

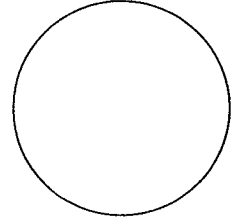


# Lesson 1

## Physics 12 – Uniform Circular Motion

Uniform circular motion is the motion of an object moving at a **constant speed in a circle** with a fixed radius. Notice that it is the **speed that is constant, not the velocity** (as the direction is constantly changing).

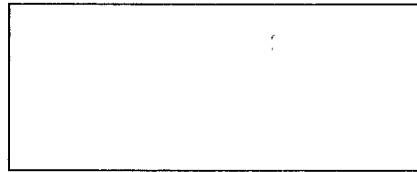
Speed is defined as:



**Time:** When dealing with circular motion, we use the **period of motion (T)** as the time it takes to complete one rotation.

**Distance:** When traveling in a circular path, the **distance travelled is equal to the circumference of that circular path.**

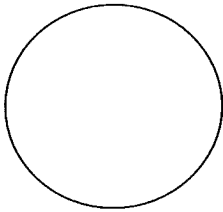
The **speed of an object moving with uniform circular motion** is therefore given by:



This formula allows us to determine the speed of the mass. Now, how do we determine the acceleration? Is the mass accelerating?

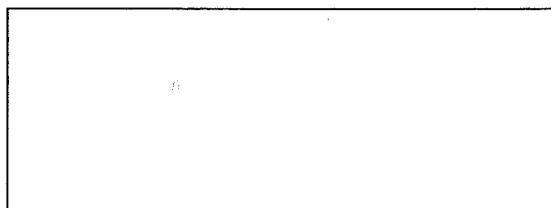
While the object is traveling at a constant speed...*the object is constantly changing direction.*

The definition of acceleration is:



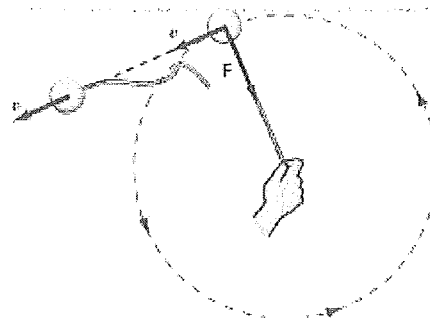
Even though the object is traveling at a constant speed, anything traveling in a circular path has acceleration due to the continual change in direction

This acceleration is found using:



In uniform circular motion, the velocity and the acceleration are continually changing direction, and are perpendicular to each other at each moment.

**Example:** The force on the stone swung on the end of a string is the force exerted inwardly by the string. When released, the stone will fly off tangentially, in the direction of the velocity it has at the moment.



## Centripetal Acceleration

As seen in the diagram, the force that is exerted on the object is directed toward the center of the circle. This means that the acceleration must also be directed toward the center of the circle.

The velocity is always directed along the tangent of the circle = perpendicular to the acceleration.

According to Newton's Laws of Motion, if an object is accelerated toward the center, then there must also be a...

This center-seeking force is called the **centripetal force**.

## Centripetal Force

Centripetal force is a name given to **any force that causes an object to move in a circle**.

*Examples include:* **Tension** through a string as in the rock example, **friction** as when a car rounds a curve on a highway, **gravity** as when the moon circles the Earth, or **electrical** as when an electron orbits a proton.

In circular motion, the force vector is always perpendicular to the velocity vector.

Centripetal Force:

In uniform circular motion, because the acceleration is uniform, the force that is causing the acceleration must also be uniform.

## Centrifugal Force?

While centripetal means "center-seeking", centrifugal means "center-fleeing". **Centrifugal force** is really just an apparent force – it **does not actually exist**. It is the apparent force that causes an object to move along a straight line. However, Newton's First Law tells us you do not need a force to keep an object moving in a straight, only to change its direction.

What we sometimes call a **centrifugal force is really the object's inertia**.

**Example:** A 0.50 kg mass hangs on a frictionless table and is attached to hanging weight. The 0.50 kg mass is whirled in a circle of radius 0.20 m at 2.3 m/s.

(A) Calculate the centripetal force acting on the mass. (B) Calculate the mass of the hanging weight.

**Example:** A car traveling at 14 m/s goes around an unbanked curve in the road that has a radius of 96 m. What is its centripetal acceleration?

What is the minimum coefficient of friction between the road and the car's tires?

**Example:** A plane makes a complete circle with a radius of 3622 m in 2.10 minutes. What is the speed of the plane?



When an object remains in a circle  $F_c$  is being counter balanced by another force ( $F_f$  or  $F_g$ ) usually.  
 eg A wheel chair traveling at  $4.0 \text{ m/s}$  speeds around an \*unbanked curve ( $r = 80 \text{ m}$ ).  
 Calculate i)  $a_c$  ii)  $\mu$

\* unbanked curve means the only force keeping the object moving in the circle is the force of friction (between the object and the surface it is traveling on).

$$\therefore \text{ii) } F_c = F_f$$

$$\frac{mv^2}{r} = \mu mg$$

$$\mu = \frac{v^2}{r a_g} = \frac{(4.0 \text{ m/s})^2}{(80 \text{ m})(9.80 \text{ m/s}^2)} = 0.020$$

$$\text{i) } a_c = \frac{v^2}{r} = \frac{(4.0 \text{ m/s})^2}{80 \text{ m}} = 0.20 \text{ m/s}^2$$

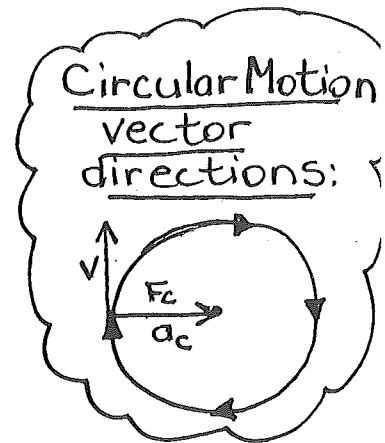
#### Additional Problems

1. Calculate speed if  $r = 3.2 \text{ m}$  and time for revolution is  $1.9 \text{ s}$ . ( $11 \text{ m/s}$ )
2. Calculate  $F_c$  on a  $555 \text{ kg}$  object,  $r = 60 \text{ m}$ ,  $v = 36 \text{ m/s}$ . ( $1.2 \times 10^4 \text{ N}$ )
3. Calculate  $F_c$  on a  $200 \text{ kg}$  object,  $r = 100 \text{ m}$ ,  $v = 20.0 \text{ m/s}$ . ( $800 \text{ N}$ )
4. Calculate  $a_c$  with  $r = 1234 \text{ m}$ , and  $t = 3.0 \text{ min}$ . ( $1.5 \text{ m/s}^2$ )

#### Summary:

- $a_c = \frac{v^2}{r}$  and  $v = \frac{2\pi r}{T} (= \frac{d}{t}) \therefore a_c = \frac{(\frac{2\pi r}{T})^2}{r} = \frac{4\pi^2 r}{T^2}$
- Since  $\Sigma F = ma$ ,  

$$F_c = \frac{mv^2}{r} \quad \text{or} \quad F_c = \frac{m 4\pi^2 r}{T^2}$$
- for unbanked curves  $F_f = F_c$

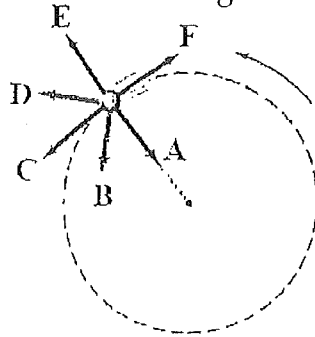


} these 2  
formulae  
not on  
formula sheet

# Lesson 1 Homework

## Uniform Circular Motion Problems

1. A ball is swung on the end of a string at a constant speed in a horizontal circle.



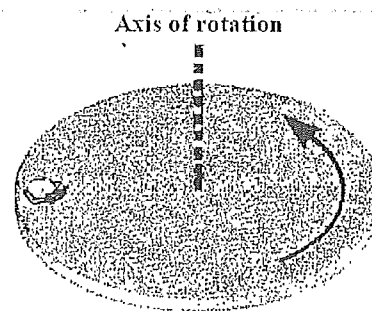
- a) Which path does the ball follow at the moment the string breaks?
- b) Which path represents the direction of the velocity of the ball?
- c) Which path represents the direction of the acceleration of the ball?
- d) Which path represents the direction of the force exerted on the ball by the string?

2. A car is about to go around a curve. Using principles of physics, explain the following questions.

- a) Can the car go around the curve with a constant speed?
- b) Can the car go around the curve with zero acceleration?
- c) Can the car go around the curve with constant acceleration?

3. A small stone rests on the edge of an antique vinyl record player which is whirling around at 33 rotations per minute.

a) Draw a diagram showing the forces acting on the stone.



b) What provides the centripetal force to keep the stone on the record?

4. Calculate the centripetal force acting on a 925 kg car as it rounds an unbanked curve with a radius of 75 m at a speed of 22 m/s. ( $6.0 \times 10^3$  N)

5. A small plane makes a complete circle with a radius of 3282 m in 2.0 minutes. What is the centripetal acceleration of the plane? ( $9.0 \text{ m/s}^2$ )

6. A car with a mass of 822 kg rounds an unbanked curve in the road at a speed of 28.0 m/s. If the radius of the curve is 105 m, what is the average centripetal force exerted on the car? ( $6.14 \times 10^3$  N)

7. An amusement park ride has a radius of 2.8 m. If the time of one revolution of a rider is 0.98 s, what is the speed of the rider?

8. An electron ( $m = 9.11 \times 10^{-31}$  kg) moves in a circle whose radius is  $2.00 \times 10^{-2}$  m. If the force acting on the electron is  $4.60 \times 10^{-14}$  N, what is the speed of the electron? ( $3.18 \times 10^7$  m/s)

9. A 925 kg car rounds an unbanked curve at a speed of 25 m/s. If the radius of the curve is 72 m, what is the minimum coefficient of friction between the car and the road required so that the car does not skid? (0.89)

10. A  $2.7 \times 10^3$  kg satellite orbits Earth at a distance of  $1.8 \times 10^7$  m from Earth's center at a speed of  $4.7 \times 10^3$  m/s. What force does Earth exert on the satellite?

11. An athlete whirls a 3.7 kg shot-put in a horizontal circle with a radius of 0.90 m. If the period of rotation is 0.30 s,

a) What is the speed of the shot-put when released? (18.8 m/s)

b) What is the centripetal force acting on the shot-put while it is rotated? ( $1.45 \times 10^3$  N)

c) How far would the shot-put travel horizontally if it is released 1.2 m above the ground? (9.3m)

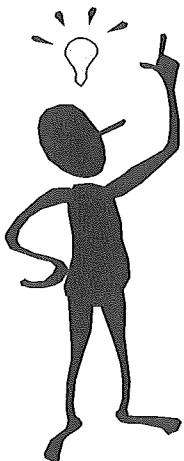
12. Calculate the speed and acceleration of a point on the circumference of a 33.3 phonograph record. The diameter of the record is 30.0 cm. (It makes 33.3 revolutions per minute). (0.52 m/s, 1.83 m/s<sup>2</sup>)

13. A string requires a 135 N force in order to break it. A 2.00 kg mass is tied to this string and whirled it in a horizontal circle with a radius of 1.10 m. What is the maximum speed that the mass can be whirled without breaking the string? (8.62 m/s)

14. A 932 kg car is traveling around an unbanked curve that has a radius of 82 m. What is the maximum speed that this car can round this curve without skidding?

a) If the coefficient of friction is 0.95? (27.6 m/s)

b) If the coefficient of friction is 0.40? (17.9 m/s)



## Checking for Understanding: Circular Motion

i.) PREDICTIONS:

**Situation 1:**

A curious physics 12 student is playing with a stopwatch. He/she twirls it **carefully** above his/her head in a **horizontal circle**. As the speed of the stopwatch is changed he/she notices the tension in the cord:

- a. stays the same
- b. increases with increasing speed
- c. decreases with increasing speed

ii) COMPARE your answers with a partner

iii) TEST your predictions

i.) PREDICTIONS:

**Situation 2:** Now the curious physics student twirls the stopwatch **carefully** in a **vertical circle**. He/she notices the tension at the **top** of the swing compared to the tension in the string at the **bottom** of the swing is:

- a. the same
- b. greater at the top than the bottom of the swing
- c. less at the top than the bottom of the swing

ii) COMPARE your answers with a partner

iii) TEST your predictions

i.) PREDICTIONS:

**Situation 3:** The curious physics student twirls the stopwatch as **slowly** as he/she possibly can while still keeping the object moving in a **vertical circle**. He/she notices the tension at the **top** of the swing:

- a. zero
- b. the same as the tension in the string at the bottom of the swing

ii) COMPARE your answers with a partner

iii) TEST your predictions

vi) EXPLAIN WHY the stop watch behaved as it did in EACH situation

Physics 12 - Circular Motion Lab Activity - Expanded version

Names: \_\_\_\_\_

**Situation 1:** A curious physics 12 student is playing with a mass on a string (as physics students like to do!) He/she twirls it carefully above his/her head in a horizontal circle. As the speed of mass is changed he/she notices the tension in the cord:

- a. Stays the same
- b. Increases with increasing speed
- c. Decreases with increasing speed

i) **DISCUSS and PREDICT** what will happen **BEFORE** you test this. Include an *explanation* for your prediction below.

ii) **TEST** your predictions and describe what you discovered.

iii) Determine the **Centripetal Force** and **Centripetal Acceleration** of your mass during its circular path. **SHOW ALL FORMULAS, VALUES AND WORK.**

$F_c =$  \_\_\_\_\_

$a_c =$  \_\_\_\_\_

**Situation 2:** Now the curious physics student twirls the mass **carefully** in a **vertical circle**. He/she notices the tension at the **top** of the swing compared to the tension in the string at the **bottom** of the swing:

- a. The same
- b. Greater at the top than at the bottom of the swing
- c. Less at the top than at the bottom

i) **DISCUSS and PREDICT** what will happen **BEFORE** you test this. Include an *explanation* for your prediction below.

ii) **TEST** your predictions and describe what you discovered.

iii) Determine the **Tension at the TOP** and the **Tension at the BOTTOM** for your string. **SHOW ALL FORMULAS, VALUES AND WORK.**

$T_{\text{top}} =$  \_\_\_\_\_

$T_{\text{bottom}} =$  \_\_\_\_\_



**Situation 3:** A curious physics 12 student is still playing with a mass. He/she twirls it as **slowly** as he/she possibly can while still keeping the mass in a **vertical** circle. He/she notices the tension at the top of the swing:

- a) Is zero
- b) Is the same as the tension in the string at the bottom of the swing

i) **DISCUSS and PREDICT** what will happen **BEFORE** you test this. Include an *explanation* for your prediction below.

ii) **TEST** your predictions and describe what you discovered.

iv) What is the **minimum speed** of the object at the top of the motion for the object to remain in its circular motion? **SHOW ALL FORMULAS, VALUES AND WORK.**

$v =$  \_\_\_\_\_

APPLY Your Understanding –

1. A stone is tied to a string and whirled around in a circle at a constant speed. Is the string more likely to break when the circle is horizontal or when it is vertical? Account for your answer with an explanation in terms of physics concepts. (You may assume that the constant speed is the same in each case)
2. Consider two people, one on the Earth's surface at the equator and the other at the North Pole. Which has the larger centripetal acceleration? Explain.

## Lesson 2

### VERTICAL CIRCULAR UNIFORM MOTION

- $F_c$  always acts toward the center of the circle.
- $F_g$  must be included and always acts down.
- **TENSION** is: the force in the rope or arm that is doing the swinging.  
always towards center of circle.

So:  $F_c = T + F_g$

you must account for **DIRECTION** of forces with this formula

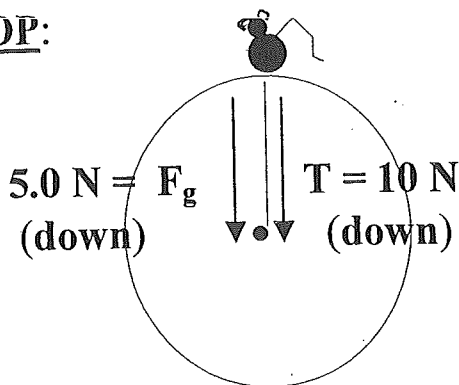
ie) - is down

+ is up

- Make a quick **diagram** to depict situation. (showing either the TOP or the BOTTOM of the swing).

Eg 1. A 5.0 N stuffed toy is swung in a vertical circle with a string tension of 10.0 N. Calc  $F_c$  at the

a) TOP:



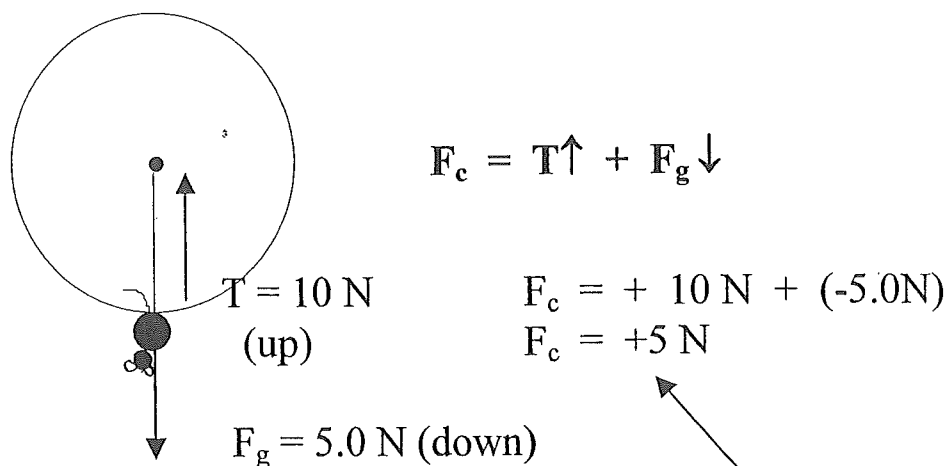
$$F_c = T\downarrow + F_g\downarrow$$

$$F_c = -10 \text{ N} + -5.0 \text{ N}$$

$$F_c = -15 \text{ N}$$

( $F_c$  is always toward the center which in this case is down)

b) BOTTOM:



• MAXIMUM SPEED occurs at the BOTTOM of the swing.

• MINIMUM SPEED (so it doesn't fall out of the loop) occurs at the TOP of the swing. AND TENSION = 0.

Therefore for minimum speed calculations:

$$F_c = F_g$$

$$\text{a.) } \frac{mv^2}{r} = ma_g \quad \text{and} \quad \frac{v^2}{r} = a_g \quad (\text{because "m" cancels})$$

So:

$$v = \sqrt{a_g r}$$

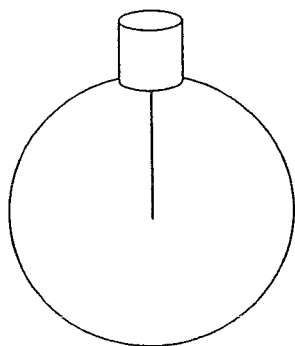
min

b.) or less commonly from  $F_c = F_g$

$$\frac{m4\pi^2 r}{T^2} = ma_g \quad (\text{again "m" cancels})$$

## Examples

1. What is the minimum speed you need to swing a pail of water to prevent the water from falling on your head, if your arm is 0.50 m long?



$$T = 0$$

$$\text{Therefore } v = \sqrt{a_g r}$$

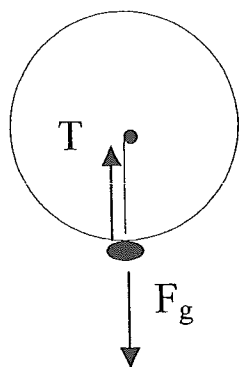
$$v = \sqrt{(9.80 \text{ m/s}^2)(0.50 \text{ m})}$$

$$v = 2.2 \text{ m/s}$$

note: do not use the sign on  $a_g$  here due to the square root sign.

**minimum velocity or speed does not depend on mass!**

2. A nylon rope requires 325 N to break. You tie a 2.00 kg toy plane to it and whirl it in a vertical circle of radius 3.20 m. What is the plane's MAX SPEED without breaking the rope.



Unknown:  $v_{\text{max}}$

Knowns:  $T = + 325 \text{ N}$

$$m = 2.00 \text{ kg}$$

$$r = 3.20 \text{ m}$$

$$a_g = -9.80 \text{ m/s}^2$$

Formulae: (1)  $F_g = ma_g$

$$(2) F_c = T + F_g$$

$$(3) F_c = mv^2/r$$

Sub, Cal, R.O.:  $F_g = ma_g = (2.00 \text{ kg}) (-9.80 \text{ m/s}^2) = -19.6 \text{ N}$

$$F_c = T + F_g$$

$$F_c = +325 \text{ N} + (-19.6 \text{ N}) = +305.4 \text{ N} = +305 \text{ N}$$

$$v = \sqrt{\frac{F_c r}{m}} \quad v = \sqrt{\frac{(305.4 \text{ N}) (3.20 \text{ m})}{2.00 \text{ kg}}} \quad v = 22.1 \text{ m/s}$$

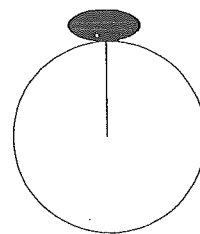
**Max speed occurs at the bottom of the swing**

3. A 1.2 kg toy is swinging in a vertical circle of radius 0.80 m. If the time for 1 revolution is 0.77 s what is the tension in the string at:

a. the top?

Find: T

Given:  $m = 1.2 \text{ kg}$   
 $r = 0.80 \text{ m}$   
 $T = 0.77 \text{ s}$   
 $a_g = -9.80 \text{ m/s}^2$



$$(RPS = \frac{1}{T})$$

$$RPM = \frac{60 \text{ sec}}{1 \text{ rev} \cdot 0.77 \text{ s}}$$

Relationships: 3.  $F_c = T + F_g$

1.  $F_g = ma_g$

2.  $F_c = \frac{4\pi^2 r m}{T^2}$

Solve:  $T = F_c - F_g$

Sub, Cal, R.O.:  $F_g = ma_g = (1.2 \text{ kg}) (-9.80 \text{ m/s}^2) =$   
 $-11.76 \text{ N} = -12 \text{ N}$

$$F_c = \frac{4\pi^2 r m}{T^2} = \frac{4\pi^2 (0.80 \text{ m}) (1.2 \text{ kg})}{(0.77)^2} = -64 \text{ N}$$

$$T = F_c - F_g$$

here 64N is  
negative as it is down

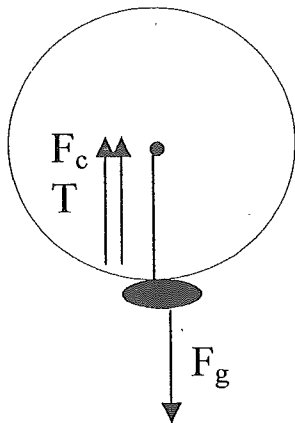
$$T = -64 \text{ N} - (-12 \text{ N})$$

$$T = -52 \text{ N} \text{ or } 52 \text{ N toward the center}$$

b. the bottom ?

$$F_g = -12 \text{ N}$$

$$F_c = +64 \text{ N} \text{ } F_c \text{ here is up (hence } +64 \text{ N)}$$



$$T = F_c - F_g$$

$$T = +64 \text{ N} - (-12 \text{ N})$$

$$T = +76 \text{ N}$$

### HILL CRESTS AND DIPS

Hill crests and dips are:

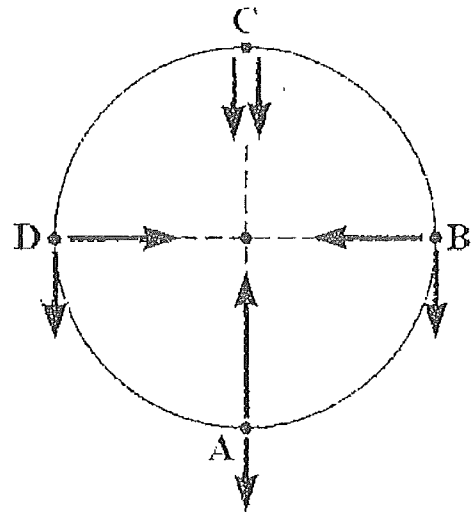
a special case of  $F_c = F_g + T$  where  $F_N$  replaces  $T$ !

## Physics 12 – Vertical Circular Motion

When the motion of an object is in a vertical circular path, the centripetal force may be different at varying points of the motion.

Why do the forces vary from point to point?

Top Position (C):



Bottom Position (A):

Side Positions (B and D):

→ since  $F_g$  is tangent to the circle in both cases, it has no component toward the center. Therefore it does not contribute to the centripetal force in these positions.

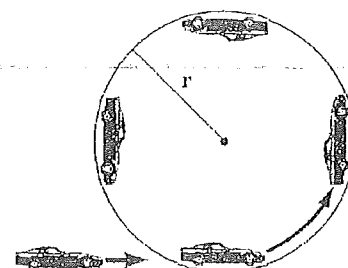
**Example:** A 1.7 kg object is swung from the end of a 0.60 m string in a vertical circle. If the time of one revolution is 1.1 s, what is the tension in the string? Assume uniform speed.

a) When it is at the top?

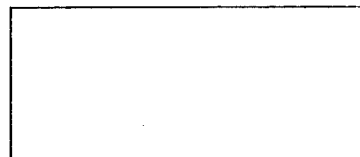
b) When it is at the bottom?



When determining the minimum speed required so that the object does not leave the circle (very useful for a roller coaster design!), the tension or normal force (if there are two surfaces) are equal to zero.



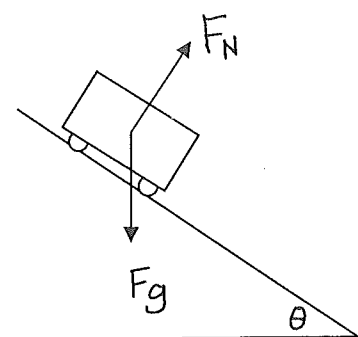
We say that the mass at the peak of the arc is *weightless*, because the net force working on it is only gravity. This is the same as an object in free fall.



**Example:** An object is swung in a vertical circle with a radius of 0.75 m. What is the minimum speed of the object at the top of the motion for the object to remain in its circular motion?

## Banked Curves

Engineers do not always rely on friction alone to provide the centripetal force necessary for a car to round a curve safely. These **curves are usually banked** which allow cars, regardless of their mass, to round the curve safely at a certain speed even if the road is frictionless.



