force produce a clockwise torque around the bolt?

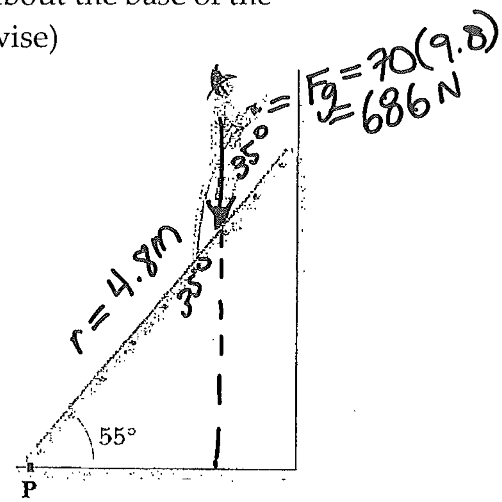
a) $F_C$ = longest lever arm produces greatest torque
b) $F_A$ = shortest lever arm will require the most force ($F_D$ will not produce a $\perp F$ so it will not produce any torque
c) $F_E$ will produce $\tau_C$



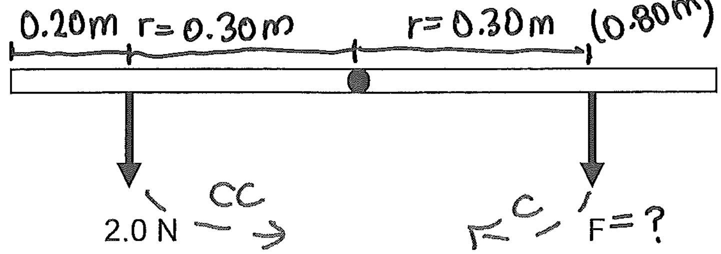
5. A 70 kg firefighter stands 4.8 m from the bottom of the 6.0 m ladder as shown in the diagram. What are the magnitude and direction of the torque about the base of the ladder at P produced by the firefighter? ($1.89 \times 10^3$ N•m clockwise)

only interested in ↗

$\tau_C = F \cdot r \cdot \sin\theta$
$= 686 \cdot 4.8 \cdot \sin 35$
$= -1889$ N·m
↓
$1.89 \times 10^3$ N·m clockwise



6. A 100 cm meter rule is pivoted at its middle point (that is, at the 50 cm point). If a weight of 2.0 N is hanged from the 20 cm point, prove that amount of weight needed to be applied at the 80 cm mark so as to keep it in a balanced position is 2.0 N as well.

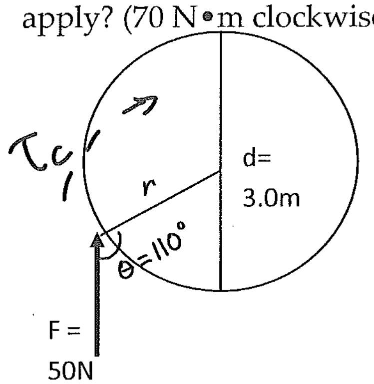



$$\Sigma\tau = 0\ N\cdot m = \tau_{cc} + \tau_c = 0$$

$$= 0.30\cdot 2.0\cdot \sin 90^\circ + (-0.30\cdot F\cdot \sin 90^\circ) = 0$$

$$F = 2.0\ N$$

7. Several children are playing in the park. One child pushes the merry-go-round with a force of 50 N. The diameter of the merry-go-round is 3.0 m. What torque does the child apply? (70 N•m clockwise)



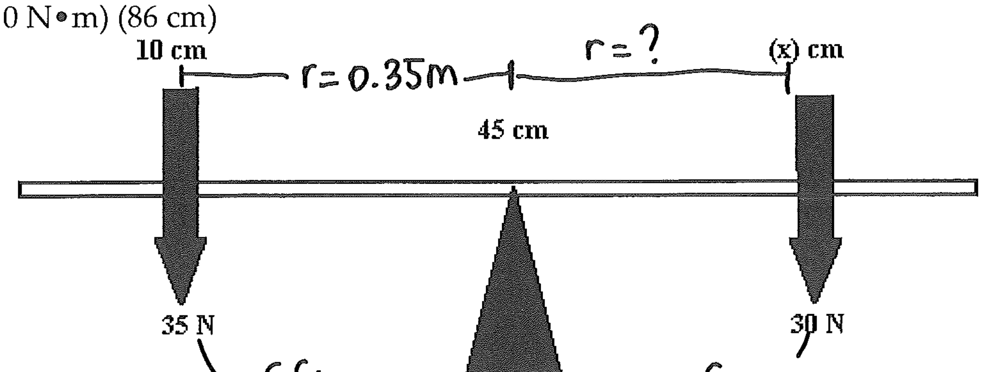

$$\tau_c = F\cdot r\cdot \sin\theta$$

$$= 50\cdot 1.5\cdot \sin 110$$

$$= 70\ N\cdot m \text{ clockwise}$$

$$r = \frac{3.0}{2} = 1.5\,m$$

8. Determine the location of the 30N Force if this system is in rotational equilibrium ($\Sigma\tau = 0$ N•m) (86 cm)

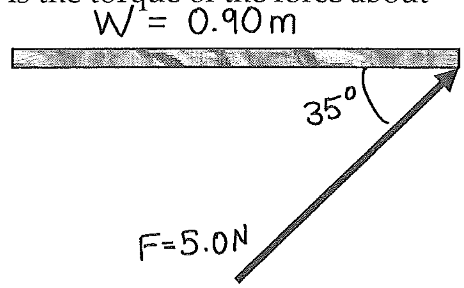



$$\Sigma\tau = 0 = \tau_{cc} + \tau_c$$

$$= 35\cdot 0.35\cdot \sin 90 + (-30\cdot r\cdot \sin 90) = 0 \qquad r = 0.41\,m$$

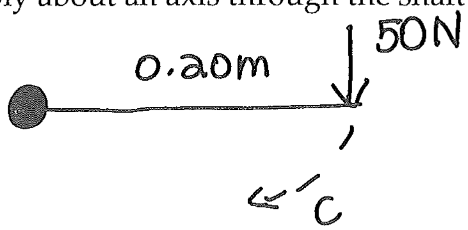
$$x = 0.45 + 0.41 \rightarrow 0.86\,m \rightarrow \underline{86\ cm}$$

9. A person pushes on the edge of a 0.90 m wide door with a horizontal force of 5.0 N acting at an angle of 35° to the plane of the door. What is the torque of the force about the door hinge? (2.6 N•m counter-clockwise) cc




$$\tau = F \cdot r \cdot \sin\theta$$
$$= 5.0 \cdot 0.90 \cdot \sin 35$$
$$= +2.6\ N \cdot m$$

10. The crank handle on an ice cream freezer is 0.20 m from the shaft. At some point in making ice cream, a boy must exert 50 N to turn the crank. How much torque does the boy apply about an axis through the shaft? (10 N•m clockwise)




$$\tau = 50 \cdot 0.20 \cdot \sin 90$$
$$= -10\ N \cdot m$$

Physics 12 – **<u>Rotational Equilibrium</u>**

An object is in **translational equilibrium** when - $\Sigma F_x = 0 \quad \Sigma F_y = 0$

An object is in **rotational equilibrium** when - $\Sigma \tau = 0$

An object is rotational equilibrium will <u>rotate</u> with constant angular velocity, which could be zero.

