Chapter 1- Unit Pricing and Currency Exchange NOTES:

REVIEW – SIMPLIFYING FRACTIONS

To simplify a fraction, divide the numerator and denominator by the same number called a **common factor**. Easy common factors to start with are 2 for even numbers, 3, or 5. If the fraction that is the answer cannot be divided by any other common factor (Solution A, below), then it is in **lowest terms** or **simplest form**. If it can be divided again by another common factor (Solution B, below), keep repeating this process until it is in lowest terms.

Example 1: Simplify $\frac{18}{48}$ ← numerator
← denominator

Solution A: $\frac{18 \div 6}{48 \div 6} = \frac{3}{8}$ Simplify, using a factor of 6

Solution B: $\frac{18 \div 2}{48 \div 2} = \frac{9 \div 3}{24 \div 3} = \frac{3}{8}$ Simplify, using a factor of 2, and then 3

PROPORTIONAL REASONING

A **ratio** is *a comparison between two numbers measured in the same units*.

A ratio can be expressed in three ways as shown below:

as a fraction $\frac{9}{16}$

in words by using the word "**to**" **9 to 16**

a notation using colon **: 9 : 16**

Ratios, like fractions, can be simplified. For example, the ratio **150 : 15** can also be expressed

$$\frac{150}{15}$$

which can be simplified $\frac{150 \div 15}{15 \div 15} = \frac{10}{1}$

Notice that the numerator of the fraction is larger than the denominator. This can be common with ratios.

If two ratios are equivalent (equal), the first (top) number of each ratio compares to the second (bottom) number in an identical manner. You can represent this equivalence in the two ratios shown here:

$$\frac{150}{15} = \frac{10}{1}$$

An equation showing equivalent ratios is called a **proportion**.

Cross Multiply and Divide

When two fractions are equal to each other, any unknown numerator or denominator can be found. The following example shows the process.

Example 1: Solve for x when $\frac{x}{3} = \frac{2.1}{4}$

Solution: Cross multiply means multiply the numbers across the equals sign (the arrow). The divide part means divide that result by the number opposite the unknown (x) as shown below.

$$\frac{x}{3} \rightleftarrows \frac{2.1}{4}$$

This gives the result $x = 3 \times 2.1 \div 4$

In other words, if $\frac{x}{3} = \frac{2.1}{4}$, then $x = 3 \times 2.1 \div 4 = 1.575$

It does not matter where the unknown (x) is in the proportion, This process works for all situations.

This process can also be used when one side of the equal sign is not in fraction form.

Example 2: Find x when $27 = \frac{x}{3}$

Solution:

Step 1. The number 27 is the same as $\frac{27}{1}$. So, place a 1 under the 27 to get:

$$\frac{27}{1} = \frac{x}{3}$$

Step 2. Cross multiply and divide as above $\frac{27}{1} \rightleftarrows \frac{x}{3}$ to solve.

So: $x = 27 \times 3 \div 1$

$x = 81$

WORKING WITH RATIO

Ratios can be used in word problems to express the relationship between parts.

Example: Charlie works as a cook in a restaurant. His chicken soup recipe contains:

- 11 cups of seasoned broth
- 5 cups of diced vegetables
- 3 cups of rice
- 3 cups of chopped chicken

Write the ratios for each of the following relationships.

a) vegetables to chicken
b) broth to vegetables
c) chicken to rice
d) chicken to the total ingredients in the recipe

Solution: The ratios are the numbers for the items *in the order they are asked for*.

a) vegetables to chicken is 5:3
b) broth to vegetables is 11:5
c) chicken to rice is 3:3 or 1:1
d) chicken to the total ingredients in the recipe is 3:22 (11 + 5 + 3 + 3)

WORKING WITH PROPORTION

When given a ratio and one of the parts, write a proportion to solve using cross multiply and divide. **Use a letter or English words in the set-up to represent the parts** to put the numbers in the correct location.

Example 1: For a painting, Greg mixes inks to get the tint he wants. He uses a ratio of yellow ink to white ink of 3:1. How many mL of yellow ink would he use if he used 500 mL of white ink?