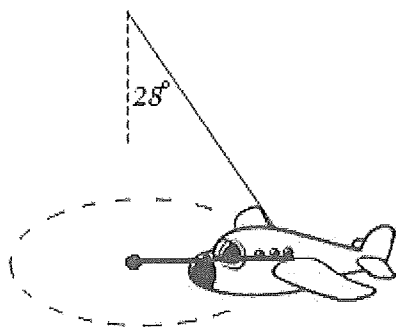
Consider a car traveling at a constant speed around a frictionless banked corner.

On the frictionless corner, the only forces acting on the car are:

In this case,  $F_N$  both accelerates the car inwards and matches the  $F_g$ . Therefore, the sum of  $F_N$  and  $F_g$  must equal \_\_\_\_\_.

**Example:** Calculate the angle at which a frictionless curve must be banked if a car is to round it safely at a speed of 22 m/s if its radius is 475 m.

**Example:** A 0.25 kg toy plane is attached to a string so that it flies in a horizontal circle with a radius of 0.80 m. The string makes a  $28^\circ$  angle to the vertical. What is its period of rotation?



**Example:** A string requires a 135 N force in order to break it. A 2.00 kg mass is tied to this string and whirled in a vertical circle with a radius of 1.10m. What is the maximum speed that this mass can be whirled without breaking the string?

**Example:** An 826 kg car travelling a speed of 14.0 m/s goes over a hill. If the radius of this curve is 61.0 m, what is the force exerted on the car by the road at the crest of the hill to keep the car in equilibrium?

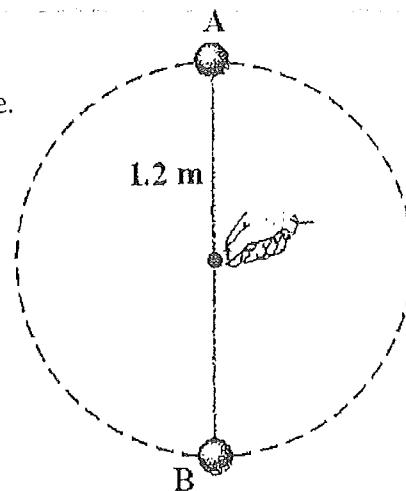
# Lesson 2 homework

## Vertical Circular Motion Problems:

1. A 0.20 kg ball on the end of a string is swung in a vertical circle.

a) Find the minimum speed the ball must have at the top of the circle to remain in a circle. (3.4 m/s)

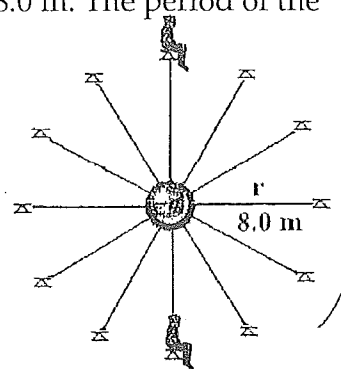
b) If the speed of the ball is 5.0 m/s at the bottom of the circle, find the tension in the string at the bottom. (6.13 N)



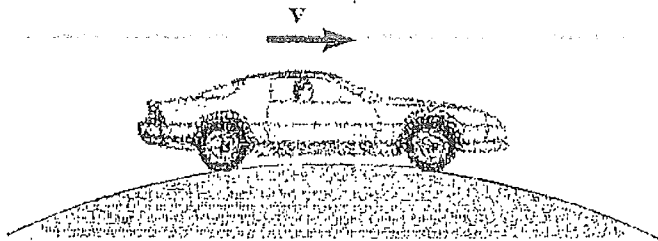
2. A 40 kg girl on a Ferris wheel moves in a vertical circle of radius 8.0 m. The period of the Ferris wheel is 12 s.

a) Draw a diagram showing the forces acting on the girl at the top and then at the bottom.

b) Find the forces exerted on the girl by the seat at the top and bottom of the circle. ( $F_g = -392\text{ N}$ , Top:  $F_c = -87.7\text{ N}$ ,  $F_N = +304\text{ N}$  Bottom:  $F_c = +87.7\text{ N}$ ,  $F_N = +480\text{ N}$ )

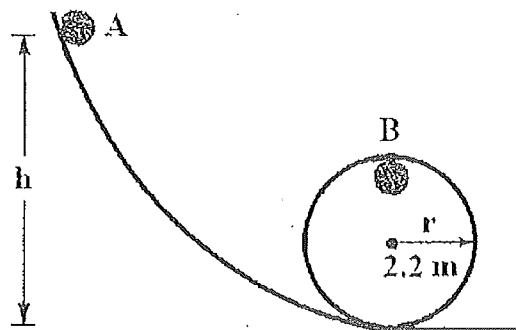


3. A 1200 kg vehicle travels at a constant speed on a hill of radius 48 m as shown in the diagram.



- Draw a diagram showing the forces acting on the vehicle.
- If the vehicle travels at the top of the hill at 14 m/s, what is the force exerted by the road on the vehicle? (6860 N)
- What is the maximum speed the vehicle can have as it passes the top of the hill before losing contact with the road? (21.7 m/s)

4. A 0.25 kg sphere rolls down without friction along the loop-the-loop track with a radius of 2.2 m as shown in the diagram.



- From what minimum height,  $h$ , must the sphere be released so that it remains on the circular track at all times, even at the top of the loop (point B)?

b) If the release height is  $2h$ , find the speed of the sphere and the normal force exerted by the track at point B. ( $v = 11.4 \text{ m/s}$ ,  $F_N = 12.3 \text{ N}$ )

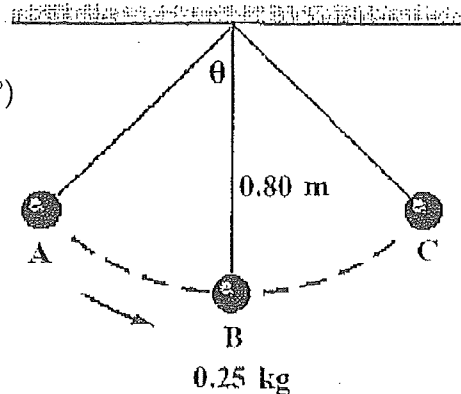
c) If the release height is  $2h$  and the radius of the loop is  $1.1 \text{ m}$ , find the speed of the sphere and the normal force exerted by the track at point B. ( $v = 13.1 \text{ m/s}$ ,  $F_N = 37 \text{ N}$ )

5. A  $0.25 \text{ kg}$  object is attached to a  $0.80 \text{ m}$  string. The object is pulled to point A, and then released as shown in the diagram. The object reaches a speed of  $3.0 \text{ m/s}$  at the bottom (point B) of its swing.

a) What is the angle  $\theta$  when the object is at point B? ( $65^\circ$ )

b) What is the tension in the string at point C? ( $1.04 \text{ N}$ )

c) What is the maximum tension in the string? ( $5.26 \text{ N}$ )



6. You are riding your bike on a track that forms a vertical circular loop as shown in the diagram. If the diameter of the loop is 10.0 m, how fast would you have to be travelling when you reached the top of the loop so that you would not fall? (7.0 m/s)

7. What is the correct speed for a car rounding a 125 m curve in the highway under very icy conditions if the banking angle is  $20.0^\circ$ ?

8. A student has a weight of 655 N. While riding on a roller coaster, this same student has an apparent weight of  $1.96 \times 10^3$  N at the bottom of the dip that has a radius of 18.0 m. What is the speed of this roller coaster? (18.8 m/s)

9. A 745 m curve on a racetrack is to be banked for cars travelling at 90.0 m/s. At what angle should it be banked if it is going to be used under very icy conditions? ( $48^\circ$ )

10. A string requires a 186 N force in order to break. A 1.50 kg mass is tied to this string and whirled in a vertical circle with a radius of 1.90 m. What is the maximum speed that this mass can be whirled without breaking the string? (14.7 m/s)

11. A 2.2 kg object is whirled in a vertical circle whose radius is 1.0 m. If the time of one revolution is 0.97 s, what is the tension in the string? (assume uniform speed)

a) When it is at the top? (-71N)

b) When it is at the bottom? (114N)

12. A wheel shaped space station whose radius is 48 m produces artificial gravity by rotating. How fast must this station rotate so that the crew members have the same apparent weight in this station as they have on Earth? (21.7 m/s)

13. An airplane travelling at a speed of 115 m/s makes a complete horizontal turn in 120 s. What is the banking angle? ( $32^\circ$ )

14. A 2.5 kg ball is tied to a 0.75 m string and whirled in a vertical circle (assume a constant speed of 12 m/s).

a) Why is the tension in the string greater at its low point than at its high point?

b) What is the tension in the string at its-

i) low point? (505 N)

ii) high point? (-456 N)

## Lesson 3

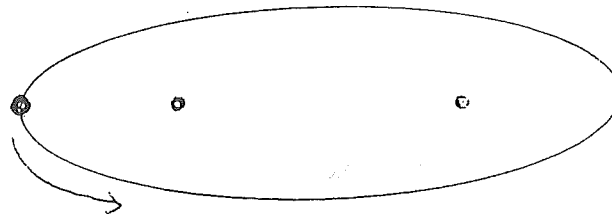
### THE EFFECTS OF GRAVITY

•1543 Copernicus developed the Copernican or **heliocentric** theory stating that the sun was the center of the solar system and the planets revolved around the sun.

•Tycho Brahe made **naked-eye observations** of the planets (in late 1500's) and his assistant Johann Kepler modified Copernicus' theory and using Tycho's data, described the motion of the planets in 3  
KEPLER LAWS:

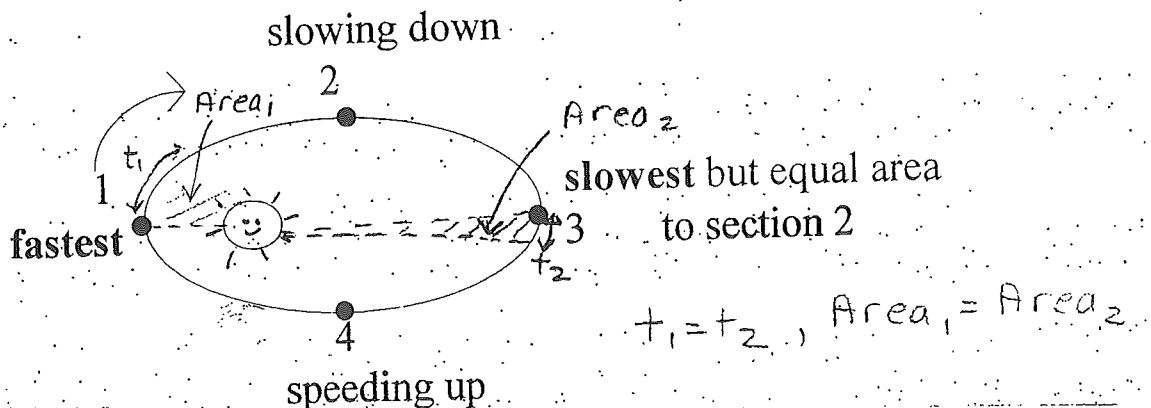
#### 1. LAW OF ELLIPTICAL ORBITS

•Each planet moves in an **ELLIPTICAL** orbit with the **SUN** as one of the focal points.



#### 2. LAW OF EQUAL AREAS

During **equal time** intervals, a straight line drawn to the sun, will **weep out equal areas** this means the closer a planet is to the sun, in its orbital path, the faster it goes to cover the same area in the same time.





### 3. LAW OF PERIODS

- The period of revolution is related to its radius of revolution (distance from the center object, in this case, the sun).

$T^2$  varies with  $R^3$   
 $\swarrow$   $\searrow$   
**period** **ORBITAL radius**  
 (time for 1 revolution) (eg. Average distance to the sun)

$$\frac{T^2}{R^3} = K \quad \swarrow \text{Kepler's Constant}$$

Therefore:

$$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3} \quad \begin{array}{l} 1 \rightarrow \text{first planet (satellite)} \\ 2 \rightarrow \text{second planet (satellite)} \end{array}$$

or

$$\frac{(T_1)^2}{(T_2)^2} = \frac{(R_1)^3}{(R_2)^3}$$

Remember: T & R are for the ORBITING objects  
NOT the center object.

#### Examples

1. Given earth's orbital radius as  $1.5 \times 10^{11}$  m, calculate the period of revolution (in years) of Neptune given its orbital radius is  $4.5 \times 10^{12}$  m. (see page 159 of text Merrill-text for these values!!).

$$\frac{T_e^2}{R_e^3} = \frac{T_n^2}{R_n^3} \rightarrow T_n = \sqrt{\frac{T_e^2 \times R_n^3}{R_e^3}} = \sqrt{\frac{(1.00a)^2 (4.5 \times 10^{12})^3}{(1.5 \times 10^{11})^3}}$$

## Satellites and Energy Examples

- eg. A  $7.5 \times 10^4$  kg space vehicle leaves the surface of the earth with a speed of  $1.3 \times 10^4$  m/s. What will its speed be when it is infinitely far from the earth?

$$E_i = E_f$$

conservation of energy concept  $\rightarrow E_{pi} + E_{ki} = E_{pf} + E_{kf}$

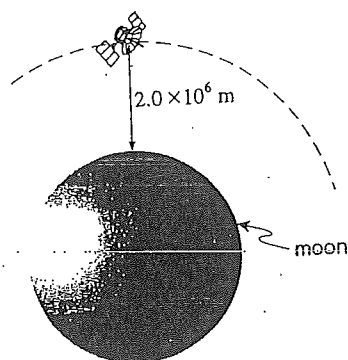
formulae  $\rightarrow -\frac{GmM}{R_E} + \frac{1}{2}mv_i^2 = 0 + \frac{1}{2}mv_f^2$

substitution  $\rightarrow \left[ \frac{-6.67 \times 10^{-11} (5.98 \times 10^{24})}{6.38 \times 10^6} \right] + \left[ \frac{1}{2} (1.3 \times 10^4)^2 \right] = \frac{1}{2} v_f^2$

$$-6.25 \times 10^7 + 8.45 \times 10^7 = \frac{1}{2} v_f^2$$

$$\frac{1}{2} v_f^2 = 2.2 \times 10^7 \longrightarrow v_f^2 = 4.4 \times 10^7 \longrightarrow v_f = 6.6 \times 10^3 \text{ m/s}$$

- eg. A stationary  $1.60 \times 10^3$  kg vehicle is taken from the surface of the moon and placed into a circular orbit at a height of  $2.0 \times 10^6$  m above the surface of the moon. Its speed in this orbit is  $1.15 \times 10^3$  m/s. How much work is required for this process?



• work is CHANGE in energy

$$W = \Delta E = E_f - E_i \text{ OR } W = \Delta E_K + \Delta E_P$$

$$\text{SO } E_i + W = E_f$$

maybe comprised of kinetic and or potential energies

work  
concept

$$E_{p \text{ surface}} + W = E_{p \text{ altitude}} + E_{k \text{ orbital}}$$

$$-\frac{GmM}{R_1} + W = -\frac{GmM}{R_2} + \frac{1}{2}mv^2$$

$$\left[ \frac{-6.67 \times 10^{-11} (1.60 \times 10^3) (7.35 \times 10^{22})}{1.74 \times 10^6} \right] + W = \left[ \frac{-6.67 \times 10^{-11} (1.60 \times 10^3) (7.35 \times 10^{22})}{3.74 \times 10^6} + \frac{1}{2} (1.60 \times 10^3) (1.15 \times 10^3)^2 \right]$$

$$[-4.51 \times 10^9] + W = [-2.10 \times 10^9] + [1.06 \times 10^9]$$

$$-4.51 \times 10^9 + W = [-1.04 \times 10^9]$$

$$W = 3.5 \times 10^9 \text{ J}$$

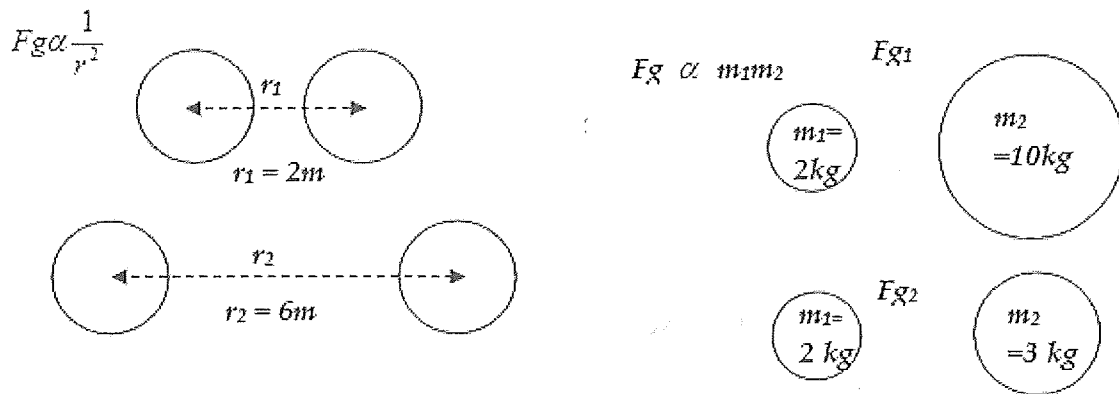
## Physics 12 – Universal Gravitation and Satellite Motion

Newton explained the dynamics of the solar system in his Law of Universal Gravitation. He explained through his first law of motion that if a planet is moving in a circular path, it has centripetal acceleration and therefore must be centripetal force acting on it. Newton showed that this force must be caused by the sun.

Using Kepler's three laws of planetary motion and Newton's three laws of motion, Newton formulated **the Law of Universal Gravitation**. (The derivation of this will be posted my portal site)

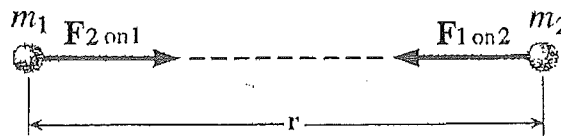
The basics of this law are:

- A. Every particle in the universe exerts an attractive force on every other particle.
- B. The gravitational force between any two objects is
  - a. directly proportional to the product of their masses –
  - b. inversely proportional to the square of the distance between their centers –



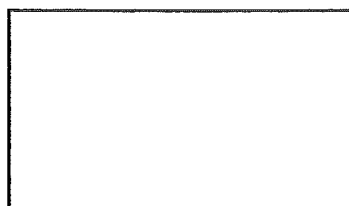
- C. The gravitational force between two objects:

\*  $F_g$  is exerted along a line connecting the centers of the two objects.



\*  $F_g$  on  $m_1$  and  $m_2$  is equal in size but opposite in direction to the  $F_g$  on  $m_2$  by  $m_1$ .

- D. When combined mathematically, the formula found is:



**Example:** Calculate the force of gravity between two 75 kg students if their centers of mass are 0.95 m apart.

**Example:** A satellite weighs 9000 N on Earth's surface. How much does it weigh if its mass is tripled and its orbital radius is doubled?

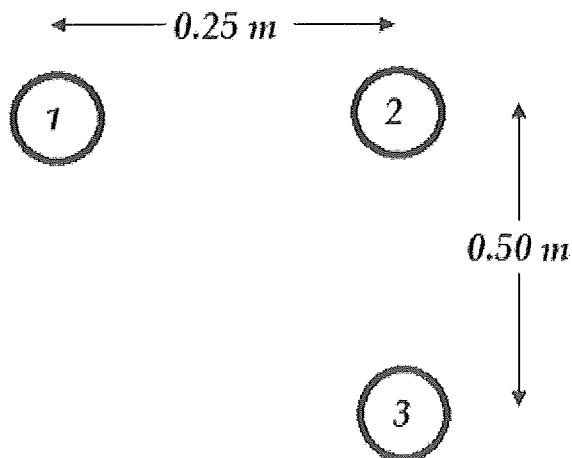
**MASS VERSUS WEIGHT:** This is a very common misconception!

Mass:

Weight:

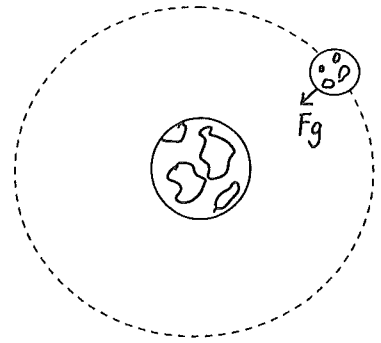
### Net Gravitational Force

Three equal 0.75 kg mass balls are placed as shown in the diagram. Determine the net gravitational force on "2" due to the presence of "1" and "3".



## Satellites in Orbit

A satellite of Earth, such as the moon, is constantly falling. But it does not fall towards the Earth... it falls **around** the Earth. Just as if you were in an elevator that was falling towards the Earth you would feel weightless if you were on an artificial satellite falling around the Earth.



**Example:** A 4500 kg Earth satellite has an orbital radius of  $8.50 \times 10^7$  m. At what *speed* does it travel?

## Lesson 3 homework

### Law of Gravitation and Satellite Motion Problems:

1. The mass of the planet Jupiter is  $1.9 \times 10^{27}$  kg, and the average radius is  $7.2 \times 10^7$  m.

a) If a 3200 kg space probe is sitting on the surface of Jupiter, what is the gravitational force between the space probe and Jupiter? ( $7.8 \times 10^4$  N)

b) If the 3200 kg space probe is to accelerate upwards at  $2.8 \text{ m/s}^2$  as it leaves Jupiter, what is the thrust force exerted by the engine? ( $8.7 \times 10^4$  N)

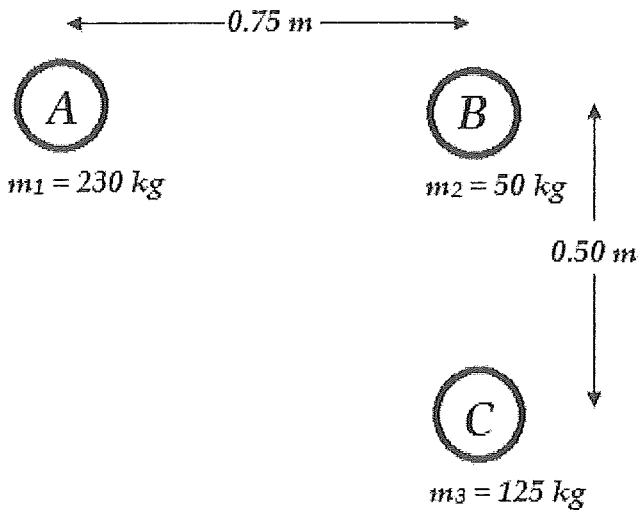
c) If an astronaut experiences a gravitational force of 1600 N at an altitude of  $3.2 \times 10^6$  m above Jupiter, what is the mass of the astronaut? (71 kg)

2. A 30 kg box is 2.0 m away from a 1200 kg car. Which exerts a greater gravitational force on the other? Explain your answer. (neither)

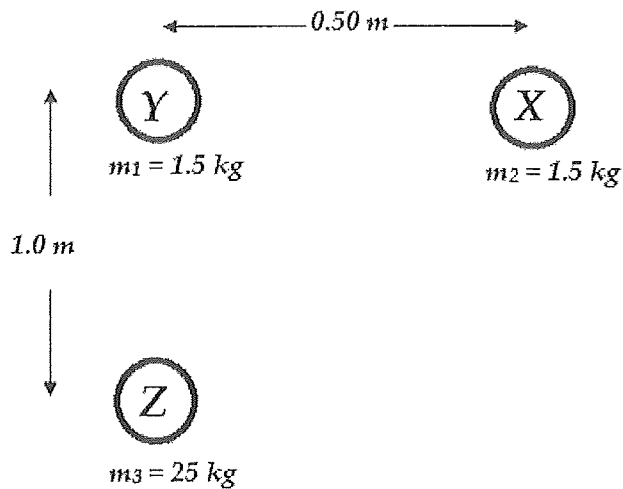
3. What gravitational force does the moon produce if the centers of the Earth and the moon are  $3.84 \times 10^8$  m apart and the moon has a mass of  $7.35 \times 10^{22}$  kg? ( $1.99 \times 10^{20}$  N)

5. If the gravitational force between two objects of equal mass is  $4.50 \times 10^{-6} \text{ N}$  when the objects are 7.0 m apart, what is the mass of each object? ( $1.8 \times 10^3 \text{ kg}$ )
6. Calculate the gravitational force on a  $6.50 \times 10^2 \text{ kg}$  spacecraft that is  $4.15 \times 10^6 \text{ m}$  above the surface of the Earth. ( $2.35 \times 10^3 \text{ N}$ )
7. The gravitational force between two objects that are 0.33 m apart is  $3.2 \times 10^{-5} \text{ N}$ . If the mass of one object is 60 kg, what is the mass of the other object? (871 kg)
8. Calculate the speed of the moon in its orbit around Earth. (Radius of moon's orbit =  $3.84 \times 10^8 \text{ m}$ ; moon's mass =  $7.35 \times 10^{22} \text{ kg}$ ) ( $1.02 \times 10^3 \text{ m/s}$ )
9. Calculate the speed of a satellite orbiting Earth at a height of  $4.4 \times 10^5 \text{ m}$  above the Earth's surface. ( $7.6 \times 10^3 \text{ m/s}$ )
10. Calculate the orbital speed of a satellite  $5.0 \times 10^6 \text{ m}$  above the surface of Jupiter. (Radius of Jupiter =  $7.18 \times 10^7 \text{ m}$ ; mass of Jupiter =  $1.90 \times 10^{27} \text{ kg}$ ) ( $4.06 \times 10^4 \text{ m/s}$ )
11. Calculate the speed of Earth in its orbit around the Sun. (Radius of Earth's orbit =  $1.50 \times 10^{11} \text{ m}$ ; sun's mass =  $1.98 \times 10^{30} \text{ kg}$ ) ( $2.97 \times 10^4 \text{ m/s}$ )
12. Using the formula  $T = \sqrt{\frac{4\pi^2 r^3}{Gm}}$ , calculate the time of one revolution (length of a year) on Mars. (Mar's mass =  $6.4 \times 10^{23} \text{ kg}$ ; radius of Mar's orbit =  $2.3 \times 10^{11} \text{ m}$ ) ( $6.0 \times 10^7 \text{ s}$  or 1.9 years)

13. Three masses are placed as shown in the diagram. Determine the net gravitational force on "B" due to the presence of "A" and "C". ( $2.15 \times 10^{-6} \text{ N}$  @  $39^\circ \text{ W of S}$ )



14. Three masses are placed as shown in the diagram. Determine the net gravitational force on "Y" due to the presence of "X" and "Z". ( $2.57 \times 10^{-9} \text{ N}$  @  $14^\circ \text{ E of S}$ )



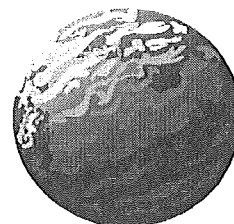
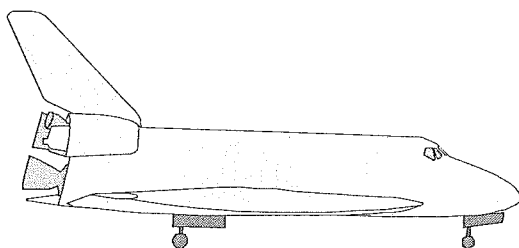


## Galilei 27 orbiting Iris 2 (part 1)

### Context

Within the scope of the EESP project (Exploration of Extrasolar Planets), we have identified three planets that are suitable for sustaining life forms that are similar to us. We are in the process of developing various mission scenarios that could eventually be carried out on these three planets. You have been selected to plan the encounter between Galilei 27 and Iris 2.