> The second condition of equilibrium is that in order to have no rotation, there must be no net torque. → rotational equilibrium
>
> Torque is defined as: **force x distance to pivot**
>
> $\tau = F \cdot r \cdot \sin\theta$

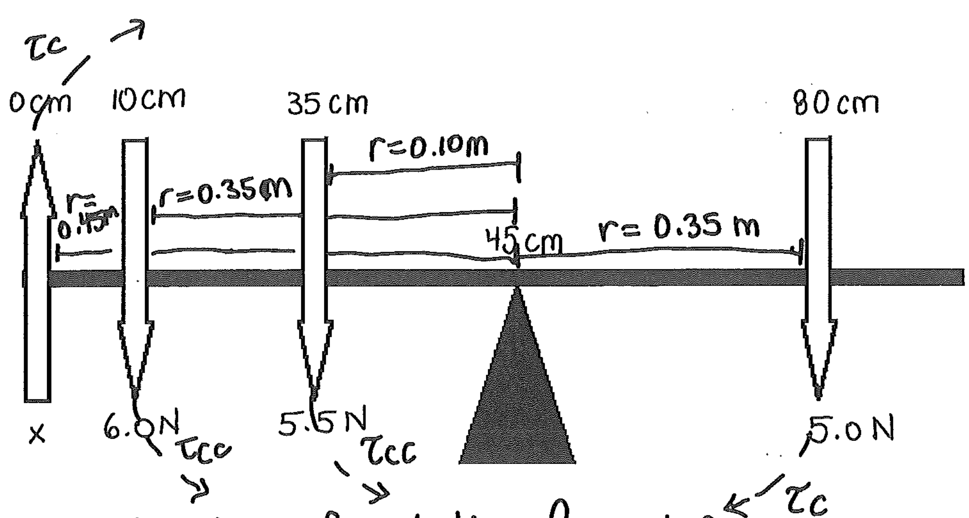
<u>Terms to know:</u>

**Center of Gravity:** where the average mass acts → where we draw Fg!

**Uniform Beam:** constant shape & density

**Arbitrary Position of Rotation:** you choose the location of pivot

With this in mind we can determine unknown measurements for a system in rotational equilibrium. Determine the force applied at x.




① determine direction of rotation for each force

② $\Sigma\tau = 0$ so... $\tau_{cc_1} + \tau_{cc_2} + (-\tau_{c_1}) + (-\tau_{c_2}) = 0$

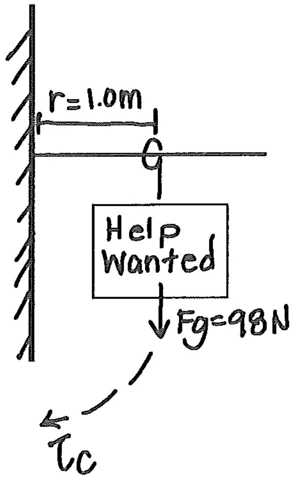
③ $(6.0 \cdot 0.35 \cdot \sin 90) + (5.5 \cdot 0.10 \cdot \sin 90) + (-x \cdot 0.45 \cdot \sin 90) + (-5.0 \cdot 0.35 \cdot \sin 90) = 0$

$2.1 + 0.55 + (-0.45x) + (-1.75) = 0$

$0.90 = 0.45x \qquad x = \underline{2.0\ N}$

We can also **combine this idea with translational equilibrium** and this allows us to determine the tension is slightly more complicated systems.

Suppose you need to hang a sign that is 10 kg 1.0 m from a rod that is not very strong. The force of gravity on the sign produces a torque around the rod's base.

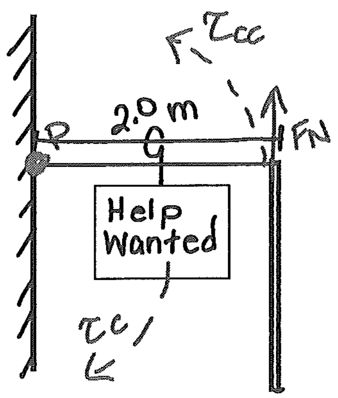



$$\tau = F \cdot r \cdot \sin\theta$$
$$= 98 \cdot 1.0 \cdot \sin 90$$
$$= 98 \text{ N}\cdot\text{m clockwise}$$
$$(-98 \text{ N}\cdot\text{m})$$

this torque acts to rotate the rod clockwise toward the ground.

How do we prevent this?

**Option One:** Place a pole under the end of the rod in order to prevent the rod from rotating. How much normal force must the pole provide to keep the system in equilibrium?




$$\Sigma\tau = 0$$
$$\underset{(c)}{\tau_{sign}} + \underset{(cc)}{\tau_{pole}} = 0$$
$$-98 + \tau_{pole} = 0$$
$$\tau_{pole} = +98 \text{ N}\cdot\text{m}$$

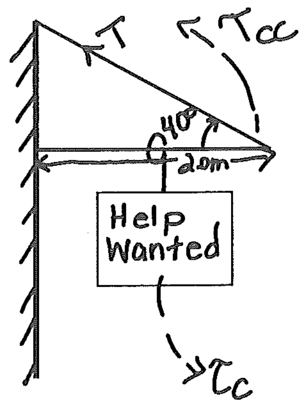
Now find $F_N$:

$$\tau = F \cdot r \cdot \sin\theta$$
$$98 = F_N \cdot 2.0 \cdot \sin 90$$

$F_N = \underline{+49\text{ N}}$

**Option Two: Support the rod with a wire.**

If the wire attaches to the end of the rod at 40°, how much tension must the wire provide to keep the system in equilibrium?




$\Sigma \tau = 0$

$\tau_{wire} + \tau_{sign} = 0$
(cc) (c)

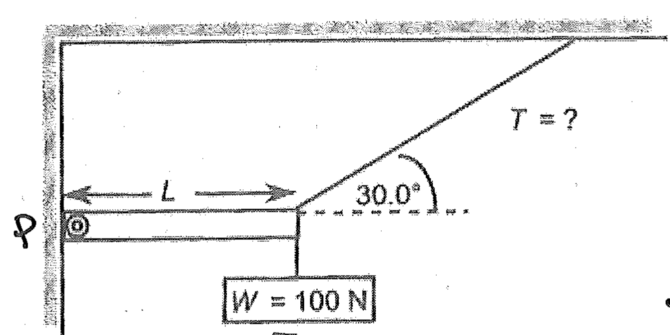
$\tau_{wire} + (-98) = 0 \qquad \tau_{wire} = +98\ N \cdot m$

$\tau_{wire} = T \cdot r \cdot \sin\theta$

$98 = T \cdot 2.0 \cdot \sin 40 \qquad T = +\underline{76\ N}$

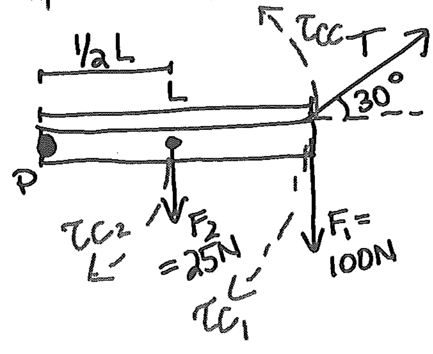
Examples:

A 25.0 N uniform beam is attached to a wall by means of a hinge. Attached to the other end of this beam is a 100 N weight. A rope also helps to support the beam as shown in the diagram. What is the tension in the rope?