Solution: Set up a proportion using the known ratio and English letters/words to represent the colours. The words are essential components of the proportion.

$$\frac{\text{yellow}}{\text{white}} \quad \frac{3}{1} = \frac{x}{500}$$

$$x = 3 \times 500 \div 1 = 1500 \text{ mL of yellow ink}$$

Example 2: For a painting, Greg mixes inks to get the tint he wants. He uses a ratio of red ink to yellow in of 2:3. How many mL of yellow ink would he need if he used 500 mL of red ink?

Solution: Set up a proportion using the known ratio and English letters/words to represent the colours.

$$\frac{\text{red}}{\text{yellow}} \quad \frac{2}{3} = \frac{500}{x}$$

$x = 3 \times 500 \div 2 = 750$ mL of yellow ink

MORE WORKING WITH PROPORTION

Sometimes, the information given in a proportion question requires you to take a different approach to solving. This occurs when two parts are mixed to make something new, and the parts are then compared to the new mixture rather than each other.

Example: Susan is making lemonade. To make her lemonade, she mixes four parts of water with one part of lemon juice. If she wants to make 15 cups of lemonade, how many cups of water and how many cups of lemon juice does she need?

Solution: Since the ratio is 4:1, this means there are 5 parts in total in the lemonade. When solving this problem, one proportion is needed for each part of the ratio, comparing that part with the final mixture.

water: $\frac{\text{water}}{\text{lemonade}} \quad \frac{4}{5} = \frac{x}{15}$

So $x = 4 \times 15 \div 5 = 12$ cups of water

lemon juice: $\frac{\text{lemon juice}}{\text{lemonade}} \quad \frac{1}{5} = \frac{x}{15}$

So $x = 1 \times 15 \div 5 = 3$ cups of lemon juice

CALCULATING UNIT PRICE

The **unit price** of an item is *the cost of one unit*. It could be the cost of one can of pop in a 6-pack, it could be the cost of one egg in a dozen, it could be the cost of one kg in 10 kg of potatoes. Whatever the items, it is always the cost of one.

To calculate the unit price, ALWAYS **take the money and divide it by the amount** of the items you have. Remember to round your answer to 2 decimal places for money!

Example: If a carton of one dozen eggs costs $3.29, how much does one egg cost?

Solution: Divide the money by the amount of the item.

$3.29 ÷ 12 eggs = $0.27 per egg

WORKING WITH UNIT PRICE

Often unit price is used to compare costs and find which choice is the better buy (lower unit price). This is done by comparing the unit price of each item. It is not necessary to round values to only 2 decimal places (money) when comparing unit cost. In these cases, more decimals is usually better and makes it easier to compare.

Example: A 48-oz can of tomatoes costs $2.99. An 18-oz can costs $1.19. Which is the better buy?

Solution: Find the unit price of 1-oz for each can and compare those costs.

Can A – 48-oz can $2.99 ÷ 48 = $0.06229 (approximately)

Can B – 18-oz can $1.19 ÷ 18 = $0.06611 (approximately)

Therefore, the 48-oz can has a lower unit price and is the better buy.

MORE WORKING WITH UNIT PRICE

Often it is necessary to use the unit price to make a further calculation. This can be done by using a proportion to compare the two different quantities. This is really using the unit price without first calculating it! We are still comparing the cost with an amount of an item, but by doing it in a proportion the actual unit price is not directly calculated.

Example: If 1.5 m of fabric costs $12.63, how much will 2.75 m cost? Remember to use the English words to help with the set-up. This is essential!

Solution: Use a proportion to find the cost of 2.75 m

$$\frac{\text{fabric}}{\text{cost}} \qquad \frac{1.5\text{ m}}{2.75\text{ m}} = \frac{\$12.63}{x}$$

$$x = \$12.63 \times 2.75 \div 1.5 = \$23.16$$

WORKING WITH UNIT RATE

A **rate** is *a ratio comparing two numbers measured in different units*. This type of question may or may not include money and is solved in exactly the same way as unit price while paying attention to the units.

Some examples of rates include

- $1.69 / 100 g for the cost of ham at the deli
- 100 km/h for how fast a car travels
- $38.00/4 h for how much you earn at work

Example: If you drive 280 km in 4 hours, how long will it take you to drive 700 km?