Galilei 27 is the name of the space ship that will transport the astronauts. Iris 2 is the name of the planet, among the suitable three, that is closest to Earth. Planet Iris 2 has a lower mass than that of the Earth but has a smaller radius. This is the information we have gathered so far:



Mass of Iris 2:  $m_I = 2.0 \times 10^{24} \text{ kg}$

Radius of Iris 2:  $r_I = 2\,000 \text{ km}$

Mass of Galilei 27:  $m_G = 5.0 \times 10^5 \text{ kg}$

Your task is to calculate certain parameters that will be needed to accomplish the technical aspects of the mission scenario. Perform the following calculations:

We should be able to approach Iris 2 without any problems. However, in order to land on its surface, we must know the attractive force that will be exerted on the space ship when it is positioned at a distance of 10 000 km from the planet's surface.

- Calculate the attractive force that will be exerted by Iris 2 on the space ship when it is positioned at that distance from the surface of the planet.
- Calculate the acceleration that Galilei 27 will experience at that distance.

Please note: this data will help us design a landing plan (the thrust the motors will need to help the spaceship brake during its landing, the adequate quantity of fuel, the time it takes to brake, etc.)

In the probable event that Galilei 27 lands on Iris 2 as anticipated, physiological factors must also be taken into account:

- Calculate the gravitational acceleration on the planet's surface.
- What force will the astronaut need to apply to lift a radiation monitor that has a mass of 2 kg to 1.5m above the surface? (2.50 J of work is done to overcome atmospheric friction)
- If the astronaut drops the monitor from a height of 1 metre, how long will it take for it to fall to the ground?

Please note: these facts will allow us to evaluate how the human body will react to gravity on Iris 2. In addition, they will help us choose the materials needed for the fabrication of space suits, and the technical equipment that the astronauts will carry when they first walk on Iris 2. They will also enable us to determine other parameters, such as resistance of the materials chosen.

**Please use a separate piece of paper and show all calculations and answers for each of the points. Make sure to label each value with its purpose. (ie. Gravitational Acceleration at Surface - ...)**

# Lesson 4

## Physics 12 - Gravitational Fields

In order to explain forces between two objects that are not in contact, scientists developed the concept of fields. **Fields are defined as spheres of influence** and can be classified as scalar or vector. Scalar field examples would be sound fields and heat fields.

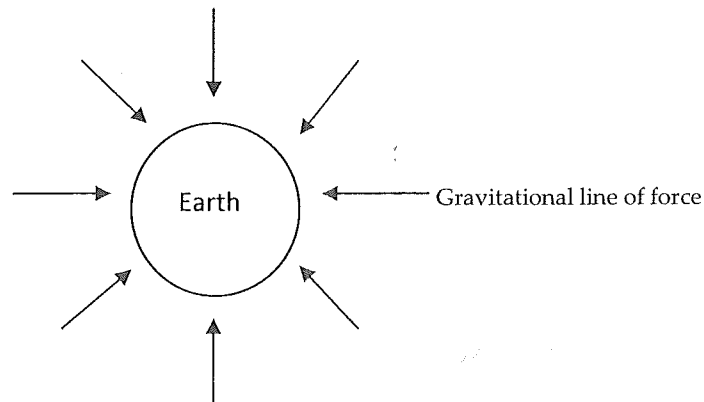
To help visualize how a field works...consider a **campfire** and the *heat field* that it emits.

As you approach the fire...

As you increase the size of the fire...



A gravitational field is a vector field due to the fact that gravity is a force and forces are **vector** quantities.



**Gravitational Field Strength** is the acceleration due to gravity and it will vary depending on the masses involved and the distance of separation between the centers of the two objects.

On Earth, the gravitational field strength is  $9.80 \text{ N/kg}$

There are two methods to determine the field strength between two objects:

*\*recall that weight is defined as the gravitational force ( $F_g$ ) between a planet and an object on the surface of that planet.*

This formula works well if we stay on the surface of the Earth or other planetary mass.

However, once we leave the surface it does not work very well because...

Therefore a more useful formula takes that into account:

**Example:** What is the gravitational field strength on the surface of the moon?

Mass of moon =  $7.35 \times 10^{22}$  kg

Radius of moon =  $1.74 \times 10^6$  m

**Example:** A satellite orbits the Earth at a radius of  $2.20 \times 10^7$  m. What is its orbital period?

**Geosynchronous Orbit** – The orbital speed of a satellite will depend on the strength of the gravitational field at the orbital radius.

The orbital period of the satellite depends only on the mass of the planet and the orbital radius of the satellite. Therefore it makes sense that at a certain orbital distance, the orbital period will match the rotational period of the planet. This satellite is then in **geosynchronous (or geostationary) orbit.**

**Example:** Find the orbital radius of a satellite that is geostationary above the Earth's equator.

What is the speed of this satellite?

## Lesson 4 homework

### Gravitational Field Strength and Problems:

1. The mass of the planet Jupiter is  $1.9 \times 10^{27}$  kg, and the average radius is  $7.2 \times 10^7$  m.

a) What is the gravitational field strength on the surface of Jupiter? (24 N/kg)

2. A 2.0 kg rock dropped near the surface of a planet reaches a speed of 15 m/s in 3.0 s.

a) What is the acceleration due to gravity near the surface of the planet? (5.0 m/s<sup>2</sup>)

b) The planet has an average radius of  $4.8 \times 10^6$  m. What is the mass of the planet? ( $1.73 \times 10^{24}$  kg)

3. Calculate the gravitational field strength on the surface of Mars. Mars has a radius of  $3.43 \times 10^6$  m and a mass of  $6.37 \times 10^{23}$  kg. (3.61 N/kg)

4. At what distance from the Earth's surface is the gravitational field strength 7.33 N/kg? ( $1.00 \times 10^6$  m)

5. What is the gravitational field strength  $1.276 \times 10^7$  m above the Earth's surface? (1.09 N/kg)

6. A 2400 kg satellite is in a stable circular orbit around the Earth with a radius of  $6.8 \times 10^6$  m.

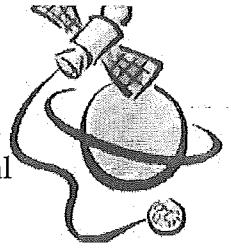
- a) What provides the centripetal force to keep the satellite in a circular orbit?
- b) Find the speed of the satellite in this orbit. ( $7.66 \times 10^3$  m/s)
- c) Find the period of the satellite. ( $5.58 \times 10^3$  s)

7. A satellite in synchronous orbit far above the equator of the Earth appears to be stationary over a position to an observer on Earth. Explain why the satellite appears to be stationary.

8. A satellite travels around a  $5.20 \times 10^{23}$  kg planet with an orbital radius of  $1.1 \times 10^7$  m. What is the orbital period of the satellite? ( $3.89 \times 10^4$  s)

# Lesson 5 (part 1)

## Physics 12 – Work and Gravitational Potential Energy

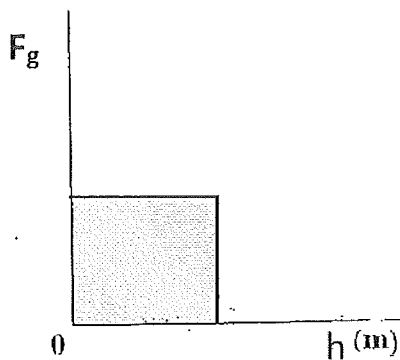


Up to this point, we have used the formula  $E_p = mgh$  to determine gravitational potential energy. While this equation works well when finding gravitational potential energy near the Earth's surface, it does not work when larger distances are involved.

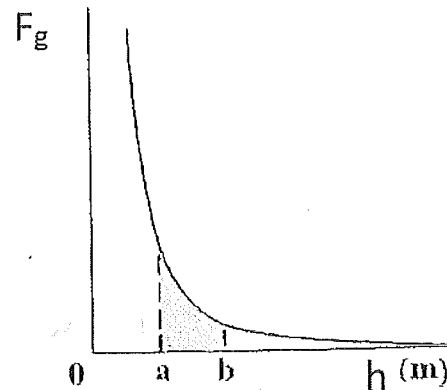
As we saw when calculating gravitational field strength in the last lesson –

Remember that the  $E_p$  of a mass is the work done in lifting it a certain height.

When  $F_g$  is constant –



When  $F_g$  is changing with distance –

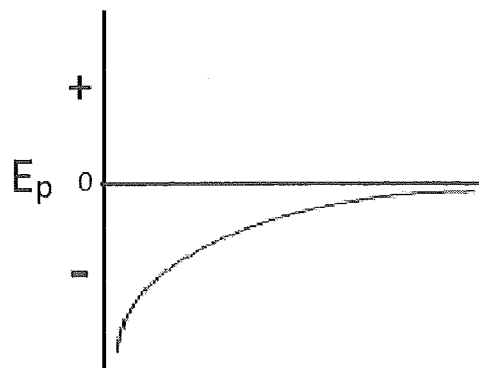
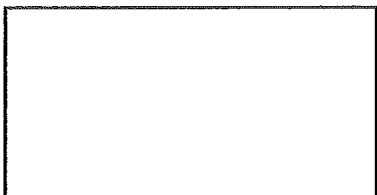


As we saw above, the gravitational potential energy was determined to be:

However, that is not the whole picture (why we need calculus). When dealing with gravitational potential energy, we use a reference point. At this point the  $PE_g$  is zero. When we are finding the gravitational potential energy on a mass provided by the gravitational force generated by a second mass, we assign the ZERO reference point when the distance between the objects is infinite ( $r = \infty$ ).

This means that when the objects get closer together, the gravitational potential energy between them decreases.

Because of this, the formula becomes:



## Calculating Work Done:

The work done on an object can be equated to the change in gravitational potential energy of an object. This comes from:  $\Delta PE = (PE)_f - (PE)_0$

$$\Delta PE_g = Gm_1m_2 \left( \frac{1}{r_0} - \frac{1}{r_f} \right)$$

## Energy Conservation:

Kinetic energy will be obtained as an object falls to earth and increases in velocity. Therefore, the potential energy will be converted to kinetic energy following the law of conservation (without any external forces acting on the object).

Law of Conservation of Mechanical Energy  $\rightarrow$

**Example One** – A  $2.50 \times 10^3$  kg geostationary satellite (a satellite that remains in the same position above the Earth's surface) is in an orbit that is  $3.60 \times 10^7$  m above Earth's surface. What is the gravitational potential energy of this satellite due to the gravitational force cause by the Earth?

**Example Two** – How much work is needed to lift a  $1.25 \times 10^3$  kg satellite from the Earth's surface to a height of  $4.00 \times 10^6$  m above the Earth's surface?

**Example Three** – A  $1.10 \times 10^3 \text{ kg}$  object is dropped from a distance of  $2.00 \times 10^5 \text{ m}$  onto the Moon's surface. How fast is the object travelling when it hits the Moon's surface?

**Example Four:** The period of a  $1000 \text{ kg}$  satellite orbiting the earth is  $6.3 \times 10^3 \text{ s}$ .

- a. Find the altitude of the satellite.
- b. Find the total energy of the satellite.



## Lesson 5 homework

### Work and Gravitational Potential Energy Problems:

1. A 2900 kg space probe is  $3.8 \times 10^5$  m from the center of a planet.
  - a) If the gravitational potential energy of the space probe is  $-1.7 \times 10^{11}$  J relative to zero at infinity, what is the mass of the planet? ( $3.3 \times 10^{23}$  kg)
  
  - b) If the space probe falls to the planet and has a speed of  $6.2 \times 10^3$  m/s just before impact with the surface of the planet, what is the radius of the planet? ( $2.8 \times 10^5$  m)
  
2. A  $5.8 \times 10^4$  kg meteor falls from an altitude of  $7.2 \times 10^5$  m above the Earth's surface. Ignore air resistance.
  - a) Find the change in gravitational potential energy of the meteor. ( $-3.7 \times 10^{11}$  J)
  
  - b) Find the speed of the meteor just before impact with the surface of the Earth (Assume initial velocity is zero) ( $3.6 \times 10^3$  m/s)

- c) If the meteor had an initial speed of  $1.8 \times 10^3$  m/s, find the kinetic energy of the meteor just before impact with the Earth's surface. ( $4.6 \times 10^{11}$  J)
3. What minimum energy is required to take a stationary 600 kg satellite from the surface of the moon to an altitude of  $9.2 \times 10^5$  m and put it into orbit with an orbital speed of  $1.36 \times 10^3$  m/s? ( $1.1 \times 10^9$  J)
4. What is the gravitational potential energy of a  $5.00 \times 10^3$  kg satellite that has an orbital radius of  $9.90 \times 10^6$  m around the Earth? (Use  $PE_g = 0$  at  $r = \infty$ ). ( $-2.0 \times 10^{11}$  J)
5. What is the work done against gravity on the satellite in problem 4 in lifting it into its orbit? ( $1.1 \times 10^{11}$  J)
6. What is the change in gravitational potential energy of the satellite in problem 4 as it is lifted from Earth's surface to its orbit? ( $1.1 \times 10^{11}$  J)

7. What is the speed of a 1750 kg meteorite when it hits the moon's surface? This meteorite has a velocity of  $1.00 \times 10^3$  m/s heading directly toward the moon when it was 15000m above the moon's surface (no friction). ( $1.02 \times 10^3$  m/s)

8. What is the gravitational potential energy of a 10.0 kg object when it is sitting on the Earth's surface? (Use  $PE_g = 0$  at  $r = \infty$ ) ( $-6.25 \times 10^8$  J)

9. What is the change in the gravitational potential energy of a  $2.50 \times 10^3$  kg satellite as it is lifted vertically into a circular orbit (radius =  $6.90 \times 10^6$  m) around Earth? ( $1.18 \times 10^{10}$  J)

10. What minimum energy is required to take a stationary 1200 kg satellite from the surface of the earth to an altitude of  $3.0 \times 10^6$  m and put it into an orbit with an orbital speed of  $2.0 \times 10^3$  m/s? ( $2.64 \times 10^{10}$  J)

11. The period of an 800 kg satellite orbiting the earth is  $5.9 \times 10^3$  s.

- a. Find the altitude of the satellite. ( $6.2 \times 10^5$  m or  $6.8 \times 10^5$  m with no rounding)
- b. Find the gravitational field strength at the altitude of the satellite. (8.1 N/kg)
- c. Find the gravitational potential energy of the satellite. ( $-4.6 \times 10^{10}$  J)
- d. Find the total energy of the satellite. ( $-2.3 \times 10^{10}$  J)

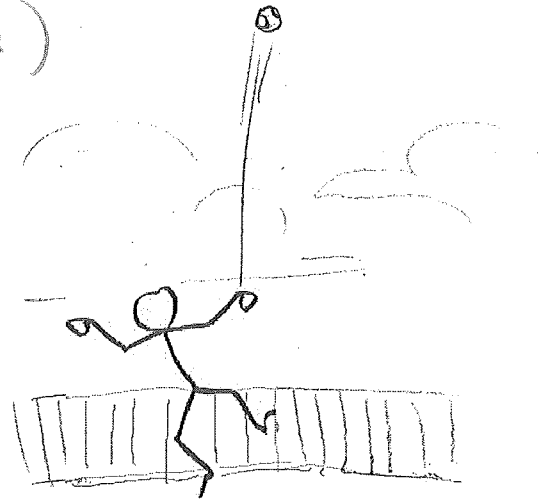
## Lesson 5 (part 2)

### Physics 12 – Escape Velocity

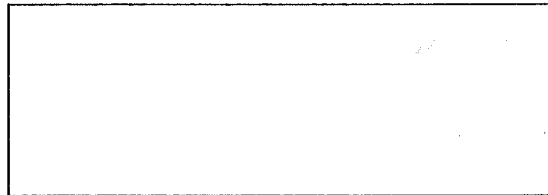
Escape velocity is the minimum speed an object requires in order to break away from the Earth's pull.

It should make sense that if an object is going to be completely freed from the Earth's gravitational pull, we need to supply it with enough KINETIC ENERGY to match its POTENTIAL ENERGY at infinity.

In terms of equations this means that:



*What goes up must come down, unless  
we throw is really, REALLY hard.*



**Example:** At what speed do you need to throw a 1.0 kg rock in order for it to break away from the Earth's pull?

Does the mass of the rock matter?

### Escape Velocity Problems:

1. What is the escape speed at the moon's surface? ( $2.4 \times 10^3$  m/s)
2. What is the mass of a planet that has an escape velocity of  $9.0 \times 10^3$  m/s and a radius of  $7.2 \times 10^6$  m? ( $4.37 \times 10^{24}$  kg)
3. What is the mass of a planet that has a radius of  $2.57 \times 10^6$  m and an escape of  $2.92 \times 10^3$  m/s? ( $1.64 \times 10^{23}$  kg)
4. What is the escape velocity from a planet with a mass of  $3.46 \times 10^{25}$  kg and a radius of  $2.75 \times 10^6$  m? ( $4.1 \times 10^4$  m/s)
5. What is the radius of a planet whose escape velocity is  $1.16 \times 10^3$  m/s and a mass of  $4.5 \times 10^{23}$  kg? ( $4.45 \times 10^7$  m)

**COMPLETE THE PARTNER ESCAPE VELOCITY ACTIVITY (Hand-in when complete).**

## Physics 12 – Escape Velocity

Name: \_\_\_\_\_

This is an **individual activity** in which you will calculate the escape velocity of planets, satellites and the Sun.

### Procedure:

You have learned that the **escape velocity** ( $v_{esc}$ ) of a body depends on the **mass** ( $M$ ) and the **radius** ( $r$ ) of the given body. The formula which relates these quantities is:

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

You will calculate the escape velocity for a number of bodies using the **MKS** system where the units for **distance** are **meters**, the units for **mass** are **kilograms**, and the units for **time** are **seconds**. In this system, the gravitational constant has the value:

As an example, the mass  $M$  of the Earth is  $5.98 \times 10^{24}$  kilograms. The radius  $r$  of the Earth is 6378 kilometers, which is equal to  $6.378 \times 10^6$  meters. The escape velocity at the surface of the Earth can therefore be calculated by:

$$v_{esc} = \sqrt{\frac{2GM}{r}} \quad v_{esc} = \sqrt{\frac{2(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.378 \times 10^6)}} \quad v_{esc} = 1.12 \times 10^4 \text{ m/s}$$

So, as with surface gravity, a simple Physics equation can be used to calculate the escape velocity for a body (in this case the Earth) if you know the mass of the body and its radius! The assumption in using this formula is that the body is spherical, but this is a pretty good assumption. If the radius of a body at its equator and pole are very different, then the escape velocity is different at those places and should be calculated separately.

The escape velocity for the Earth is therefore  $1.12 \times 10^4$  meter per second. This is the **velocity** that an object (or gas molecule!) needs at the surface of the Earth to be able to overcome the gravitational attraction of the Earth and escape to space.

A table of masses and radii is given on the next page for many bodies in the Solar System. Make sure to convert the radii from kilometers to meters when making the calculation, and make sure that you can **calculate the escape velocity of the Earth** correctly. Then, **calculate the escape velocity at each of the other bodies**.

**SHOW ALL WORK ON A SEPARATE PIECE OF PAPER AND STAPLE TO THIS SHEET TO HAND IN WHEN COMPLETE.**

Body	Mass (kg)	Radius (km)	Escape Velocity (m/s)
Earth	$5.98 \times 10^{24}$	6378	
Mercury	$3.30 \times 10^{23}$	2439	
Venus	$4.87 \times 10^{24}$	6051	
Mars	$6.42 \times 10^{23}$	3393	
Jupiter	$1.90 \times 10^{27}$	71492	
Saturn	$5.69 \times 10^{26}$	60268	
Uranus	$8.68 \times 10^{25}$	25559	
Neptune	$1.02 \times 10^{26}$	24764	
Pluto	$1.29 \times 10^{22}$	1150	
Moon	$7.35 \times 10^{22}$	1738	
Ganymede	$1.48 \times 10^{23}$	2631	
Titan	$1.35 \times 10^{23}$	2575	
Sun	$1.99 \times 10^{30}$	696000	

Note that the Gas Giant planets (Jupiter, Saturn, Uranus and Neptune) do not have solid surfaces. The radii of these planets are specified at the point where the pressure in their atmosphere is approximately equal to that at the surface of the Earth.

### Final Problem:

Convert the escape velocities of the Earth, Mercury and Venus to **km/h** to get a good feeling for how much initial velocity an object must really have in order to escape the gravitational force of our planet!

Earth - \_\_\_\_\_

Mercury - \_\_\_\_\_

Venus - \_\_\_\_\_



**PHYSICS 12 ADDITIONAL PROBLEMS – CIRCULAR MOTION AND GRAVITATION****Uniform Circular Motion:**

1. A car travels around a curved path that has a radius of 225 m at a constant speed of 26.0 m/s. What is the centripetal acceleration of the car?

(3.00 m/s<sup>2</sup>)

2. How fast can a  $1.6 \times 10^3$  kg car round an unbanked curve (radius 55 m) if the coefficient of friction between the car and the road is 0.60?

(18 m/s)

3. A 0.150 kg mass is twirled in a horizontal circle (radius = 0.750 m) at a rate of 2.50 RPS (revolutions per second) from the end of a string. Assuming the string is also horizontal, what is the tension in the string?

(27.8 N)

***Vertical Circular Motion:***

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4. You are riding your bike (total mass = 95.0 kg) over a rise (radius of curve = 10.0 m) on a bike path. How fast must you be travelling so that your bike loses contact with the path?

(9.90 m/s)

5. A student (mass = 50.0 kg) is riding through a dip (radius of curve is 15.0 m) on a roller coaster at a speed of 10.0 m/s. What will be the student's apparent weight (in Newtons) at the bottom of the dip?

(823 N)

6. A 1.75 kg mass is swung in a vertical circle (radius = 1.10 m) using a cord that will break if it is subjected to a force greater than 262 N. What is the maximum speed that this mass can travel as it passes through the bottom of its circle?

(12.4 m/s)

**Banked Curves:**

7. A car travels around a curve (radius = 60.0 m) at a speed of 22.0 m/s. At what angle must the curve be banked so that the car does not have to rely on friction to remain on the road?

(39.5°)

8. An engineer is to design a curved exit ramp from a freeway for traffic with a speed of 20.0 m/s. If she decides to bank the curve at an angle of 20.0°, at what radius of curve must it be built if the vehicles are not to rely on friction?

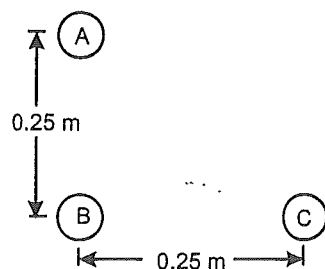
(112 m)

9. A car travels on a circular banked track (radius = 255 m,  $\theta = 25.0^\circ$ ). What is the time of one lap (revolution) of the car if it is not to rely on friction to hold it on the track?

(46.9 s)

**Newton's Law of Universal Gravitation:**

10.



Three equal balls (masses = 0.35 kg) are placed at corners of a right angle triangle as shown in the diagram. What is the net gravitational force on B due to the presence of the other two balls?

( $1.85 \times 10^{-10}$  N 45.0° N of E)

11. On the surface of Planet T, which has a mass of  $7.90 \times 10^{25}$  kg, an object has a weight of 112 N and a mass of 75.0 kg. What is the radius of this planet?

( $5.94 \times 10^7$  m)

12. An object (mass = 525 kg) is  $3.0 \times 10^3$  km above the earth's surface. This object is falling toward the earth because of the earth's gravitational force on it. What is the rate of acceleration when it is at this distance?  
(mass of earth =  $5.98 \times 10^{24}$  kg, radius of earth =  $6.38 \times 10^6$  m)

( $4.53 \text{ m/s}^2$ )

**Gravitational Fields:**

13. How far from the surface of the earth ( $m_E = 5.98 \times 10^{24}$  kg,  $r_E = 6.38 \times 10^6$  m) is the gravitational field strength  $6.13 \times 10^{-1}$  N/kg?

( $1.91 \times 10^7$  m)

14. Calculate the gravitational field strength at a point in space where the weight of an object is  $7.22 \times 10^2$  N and its mass is  $1.10 \times 10^2$  kg.

(6.56 N/kg)

15. If an object has a weight of  $7.00 \times 10^2$  N on the surface of the earth (mass =  $5.98 \times 10^{24}$  kg, radius =  $6.38 \times 10^6$  m), what is its weight at a distance of  $6.38 \times 10^6$  m above the earth's surface?

(175 N)

## Circular Motion and Gravitation

### Launching:

16. You find yourself on a planet where the potential energy of a  $2.00 \text{ kg}$  mass with respect to the centre of the planet is  $3.64 \times 10^6 \text{ J}$ . What is the escape velocity on this planet?

$(1.91 \times 10^3 \text{ m/s})$

17. What is the mass of an a planet (radius =  $5.85 \times 10^7 \text{ m}$ ) which has an escape velocity of  $2.54 \times 10^4 \text{ m/s}$ ?

$(2.83 \times 10^{26} \text{ kg})$

18. What is the escape velocity on a planet that has a mass of  $3.18 \times 10^{23} \text{ kg}$  and a radius of  $2.43 \times 10^6 \text{ m}$ ?