This beam would rotate around (P) due to the weight of the beam and the 100N.

so ... we use $\Sigma \tau = 0\ N \cdot m$




$\Sigma \tau = \tau_{cc} + (-\tau_{c_1}) + (-\tau_{c_2}) = 0$

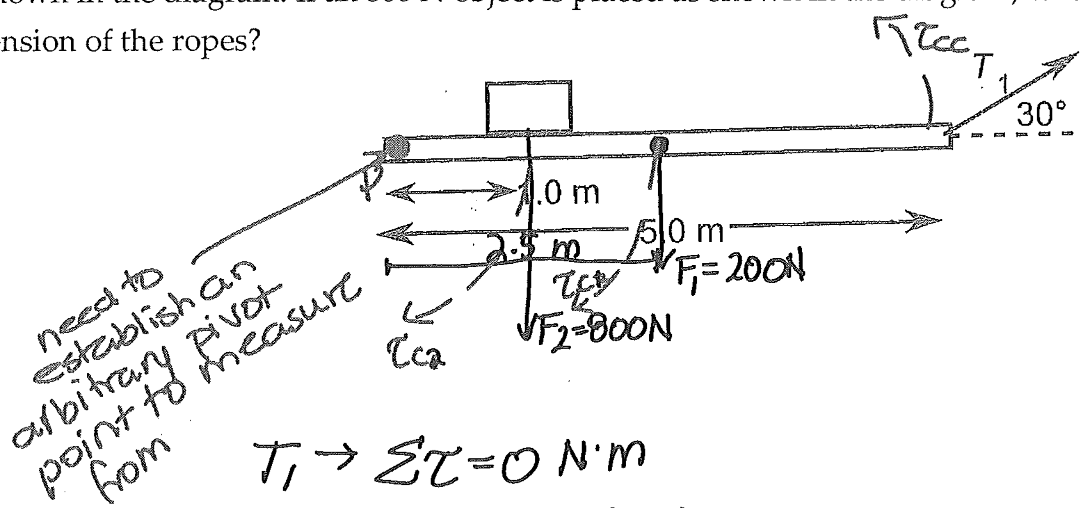
$= (T \cdot \cancel{L} \cdot \sin 30) + (-100 \cdot \cancel{L} \cdot \sin 90) + (-25 \cdot \frac{1}{2} \cdot \cancel{L} \cdot \sin 90) = 0$

*L is the same & can be cancelled

$= 0.5T - 100 - 12.5 = 0$

$0.5T = 112.5 \qquad T = \underline{225\ N}$

A uniform beam 5.0 m long has a weight of 200 N and is suspended by three ropes as shown in the diagram. If an 800 N object is placed as shown in the diagram, what is the tension of the ropes?




$T_1 \rightarrow \Sigma\tau = 0 \text{ N}\cdot\text{m}$

$= \tau_{cc} + (-\tau_{c_1}) + (-\tau_{c_2}) = 0$

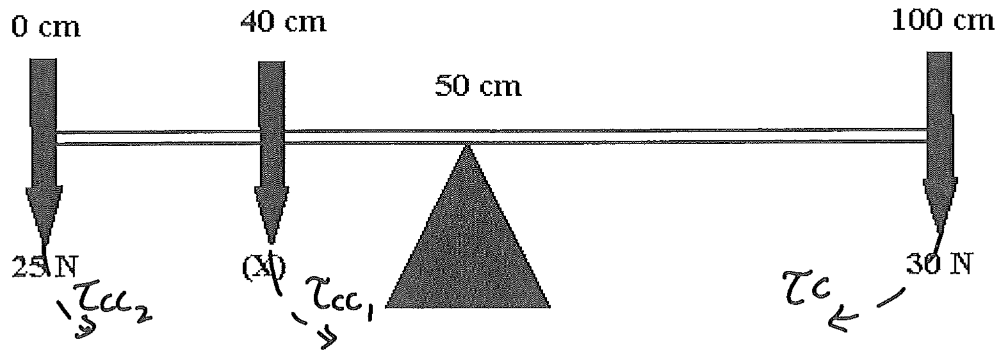
$= (T_1 \cdot 5.0 \cdot \sin 30) + (-200 \cdot 2.5 \cdot \sin 90) + (-800 \cdot 1.0 \cdot \sin 90) = 0$

$= 2.5T_1 - 500 - 800 = 0$

$2.5T_1 = 1300 \qquad T_1 = \underline{520 \text{ N}}$

**Rotational Equilibrium Problems:**

1. Determine the force at 40 cm. (25 N)




$\Sigma\tau = 0$

$= \tau_{cc_1} + \tau_{cc_2} + (-\tau_c) = 0$

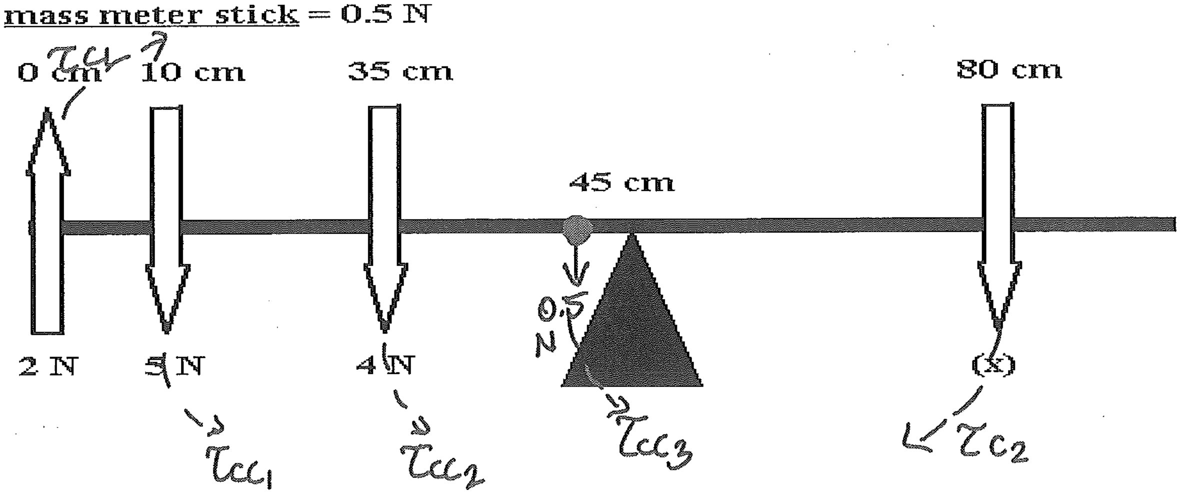
$= (X \cdot 0.10 \cdot \sin 90) + (25 \cdot 0.5 \cdot \sin 90) + (-30 \cdot 0.5 \cdot \sin 90) = 0$