$(4.18 \times 10^3 \text{ m/s})$

**Satellites in Orbit:**

19. What is the speed of an artificial satellite (mass = 625 kg) which is placed in an orbit  $1.00 \times 10^6$  m above the surface of a planet? ( $m_p = 3.18 \times 10^{23}$  kg,  $r_p = 2.43 \times 10^6$  m)

( $2.49 \times 10^3$  m/s)

20. An artificial satellite (mass = 572 kg) is put into a circular orbit about the earth (mass =  $5.98 \times 10^{24}$  kg). If the radius of this orbit is  $1.2 \times 10^7$  m, how long will it take to make one revolution?

( $1.3 \times 10^4$  s)

21. An artificial satellite (mass = 611 kg) is put into a circular orbit around Jupiter (mass =  $1.90 \times 10^{27}$  kg, radius =  $6.99 \times 10^7$  m). If this satellite has an orbital velocity of  $3.12 \times 10^4$  m/s, how far above Jupiter's surface is the satellite?

( $6.03 \times 10^7$  m)

***Gravitational Potential Energy:***

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22. What is the gravitational potential energy of the moon with respect to the earth?

$(-7.63 \times 10^{28} \text{ J})$

23. What is the change in the gravitational potential energy of a 10.0 kg object when it is lifted vertically to a height of  $6.38 \times 10^6 \text{ m}$  above the surface of the earth?

$(3.13 \times 10^8 \text{ J})$

\* \* \* \* \*

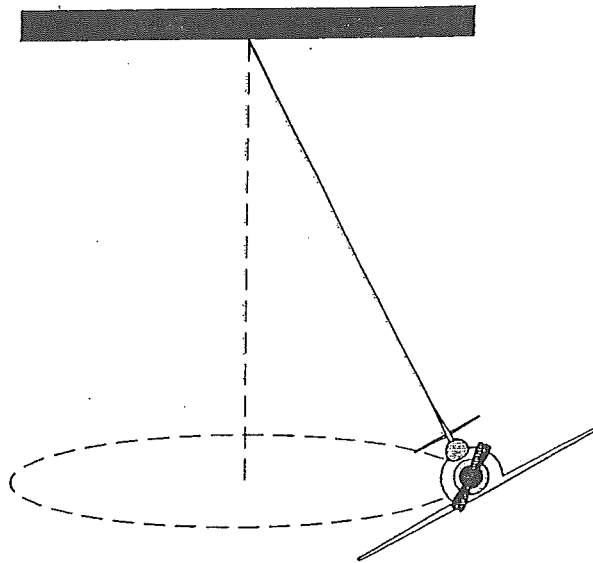


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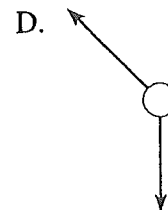
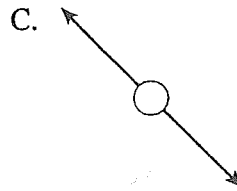
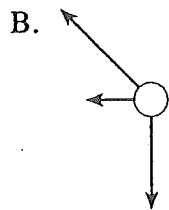
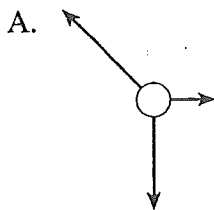
# Lesson 7

## Provincial Exam Questions: Circ. Motion + Univ. Gravitation

1. A small toy airplane suspended as shown below flies in a circular path.

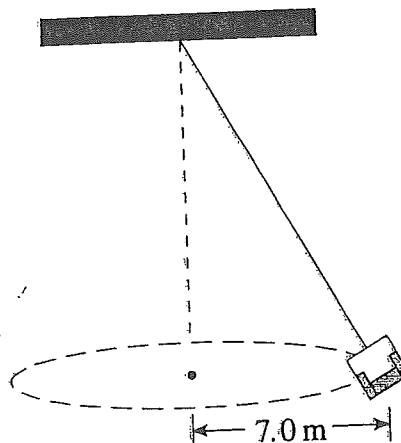


Which of the following free body diagrams best describes the forces acting on the airplane at the position shown?



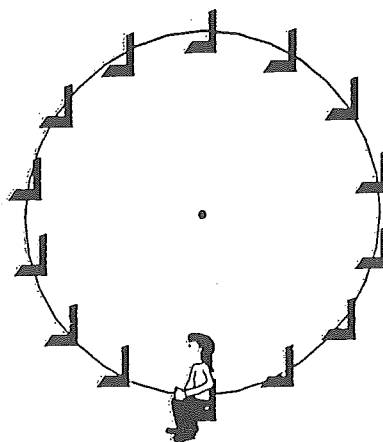
2. A 1.5 kg object is in uniform circular motion with a period of 3.0 s. If the radius of the path is 4.0 m, what is the centripetal force on the object?

3. An empty 12 kg seat on a swing-type ride at the fairgrounds has a kinetic energy of 480 J.



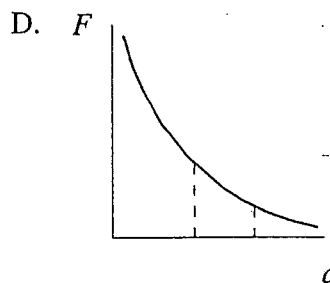
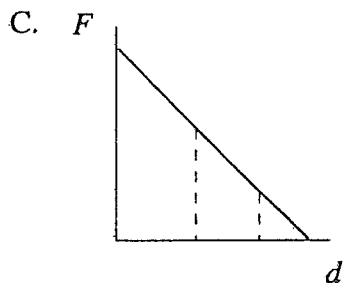
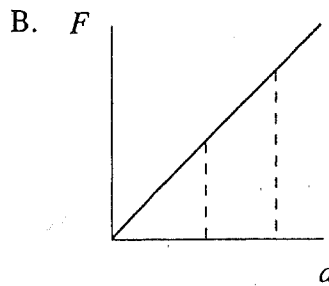
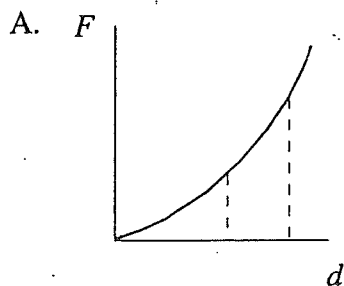
What is the centripetal force on the empty seat?

4. A 75 kg person rides a Ferris wheel which is rotating uniformly. The centripetal force acting on the person is 45 N.



What force does the seat exert on the rider at the top and at the bottom of the ride?

5. Which of the following illustrates the work required to move an object in a gravitational field?



6. A 1 500 kg satellite orbits the earth at 2 500 m/s. What is the satellite's centripetal acceleration?
- A.  $0.098 \text{ m/s}^2$
  - B.  $0.98 \text{ m/s}^2$
  - C.  $9.8 \text{ m/s}^2$
  - D.  $1.5 \times 10^2 \text{ m/s}^2$

7.

A 1 500 kg satellite travels around the earth in a stable orbit with a radius of  $1.3 \times 10^7$  m.

a) What is the speed of the satellite in this orbit?

(5 marks)

b) The satellite is then moved to a new orbit with twice the radius of the first orbit. The speed in this orbit is

☐ the same as

☐ less than

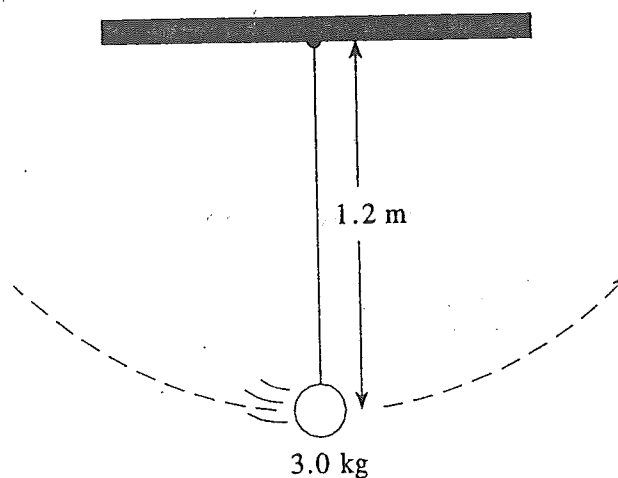
☐ more than

the speed in the first orbit. (Check one response.)

c) Using principles of physics, explain your answer to b).

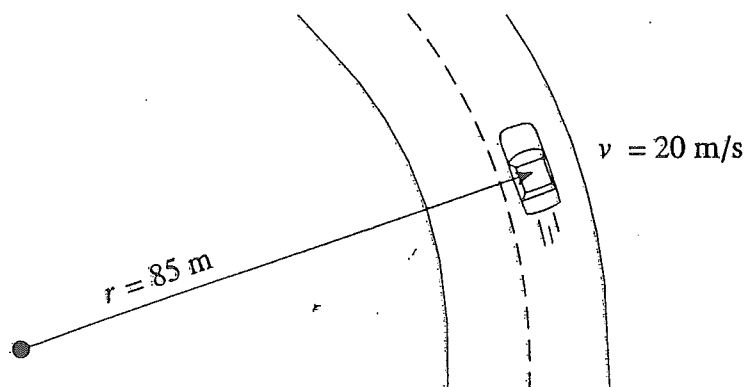
(3 marks)

8. A 1.2 m long pendulum reaches a speed of 4.0 m/s at the bottom of its swing.



What is the tension in the string at this position?

9. A 1 200 kg car rounds a flat circular section of road at 20 m/s as shown in the diagram.



The coefficient of friction between the car tires and the road surface is 0.65. What minimum friction force is required for the car to follow this curve?

10.

A satellite's orbit is maintained by a

- A. normal force.
- B. frictional force.
- C. centrifugal force.
- D. gravitational force.

11. What is the gravitational field strength on the surface of a planetoid with a mass of  $7.4 \times 10^{22}$  kg and a radius of  $1.7 \times 10^6$  m.

12. A 1 500 kg satellite is in a stable orbit at an altitude of  $4.0 \times 10^5$  m above Earth's surface. What is the satellite's total energy in this orbit?

13. The moon Titan orbits the planet Saturn with a period of  $1.4 \times 10^6$  s. The average radius of this orbit is  $1.2 \times 10^9$  m.

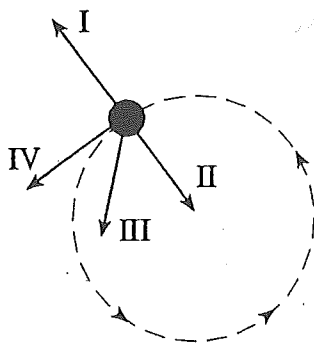
a) What is Titan's centripetal acceleration?

(2 marks)

b) Calculate Saturn's mass.

(5 marks)

14. A satellite moves in a circular path at a constant speed. Which vector in the diagram below best represents the satellite's acceleration?



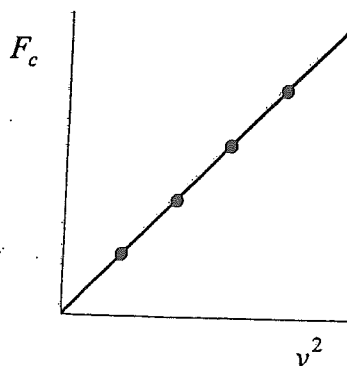
15. A 2.5 kg object moves at a constant speed of 8.0 m/s in a 5.0 m radius circle. What is the object's acceleration?

16. What is the magnitude of Earth's centripetal acceleration as it orbits the Sun?

17.

A satellite orbits Earth at a velocity of  $3.1 \times 10^3$  m/s. What is the radius of this orbit?

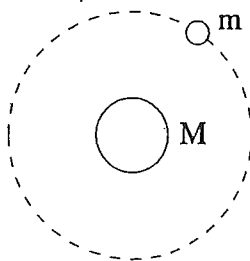
18. A student plots a graph of centripetal force  $F_c$  versus the square of velocity  $v^2$  for an object in uniform circular motion.



What is the slope of this graph?

- A.  $\frac{m}{r}$
- B.  $\frac{r}{m}$
- C.  $\frac{4\pi^2 r}{T^2}$
- D.  $\frac{T^2}{4\pi^2 r}$

19. Which of the following is a correct expression for the total energy of the orbiting satellite shown below?



- A.  $E_T = -G \frac{Mm}{r}$
- B.  $E_T = G \frac{Mm}{r}$
- C.  $E_T = \frac{1}{2}mv^2 + mgr$
- D.  $E_T = \frac{1}{2}mv^2 + \left(-G \frac{Mm}{r}\right)$

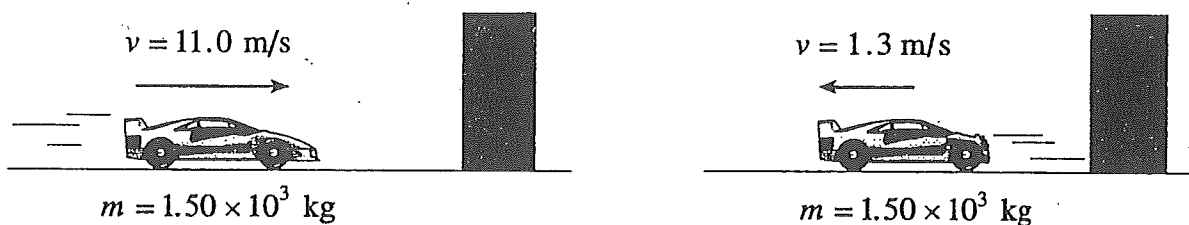
20. An electron orbits the nucleus of an atom with velocity  $v$ . If this electron were to orbit the same nucleus with twice the previous orbital radius, its orbital velocity would now be

A.  $\frac{v}{2}$   
 B.  $\frac{v}{\sqrt{2}}$   
 C.  $v$   
 D.  $2v$

21. A 1 500 kg satellite travels in a stable circular orbit around the earth. The orbital radius is  $4.2 \times 10^7$  m. What is the satellite's kinetic energy? (7 marks)

22. What is the minimum work done when a 65 kg student climbs an 8.0 m-high stairway in 12 s?

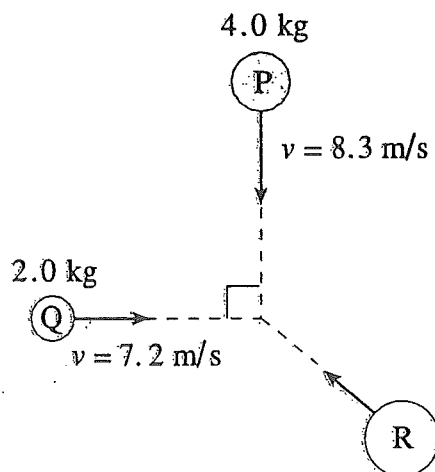
23. A  $1.50 \times 10^3$  kg car travelling at 11.0 m/s collides with a wall as shown.



The car rebounds off the wall with a speed of 1.3 m/s. If the collision lasts for 1.7 s, what force does the wall apply to the car during the collision?

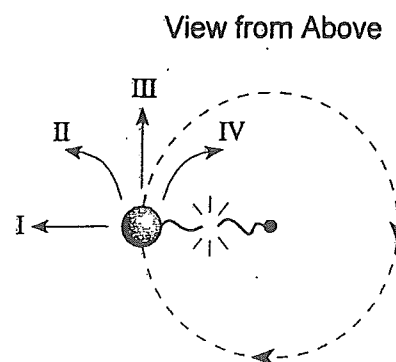
24. A 1 500 kg car travelling at 25 m/s collides with a 2 500 kg van stopped at a traffic light. As a result of the collision the two vehicles become entangled. With what initial speed will the entangled mass move off, and is the collision elastic or inelastic?

25. Three objects travel as shown.



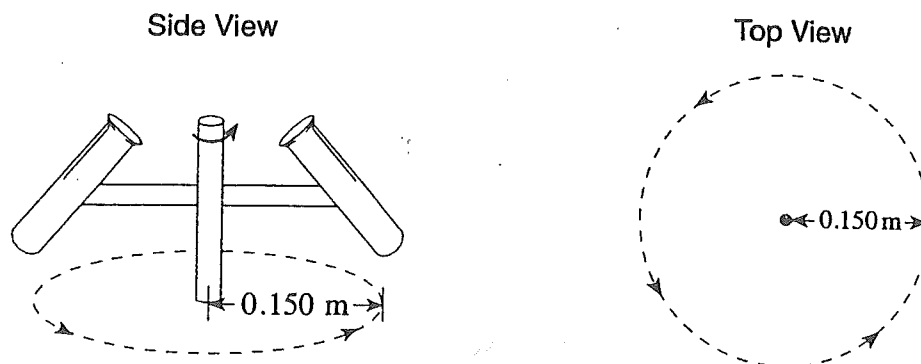
What is the magnitude of the momentum of object R so that the combined masses remain stationary after they collide?

26. A ball attached to a string is swung in a horizontal circle.



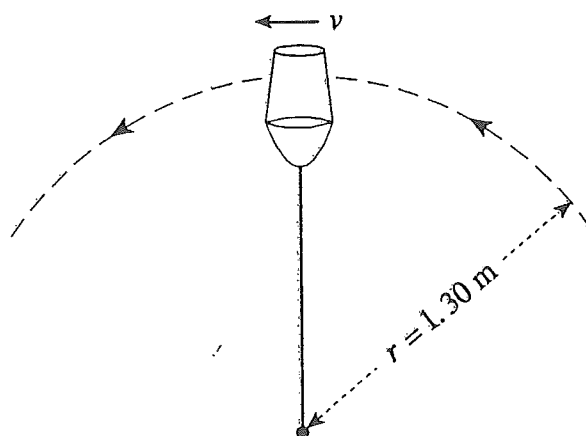
Which path will the ball follow at the instant the string breaks?

27. A test tube rotates in a centrifuge with a period of  $1.20 \times 10^{-3}$  s. The bottom of the test tube travels in a circular path of radius 0.150 m.



What is the centripetal force exerted on a  $2.00 \times 10^{-8}$  kg amoeba at the bottom of the tube?

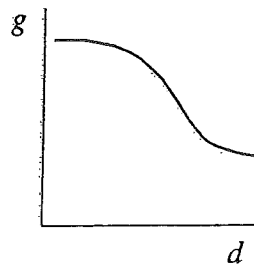
28. A physics student swings a 5.0 kg pail of water in a vertical circle of radius 1.3 m.



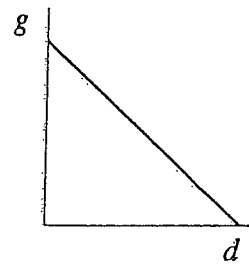
What is the minimum speed,  $v$ , at the top of the circle if the water is not to spill from the pail?

29. Which of the following is a correct graph for gravitational field strength,  $g$ , versus the distance,  $d$ ?

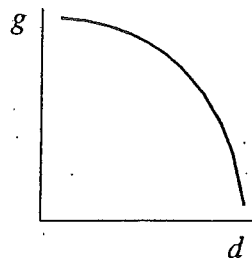
A.



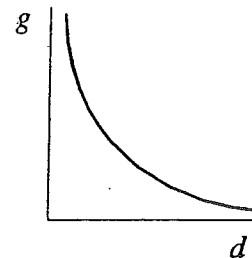
B.



C.



D.



30. Sputnik 1, Earth's 1st. artificial satellite, had an orbital period of 5760 s. What was the average orbital radius of Sputnik's orbit?

31. A 620 kg satellite orbits the earth where the acceleration due to gravity is  $0.233 \text{ m/s}^2$ . What is the kinetic energy of this orbiting satellite?

32. A 5.0 kg rock dropped near the surface of Mars reaches a speed of 15 m/s in 4.0 s.

a) What is the acceleration due to gravity near the surface of Mars? (2 marks)

b) Mars has an average radius of  $3.38 \times 10^6 \text{ m}$ . What is the mass of Mars? (5 marks)

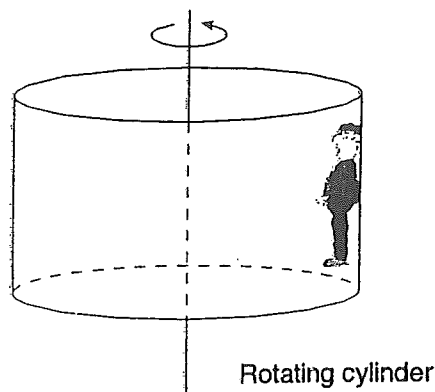
33. A 0.500 kg ball is swung in a horizontal circle of radius 1.20 m with a period of 1.25 s. What is the centripetal force on the ball?

34. A rock drops from a very high altitude towards the surface of the moon. Which of the following is correct about the changes that occur in the rock's mass and weight?

	MASS	WEIGHT
A.	decreases	decreases
B.	decreases	increases
C.	remains constant	decreases
D.	remains constant	increases



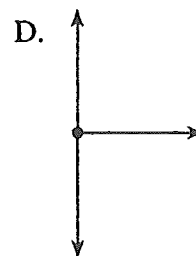
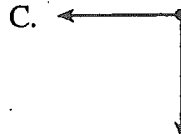
35. In a popular amusement park ride, a large cylinder is set in rotation. The floor then drops away leaving the riders suspended against the wall in a vertical position as shown.



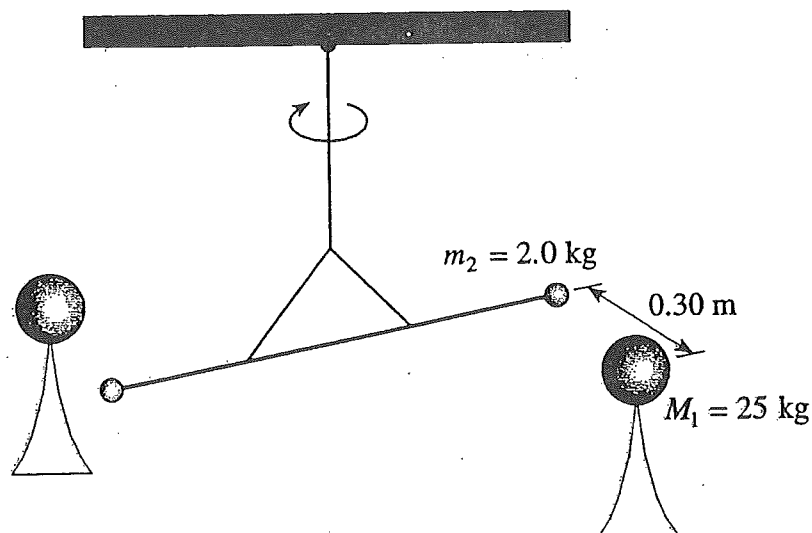
Which of the following is the correct free-body diagram for the person at the position shown?



B.



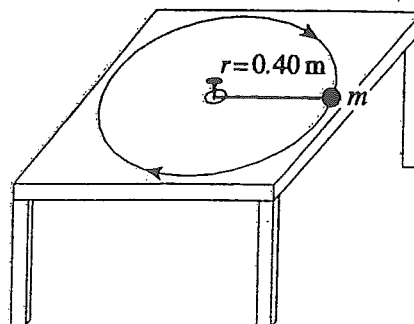
36. Cavendish's historic experiment is set up as shown to determine the force between two identical sets of masses. What would be the net force of attraction between **one** set of masses?



37. A 1 570 kg satellite orbits a planet in a circle of radius  $5.94 \times 10^6$  m. Relative to zero at infinity the gravitational potential energy of this satellite is  $-9.32 \times 10^{11}$  J. What is the mass of the planet?

38.

An object is attached to a string that can withstand a maximum tension force of 6.3 N. The object travels in a circular path of radius 0.40 m with a period of 2.1 s.



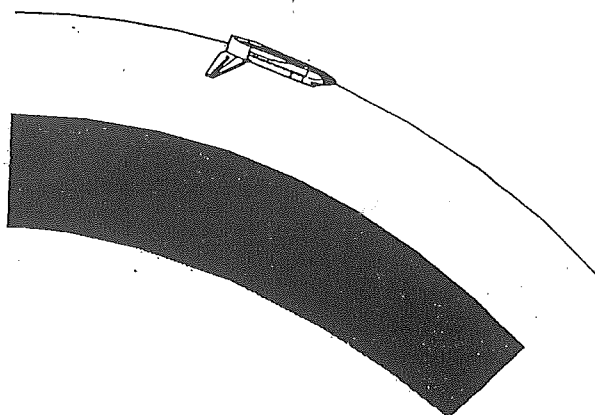
What is the maximum mass of the object?

39.

A 65 kg pilot in a stunt plane performs a vertical loop with a 700 m radius. The plane reaches a speed of 210 m/s at the bottom of the loop. What is the upward force on the pilot at the bottom of the loop?

40.

A space shuttle is placed in a circular orbit at an altitude of  $3.00 \times 10^5 \text{ m}$  above Earth's surface.



a) What is the shuttle's orbital speed?

(3 marks)

b) The space shuttle is then moved to a higher orbit in order to capture a satellite.

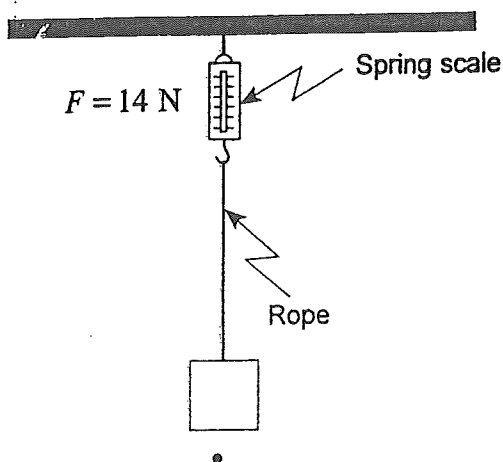
The shuttle's speed in this new higher orbit will have to be (check one)

- ☐ greater than
  - ☐ less than
  - ☐ the same as
- the speed in the lower orbit

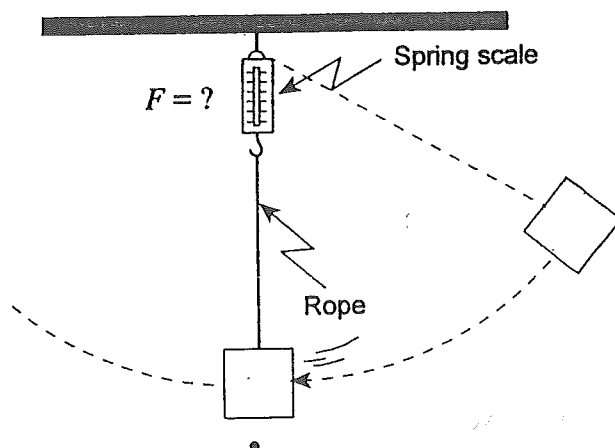
c) Using principles of physics, explain your answer to b).

(3 marks)

41. A mass is suspended by a string attached to a spring scale that initially reads 14 N as shown in Diagram 1.

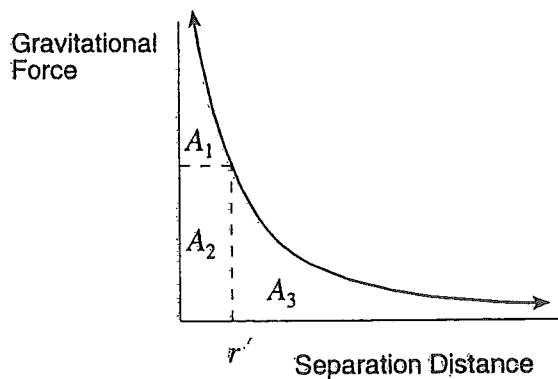


The mass is pulled to the side and then released as shown in Diagram 2.



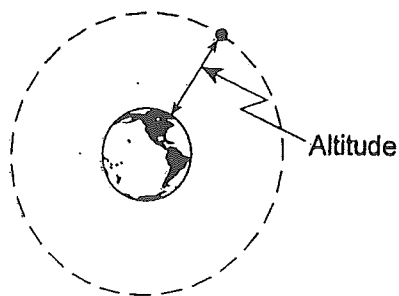
As the mass passes point Q, how will the reading on the spring scale compare to the previous value of 14 N? Using principles of physics, explain your answer. (4 marks)

42. Which of the indicated areas of the graph represent the work needed to send an object from separation distance  $r$  to infinity?



- A.  $A_1 + A_2$
- B.  $A_2$
- C.  $A_2 + A_3$
- D.  $A_3$

43. A satellite experiences a gravitational force of 228 N at an altitude of  $4.0 \times 10^7$  m above Earth.



What is the mass of this satellite?

44. A  $4.00 \times 10^3$  kg object is lifted from the earth's surface to an altitude of  $3.2 \times 10^5$  m. How much work does this require? (7 marks)

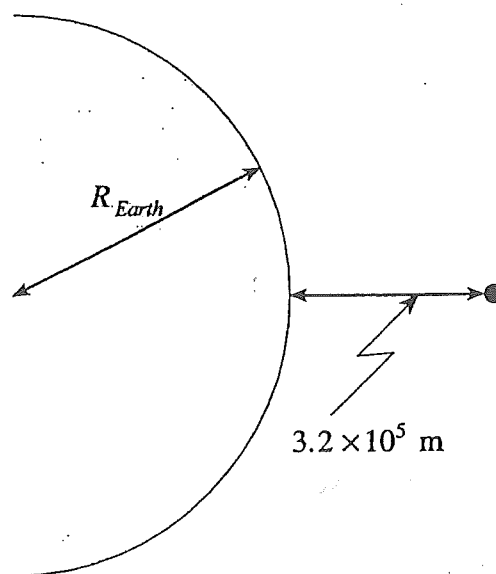
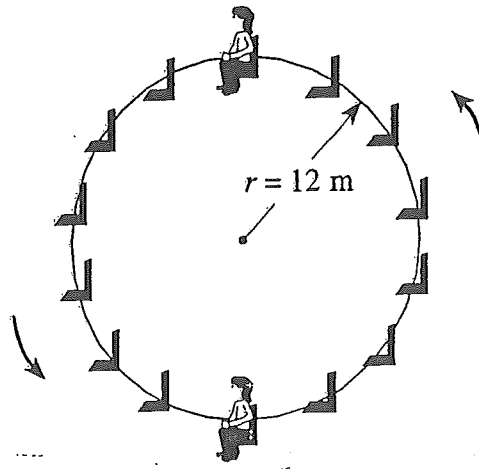


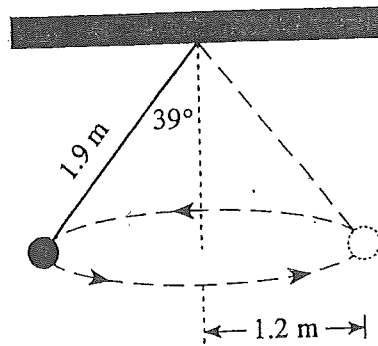
diagram not to scale

45. An object travels along a circular path with a constant speed  $v$  when a force  $F$  acts on it. How large a force is required for this object to travel along the same path at twice the speed ( $2v$ )?
- A.  $\frac{1}{2} F$
  - B.  $F$
  - C.  $2F$
  - D.  $4F$
46. The equation  $E_p = mgh$ , in which  $g$  is  $9.8 \text{ m/s}^2$ , can not be used for calculating the gravitational potential energy of an orbiting Earth satellite because
- A. the Earth is rotating.
  - B. of the influence of other astronomical bodies.
  - C. the Earth's gravity disappears above the atmosphere.
  - D. the Earth's gravitational field strength varies with distance.

47. The diagram shows a 52 kg child riding on a Ferris wheel of radius 12 m and period 18 s. What force (normal force) does the seat exert on the child at the top and bottom of the ride?



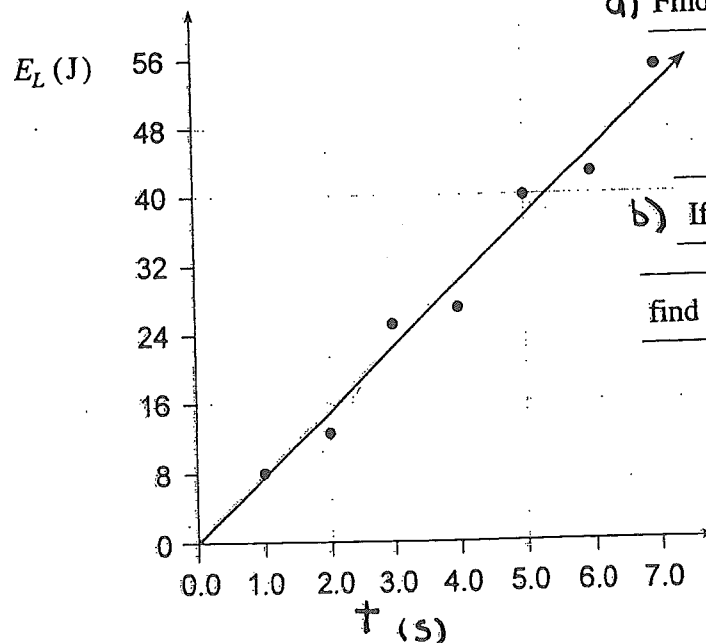
48. The diagram shows an object of mass 3.0 kg travelling in a circular path of radius 1.2 m while suspended by a piece of string of length 1.9 m. What is the centripetal force on the mass?



49. a) Mars has a mass of  $6.37 \times 10^{23}$  kg and a radius of  $3.43 \times 10^6$  m. What is the gravitational field strength on its surface? (4 marks)
- b) What thrust force must the rocket engine of a Martian lander exert if the 87.5 kg spacecraft is to accelerate upwards at  $1.20 \text{ m/s}^2$  as it leaves the surface of Mars? (3 marks)

50. The graph shows the light energy  $E_L$  emitted by a bulb versus time  $t$ .

(previous unit)



- a) Find the power output of the bulb. (2 marks)
- b) If this bulb is 20% efficient, find the power delivered to the bulb. (3 marks)

## Circular Motion and Universal Gravitation

1. D

2. 26 N

3.  $1.4 \times 10^2 \text{ N}$

4.  $F_{\text{top}} = 6.9 \times 10^2 \text{ N}$ ,  $F_{\text{bottom}} = 7.8 \times 10^2 \text{ N}$

5. D

6.  $0.098 \text{ m/s}^2$

7. a.  $5.5 \times 10^3 \text{ m/s}$  b) less than

C. The satellite's speed in a stable orbit is inversely proportional to the square root of orbit radius:  $v \propto \frac{1}{\sqrt{r}}$ . Therefore, in an orbit with twice the radius of the first, the satellite speed will be lower.

8. 69 N

9.  $5.6 \times 10^3 \text{ N}$

10. D

11. 1.7 N/kg

12.  $4.4 \times 10^{10} \text{ J}$

13.  $2.4 \times 10^{-2} \text{ m/s}^2$

b.  $F_{\text{net}} = ma_c$

$$\frac{Gm_s m_T}{r^2} = \frac{m_T 4\pi^2 r}{T^2} \quad \text{OR} \quad \frac{GmM}{r^2} = ma_c \quad \text{OR} \quad \text{Kepler's: } \frac{r^3}{T^2} = \frac{GM}{4\pi^2} \quad \leftarrow 3 \text{ marks}$$

$$m_s = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4\pi^2 (1.2 \times 10^9 \text{ m})^3}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 (1.4 \times 10^6 \text{ s})^2} \quad \leftarrow 1 \text{ mark}$$

$$= 5.2 \times 10^{26} \text{ kg} \quad \leftarrow 1 \text{ mark}$$

14. II

15.  $13 \text{ m/s}^2$

16.  $5.9 \times 10^{-3} \text{ m/s}^2$

17.  $4.2 \times 10^7 \text{ m}$

18. A

19. D

20. B

21.

\* note how the marks are awarded \*

$$F_{net} = ma_c$$

$$\frac{Gm_E m_s}{r^2} = \frac{m_s \cdot v^2}{r}$$

← 2 marks

$$\therefore v = \left( \frac{Gm_E}{r} \right)^{\frac{1}{2}}$$

← 1 mark

$$= \left( \frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \cdot 5.98 \times 10^{24} \text{ kg}}{4.2 \times 10^7 \text{ m}} \right)^{\frac{1}{2}}$$

← 1 mark

$$= 3.1 \times 10^3 \text{ m/s}$$

← 1 mark

$$E_k = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \cdot 1500 \text{ kg} (3.1 \times 10^3 \text{ m/s}^2)^2$$

$$= 7.1 \times 10^9 \text{ J}$$

← 2 marks

22.  $5.1 \times 10^3 \text{ J}$

23.  $1.1 \times 10^4 \text{ N}$

24.  $9.4 \text{ m/s}$ , inelastic

25.  $36 \text{ kgm/s}$

26. III

27.  $8.22 \times 10^{-2} \text{ N}$

28.  $3.6 \text{ m/s}$

29. D

30.  $6.95 \times 10^6 \text{ m}$

31.  $2.99 \times 10^9 \text{ J}$

32. a.  $3.8 \text{ m/s}^2$  b.

again, note  
how formula  
manipulation is  
required in  
DETAIL for  
full marks!

$$F_g = \frac{GMm}{R^2}$$

← 1 mark

$$mg = \frac{GMm}{R^2}$$

← 1 mark

$$\therefore M = \frac{gR^2}{G}$$

← 1 mark

$$= \frac{3.8 \times (3.38 \times 10^6)^2}{6.67 \times 10^{-11}}$$

← 1 mark

$$= 6.5 \times 10^{23} \text{ kg}$$

← 1 mark

33. 15.2N

34. D

35. B

36.  $3.7 \times 10^{-8} \text{ N}$

37.  $5.29 \times 10^{25} \text{ kg}$

38. 1.8 kg

39.  $4.7 \times 10^3 \text{ N}$

40.

↑ a)

$$F_c = F_g$$

(1 mark)

(2 marks)

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.68 \times 10^6}}$$

← 1 mark

$$v = 7.73 \times 10^3 \text{ m/s}$$

← 1 mark

$$m \frac{v^2}{R} = \frac{GMm}{R^2}$$

$$v^2 = \frac{GM}{R}$$

$$v = \sqrt{\frac{GM}{R}}$$

40 b. less than

c.

As the space shuttle moves further away from the earth's centre the force of gravity acting on the shuttle decreases. Since the centripetal force is provided by the force of gravity, it must decrease as well. ← 2 marks

The smaller centripetal force generates a smaller centripetal acceleration ← 1 mark which in turn requires a smaller orbital velocity.

41.

The reading will be greater than 14 N.  $\left( \text{by } \frac{mv^2}{r} \right)$  ← 1 mark

Initially, the net force is zero, so the spring scale reads the weight of the mass. When moving, there is a net (centripetal) force provided by the spring scale (tension in the rope) which exceeds the weight (force of gravity) of the mass so that the mass goes in a vertical circle. ← 3 marks

42.

D

43.

$$1.2 \times 10^3 \text{ kg}$$

44.

$$R_1 = 6.38 \times 10^6 \text{ m}$$

$$R_2 = 6.38 \times 10^6 \text{ m} + 3.2 \times 10^5 \text{ m}$$

$$= 6.70 \times 10^6 \text{ m}$$

← 1 mark

$$W = \Delta E$$

← 1 mark

$$\Delta E_p = E_{p2} - E_{p1}$$

$$= \frac{-GMm}{R_2} - \left( \frac{-GMm}{R_1} \right)$$

← 1 mark

$$= \frac{-6.67 \times 10^{-11} \cdot 5.98 \times 10^{24} \cdot 4.00 \times 10^3}{6.70 \times 10^6} - \frac{-6.67 \times 10^{-11} \cdot 5.98 \times 10^{24} \cdot 4.00 \times 10^3}{6.38 \times 10^6}$$

← 2 marks

$$= -2.38 \times 10^{11} \text{ J} - (-2.50 \times 10^{11} \text{ J})$$

← 1 mark

$$\Delta E_p = 1.2 \times 10^{10} \text{ J}$$

← 1 mark

45. D

46. D

$$47. F_{\text{Top}} = 4.3 \times 10^2 \text{ N}, F_{\text{Bottom}} = 5.9 \times 10^2 \text{ N}$$

$$48. 24 \text{ N}$$

$$49. 3.61 \text{ N/kg}$$

$$(50. a) \approx 7.6 \text{ W} \quad b) \approx 38 \text{ W}$$

review of efficiency formula)

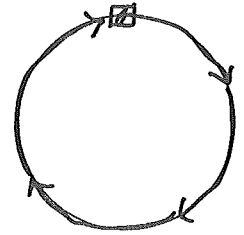


# Circular Motion Key

## Physics 12 – Uniform Circular Motion

Uniform circular motion is the motion of an object moving at a **constant speed in a circle** with a fixed radius. Notice that it is the **speed that is constant, not the velocity** (as the direction is constantly changing).

Speed is defined as:  $v = \frac{d}{t}$



**Time:** When dealing with circular motion, we use the period of motion (T) as the time it takes to complete one rotation.

**Distance:** When traveling in a circular path, the distance travelled is equal to the circumference of that circular path.

$$C = 2\pi r$$

The speed of an object moving with uniform circular motion is therefore given by:

$$v = \frac{2\pi r}{T} \quad \begin{matrix} \text{(distance)} \\ \text{(time = period)} \end{matrix}$$

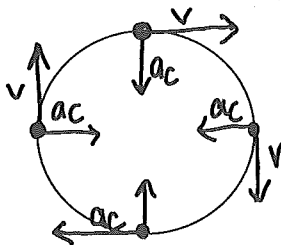
This formula allows us to determine the speed of the mass. Now, how do we determine the acceleration? Is the mass accelerating? *yes*

While the object is traveling at a constant speed... *the object is constantly changing direction.*

The definition of acceleration is:

$$\vec{a} = \frac{\Delta \vec{v}}{t}$$

*= continual change in velocity*



*\*the direction of the velocity at any time is tangent to the circle.*

Even though the object is traveling at a constant speed, anything traveling in a circular path has acceleration due to the continual change in direction

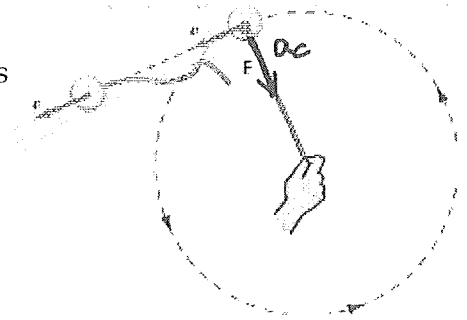
This acceleration is found using:

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$\begin{aligned} \frac{v^2}{r} &= \frac{(2\pi r)^2}{T^2} \\ &= \frac{4\pi^2 r^2}{T^2} \cdot \frac{1}{r} \end{aligned}$$

In uniform circular motion, the velocity and the acceleration are continually changing direction, and are perpendicular to each other at each moment.

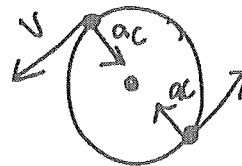
**Example:** The force on the stone swung on the end of a string is the force exerted inwardly by the string. When released, the stone will fly off tangentially, in the direction of the velocity it has at the moment.



## Centripetal Acceleration

As seen in the diagram, the force that is exerted on the object is directed toward the center of the circle. This means that the acceleration must also be directed toward the center of the circle.

= centripetal means  
"center-seeking"



The velocity is always directed along the tangent of the circle = perpendicular to the acceleration.

According to Newton's Laws of Motion, if an object is accelerated toward the center, then there must also be a...  $\Sigma F$  in the same direction

This center-seeking force is called the centripetal force.

Centripetal Force =  $F_c$

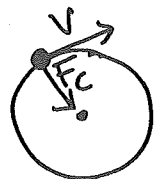
Centripetal force is a name given to any force that causes an object to move in a circle.

Examples include: Tension through a string as in the rock example, friction as when a car rounds a curve on a highway, gravity as when the moon circles the Earth, or electrical as when an electron orbits a proton.

In circular motion, the force vector is always perpendicular to the velocity vector.

Centripetal Force:

$$F_c = ma_c = \frac{mv^2}{r} = m \frac{4\pi^2 r}{T^2}$$



\* same  
direction as  
ac

In uniform circular motion, because the acceleration is uniform, the force that is causing the acceleration must also be uniform.

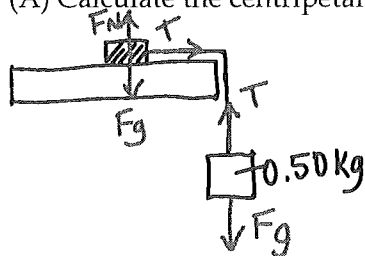
## Centrifugal Force?

While centripetal means "center-seeking", centrifugal means "center-fleeing". Centrifugal force is really just an apparent force – it does not actually exist. It is the apparent force that causes an object to move along a straight line. However, Newton's First Law tells us you do not need a force to keep an object moving in a straight, only to change its direction.

What we sometimes call a centrifugal force is really the object's inertia.

**Example:** A 0.50 kg mass hangs on a frictionless table and is attached to hanging weight. The 0.50 kg mass is whirled in a circle of radius 0.20 m at 2.3 m/s.

(A) Calculate the centripetal force acting on the mass. (B) Calculate the mass of the hanging weight.



$$F_c = \frac{mv^2}{r}$$

$$= \frac{(0.50)(2.3)^2}{0.20}$$

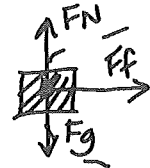
$$F_c = 13.2 \text{ N}$$

$$F_g = T = F_c$$

$$13.2 = m(9.8)$$

$$m = 1.35 \text{ kg}$$

**Example:** A car traveling at 14 m/s goes around an unbanked <sup>flat</sup> curve in the road that has a radius of 96 m. What is its centripetal acceleration?



$$a_c = \frac{v^2}{r} = \frac{14^2}{96} = 2.04 \text{ m/s}^2$$

What is the minimum coefficient of friction between the road and the car's tires?

friction acts as the  $F_c$  holding it in circular path

$$F_c = F_f = ma_c$$

$$\uparrow$$

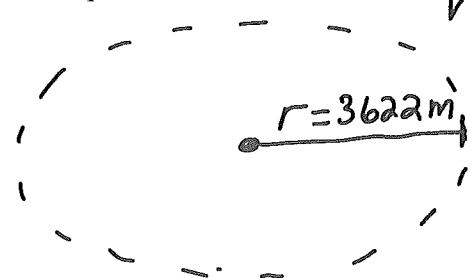
$$= \mu F_N \text{ or } \mu \cdot m \cdot g$$

$$\text{so... } \mu \cdot m \cdot g = m \cdot a_c$$

$$\mu(9.8) = 2.04$$

$$\mu = 0.21$$

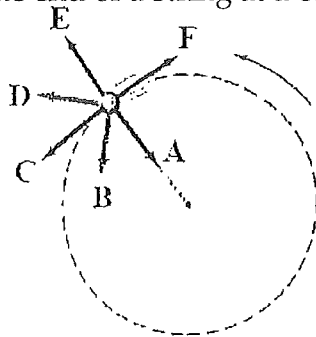
**Example:** A plane makes a complete circle with a radius of 3622 m in 2.10 minutes. What is the speed of the plane?



$$v = \frac{2\pi r}{T} = \frac{2(3.14)(3622)}{(2.10)(60)} = 181 \text{ m/s}$$

## Uniform Circular Motion Problems

1. A ball is swung on the end of a string at a constant speed in a horizontal circle.



- a) Which path does the ball follow at the moment the string breaks?

C → when the string breaks, the ball flies off in the direction of the velocity it has at that moment.

- b) Which path represents the direction of the velocity of the ball?

C → tangent to the circular path

- c) Which path represents the direction of the acceleration of the ball?

A → in uniform circular motion, the acceleration of the object is directed toward the center of the circle = centripetal

- d) Which path represents the direction of the force exerted on the ball by the string?

A → toward center = centripetal

2. A car is about to go around a curve. Using principles of physics, explain the following questions.

- a) Can the car go around the curve with a constant speed?

yes → speed is scalar & may remain constant

- b) Can the car go around the curve with zero acceleration?

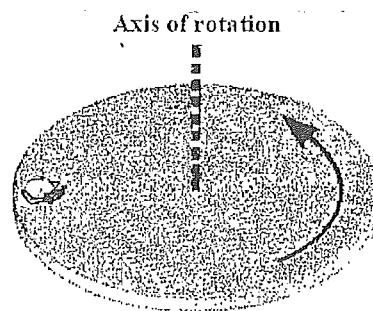
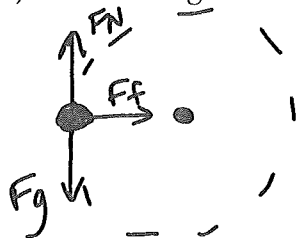
no → if direction changes = velocity changes = accel.

- c) Can the car go around the curve with constant acceleration?

no → direction change = changing accel

3. A small stone rests on the edge of an antique vinyl record player which is whirling around at 33 rotations per minute.

a) Draw a diagram showing the forces acting on the stone.



b) What provides the centripetal force to keep the stone on the record?

static  $F_f$  between the record and the stone.

4. Calculate the centripetal force acting on a 925 kg car as it rounds an unbanked curve with a radius of 75 m at a speed of 22 m/s. ( $6.0 \times 10^3$  N)

$$F_c = \frac{mv^2}{r} = \frac{(925)(22)^2}{75} = \underline{6.0 \times 10^3 \text{ N}}$$

5. A small plane makes a complete circle with a radius of 3282 m in 2.0 minutes. What is the centripetal acceleration of the plane? ( $9.0 \text{ m/s}^2$ )

$$a_c = \frac{v^2}{r} \quad v = \frac{2\pi r}{T} = \frac{2\pi(3282)}{2.0(60)} = 172 \text{ m/s}$$

$$= \frac{(172)^2}{3282} \quad a_c = \underline{9.0 \text{ m/s}^2}$$

6. A car with a mass of 822 kg rounds an unbanked curve in the road at a speed of 28.0 m/s. If the radius of the curve is 105 m, what is the average centripetal force exerted on the car?

( $6.14 \times 10^3$  N)  $F_c = \frac{mv^2}{r} = \frac{(822)(28)^2}{105} = \underline{6.14 \times 10^3 \text{ N}}$

7. An amusement park ride has a radius of 2.8 m. If the time of one revolution of a rider is 0.98 s, what is the speed of the rider?

$$v = \frac{2\pi r}{T} \left( \frac{\text{distance}}{\text{time}} \right) = \frac{2\pi(2.8)}{0.98} = \underline{18 \text{ m/s}}$$

8. An electron ( $m = 9.11 \times 10^{-31}$  kg) moves in a circle whose radius is  $2.00 \times 10^{-2}$  m. If the force acting on the electron is  $4.60 \times 10^{-14}$  N, what is the speed of the electron? ( $3.18 \times 10^7$  m/s)

$$F_c = \frac{mv^2}{r} \quad 4.60 \times 10^{-14} = \frac{(9.11 \times 10^{-31})v^2}{2.00 \times 10^{-2}} \quad v = 3.18 \times 10^7 \text{ m/s}$$

9. A 925 kg car rounds an unbanked curve at a speed of 25 m/s. If the radius of the curve is 72 m, what is the minimum coefficient of friction between the car and the road required so that the car does not skid? (0.89)

$$F_c = F_f \quad F_c = \frac{mv^2}{r} = \frac{(925)(25)^2}{72} = 8030 \text{ N}$$

$$8030 = \mu(925 \cdot 9.8)$$

$$\mu = 0.89$$

10. A  $2.7 \times 10^3$  kg satellite orbits Earth at a distance of  $1.8 \times 10^7$  m from Earth's center at a speed of  $4.7 \times 10^3$  m/s. What force does Earth exert on the satellite?

$$F_c = \frac{mv^2}{r} = \frac{(2700)(4700)^2}{1.8 \times 10^7} = 3.3 \times 10^3 \text{ N}$$

11. An athlete whirls a 3.7 kg shot-put in a horizontal circle with a radius of 0.90 m. If the period of rotation is 0.30 s,

a) What is the speed of the shot-put when released? (18.8 m/s)

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.90)}{0.30} = 18.8 \text{ m/s}$$

b) What is the centripetal force acting on the shot-put while it is rotated? ( $1.45 \times 10^3$  N)

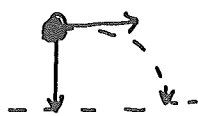
$$F_c = \frac{mv^2}{r} = \frac{(3.7)(18.8)^2}{0.90} = 1.45 \times 10^3 \text{ N}$$

c) How far would the shot-put travel horizontally if it is released 1.2 m above the ground?