$= 0.10X - 2.5$ $\qquad X = \underline{25\ N}$

$0.10X = 2.5$

2. Determine the force at 80 cm. (3.64 N)

mass meter stick = 0.5 N




$\Sigma\tau = \tau_{cc_1} + \tau_{cc_2} + \tau_{cc_3} + (-\tau_{c_1}) + (-\tau_{c_2}) = 0$

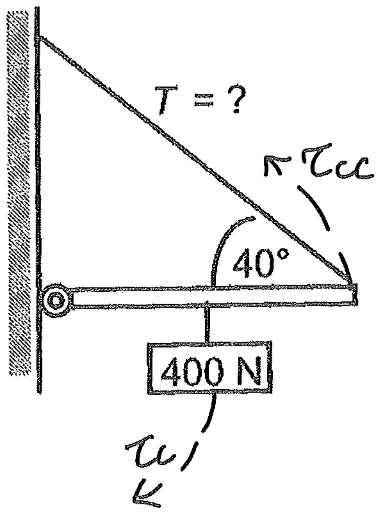
$= (5 \cdot 0.35 \cdot \sin 90) + (4 \cdot 0.10 \cdot \sin 90) + (0.5 \cdot 0.05 \cdot \sin 90) + (-2 \cdot 0.45 \cdot \sin 90) + (-X \cdot 0.35 \cdot \sin 90) =$

$1.75 + 0.40 + 0.025 - 0.90 - 0.35X = 0$

$0.35X = 1.275 \qquad X = \underline{3.64\ N}$

3. A beam of negligible mass is attached to a wall by means of a hinge. Attached to the center of the beam is a 400 N weight. A rope also helps to support this beam as shown in the diagram.

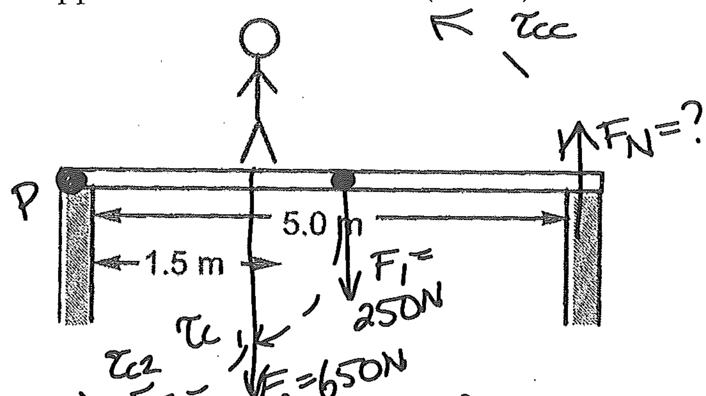
What is the tension in the rope? (311 N)




$$\Sigma\tau = \tau_{cc} + \tau_c = 0$$
$$= T \cdot \cancel{L} \cdot \sin 40 + (-400 \cdot \tfrac{1}{2} \cdot \cancel{L} \cdot \sin 90) = 0$$
$$= 0.643T - 200 = 0$$
$$0.643T = 200 \qquad T = \underline{311\ N}$$

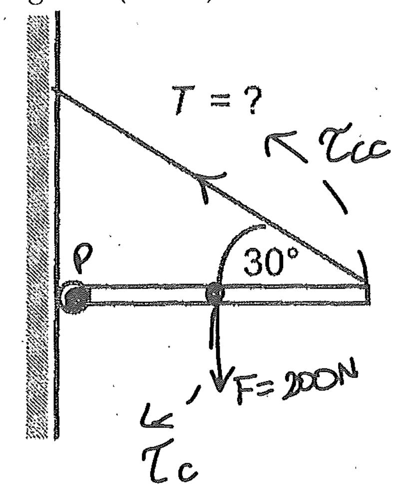
4. A 650 N student stands on a 250 N uniform beam that is supported by two supports as shown in the diagram. If the supports are 5.0 m apart and the student stands 1.5 m from the left support, what is the force that the right support exerts on the beam? (+320N)

$$\Sigma\tau = 0$$
$$= \tau_{c_1} + \tau_{c_2} + \tau_{cc} = 0$$




$$= (-250 \cdot 2.5 \cdot \sin 90) + (-650 \cdot 1.5 \cdot \sin 90) + (F_N \cdot 5.0 \cdot \sin 90) = 0$$
$$= -625 - 975 + 5.0\,F_N$$
$$F_N = +320N$$

5. Find the tension in the rope supporting the 200 N hinged uniform beam shown in the diagram. (200 N)

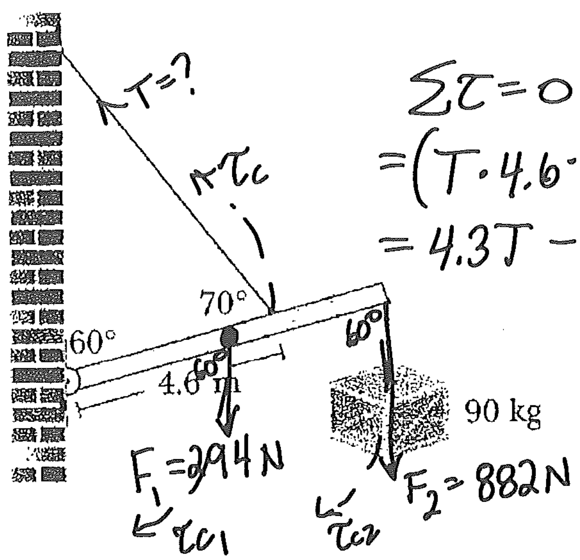



$\Sigma\tau = (T \cdot \cancel{L} \cdot \sin 30) + (-200 \cdot \frac{1}{2} \cdot \cancel{L} \sin 90) =$

$0.5T = 100$

$\underline{T = 200N}$

6. A 7.0 m uniform beam of mass 30 kg is attached to a vertical wall by a cable as shown in the diagram. A 90 kg crate hands from the far end of the beam. Find the tension in the cable connected to the wall. (1450 N)



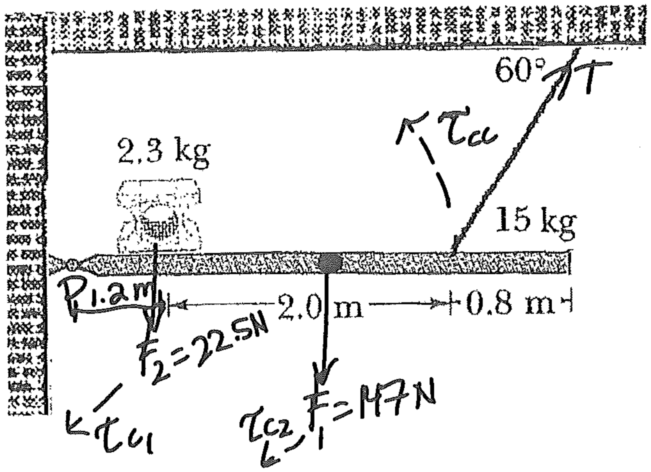

$\Sigma\tau = 0$

$= (T \cdot 4.6 \cdot \sin 70) + (-294 \cdot 3.5 \cdot \sin 60) + (-882 \cdot 7.0 \cdot \sin 60)$