(9.3m)

$$v = \frac{d}{t}$$

$$18.8 = \frac{d}{0.495}$$



$$d = 9.3 \text{ m}$$

$$t: -1.2 = 0 + \frac{1}{2}(-9.8)t^2 \quad t = 0.495 \text{ s}$$

\* projectile motion

12. Calculate the speed and acceleration of a point on the circumference of a 33.3 phonograph record. The diameter of the record is 30.0 cm. (It makes 33.3 revolutions per minute). (0.52

m/s, 1.83 m/s<sup>2</sup>)

$$v = \frac{2\pi r}{T} = \frac{2\pi (0.15)}{1.80} \quad v = 0.52 \text{ m/s} \quad T = \frac{60}{33.3} = 1.80 \text{ s}$$

$$a_c = \frac{v^2}{r} = \frac{(0.52)^2}{0.15} \quad a_c = 1.83 \text{ m/s}^2$$

13. A string requires a 135 N force in order to break it. A 2.00 kg mass is tied to this string and whirled it in a horizontal circle with a radius of 1.10 m. What is the maximum speed that the mass can be whirled without breaking the string? (8.62 m/s)

$$F_c = \frac{mv^2}{r} \quad 135 = \frac{2.00v^2}{1.10} \quad v = 8.62 \text{ m/s}$$

14. A 932 kg car is traveling around an unbanked curve that has a radius of 82 m. What is the maximum speed that this car can round this curve without skidding?

a) If the coefficient of friction is 0.95? (27.6 m/s)  $a) F_c = F_f = (0.95)(932 \cdot 9.8) = 8677 \text{ N}$

b) If the coefficient of friction is 0.40? (17.9 m/s)

$$b) F_c = (0.40)(932 \cdot 9.8) = 3653 \text{ N} \quad F_c = \frac{mv^2}{r} = \frac{(932)(v^2)}{82} = 8677$$

$$F_c = \frac{mv^2}{r} = \frac{(932)v^2}{82} = 3653 \quad v = 27.6 \text{ m/s}$$

$$v = 17.9 \text{ m/s}$$



## Physics 12 – Vertical Circular Motion

When the motion of an object is in a vertical circular path, the centripetal force may be different at varying points of the motion.

Why do the forces vary from point to point?

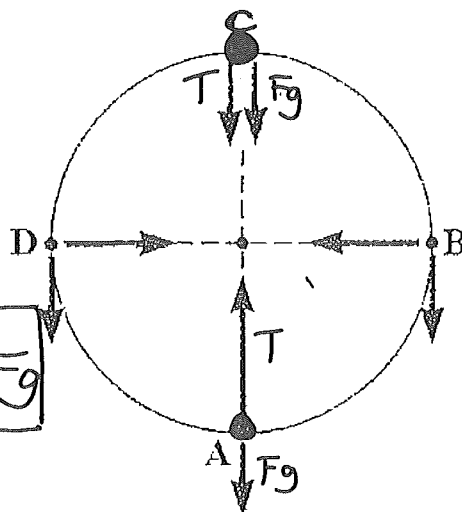
Top Position (C):



\* Forces are in the same direction

so...

$$-F_c = -T + (-F_g) \text{ or } \boxed{F_c = T + F_g}$$



Bottom Position (A):

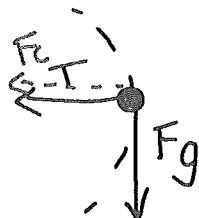
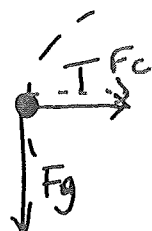


\* Forces are in the opposite direction

so...  $F_c = T + (-F_g)$  or

$$\boxed{F_c = T - F_g}$$

Side Positions (B and D):



→ since  $F_g$  is tangent to the circle in both cases, it has no component toward the center. Therefore it does not contribute to the centripetal force in these positions.

$$\boxed{F_c = T}$$

**Example:** A 1.7 kg object is swung from the end of a 0.60 m string in a vertical circle. If the time of one revolution is 1.1 s, what is the tension in the string? Assume uniform speed.

a) When it is at the top?

$$F_c = T + F_g$$

$$33.3 = T + (1.7 \cdot 9.8)$$

$$\underline{T = 17 \text{ N}}$$

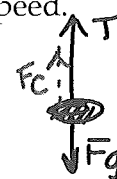


b) When it is at the bottom?

$$F_c = T - F_g$$

$$33.3 = T - (1.7 \cdot 9.8)$$

$$\underline{T = 50 \text{ N}}$$



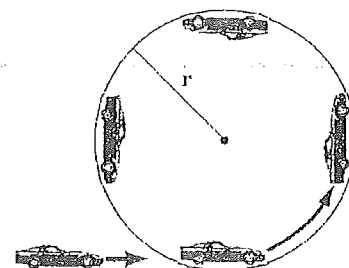
Need to find  $F_c$ : 
$$F_c = \frac{m 4 \pi^2 r}{T^2} = \frac{(1.7)(4)\pi^2(0.60)}{1.1^2} \quad F_c = 33.3 \text{ N}$$

When determining the minimum speed required so that the object does not leave the circle (very useful for a roller coaster design!), the tension or normal force (if there are two surfaces) are equal to zero.

$$\text{so} \rightarrow F_C = \cancel{T} + F_g \quad \left| \quad \frac{mv^2}{r} = \cancel{mg} \right.$$

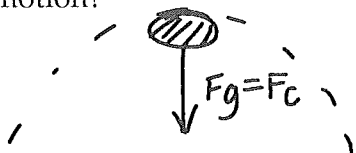
$$F_C = F_g$$

We say that the mass at the peak of the arc is *weightless*, because the net force working on it is only gravity. This is the same as an object in free fall.



$$v = \sqrt{rg}$$

**Example:** An object is swung in a vertical circle with a radius of 0.75 m. What is the minimum speed of the object at the top of the motion for the object to remain in its circular motion?



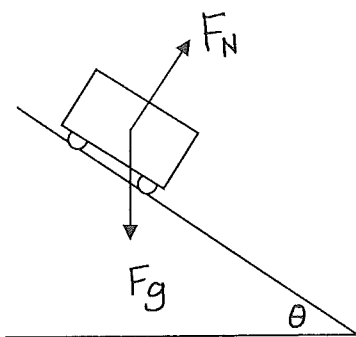
$$v = \sqrt{rg}$$

$$= \sqrt{0.75 \cdot 9.8}$$

$$v = 2.7 \text{ m/s}$$

## Banked Curves

Engineers do not always rely on friction alone to provide the centripetal force necessary for a car to round a curve safely. These **curves are usually banked** which allow cars, regardless of their mass, to round the curve safely at a certain speed even if the road is frictionless.

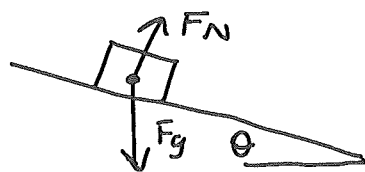


Consider a car traveling at a constant speed around a frictionless banked corner.

On the frictionless corner, the only forces acting on the car are:  $F_N$  &  $F_g$

In this case,  $F_N$  both accelerates the car inwards and matches the  $F_g$ . Therefore, the sum of  $F_N$  and  $F_g$  must equal  $F_C$ .

**Example:** Calculate the angle at which a frictionless curve must be banked if a car is to round it safely at a speed of 22 m/s if its radius is 475 m.



translate to diagram:

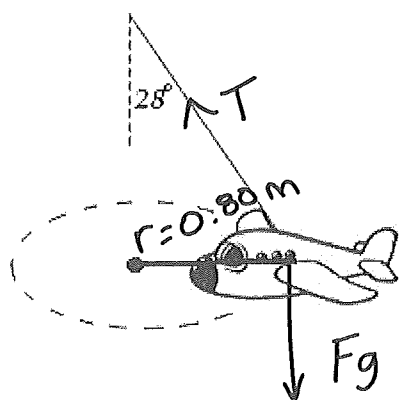
$$\tan \theta = \frac{F_C}{F_g} = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg}$$

$$\tan \theta = \frac{v^2}{rg}$$

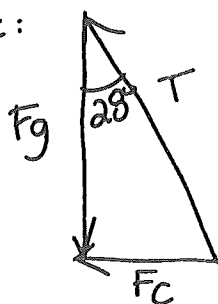
$$\tan^{-1} \left( \frac{22^2}{(475)(9.8)} \right)$$

$$= 6.0^\circ$$

**Example:** A 0.25 kg toy plane is attached to a string so that it flies in a horizontal circle with a radius of 0.80 m. The string makes a 28° angle to the vertical. What is its period of rotation?



translate:



$$\text{Find } F_c: \tan \theta = \frac{F_c}{F_g}$$

$$\tan 28 = \frac{F_c}{(0.25)(9.8)}$$

$$F_c = 1.30 \text{ N}$$

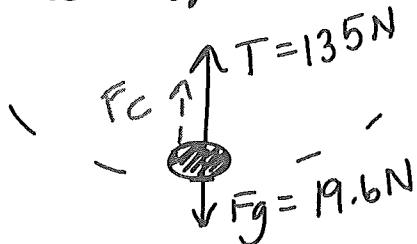
looking for period (T):  $F_c = \frac{m4\pi^2 r}{T^2}$

$$1.30 = \frac{(0.25)4\pi^2(0.80)}{T^2}$$

$$T = 2.5 \text{ s}$$

**Example:** A string requires a 135 N force in order to break it. A 2.00 kg mass is tied to this string and whirled in a vertical circle with a radius of 1.10 m. What is the maximum speed that this mass can be whirled without breaking the string?

choose top or bottom:



$$F_c = T - F_g$$

$$= 135 - 19.6 = 115.4 \text{ N}$$

$$F_c = \frac{mv^2}{r}$$

$$115.4 = \frac{(2.00)v^2}{1.10}$$

$$v = 7.97 \text{ m/s}$$

**Example:** An 826 kg car travelling a speed of 14.0 m/s goes over a hill. If the radius of this curve is 61.0 m, what is the force exerted on the car by the road at the crest of the hill?



$$F_c = F_N + F_g$$

$$F_c = \frac{mv^2}{r} = \frac{(826)(14)^2}{61.0}$$

$$F_c = 2664 \text{ N}$$

$$F_g = mg = 8095 \text{ N}$$

$$2664 + F_N + (-8095) = 0$$

$$F_N = 5.43 \times 10^3 \text{ N}$$

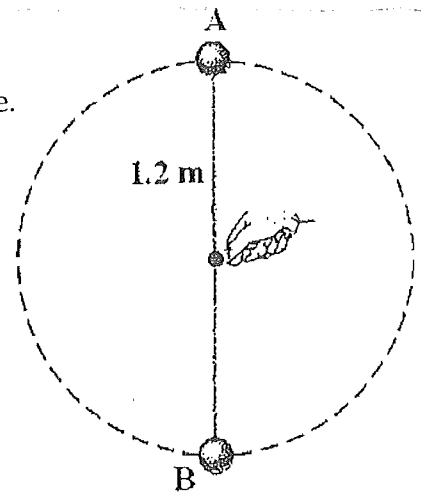
$F_c$  &  $F_g$  are both down

### Vertical Circular Motion Problems:

1. A 0.20 kg ball on the end of a string is swung in a vertical circle.

a) Find the minimum speed the ball must have at the top of the circle to remain in a circle. (3.4 m/s)

b) If the speed of the ball is 5.0 m/s at the bottom of the circle, find the tension in the string at the bottom. (6.13 N)



$$a) F_c = F_g = (0.20)(9.8) = 1.96 \text{ N}$$

$$1.96 = \frac{(0.20)v^2}{1.2} \quad v = 3.4 \text{ m/s}$$

$$b) F_c = \frac{(0.20)(5.0)^2}{1.2} = 4.17 \text{ N}$$

$$F_c = T - F_g \text{ (bottom)}$$

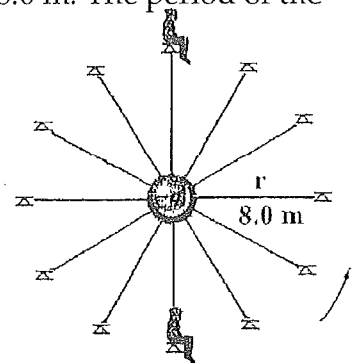
$$4.17 = T - (0.20 \cdot 9.8)$$

$$T = 6.13 \text{ N}$$

2. A 40 kg girl on a Ferris wheel moves in a vertical circle of radius 8.0 m. The period of the Ferris wheel is 12 s.

a) Draw a diagram showing the forces acting on the girl at the top and then at the bottom.

b) Find the forces exerted on the girl by the seat at the top and bottom of the circle. ( $F_g = -392 \text{ N}$ , Top:  $F_c = -87.7 \text{ N}$ ,  $F_N = +304 \text{ N}$   
Bottom:  $F_c = +87.7 \text{ N}$ ,  $F_N = +480 \text{ N}$ )



a) top:

bottom:

$$b) F_c = m \frac{4\pi^2 r}{T^2} = (40) \frac{4\pi^2 (8.0)}{(12)^2} = 87.7 \text{ N}$$

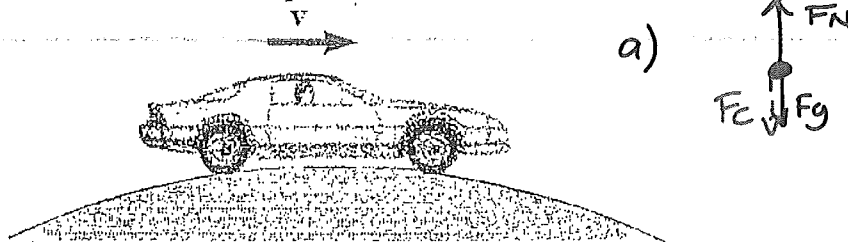
top  $\rightarrow F_c = F_N + F_g$

$$\begin{array}{l} \underline{F_g}: \\ -87.7 = F_N + (-392) \end{array} \quad \underline{F_N}: \quad F_N = 304 \text{ N}$$

bottom  $\rightarrow F_c = F_N + F_g$

$$+87.7 = F_N + (-392) \quad F_N = 480 \text{ N}$$

3. A 1200 kg vehicle travels at a constant speed on a hill of radius 48 m as shown in the diagram.



a) Draw a diagram showing the forces acting on the vehicle.

b) If the vehicle travels at the top of the hill at 14 m/s, what is the force exerted by the road on the vehicle? (6860 N)

c) What is the maximum speed the vehicle can have as it passes the top of the hill before losing contact with the road? (21.7 m/s)

$$b) \text{ Find } F_N: F_C = \frac{(1200)(14)^2}{48} = 4900 \text{ N}$$

$$F_C = F_N + F_g \quad -4900 = F_N + (-9.8 \cdot 1200) \quad F_N = \underline{6860 \text{ N}}$$

$$c) F_C = F_g = 11760 = \frac{(1200)v^2}{48} \quad v = \underline{21.7 \text{ m/s}}$$

4. A 0.25 kg sphere rolls down without friction along the loop-the-loop track with a radius of 2.2 m as shown in the diagram.

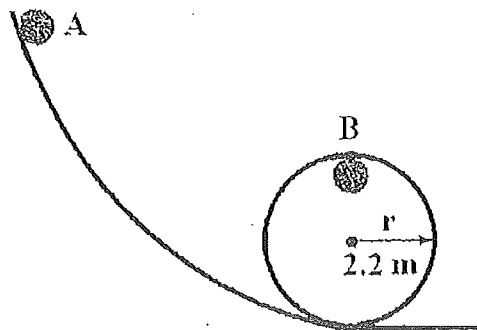
$$a) F_C = F_g = (0.25 \cdot 9.8) = 2.45 \text{ N}$$

$$2.45 = \frac{(0.25)v^2}{2.2} \quad v = 4.64 \text{ m/s}$$

$$\Delta KE = \Delta PE = \frac{1}{2}(0.25)(4.64)^2 = 2.70 \text{ J}$$

$$2.70 = (0.25)(9.8)\Delta h \quad \Delta h = 1.10 \text{ m}$$

$$h = \Delta h + 2(2.2) = 1.10 + (4.4) = \underline{5.5 \text{ m}}$$



a) From what minimum height,  $h$ , must the sphere be released so that it remains on the circular track at all times, even at the top of the loop (point B)?

b) If the release height is  $2h$ , find the speed of the sphere and the normal force exerted by the track at point B. ( $v = 11.4 \text{ m/s}$ ,  $F_N = -12.3 \text{ N}$ )

$$2(5.5) = 11.0 \text{ m} \rightarrow \Delta h = 11.0 - 4.4 = 6.6 \text{ m}$$

$$\Delta KE = \Delta PE \quad \frac{1}{2}(v_F^2 - 0^2) = -9.8(-6.6) \quad \underline{v_F = 11.4 \text{ m/s}}$$

$$F_C = \frac{(0.25)(11.4)^2}{2.2} = 14.7 \text{ N}$$

$$F_C = F_N + F_g$$

$$-14.7 = F_N + (-2.45) \quad \underline{F_N = -12.3 \text{ N}}$$

c) If the release height is  $2h$  and the radius of the loop is  $1.1 \text{ m}$ , find the speed of the sphere and the normal force exerted by the track at point B. ( $v = 13.1 \text{ m/s}$ ,  $F_N = -37 \text{ N}$ )

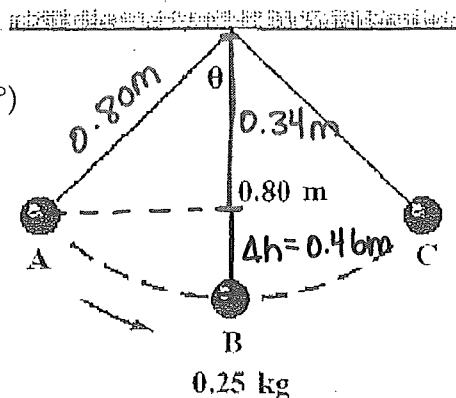
$$\Delta h = 11.0 - 2.2 = 8.8 \text{ m} \quad \frac{1}{2}(v_F^2 - 0^2) = -9.8(-8.8)$$

$$\underline{v_F = 13.1 \text{ m/s}}$$

$$F_C = \frac{(0.25)(13.1)^2}{1.1} = 39 \text{ N}$$

$$F_C = F_N + F_g \quad -39 = F_N + (-2.45) \quad \underline{F_N = -37 \text{ N}}$$

5. A  $0.25 \text{ kg}$  object is attached to a  $0.80 \text{ m}$  string. The object is pulled to point A, and then released as shown in the diagram. The object reaches a speed of  $3.0 \text{ m/s}$  at the bottom (point B) of its swing.



a) What is the angle  $\theta$  when the object is at point A? ( $65^\circ$ )

b) What is the tension in the string at point C? ( $1.04 \text{ N}$ )

c) What is the maximum tension in the string? ( $5.26 \text{ N}$ )

$$a) \Delta KE = \Delta PE$$

$$\frac{1}{2}(3.0^2 - 0^2) = -9.8 \Delta h \quad \Delta h = -0.46 \text{ m}$$

$$\cos^{-1}\left(\frac{0.46}{0.80}\right) = 65^\circ = \theta$$

$$b) v = 0 \text{ m/s} \text{ so } a_c = 0 \text{ m/s}^2 \rightarrow \Sigma F_c = ma_c = 0 \text{ N}$$

$$F_C = T + F_g$$

$$0 = T + (-0.25)(9.8)(\cos 65^\circ)$$

$$\underline{T = 1.04 \text{ N}}$$

c) max.  $a_c$  is at point B  $\rightarrow$  max.  $F_c$  will be at point B

$$F_C = \frac{(0.25)(3.0)^2}{0.80} = 2.81 \text{ N}$$

$$F_C = T + F_g \quad 2.81 = T + (-0.25 \times 9.8)$$

$$\underline{T = 5.26 \text{ N}}$$


6. You are riding your bike on a track that forms a vertical circular loop as shown in the diagram. If the diameter of the loop is 10.0 m, how fast would you have to be travelling when you reached the top of the loop so that you would not fall? (7.0 m/s)

$$F_c = F_g \quad \frac{mv^2}{r} = mg \quad v = \sqrt{rg} = \sqrt{(5.0)(9.8)} = \underline{7.0 \text{ m/s}}$$

7. What is the correct speed for a car rounding a 125 m curve in the highway under very icy conditions if the banking angle is  $20.0^\circ$ ?

$$\tan \theta = \frac{v^2}{rg} \quad \tan 20 = \frac{v^2}{(125)(9.8)} \quad v = \underline{21.1 \text{ m/s}}$$

8. A student has a weight of 655 N. While riding on a roller coaster, this same student has an apparent weight of  $1.96 \times 10^3 \text{ N}$  at the bottom of the dip that has a radius of 18.0 m. What is the speed of this roller coaster? (18.8 m/s)



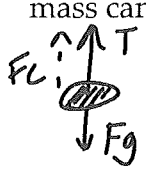
$$F_c = F_N + F_g = 1960 + (-655) = 1305 \text{ N}$$

$$1305 = \frac{(66.8)(v^2)}{18.0} \quad v = \underline{18.8 \text{ m/s}}$$

9. A 745 m curve on a racetrack is to be banked for cars travelling at 90.0 m/s. At what angle should it be banked if it is going to be used under very icy conditions? ( $48^\circ$ )

$$\tan \theta = \frac{v^2}{rg} \quad \tan^{-1} \left( \frac{90^2}{(745)(9.8)} \right) = \underline{48^\circ}$$

10. A string requires a 186 N force in order to break. A 1.50 kg mass is tied to this string and whirled in a vertical circle with a radius of 1.90 m. What is the maximum speed that this mass can be whirled without breaking the string? (14.7 m/s)



$$F_c = T + F_g = 186 + (-1.5 \cdot 9.8) = 171.3 \text{ N}$$

$$171 = \frac{(1.50)(v^2)}{1.9} \quad v = \underline{14.7 \text{ m/s}}$$

11. A 2.2 kg object is whirled in a vertical circle whose radius is 1.0 m. If the time of one revolution is 0.97 s, what is the tension in the string? (assume uniform speed)

a) When it is at the top? (-71 N)  $F_c = T + F_g$   $F_c = \frac{m4\pi^2 r}{T^2}$

b) When it is at the bottom? (114 N)  $-92.3 = T + (-21.6)$   $= \frac{(2.2)(4\pi^2)(1.0)}{(0.97)^2}$

$92.3 = T + (-21.6)$   $T_{\text{top}} = \underline{-70.7 \text{ N}}$   $= \underline{92.3 \text{ N}}$

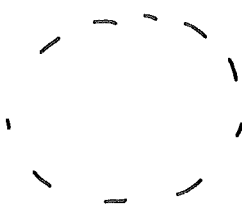
$T_{\text{bottom}} = \underline{114 \text{ N}}$

12. A wheel shaped space station whose radius is 48 m produces artificial gravity by rotating. How fast must this station rotate so that the crew members have the same apparent weight in this station as they have on Earth? (21.7 m/s)

$$F_c = F_g$$

$$v = \sqrt{r \cdot g} = \sqrt{48 \cdot 9.8} = 21.7 \text{ m/s}$$

13. An airplane travelling at a speed of 115 m/s makes a complete horizontal turn in 120 s. What is the banking angle? (32°)

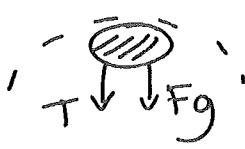
  $v = \frac{d}{t}$   $115 = \frac{d}{120}$   $d = 13800 \text{ m} = \text{circumference, (distance travelled)}$

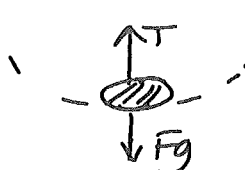
$C = 2\pi r$   $13800 = 2\pi r$   $r = 2197 \text{ m}$

$\tan^{-1}\left(\frac{115^2}{(2197)(9.8)}\right)$   $\theta = 32^\circ$

14. A 2.5 kg ball is tied to a 0.75 m string and whirled in a vertical circle (assume a constant speed of 12 m/s).

a) Why is the tension in the string greater at its low point than at its high point?

high  $\rightarrow$    $F_c = T + F_g$   
 $T = F_c - F_g$

low  $\rightarrow$    $F_c = T + (-F_g)$   
 $T = F_c + F_g$

\*different directions for tension

b) What is the tension in the string at its-

i) low point? (505 N)

ii) high point? (-456 N)

$$F_c = \frac{(2.5)(12)^2}{0.75} = 480 \text{ N}$$

low:  $480 = T + (-24.5)$   $T = 505 \text{ N}$   
 high:  $-480 = T + (-24.5)$   $T = -456 \text{ N}$



## Physics 12 – Universal Gravitation and Satellite Motion

Newton explained the dynamics of the solar system in his Law of Universal Gravitation. He explained through his first law of motion that if a planet is moving in a circular path, it has centripetal acceleration and therefore must be centripetal force acting on it. Newton showed that this force must be caused by the sun.

Using Kepler's three laws of planetary motion and Newton's three laws of motion, Newton formulated **the Law of Universal Gravitation**. (The derivation of this will be posted my portal site)

The basics of this law are:

A. Every particle in the universe exerts an attractive force on every other particle.

B. The gravitational force between any two objects is

a. directly proportional to the product of their masses –  $F_g \propto m_1 m_2$

b. inversely proportional to the square of the distance between their centers –  $F_g \propto \frac{1}{r^2}$

$F_g \propto \frac{1}{r^2}$

$r_1 = 2m$

$r_2 = 6m$

$F_g \propto \frac{1}{2^2}$

$= 0.25 N$

$F_g \propto \frac{1}{6^2}$

$= 0.028 N$

*\*closer = stronger*

$F_g$

$F_g \propto m_1 m_2$

$F_{g1}$

$m_1 = 2kg$

$m_2 = 10kg$

$F_g \propto (2)(10)$

$= 20 N$

$F_{g2}$

$m_1 = 2kg$

$m_2 = 3kg$

$F_g \propto (2)(3)$

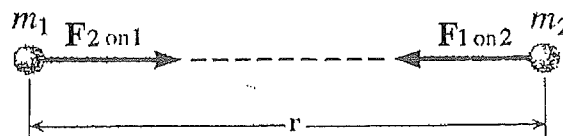
$= 6.0 N$

*\*larger masses = stronger*

$F_g$

C. The gravitational force between two objects:

\*  $F_g$  is exerted along a line connecting the centers of the two objects.



\*  $F_g$  on  $m_1$  and  $m_2$  is equal in size but opposite in direction to the  $F_g$  on  $m_2$  by  $m_1$ .

D. When combined mathematically, the formula found is:

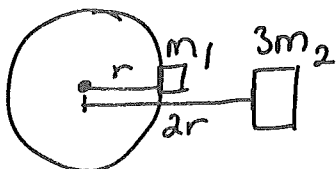
$$F_g = G \frac{m_1 m_2}{r^2}$$

$G \rightarrow$  gravitational constant  
 $= 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$

**Example:** Calculate the force of gravity between two 75 kg students if their centers of mass are 0.95 m apart.

$$F_g = \frac{(6.67 \times 10^{-11})(75)(75)}{(0.95)^2} \quad F_g = \underline{4.15 \times 10^{-7} \text{ N}}$$

**Example:** A satellite weighs 9000 N on Earth's surface. How much does it weigh if its mass is tripled and its orbital radius is doubled?



$$F_{g1} = \frac{Gm_1m_2}{r} = 9000 \text{ N}$$

$$F_{g2} = \frac{Gm_1 3m_2}{(2r)^2}$$

$$F_{g2} = \frac{3}{4} \frac{Gm_1m_2}{r^2}$$

$$\text{so... } F_{g2} = \frac{3}{4}(9000) = \underline{6750 \text{ N}}$$

**MASS VERSUS WEIGHT:** This is a very common misconception!

Mass:

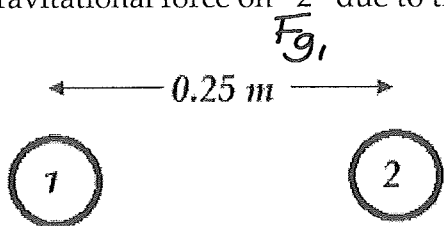
- amount of matter
- constant everywhere

Weight:

- gravitational attraction ( $F_g$ )
- changes depending on location

### Net Gravitational Force

Three equal 0.75 kg mass balls are placed as shown in the diagram. Determine the net gravitational force on "2" due to the presence of "1" and "3".

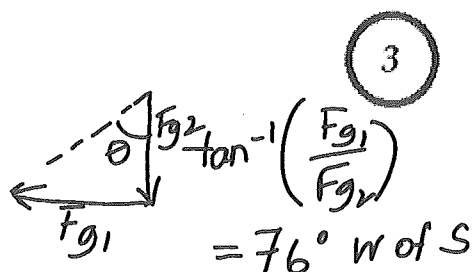


$$F_{g1} = \frac{(6.67 \times 10^{-11})(0.75)(0.75)}{(0.25)^2}$$

$$F_{g1} = \underline{6.00 \times 10^{-10} \text{ N}}$$

$$F_{g2} = \frac{(6.67 \times 10^{-11})(0.75)(0.75)}{(0.50)^2}$$

$$F_{g2} = \underline{1.50 \times 10^{-10} \text{ N}}$$

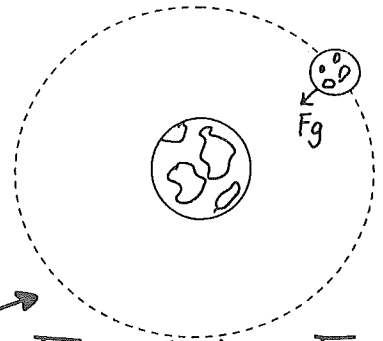


$$R = \sqrt{(1.5 \times 10^{-10})^2 + (6.0 \times 10^{-10})^2}$$

$$= 6.18 \times 10^{-10} \text{ N @ } 76^\circ \text{ W of S}$$

## Satellites in Orbit

A satellite of Earth, such as the moon, is constantly falling. But it does not fall towards the Earth... it falls **around** the Earth. Just as if you were in an elevator that was falling towards the Earth you would feel weightless if you were on an artificial satellite falling around the Earth.



\* Circular motion →

$$F_c \text{ exists} = F_g$$

**Example:** A 4500 kg Earth satellite has an orbital radius of  $8.50 \times 10^7$  m. At what speed does it travel?

$$m_1 = 5.98 \times 10^{24} \text{ kg}$$

$$m_2 = 4500 \text{ kg}$$

$$r = 8.50 \times 10^7 \text{ m}$$

$$F_c = F_g$$

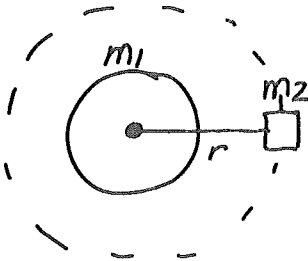
$$\cancel{m_2} v^2 = G \frac{m_1 \cancel{m_2}}{r^2}$$

$$v = \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(8.50 \times 10^7)^2}} \quad v^2 = \frac{G m_1}{r} \quad \text{or}$$

$$v = 2166 \text{ m/s} \\ (2.17 \times 10^3 \text{ m/s})$$

orbital velocity:
$v = \sqrt{\frac{G m_1}{r}}$

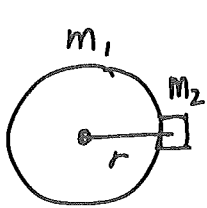
( $m_1$  = body causing the orbital attraction)



## Law of Gravitation and Satellite Motion Problems:

1. The mass of the planet Jupiter is  $1.9 \times 10^{27}$  kg, and the average radius is  $7.2 \times 10^7$  m.

a) If a 3200 kg space probe is sitting on the surface of Jupiter, what is the gravitational force between the space probe and Jupiter? ( $7.8 \times 10^4$  N)

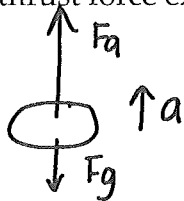


$$F_g = ?$$

$$F_g = \frac{(6.67 \times 10^{-11})(1.9 \times 10^{27})(3200)}{(7.2 \times 10^7)^2}$$

$$F_g = \underline{7.8 \times 10^4 \text{ N}}$$

b) If the 3200 kg space probe is to accelerate upwards at  $2.8 \text{ m/s}^2$  as it leaves Jupiter, what is the thrust force exerted by the engine? ( $8.7 \times 10^4$  N)

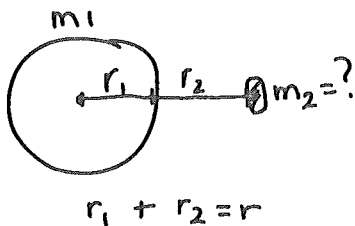


$$ma = F_a + F_g$$

$$(3200)(2.8) = F_a + (-7.8 \times 10^4)$$

$$F_a = \underline{8.7 \times 10^4 \text{ N}}$$

c) If an astronaut experiences a gravitational force of 1600 N at an altitude of  $3.2 \times 10^6$  m above Jupiter, what is the mass of the astronaut? (71 kg)



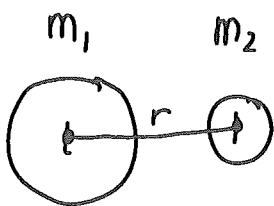
$$F_g = \frac{(6.67 \times 10^{-11})(1.9 \times 10^{27})m_2}{(3.2 \times 10^6 + 7.2 \times 10^7)^2}$$

$$m_2 = \underline{71 \text{ kg}}$$

2. A 30 kg box is 2.0 m away from a 1200 kg car. Which exerts a greater gravitational force on the other? Explain your answer. (neither)

neither  $\rightarrow F_g$  is equal & opposite  
magnitude direction

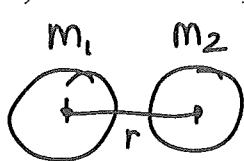
3. What gravitational force does the moon produce if the centers of the Earth and the moon are  $3.84 \times 10^8$  m apart and the moon has a mass of  $7.35 \times 10^{22}$  kg? ( $1.99 \times 10^{20}$  N)



$$F_g = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(7.35 \times 10^{22})}{(3.84 \times 10^8)^2}$$

$$= \underline{1.99 \times 10^{20} \text{ N}}$$

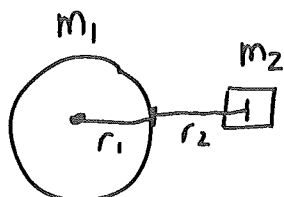
5. If the gravitational force between two objects of equal mass is  $4.50 \times 10^{-6} \text{ N}$  when the objects are  $7.0 \text{ m}$  apart, what is the mass of each object? ( $1.8 \times 10^3 \text{ kg}$ )



$$4.50 \times 10^{-6} = \frac{(6.67 \times 10^{-11}) m^2}{(7.0)^2} \quad m = 1818 \text{ Kg}$$

$$= 1.8 \times 10^3 \text{ Kg}$$

6. Calculate the gravitational force on a  $6.50 \times 10^2 \text{ kg}$  spacecraft that is  $4.15 \times 10^6 \text{ m}$  above the surface of the Earth. ( $2.35 \times 10^3 \text{ N}$ )

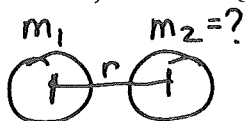


$$r = r_1 + r_2$$

$$F_g = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(650)}{(6.38 \times 10^6 + 4.15 \times 10^6)^2}$$

$$F_g = 2.35 \times 10^3 \text{ N}$$

7. The gravitational force between two objects that are  $0.33 \text{ m}$  apart is  $3.2 \times 10^{-5} \text{ N}$ . If the mass of one object is  $60 \text{ kg}$ , what is the mass of the other object? ( $871 \text{ kg}$ )



$$3.2 \times 10^{-5} = \frac{(6.67 \times 10^{-11})(60)(m_2)}{(0.33)^2} \quad m_2 = 871 \text{ Kg}$$

8. Calculate the speed of the moon in its orbit around Earth. (Radius of moon's orbit =  $3.84 \times 10^8 \text{ m}$ ; moon's mass =  $7.35 \times 10^{22} \text{ kg}$ ) ( $1.02 \times 10^3 \text{ m/s}$ )

$$v = \sqrt{\frac{Gm_1}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{3.84 \times 10^8}} \quad v = 1019 \text{ m/s}$$

$$(1.02 \times 10^3 \text{ m/s})$$

9. Calculate the speed of a satellite orbiting Earth at a height of  $4.4 \times 10^5 \text{ m}$  above the Earth's surface. ( $7.6 \times 10^3 \text{ m/s}$ )

$$v = \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.38 \times 10^6 + 4.4 \times 10^5)}} \quad v = 7.6 \times 10^3 \text{ m/s}$$

10. Calculate the orbital speed of a satellite  $5.0 \times 10^6 \text{ m}$  above the surface of Jupiter. (Radius of Jupiter =  $7.18 \times 10^7 \text{ m}$ ; mass of Jupiter =  $1.90 \times 10^{27} \text{ kg}$ ) ( $4.06 \times 10^4 \text{ m/s}$ )

$$v = \sqrt{\frac{(6.67 \times 10^{-11})(1.90 \times 10^{27})}{(7.18 \times 10^7 + 5.0 \times 10^6)}} \quad v = 4.06 \times 10^4 \text{ m/s}$$

11. Calculate the speed of Earth in its orbit around the Sun. (Radius of Earth's orbit =  $1.50 \times 10^{11} \text{ m}$ ; sun's mass =  $1.98 \times 10^{30} \text{ kg}$ ) ( $2.97 \times 10^4 \text{ m/s}$ )

$$v = \sqrt{\frac{(6.67 \times 10^{-11})(1.98 \times 10^{30})}{(1.50 \times 10^{11})}} \quad v = 2.97 \times 10^4 \text{ m/s}$$

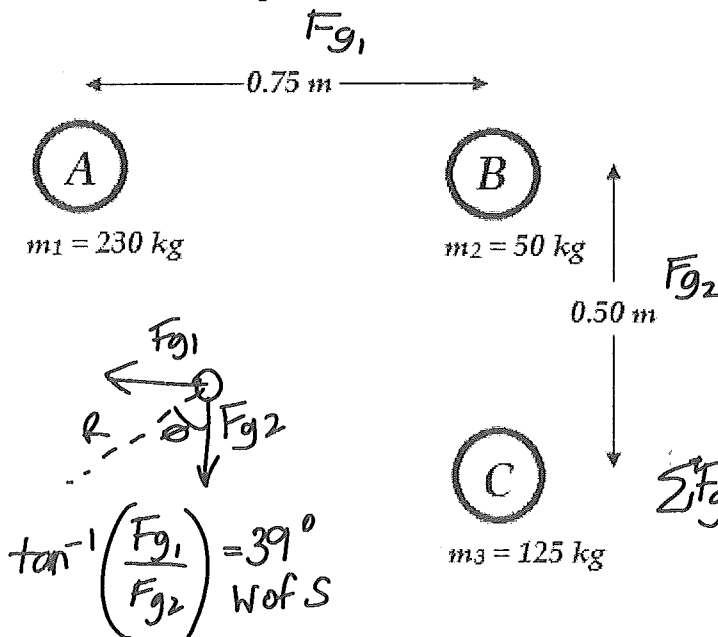
12. Using the formula  $T = \sqrt{\frac{4\pi^2 r^3}{Gm}}$ , calculate the time of one revolution (length of a year) on Mars. (Mar's mass =  $6.4 \times 10^{23}$  kg; radius of Mar's orbit =  $2.3 \times 10^{11}$  m) ( $6.0 \times 10^7$  s or 1.9 years)

$$T = \sqrt{\frac{4\pi^2 (2.3 \times 10^{11})^3}{(6.67 \times 10^{-11})(6.4 \times 10^{23})}}$$

$$T = 6.0 \times 10^7 \text{ s}$$

$$= 1.9 \text{ years}$$

13. Three masses are placed as shown in the diagram. Determine the net gravitational force on "B" due to the presence of "A" and "C". ( $2.15 \times 10^{-6}$  N @  $39^\circ$  W of S)



$$F_{g1} = \frac{(6.67 \times 10^{-11})(230)(50)}{(0.75)^2}$$

$$F_{g1} = 1.36 \times 10^{-6} \text{ N}$$

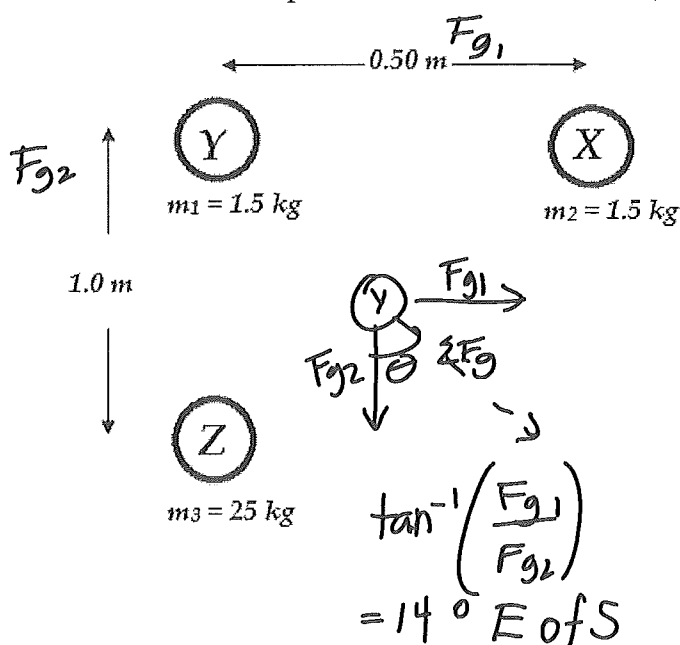
$$F_{g2} = \frac{(6.67 \times 10^{-11})(50)(125)}{(0.50)^2}$$

$$F_{g2} = 1.67 \times 10^{-6} \text{ N}$$

$$\Sigma F_g = \sqrt{(1.36 \times 10^{-6})^2 + (1.67 \times 10^{-6})^2}$$

$$= 2.15 \times 10^{-6} \text{ N @ } 39^\circ \text{ W of S}$$

14. Three masses are placed as shown in the diagram. Determine the net gravitational force on "Y" due to the presence of "X" and "Z". ( $2.57 \times 10^{-9}$  N @  $14^\circ$  E of S)



$$F_{g1} = \frac{(6.67 \times 10^{-11})(1.5)(1.5)}{(0.50)^2}$$

$$= 6.0 \times 10^{-10} \text{ N}$$

$$F_{g2} = \frac{(6.67 \times 10^{-11})(1.5)(25)}{(1.0)^2}$$

$$= 2.5 \times 10^{-9} \text{ N}$$

$$\Sigma F_g = \sqrt{(6.0 \times 10^{-10})^2 + (2.5 \times 10^{-9})^2}$$

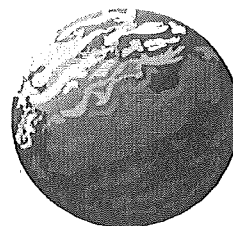
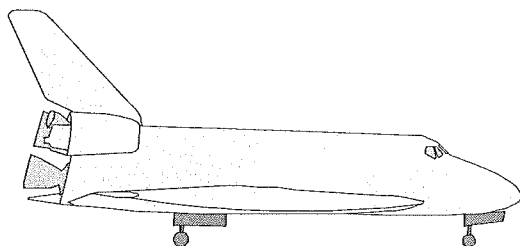
$$= 2.57 \times 10^{-9} \text{ N @ } 14^\circ \text{ E of S}$$

## Galilei 27 orbiting Iris 2 (part 1)

### Context

Within the scope of the EESP project (Exploration of Extrasolar Planets), we have identified three planets that are suitable for sustaining life forms that are similar to us. We are in the process of developing various mission scenarios that could eventually be carried out on these three planets. You have been selected to plan the encounter between Galilei 27 and Iris 2.

Galilei 27 is the name of the space ship that will transport the astronauts. Iris 2 is the name of the planet, among the suitable three, that is closest to Earth. Planet Iris 2 has a lower mass than that of the Earth but has a smaller radius. This is the information we have gathered so far:



Mass of Iris 2:  $m_I = 2.0 \times 10^{24} \text{ kg}$

Radius of Iris 2:  $r_I = 2\,000 \text{ km}$

Mass of Galilei 27:  $m_G = 5.0 \times 10^5 \text{ kg}$

Your task is to calculate certain parameters that will be needed to accomplish the technical aspects of the mission scenario. Perform the following calculations:

We should be able to approach Iris 2 without any problems. However, in order to land on its surface, we must know the attractive force that will be exerted on the space ship when it is positioned at a distance of 10 000 km from the planet's surface.

- Calculate the attractive force that will be exerted by Iris 2 on the space ship when it is positioned at that distance from the surface of the planet.
- Calculate the acceleration that Galilei 27 will experience at that distance.

Please note: this data will help us design a landing plan (the thrust the motors will need to help the spaceship brake during its landing, the adequate quantity of fuel, the time it takes to brake, etc.)

In the probable event that Galilei 27 lands on Iris 2 as anticipated, physiological factors must also be taken into account:

- Calculate the gravitational acceleration on the planet's surface.
- What force will the astronaut need to apply to lift a radiation monitor that has a mass of 2 kg to 1.5m above the surface? (2.50 J of work is done to overcome atmospheric friction)
- If the astronaut drops the monitor from a height of 1 metre, how long will it take for it to fall to the ground?

Please note: these facts will allow us to evaluate how the human body will react to gravity on Iris 2. In addition, they will help us choose the materials needed for the fabrication of space suits, and the technical equipment that the astronauts will carry when they first walk on Iris 2. They will also enable us to determine other parameters, such as resistance of the materials chosen.

**Please use a separate piece of paper and show all calculations and answers for each of the points. Make sure to label each value with its purpose. (ie. Gravitational Acceleration at Surface - ...)**

1.  $F_g$  at 10000 km above surface  $\rightarrow$

$$F_g = \frac{(6.67 \times 10^{-11})(2.0 \times 10^{24})(5.0 \times 10^5)}{(2.0 \times 10^6 + 1.0 \times 10^7)^2}$$

$$F_g = \boxed{4.63 \times 10^5 \text{ N}}$$

2. ' $g$ ' at 10000 km above surface  $\rightarrow$

$$g = \frac{(6.67 \times 10^{-11})(2.0 \times 10^{24})}{(2.0 \times 10^6 + 1.0 \times 10^7)^2}$$

$$g = 0.926 \text{ N/kg}$$
$$a = 0.926 \text{ m/s}^2$$

3. ' $g$ ' at surface  $\rightarrow$

$$g = \frac{(6.67 \times 10^{-11})(2.0 \times 10^{24})}{(2.0 \times 10^6)^2}$$

$$g = 33.4 \text{ N/kg}$$
$$a = 33.4 \text{ m/s}^2$$

4. Force applied  $\rightarrow W = \sum F \cdot d$

$$W_f = F_f \cdot d$$
$$2.5 = F_f \cdot 1.5 \quad F_f = 1.67 \text{ N}$$

$$\Delta PE = \sum F \cdot d$$
$$(2.0)(33.4)(1.5) = \sum F \cdot 1.5$$

$$\sum F = 66.8 \text{ N}$$

$$\sum F = F_a + F_f$$
$$66.8 = F_a + (-1.67)$$

$$F_a = \boxed{68.5 \text{ N}}$$

5. ✗ with air friction

$$ma = F_g + F_{f \text{ air}}$$

$$(2.0)a = (2.0)(33.4) + \left(-\frac{2.5}{1.5}\right)$$

$$a = 32.6 \text{ m/s}^2 \text{ (down)}$$

$$-1.0 = 0 + \frac{1}{2}(-32.6)t^2$$

$$t = \boxed{0.248 \text{ s}}$$



## Physics 12 - Gravitational Fields

In order to explain forces between two objects that are not in contact, scientists developed the concept of fields. Fields are defined as spheres of influence and can be classified as scalar or vector. Scalar field examples would be sound fields and heat fields.

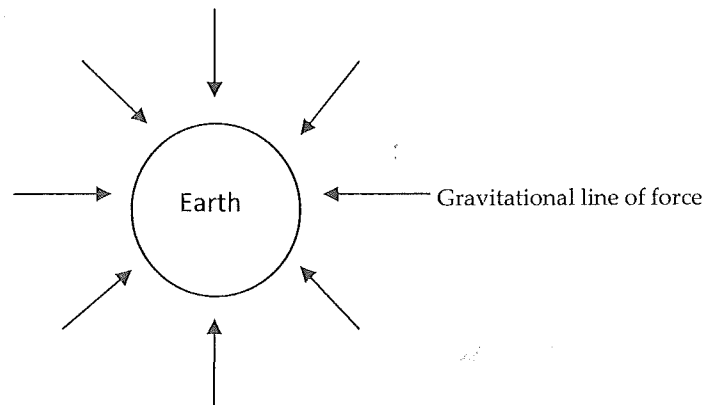
To help visualize how a field works...consider a **campfire** and the *heat field* that it emits.

As you approach the fire... *the field strength increases ( $\uparrow$  temp.)*

As you increase the size of the fire...  
" "



A gravitational field is a vector field due to the fact that gravity is a force and forces are vector quantities.



Gravitational Field Strength is the acceleration due to gravity and it will vary depending on the masses involved and the distance of separation between the centers of the two objects.

On Earth, the gravitational field strength is 9.80 N/kg

$$F_g = mg \quad \leftarrow \text{"field strength"}$$

There are two methods to determine the field strength between two objects:

symbol for field strength  $\rightarrow$

$$g = \frac{F_g}{m}$$

weight of test object (N)  $\leftarrow$

mass (kg)  $\leftarrow$

\*recall that weight is defined as the gravitational force ( $F_g$ ) between a planet and an object on the surface of that planet.

This formula works well if we stay on the surface of the Earth or other planetary mass.

However, once we leave the surface it does not work very well because... 'g' varies with distance  
Therefore a more useful formula takes that into account:

$$g = \frac{Gm_1}{r^2}$$

$m_1$  = mass of object producing the field (ie. planet)

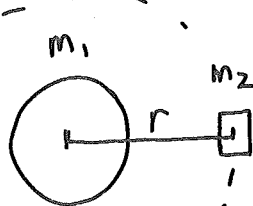
**Example:** What is the gravitational field strength on the surface of the moon?

Mass of moon =  $7.35 \times 10^{22}$  kg

Radius of moon =  $1.74 \times 10^6$  m

$$g = \frac{Gm_1}{r^2} = \frac{(6.67 \times 10^{-11})(7.35 \times 10^{22})}{(1.74 \times 10^6)^2} = 1.62 \text{ N/kg}$$

**Example:** A satellite orbits the Earth at a radius of  $2.20 \times 10^7$  m. What is its orbital period?



circular motion

$$F_c = F_g$$

$$ma_c = mg$$

$$a_c = g$$

$$\frac{4\pi^2 r}{T^2} = \frac{Gm_1}{r^2}$$

simplified:

$$T = \sqrt{\frac{4\pi^2 r^3}{Gm_1}}$$

$$T = \sqrt{\frac{4\pi^2 (2.20 \times 10^7)^3}{(6.67 \times 10^{-11})(5.98 \times 10^{24})}}$$

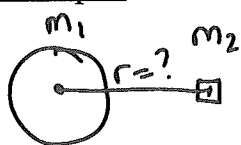
$$T = 3.25 \times 10^4 \text{ s}$$

**Geosynchronous Orbit** – The orbital speed of a satellite will depend on the strength of the gravitational field at the orbital radius.

$$v = \sqrt{\frac{Gm_1}{r}}$$

The orbital period of the satellite depends only on the mass of the planet and the orbital radius of the satellite. Therefore it makes sense that at a certain orbital distance, the orbital period will match the rotational period of the planet. This satellite is then in geosynchronous (or geostationary) orbit.