$= 4.3T - 891 - 5347 = 0$

$4.3T = 6238 \qquad T = 1450N$

7. A uniform 15 kg plank of length 4.0 m holding a 2.3 kg block is attached by a rope to a ceiling as shown in the diagram. What is the tension in the rope? (116 N)

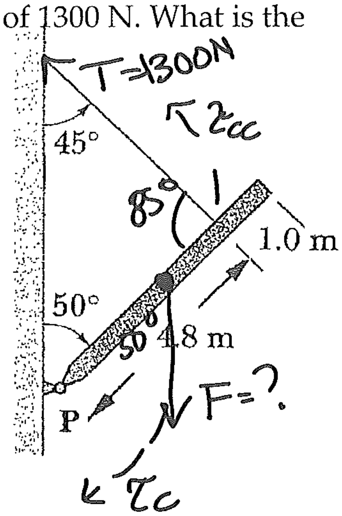



$$\Sigma\tau = (T \cdot 3.2 \cdot \sin 60) + (-147 \cdot 2.0 \cdot \sin 90) + (-22.5 \cdot 1.2 \cdot \sin 90) = 0$$

$$2.77T = 321$$

$$T = \underline{116\ N}$$

8. A 5.8 m uniform beam is supported by a cable having a tension of 1300 N. What is the mass of this beam? (286 kg)



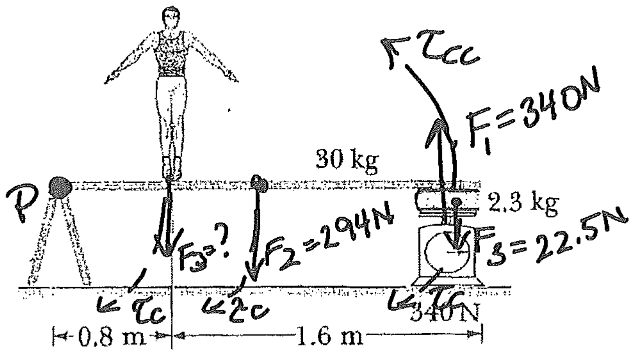

$$\Sigma\tau = (1300 \cdot 4.8 \cdot \sin 85) + (-F \cdot 2.9 \cdot \sin 50) = 0$$

$$= 6216 + (-2.22F) = 0$$

$$6216 = 2.22F \qquad F = \underline{2800N}$$

$$m = \frac{F_g}{9.8} = \frac{2800}{9.8} = 286\ kg$$

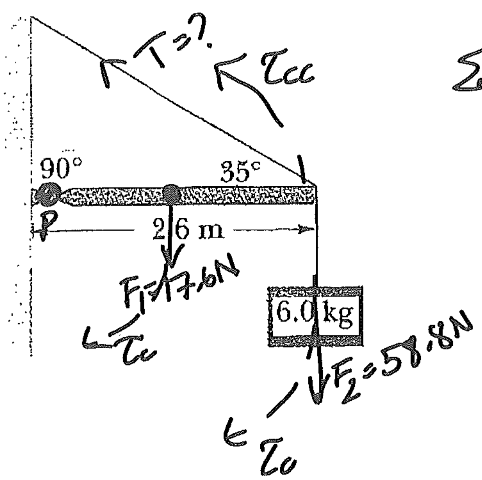
9. A gymnast stands on a uniform 2.4 m beam with a mass of 30 kg. The beam is held level by a 2.3 kg book resting on a scale at the other end. The scale on the right end reads 340 N. What is the mass of the gymnast? (52 kg)




$$\Sigma\tau=(340\cdot2.4\cdot\sin90)+(-22.5\cdot2.4\cdot\sin90)+(-294\cdot1.2\cdot\sin90)+(-F\cdot0.8\cdot\sin90)=0$$

$0.8F=409.2 \quad F=511.5\,N \quad \text{mass}=\frac{511.5}{9.8}=\underline{52.2\,kg}$

10. A 6.0 kg box is supported by a uniform 2.6 m beam as shown in the diagram. The beam has a mass of 1.8 kg. Find the tension in the cable connected to the wall. (118 N)




$$\Sigma\tau\ (T\cdot2.6\cdot\sin35)+(-17.6\cdot1.3\cdot\sin90)+(-58.8\cdot2.6\cdot\sin90)=0$$

$1.49T=175.76 \quad T=\underline{118\,N}$

## Physics 12 – Static Equilibrium

**An object is in static equilibrium if it is at rest.** In static equilibrium, the sum of the external forces is zero, and the sum of the external torques about any pivot point is zero.

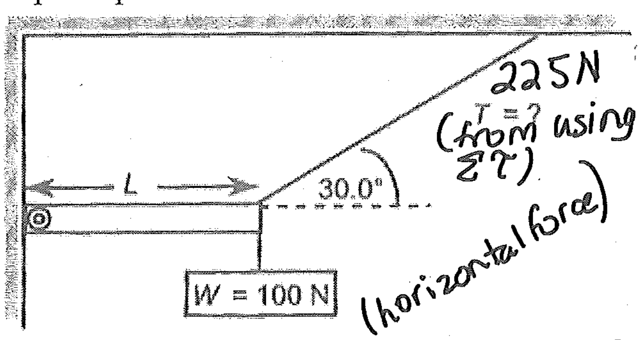
**<u>Both conditions of equilibrium must be met</u>** to be in static equilibrium.

$$\Sigma F_x = 0 \quad , \quad \Sigma F_y = 0 \quad , \quad \Sigma \tau = 0$$

By using **both** conditions together we can determine even more about the forces acting on a structure (very helpful when designing a structure!) or a system.

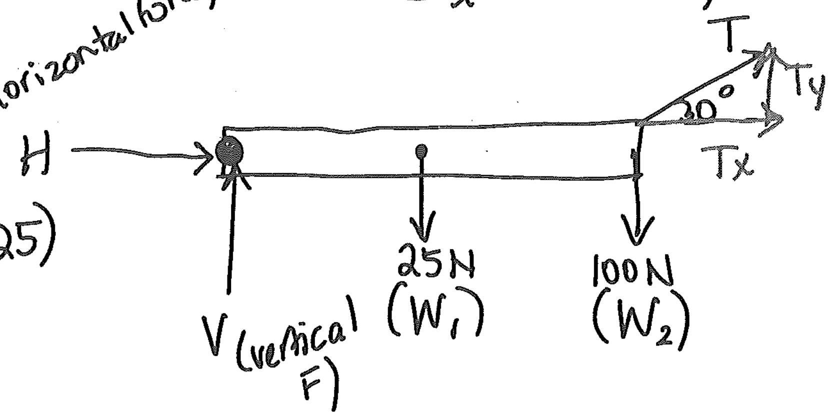
Last lesson we determined the tension for the following situation using the second condition of equilibrium - $\Sigma \tau = 0$.