**Example:** Find the orbital radius of a satellite that is geostationary above the Earth's equator.



$$T_{\text{earth}} = 24 \text{ hr} \times 60 \text{ min} \times 60 \text{ s} = 86400 \text{ s} \quad (1 \text{ rotation} = 1 \text{ day})$$

$$\frac{4\pi^2 r}{T^2} = \frac{Gm_1}{r^2} \quad \frac{4\pi^2 r}{(86400)^2} = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{r^2}$$

$$r^3 = 7.54 \times 10^{22} \quad r = 4.23 \times 10^7 \text{ m}$$

What is the speed of this satellite?

$$v = \frac{2\pi r}{T} = \frac{2\pi (4.23 \times 10^7)}{86400} = 3073 \text{ m/s}$$

## Gravitational Field Strength and Problems:

1. The mass of the planet Jupiter is  $1.9 \times 10^{27}$  kg, and the average radius is  $7.2 \times 10^7$  m.

a) What is the gravitational field strength on the surface of Jupiter? (24 N/kg)

$$g = \frac{Gm_1}{r^2} = \frac{(6.67 \times 10^{-11})(1.9 \times 10^{27})}{(7.2 \times 10^7)^2} = 24.4 \text{ N/m}$$

2. A 2.0 kg rock dropped near the surface of a planet reaches a speed of 15 m/s in 3.0 s.

a) What is the acceleration due to gravity near the surface of the planet? (5.0 m/s<sup>2</sup>)

$$\vec{a} = \frac{-15 - 0}{3.0} \quad \vec{a} = -5.0 \text{ m/s}^2 \quad (\text{also 'g'})$$

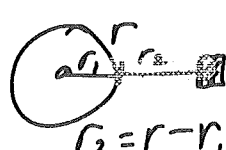
b) The planet has an average radius of  $4.8 \times 10^6$  m. What is the mass of the planet? ( $1.73 \times 10^{24}$  kg)

$$g = \frac{Gm_1}{r^2} \quad 5.0 = \frac{(6.67 \times 10^{-11})m_1}{(4.8 \times 10^6)^2} \quad m_1 = 1.73 \times 10^{24} \text{ kg}$$

3. Calculate the gravitational field strength on the surface of Mars. Mars has a radius of  $3.43 \times 10^6$  m and a mass of  $6.37 \times 10^{23}$  kg. (3.61 N/kg)

$$g = \frac{Gm_1}{r^2} = \frac{(6.67 \times 10^{-11})(6.37 \times 10^{23})}{(3.43 \times 10^6)^2} = 3.61 \text{ N/kg}$$

4. At what distance from the Earth's surface is the gravitational field strength 7.33 N/kg? ( $1.00 \times 10^6$  m)

$$7.33 = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{r^2}$$


$$r = 7.38 \times 10^6 \text{ m} - 6.38 \times 10^6 \text{ m} = 1.0 \times 10^6 \text{ m}$$

$r_2 = r - r_1$

5. What is the gravitational field strength  $1.276 \times 10^7$  m above the Earth's surface? (1.09 N/kg)

$$g = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.38 \times 10^6 + 1.276 \times 10^7)^2} \quad g = 1.09 \text{ N/kg}$$

6. A 2400 kg satellite is in a stable circular orbit around the Earth with a radius of  $6.8 \times 10^6$  m.

a) What provides the centripetal force to keep the satellite in a circular orbit?  $F_c = F_g$

b) Find the speed of the satellite in this orbit. ( $7.66 \times 10^3$  m/s)

c) Find the period of the satellite. ( $5.58 \times 10^3$  s)

$$c) T = \frac{\sqrt{4\pi^2 (6.8 \times 10^6)^3}}{\sqrt{(6.67 \times 10^{-11})(5.98 \times 10^{24})}}$$
$$T = 5.58 \times 10^3 \text{ s}$$

$$b) v = \sqrt{\frac{Gm}{r}}$$
$$= \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.8 \times 10^6}}$$
$$v = 7.66 \times 10^3 \text{ m/s}$$

7. A satellite in synchronous orbit far above the equator of the Earth appears to be stationary over a position to an observer on Earth. Explain why the satellite appears to be stationary.

- the synchronous orbit matches the rotation of the Earth = moves at same rate as Earth rotates

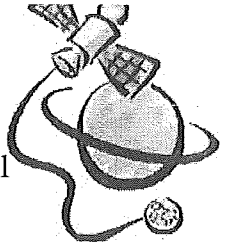
8. A satellite travels around a  $5.20 \times 10^{23}$  kg planet with an orbital radius of  $1.1 \times 10^7$  m. What is the orbital period of the satellite? ( $3.89 \times 10^4$  s)

$$T = \frac{\sqrt{4\pi^2 (1.1 \times 10^7)^3}}{\sqrt{(6.67 \times 10^{-11})(5.20 \times 10^{23})}}$$

$$T = 3.89 \times 10^4 \text{ s}$$

## Physics 12 – Work and Gravitational Potential Energy

Up to this point, we have used the formula  $E_p = mgh$  to determine gravitational potential energy. While this equation works well when finding gravitational potential energy near the Earth's surface, it does not work when larger distances are involved.



As we saw when calculating gravitational field strength in the last lesson – 'g' changes

$$g = \frac{Gm_1}{r^2}$$

• as the distance from Earth ↑, 'g' will ↓

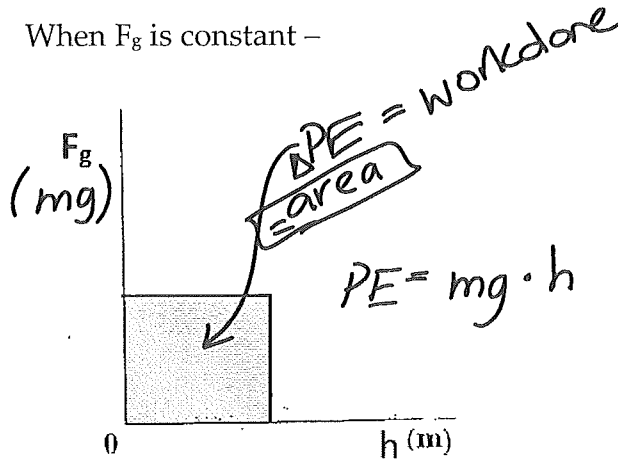
$$g = \frac{Gm_1}{r^2} \quad PE = m_2gh$$

$$\text{so... } PE = m_2 \left( \frac{Gm_1}{r^2} \right) h$$

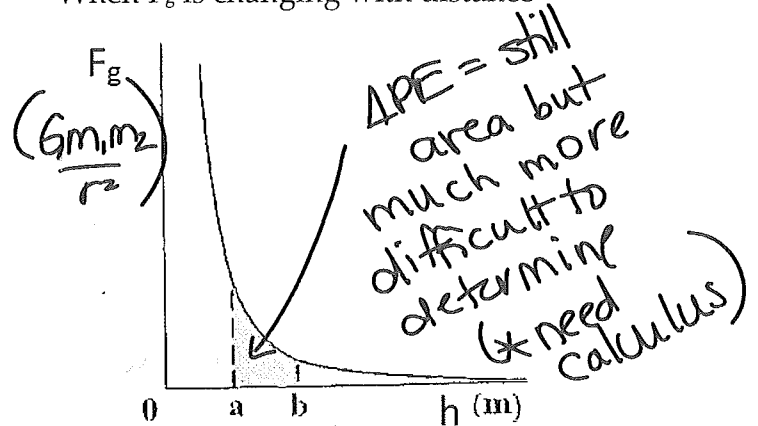
$$PE = \frac{Gm_1m_2}{r}$$

Remember that the  $E_p$  of a mass is the work done in lifting it a certain height.

When  $F_g$  is constant –



When  $F_g$  is changing with distance –



As we saw above, the gravitational potential energy was determined to be:

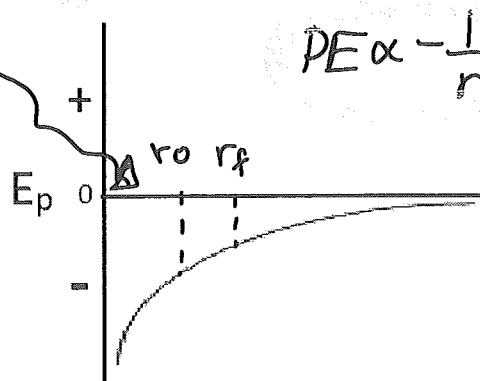
$$PE_g = \frac{Gm_1m_2}{r}$$

However, that is not the whole picture (why we need calculus). When dealing with gravitational potential energy, we use a reference point. At this point the  $PE_g$  is zero. When we are finding the gravitational potential energy on a mass provided by the gravitational force generated by a second mass, we assign the ZERO reference point when the distance between the objects is infinite ( $r = \infty$ ).

This means that when the objects get closer together, the gravitational potential energy between them decreases.

Because of this, the formula becomes:

$$PE_g = -\frac{Gm_1m_2}{r}$$



## Calculating Work Done: $W = \Delta E$

The work done on an object can be equated to the change in gravitational potential energy of an object. This comes from:  $\Delta PE = (PE)_f - (PE)_o$

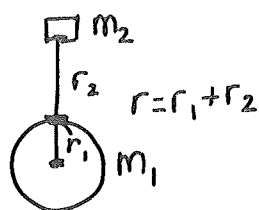
$$\Delta PE = PE_f - PE_o = -\frac{Gm_1m_2}{r_f} - \left(-\frac{Gm_1m_2}{r_o}\right) \text{ *factored/simplified} \rightarrow \Delta PE_g = Gm_1m_2 \left( \frac{1}{r_o} - \frac{1}{r_f} \right)$$

## Energy Conservation:

Kinetic energy will be obtained as an object falls to earth and increases in velocity. Therefore, the potential energy will be converted to kinetic energy following the law of conservation (without any external forces acting on the object).

Law of Conservation of Mechanical Energy  $\rightarrow KE_o + PE_o = KE_f + PE_f$

**Example One** – A  $2.50 \times 10^3$  kg geostationary satellite (a satellite that remains in the same position above the Earth's surface) is in an orbit that is  $3.60 \times 10^7$  m above Earth's surface. What is the gravitational potential energy of this satellite due to the gravitational force cause by the Earth?



$$PE_g = -\frac{Gm_1m_2}{r} = -\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(2500)}{(6.38 \times 10^6 + 3.6 \times 10^7)}$$

$$PE_g = \underline{-2.35 \times 10^{10} \text{ J}}$$

**Example Two** – How much work is needed to lift a  $1.25 \times 10^3$  kg satellite from the Earth's surface to a height of  $4.00 \times 10^6$  m above the Earth's surface?

$$\begin{aligned} W = \Delta PE &= Gm_1m_2 \left( \frac{1}{r_o} - \frac{1}{r_f} \right) \\ &= (6.67 \times 10^{-11})(5.98 \times 10^{24})(1250) \left( \frac{1}{6.38 \times 10^6} - \frac{1}{(6.38 \times 10^6 + 4.00 \times 10^6)} \right) \\ &= 4.99 \times 10^{17} (6.0 \times 10^{-8}) \\ &= \underline{3.0 \times 10^{10} \text{ J of work}} \end{aligned}$$

**Example Three** – A  $1.10 \times 10^3 \text{ kg}$  object is dropped from a distance of  $2.00 \times 10^5 \text{ m}$  onto the Moon's surface. How fast is the object travelling when it hits the Moon's surface?

$$-\Delta PE = \Delta KE \quad (\Delta PE + \Delta KE = 0)$$

$$\Delta PE_g = (6.67 \times 10^{-11}) (7.35 \times 10^{22}) (1100) \left( \frac{1}{1.74 \times 10^6 + 2.0 \times 10^5} - \frac{1}{1.74 \times 10^6} \right)$$

$$\Delta PE_g = -3.195 \times 10^8 \text{ J}$$

$$-(-3.195 \times 10^8) = \frac{1}{2} (1100) (v_F^2 - 0^2) \quad v_F = \underline{762 \text{ m/s}}$$

**Example Four:** The period of a 1000 kg satellite orbiting the earth is  $6.3 \times 10^3 \text{ s}$ .

- Find the altitude of the satellite.
- Find the total energy of the satellite.

$$a) T = \sqrt{\frac{4\pi^2 r^3}{Gm_1}} \quad 6.3 \times 10^3 = \sqrt{\frac{4\pi^2 r^3}{(6.67 \times 10^{-11})(5.98 \times 10^{24})}}$$

$$3.97 \times 10^7 = \frac{4\pi^2 r^3}{3.99 \times 10^{14}}$$

$$r = 7.37 \times 10^6 \text{ m}$$

$$\text{altitude} = 7.37 \times 10^6 - 6.38 \times 10^6 = \underline{9.95 \times 10^5 \text{ m}}$$

$$b) PE_g = -\frac{Gm_1 m_2}{r} = -\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1000)}{7.37 \times 10^6}$$

$$= -5.41 \times 10^{10} \text{ J}$$

$$v = \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{7.37 \times 10^6}} \quad v = 7357 \text{ m/s}$$

$$KE = \frac{1}{2} (1000) (7357)^2 = 2.71 \times 10^{10} \text{ J}$$

$$\text{Total } E = KE + PE_g = 2.71 \times 10^{10} + (-5.41 \times 10^{10}) = \underline{-2.70 \times 10^{10} \text{ J}}$$

## Work and Gravitational Potential Energy Problems:

1. A 2900 kg space probe is  $3.8 \times 10^5$  m from the center of a planet.

a) If the gravitational potential energy of the space probe is  $-1.7 \times 10^{11}$  J relative to zero at infinity, what is the mass of the planet? ( $3.3 \times 10^{23}$  kg)

$$PE_g = -G \frac{m_1 m_2}{r} = -\frac{(6.67 \times 10^{-11})(m_1)(2900)}{(3.8 \times 10^5)} = -1.7 \times 10^{11}$$

$$m_1 = 3.3 \times 10^{23} \text{ kg}$$

b) If the space probe falls to the planet and has a speed of  $6.2 \times 10^3$  m/s just before impact with the surface of the planet, what is the radius of the planet? ( $2.8 \times 10^5$  m)

$$KE_o + PE_o = KE_f + PE_f$$

$$0 + \left( -\frac{(6.67 \times 10^{-11})(3.3 \times 10^{23})(2900)}{3.8 \times 10^5} \right) = \frac{1}{2}(2900)(6200)^2 + \left( -\frac{(6.67 \times 10^{-11})(3.3 \times 10^{23})(2900)}{r_f} \right)$$

$$-1.68 \times 10^{11} = 5.57 \times 10^{10} - \frac{6.38 \times 10^{16}}{r_f}$$

$$-2.24 \times 10^{11} = -\frac{6.38 \times 10^{16}}{r_f}$$

$$r_f = 2.8 \times 10^5 \text{ m}$$

2. A  $5.8 \times 10^4$  kg meteor falls from an altitude of  $7.2 \times 10^5$  m above the Earth's surface. Ignore air resistance.

a) Find the change in gravitational potential energy of the meteor. ( $-3.7 \times 10^{11}$  J)

$$\Delta PE_g = G m_1 m_2 \left( \frac{1}{r_o} - \frac{1}{r_f} \right)$$

$$= (6.67 \times 10^{-11})(5.98 \times 10^{24})(5.8 \times 10^4) \left( \frac{1}{6.38 \times 10^6 + 7.2 \times 10^5} - \frac{1}{6.38 \times 10^6} \right)$$

$$= -3.7 \times 10^{11} \text{ J}$$

b) Find the speed of the meteor just before impact with the surface of the Earth (Assume initial velocity is zero) ( $3.6 \times 10^3$  m/s)

$$3.7 \times 10^{11} = \frac{1}{2}(5.8 \times 10^4) v_f^2 \quad v_f = 3.6 \times 10^3 \text{ m/s}$$



- c). If the meteor had an initial speed of  $1.8 \times 10^3$  m/s, find the kinetic energy of the meteor just before impact with the Earth's surface. ( $4.6 \times 10^{11}$  J)

$$3.7 \times 10^{11} = \frac{1}{2} (5.8 \times 10^4) (v_f^2 - (1.8 \times 10^3)^2)$$

$$1.27 \times 10^7 = v_f^2 - 3.24 \times 10^6$$

$$v_f = 4000 \text{ m/s} \quad KE = \frac{1}{2} (5.8 \times 10^4) (4000)^2 = 4.6 \times 10^{11} \text{ J}$$

3. What minimum energy is required to take a stationary 600 kg satellite from the surface of the moon to an altitude of  $9.2 \times 10^5$  m and put it into orbit with an orbital speed of  $1.36 \times 10^3$  m/s? ( $1.1 \times 10^9$  J)

$$\Delta PE = Gm_1 m_2 \left( \frac{1}{r_0} - \frac{1}{r_f} \right)$$

$$= (6.67 \times 10^{-11}) (7.35 \times 10^{22}) (600) \left( \frac{1}{1.74 \times 10^6} - \frac{1}{1.74 \times 10^6 + 9.2 \times 10^5} \right)$$

$$= 2.94 \times 10^{15} (5.74 \times 10^{-7} - 3.75 \times 10^{-7}) = 5.85 \times 10^8 \text{ J}$$

$$\Delta KE = \frac{1}{2} (600) (1360)^2 = 5.55 \times 10^8 \text{ J}$$

$$\text{Total Energy} = 5.55 \times 10^8 + 5.85 \times 10^8 = \boxed{1.1 \times 10^9 \text{ J}}$$

4. What is the gravitational potential energy of a  $5.00 \times 10^3$  kg satellite that has an orbital radius of  $9.90 \times 10^6$  m around the Earth? (Use  $PE_g = 0$  at  $r = \infty$ ). ( $-2.0 \times 10^{11}$  J)

$$PE_g = -\frac{Gm_1 m_2}{r} = -\frac{(6.67 \times 10^{-11}) (5.98 \times 10^{24}) (5000)}{9.90 \times 10^6}$$

$$= -2.0 \times 10^{11} \text{ J}$$

5. What is the work done against gravity on the satellite in problem 4 in lifting it into its orbit? ( $1.1 \times 10^{11}$  J)

$$W = \Delta PE_g = Gm_1 m_2 \left( \frac{1}{r_0} - \frac{1}{r_f} \right)$$

$$= (6.67 \times 10^{-11}) (5.98 \times 10^{24}) (5000) \left( \frac{1}{6.38 \times 10^6} - \frac{1}{9.90 \times 10^6} \right)$$

$$= 1.99 \times 10^{10} (5.5 \times 10^{-8})$$

$$= 1.1 \times 10^{11} \text{ J}$$

6. What is the change in gravitational potential energy of the satellite in problem 4 as it is lifted from Earth's surface to its orbit? ( $1.1 \times 10^{11}$  J)

$$1.1 \times 10^{11} \text{ J} \quad (W = \Delta PE_g)$$

7. What is the speed of a 1750 kg meteorite when it hits the moon's surface? This meteorite has a velocity of  $1.00 \times 10^3$  m/s heading directly toward the moon when it was 15000m above the moon's surface (no friction). ( $1.02 \times 10^3$  m/s)

$$\Delta KE = -\Delta PE$$

$$\Delta PE_g = (6.67 \times 10^{-11}) (7.35 \times 10^{22}) (1750) \left( \frac{1}{1.74 \times 10^6 + 15000} - \frac{1}{1.74 \times 10^6} \right)$$

$$= (8.58 \times 10^5) (-4.0 \times 10^{-9})$$

$$= -4.21 \times 10^7 \text{ J}$$

$$4.21 \times 10^7 = \frac{1}{2} (1750) (v_f^2 - (1000)^2)$$

$$v_f = 1023 \text{ m/s} = 1.02 \times 10^3 \text{ m/s}$$

8. What is the gravitational potential energy of a 10.0 kg object when it is sitting on the Earth's surface? (Use  $PE_g = 0$  at  $r = \infty$ ) ( $-6.25 \times 10^8$  J)

$$PE_g = - \frac{G M_1 m_2}{r} = - \frac{(6.67 \times 10^{-11}) (5.98 \times 10^{24}) (10.0)}{6.38 \times 10^6}$$

$$= -6.25 \times 10^8 \text{ J}$$

9. What is the change in the gravitational potential energy of a  $2.50 \times 10^3$  kg satellite as it is lifted vertically into a circular orbit (radius =  $6.90 \times 10^6$  m) around Earth? ( $1.18 \times 10^{10}$  J)

$$\Delta PE_g = (6.67 \times 10^{-11}) (5.98 \times 10^{24}) (2500) \left( \frac{1}{6.38 \times 10^6} - \frac{1}{6.90 \times 10^6} \right)$$

$$= (9.97 \times 10^{17}) (1.1 \times 10^{-8})$$

$$= 1.18 \times 10^{10} \text{ J}$$

10. What minimum energy is required to take a stationary 1200 kg satellite from the surface of the earth to an altitude of  $3.0 \times 10^6$  m and put it into an orbit with an orbital speed of  $2.0 \times 10^3$  m/s? ( $2.64 \times 10^{10}$  J)

$$\begin{aligned}\Delta PE_g &= (6.67 \times 10^{-11})(5.98 \times 10^{24})(1200) \left( \frac{1}{6.38 \times 10^6} - \frac{1}{6.38 \times 10^6 + 3.0 \times 10^6} \right) \\ &= (4.79 \times 10^{17}) (1.56 \times 10^{-7} - 1.06 \times 10^{-7}) \\ &= 2.40 \times 10^{10} \text{ J} \\ \Delta KE &= \frac{1}{2}(1200)(2000^2 - 0^2) = 2.40 \times 10^9 \text{ J} \\ \text{Total Energy} &= 2.40 \times 10^9 + 2.40 \times 10^{10} = \boxed{2.64 \times 10^{10} \text{ J}}\end{aligned}$$

11. The period of an 800 kg satellite orbiting the earth is  $5.9 \times 10^3$  s.

- Find the altitude of the satellite. ( $6.2 \times 10^5$  m or  $6.8 \times 10^5$  m with no rounding)
- Find the gravitational field strength at the altitude of the satellite. (8.1 N/kg)
- Find the gravitational potential energy of the satellite. ( $-4.6 \times 10^{10}$  J)
- Find the total energy of the satellite. ( $-2.3 \times 10^{10}$  J)

a)

$$5.9 \times 10^3 = \sqrt{\frac{4\pi^2 r^3}{(6.67 \times 10^{-11})(5.98 \times 10^{24})}}$$

$$3.48 \times 10^7 = \frac{4\pi^2 r^3}{3.99 \times 10^{14}}$$

$$r = 7.0 \times 10^6 \text{ m}$$

$$\text{altitude} = 7.0 \times 10^6 - 6.38 \times 10^6 = \boxed{6.2 \times 10^5 \text{ m}}$$

( $6.8 \times 10^5$  m with no rounding)

b)  $g = \frac{Gm_1}{r^2} = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(7.0 \times 10^6)^2} = \underline{8.1 \text{ N/kg}}$

c)  $PE_g = -\frac{Gm_1 m_2}{r} = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(800)}{7.0 \times 10^6} = -4.6 \times 10^{10} \text{ J}$

d)  $v = \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{7.0 \times 10^6}} \quad v = 7549 \text{ m/s} \quad KE = \frac{1}{2}(800)(7549^2)$

$$\begin{aligned}\text{Total E} &= KE + PE = 2.3 \times 10^{10} + (-4.6 \times 10^{10}) \\ &= \underline{-2.3 \times 10^{10} \text{ J}}\end{aligned}$$

$KE = 2.3 \times 10^{10} \text{ J}$

## Physics 12 – Escape Velocity

Escape velocity is the minimum speed an object requires in order to break away from the Earth's pull.

It should make sense that if an object is going to be completely freed from the Earth's gravitational pull, we need to supply it with enough KINETIC ENERGY to match its POTENTIAL ENERGY at infinity.

In terms of equations this means that:

$$\Delta PE = -\Delta KE$$

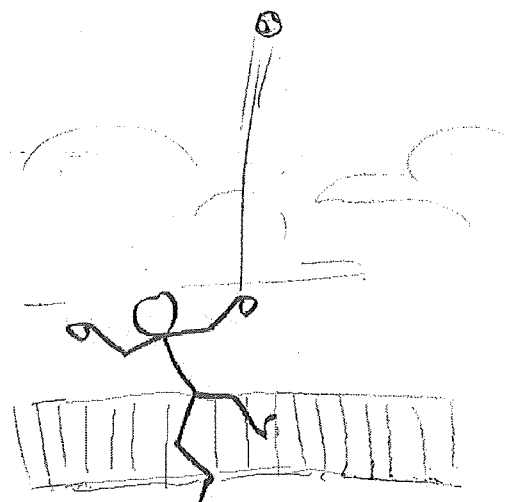
$$\cancel{PE_f} - PE_o = -(\cancel{KE_f} - KE_o)$$

$$-PE_o = KE_o$$

$$-\left(-\frac{Gm_1m_2}{r}\right) = \frac{1}{2}m_2v^2$$

$$\frac{Gm_1}{r} = \frac{1}{2}v^2$$

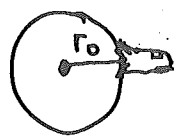
$$V_{esc} = \sqrt{\frac{2Gm_1}{r_o}}$$



What goes up must come down, unless the throw is really, REALLY hard.

$$KE_o = ?$$

$$KE_f = 0$$



$$PE_o = ?$$

$$PE_f = 0$$

**Example:** At what speed do you need to throw a 1.0 kg rock in order for it to break away from the Earth's pull?

$$V_{esc} = \sqrt{\frac{2(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.38 \times 10^6}} = \underline{1.1 \times 10^4 \text{ m/s}}$$

Does the mass of the rock matter?

No → not for determining the velocity  
yes → to determine KE

# Escape Velocity Problems:

1. What is the escape speed at the moon's surface? ( $2.4 \times 10^3$  m/s)

$$v = \sqrt{\frac{2Gm_1}{r}} = \sqrt{\frac{2(6.67 \times 10^{-11})(7.35 \times 10^{22})}{1.74 \times 10^6}} = 2.4 \times 10^3 \text{ m/s}$$

2. What is the mass of a planet that has an escape velocity of  $9.0 \times 10^3$  m/s and a radius of  $7.2 \times 10^6$  m? ( $4.37 \times 10^{24}$  kg)

$$9000 = \sqrt{\frac{2(6.67 \times 10^{-11})m_1}{7.2 \times 10^6}}$$

$$m_1 = 4.37 \times 10^{24} \text{ kg}$$

$$8.1 \times 10^7 = \frac{2(6.67 \times 10^{-11})m_1}{7.2 \times 10^6}$$

3. What is the mass of a planet that has a radius of  $2.57 \times 10^6$  m and an escape of  $2.92 \times 10^3$  m/s? ( $1.64 \times 10^{23}$  kg)

$$2.92 \times 10^3 = \sqrt{\frac{2(6.67 \times 10^{-11})m_1}{2.57 \times 10^6}}$$

$$m_1 = 1.64 \times 10^{23} \text{ kg}$$

$$8.53 \times 10^6 = \frac{2(6.67 \times 10^{-11})m_1}{2.57 \times 10^6}$$

4. What is the escape velocity from a planet with a mass of  $3.46 \times 10^{25}$  kg and a radius of  $2.75 \times 10^6$  m? ( $4.1 \times 10^4$  m/s)

$$v = \sqrt{\frac{2(6.67 \times 10^{-11})(3.46 \times 10^{25})}{(2.75 \times 10^6)}}$$

$$v = 4.1 \times 10^4 \text{ m/s}$$

5. What is the radius of a planet whose escape velocity is  $1.16 \times 10^3$  m/s and a mass of  $4.5 \times 10^{23}$  kg? ( $4.45 \times 10^7$  m)

$$1.16 \times 10^3 = \sqrt{\frac{2(6.67 \times 10^{-11})(4.5 \times 10^{23})}{r}}$$

$$1.35 \times 10^6 = \frac{2(6.67 \times 10^{-11})(4.5 \times 10^{23})}{r}$$

$$r = 4.45 \times 10^7 \text{ m}$$

COMPLETE THE PARTNER ESCAPE VELOCITY ACTIVITY (Hand-in when complete).