For the same situation, what are the horizontal and vertical forces that the wall exerts on the beam? We can no longer use torque because no torque is produced when applied to the pivot point.




*this time we will use the first condition of equilibrium:

$\Sigma F_x = 0 \quad \Sigma F_y = 0$




$T_x = \cos 30 \cdot T = \cos 30(225)$

$T_x = 195\,N$

$T_y = \sin 30(225$

$T_y = 113\,N$

$\Sigma F_x = 0 \rightarrow T_x + H = 0$

$195 + H = 0$

$H = -195\,N$

(to left)

$\Sigma F_y = 0 \rightarrow T_y + V + W_1 + W_2 = 0$

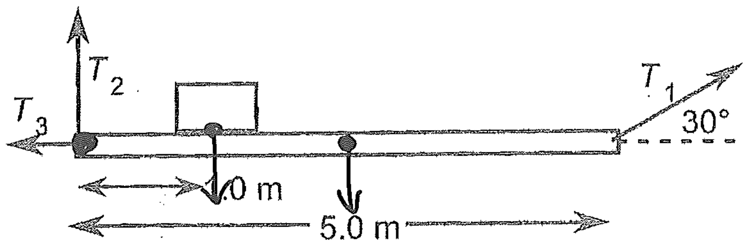
$113 + V + (-25) + (-100) = 0$

$V = +12\,N$

(up)

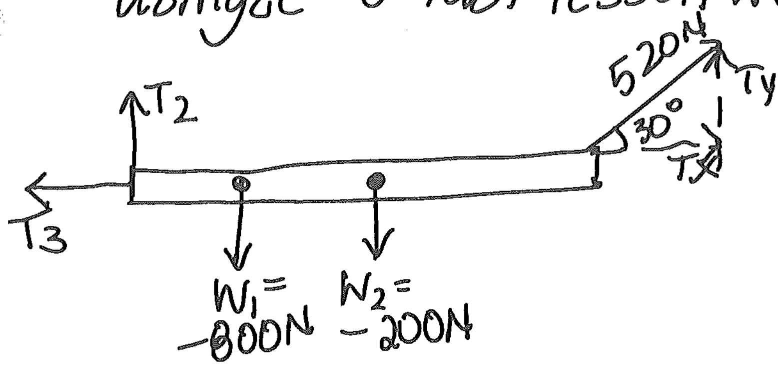
We determined $T_1$ last lesson by using the first condition of equilibrium. However, no torque is produced by $T_2$ or $T_3$ (they are applied to the pivot point). In order to determine these tensions we will now need to use the first condition of equilibrium.

A uniform beam 5.0 m long has a weight of 200 N and is suspended by three ropes as shown in the diagram. If an 800 N object is placed as shown in the diagram, what is the tension of the ropes?




~~t~~there is no torque around the pivot point so we will need to use $\Sigma F_x = 0$ $\Sigma F_y = 0$ to find $T_2$ and $T_3$

using $\Sigma \tau = 0$ last lesson we determined $T_1 = \underline{520 N}$




$T_x = \cos 30 (520)$
$T_x = 450 N$
$T_y = \sin 30 (520)$
$T_y \approx 260 N$

$\Sigma F_x = 0 \rightarrow T_x + T_3 = 0$

$450 + T_3 = 0$

$\underline{T_3 = -450 N}$ (to left)

$\Sigma F_y = 0 \rightarrow T_y + W_1 + W_2 + T_2 = 0$

$260 + (-800) + (-200) + T_2 = 0$

$T_2 = \underline{740 N}$ (up)

**Example:**

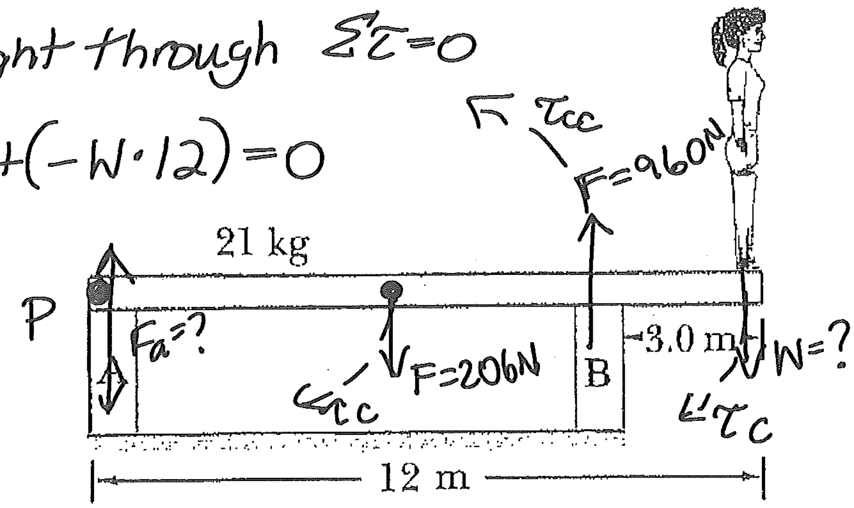
A uniform 12 m diving board with a mass of 21 kg rests on two supports as shown in the diagram. A diver is standing at the right end of the board. If support B exerts a reaction force of 960 N on the board, what reaction force does support A exert?

① find the diver's weight through $\Sigma\tau = 0$

$$\Sigma\tau = (960 \cdot 9.0) + (-206 \cdot 6) + (-W \cdot 12) = 0$$

$12W = 7404$

$W = 617\,N$




② use $\Sigma F_x = 0$, $\Sigma F_y = 0$

$\Sigma F_x = 0\,N$ (no x-directional forces)

$$\Sigma F_y = F_a + (-206) + (-617) + 960 = 0$$

$$= F_a = -137\,N$$

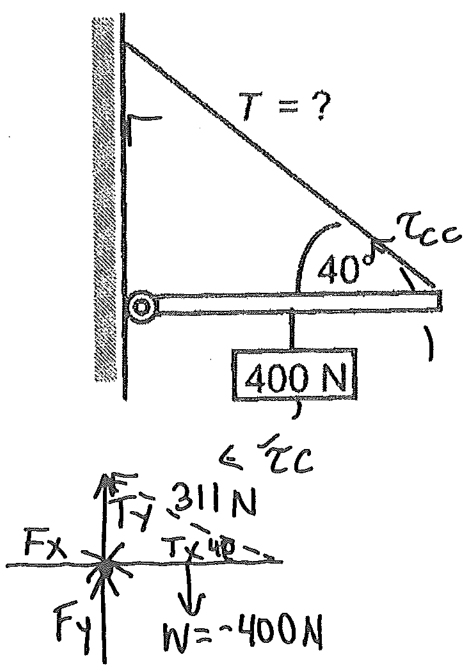
indicates that support A actually pulls downwards by means bolts and screws.

**Static Equilibrium Problems:**

1. A beam of negligible mass is attached to a wall by means of a hinge. Attached to the center of the beam is a 400 N weight. A rope also helps to support this beam as shown in the diagram.

a) What is the tension in the rope? (311 N)

b) What are the vertical and horizontal forces that the wall exerts on the beam? ($F_x$=+238N, $F_y$=+200N)




a) $\Sigma\tau=0$

$= T\cdot \cancel{L}\cdot\sin 40° + (-400\cdot\frac{1}{2}\cdot\cancel{L}\cdot\sin 90) = 0$

$= 0.643T - 200 = 0$

$T = \underline{311\text{ N}}$

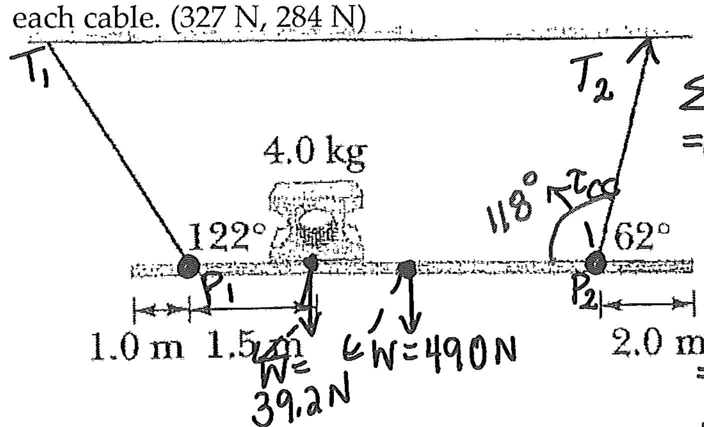
b) $\Sigma F_x = 0, \Sigma F_y = 0$

$\Sigma F_x = -\cos 40(311) + F_x = 0$ $\quad F_x = \underline{+238\text{ N}}$

$\Sigma F_y = \sin 40(311) + (-400) + F_y = 0$

$F_y = \underline{+200\text{ N}}$

2. A uniform 50 kg plank of length 10 m is suspended horizontally by two cables. A 4.0 kg block 2.5 m from the left end of the plank as shown in the diagram. Find the tension in each cable. (327 N, 284 N)




using pivot point 1 ($P_1$):

$\Sigma\tau=0$

$=(T\cdot 7.0\cdot\sin 118)+(-490\cdot 4)+(-39.2\cdot 1.5)=0$

$6.18T = 2018.8$ $\quad T_2 = \underline{327\text{ N}}$

using $P_2$:

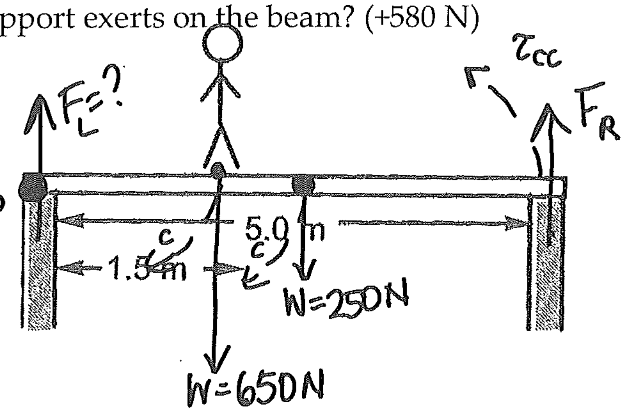
$\Sigma\tau=0$

$=(490\cdot 3)+(39.2\cdot 5.5)+(-T_1\cdot 7.0\cdot\sin 122) = 0$

$5.94\, T_1 = 1686$ $\quad T_1 = \underline{284\text{ N}}$

*use 2 pivot points as forces directly applied to a pivot point will not produce a torque.

3. A 650 N student stands on a 250 N uniform beam that is supported by two supports as shown in the diagram. If the supports are 5.0 m apart and the student stands 1.5 m from the left support, what is the force that the left support exerts on the beam? (+580 N)




$F_R$: $\Sigma\tau = 0$

$= (F_R \cdot 5.0) + (-250 \cdot 2.5) + (-650 \cdot 1.5) = 0$

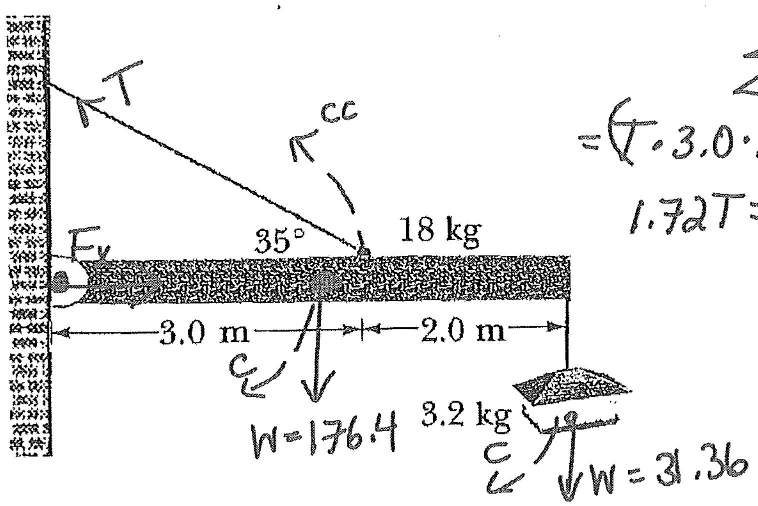
$5.0 F_R = 1600 \quad F_R = \underline{+320\,N}$

$F_L$: $\Sigma F_x = 0,\ \Sigma F_y = 0$

$\Sigma F_x = 0$

$\Sigma F_y = F_L + (-650) + (-250) + 320 = 0 \qquad F_L = \underline{+580\,N}$

4. A uniform 18 kg beam with a length of 5.0 m is mounted by a hinge on a wall. The beam is held in a horizontal position by a cable with an angle of 35° and supports a cage with a mass of 3.2 kg as shown in the diagram. Find the force that the hinge exerts on the beam. (+285 N)




$\Sigma\tau = 0$

$= (T \cdot 3.0 \cdot \sin 35) + (-176.4 \cdot 2.5) + (-31.36 \cdot 5.0) = 0$

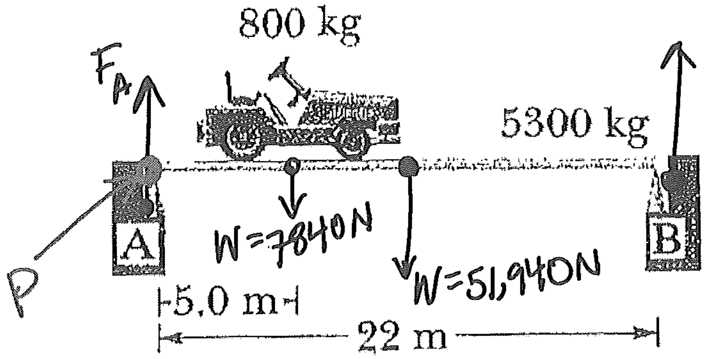
$1.72T = 597.8 \qquad T = \underline{348\,N}$

$T_x = -\cos 35(348) = -285\,N$

$T_y = +\sin 35(348) = +200\,N$

$\Sigma F_x = 0 \quad -285 + F_x = 0 \qquad F_x = \underline{+285\,N}$

5. An 800 kg automobile moves toward the right end of the bridge which has a mass of 5300 kg and a length of 22 m. Find the reaction forces exerted by supports A and B on the bridge at the position shown in the diagram. (+2.78x10⁴ N, +3.20x10⁴ N)




$\sum \tau = 0$

$= (F_B \cdot 22) + (-51940 \cdot 11) + (-7840 \cdot 5) =$

$22F_B = 610540$ $F_B = +27752N$

$(+2.78 \times 10^4 N)$

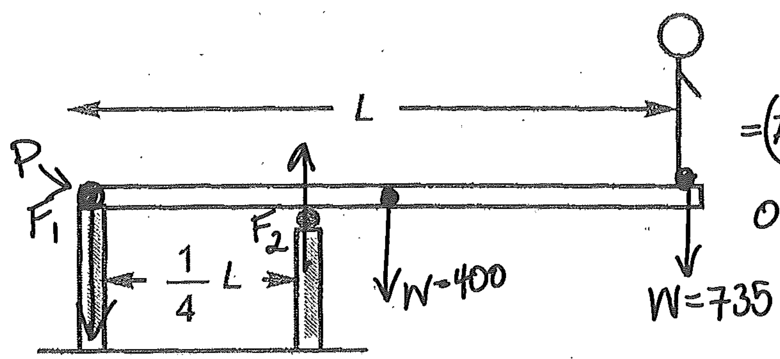
$\sum F_x = 0$

$\sum F_y = 27752 + (-51940) + (-7840) + F_1 = 0$

$F_A = +32028\ N$

$(+3.20 \times 10^4 N)$

6. A uniform 400 N diving board is supported at two points as shown below. If a 75 kg diver stands at the end of the board, what are the forces acting on each support? (+3.74x10³N, -2.61x10³N)




$\sum \tau = 0$

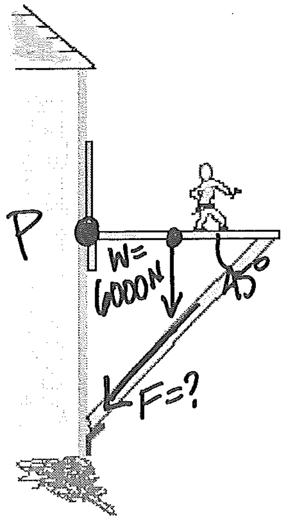
$= (F_2 \cdot \frac{1}{4} \cdot L) + (-735 \cdot L) + (-400 \cdot \frac{1}{2} L) =$

$0.25F_2 = 935$ $F_2 = +3740N$

$\sum F_x = 0$

$\sum F_y = F_1 + 3740 + (-400) + (-735) = 0$

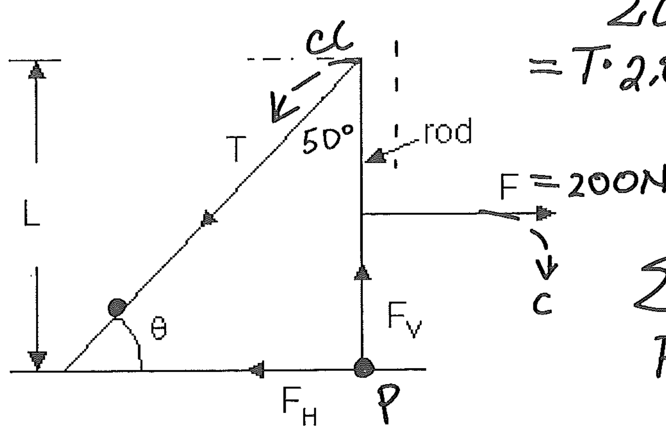
$F_1 = -2605\ N$

7. 


You've decided to build a new deck. It is to be 10 m wide and run the 30 m length of the house. You must estimate the weight of the deck, as well as, the maximum load you expect it to support to be 6000 N. The deck is to be supported where it is attached to the house and at the outside edge by support beams that make a 45° angle as shown. What will the compressive tension in the support beam be? (You can ignore the weight of the man and the beam as they are included in the max 6000N load) ($-4.24\times10^3$ N)

$$\Sigma\tau = 0$$
$$= (-F\cdot 10\cdot \sin 45) + (-6000\cdot 5) = 0$$
$$7.07F = -30000 \qquad F = \underline{-4243\text{ N}}$$

8. A rigid, vertical rod (2.0 m) of negligible mass is connected to the floor by an axle through its lower end. The rod also has a wire connected between its top end and the floor. If a horizontal force of 200 N (F) is applied at the midpoint of the rod, find a) the tension in the wire, and b) the horizontal and vertical components of force exerted by the bolt on the rod. ($F_H$ = -100N, $F_V$ = +84.2N)




$$\Sigma\tau = 0$$
$$= T\cdot 2.0\cdot \sin 50 + (-200\cdot 1.0) = 0$$
$$1.53T = 200 \qquad T = \underline{131\text{ N}}$$

$$\Sigma F_x = -\sin 50(131) + 200 - F_H = 0$$
$$F_H = \underline{-99.6\text{ N}}$$

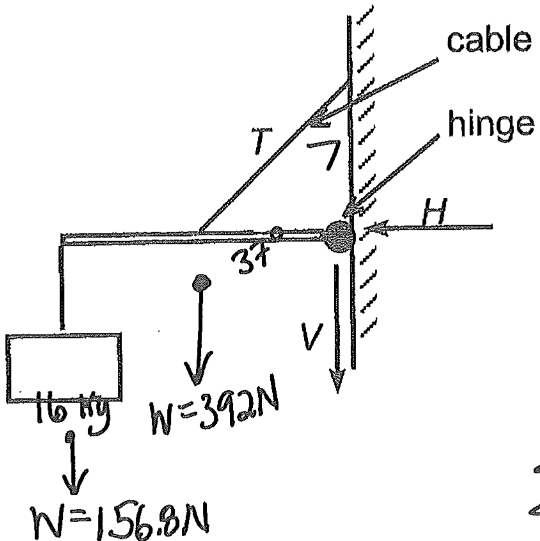
$$\Sigma F_y = -\cos 50(131) + F_V = 0$$
$$F_V = \underline{+84.2\text{N}}$$

9. A uniform beam (mass = 40 kg) is supported by a cable that is attached to the center of the beam as shown in the diagram.

a) Find the tension (T) in the cable. ($1.17\text{x}10^3$ N)

b) Find H and V (the horizontal and vertical forces acting on the hinge)
(H = -936N, V = -157N)

$\Sigma\tau = 0$




$= (-T\cdot\frac{1}{2}\cdot\cancel{L}\cdot\sin 37) + (156.8\cdot\cancel{L}) + (392\cdot\frac{1}{2}\cdot\cancel{L}) =$

$0.301T = 352.8 \qquad \underline{T = 1172\ N}$

$\Sigma F_x = \cos 37(1172) + H = 0$

$\underline{H = -936\ N}$

$\Sigma F_y = \sin 37(1172) + (-392) + V + (-156.8) =$

$V = \underline{-157\ N}$