Solution : Use a proportion to find the time to drive the 700 km. Remember to use the English words to help with the set-up. This is essential!

$$\frac{\text{hours}}{\text{km}} \qquad \frac{4\text{ h}}{280\text{ km}} = \frac{x}{700}$$

$x = 4 \times 700 \div 280 = 10$ hours

REVIEW – DECIMALS AND PERCENT

A **percent** is *a fraction out of 100*. So if you got a mark of 86% on a test that means you got the equivalent of 86 out if 100.

A) To change a percent into a decimal, simply divide the percent number by 100.

Example: What is 67% as a decimal?

Solution: 67% ÷ 100 = 0.67

B) To change a decimal to a percent, simply multiply the decimal by 100.

Example: Write the decimal 0.76 as a percent.

Solution: 0.76 × 100 = 76%

C) To change a fraction to a decimal, divide the numerator (top number) by the denominator (bottom number).

Example: Write the fraction $\frac{3}{5}$ as a decimal.

Solution: 3 ÷ 5 = 0.6

This decimal can then be changed to a percent as described in part B.

MORE DECIMALS AND PERCENT

Often we are asked to find a percent of a number. This can be done with a proportion, remembering that a percent is a number always out of 100. It can also be done by converting the percent to a decimal and multiplying. When the percentage is placed over 100, the % sign is dropped.

Example: Calculate 20% of 45

Solution A: Set up the proportion and solve. Remember to use the English words to help with the set-up. This is essential!

$$\frac{\text{part}}{\text{whole}} \qquad \frac{20}{100} = \frac{x}{45}$$

$x = 20 \times 45 \div 100 = 9$ So, 20% of 45 is 9.

Solution B: Convert 20% to a decimal and multiply to solve.

20% ÷ 100 = 0.20

0.20 × 45 = 9

NOTE: If the percentage is greater than 100 – and it can be – your answer will be larger than the number you started with.

It is also possible to find the percentage given two numbers. To calculate what percent one number is of another means you need to determine what number out of 100 is equal to your ratio. Using a proportion of simplifying a fraction are the two ways to solve this type of question.

Example: What percent is 5 of 20?

Solution A: Set up the proportion and solve.

$$\frac{\text{part}}{\text{whole}} \qquad \frac{x}{100} = \frac{5}{20}$$

$x = 5 \times 100 \div 20 = 25\%$ So, 5 of 20 is 25%

Solution B: Convert 5 of 20 to a fraction $\frac{5}{20}$, divide the numerator by the denominator, and multiply by 100 to solve.

5 ÷ 20 = 0.25

0.25 × 100 = 25%

PRICE MARKUPS

When a person owns a business, the merchandise is bought at a *wholesale cost* (what the business owner pays for the goods) and is always sold at a higher price, called the *retail price*. The difference between these two prices is called the **markup**, and it can be written in dollars or as a percentage of the wholesale price.

Example: Melanie owns a clothing store. She marks up the price of her goods by 85% of the wholesale price.

a) What is the markup, in dollars, on a coat that has a wholesale price of $125?

Solution: First, change 85% to a decimal.

85% ÷ 100 = 0.85

Second, multiply the wholesale price by the markup as a decimal.

$125 × 0.85 = $106.25 The markup is $106.25

b) What is the retail price? (The price Melanie charges her customers.)

Solution: Add the wholesale price and the markup together.

$125 + $106.25 = $231. 25 The retail cost is $231.25

TAXES

Taxes are added to the price of most articles and services purchased. Taxes are a percentage of the cost of the item. The higher the cost, the more taxes are paid. Tax rates are different in each province and territory across Canada. Some charge GST, PST, either, both, or HST. Tax rates will always be given to you for each question so you can solve the problems.

Example: Marie needs a pair of safety boots that cost $179.99.

a) How much will the taxes be if GST is 5% and PST is 6%?

Solution: These taxes can be calculated separately or combined in one calculation.

Separate taxes: 5% = 0.05
6% = 0.06

$179.99 × 0.05 = $9.00 The GST is $9.00
$179.99 × 0.06 = $10.80 The PST is $10.80

The total taxes paid will be $9.00 + $10.80 = $19.80

Combined taxes: 5% + 6% = 11%

$179.99 × 0.11 = $19.80 The taxes are $19.80

b) What is the final price of the boots?

Solution: The final price is calculated by adding the tax amounts to the retail price.

$179.99 + $19.80 = $199.75

6) The markup on a restaurant meal is 250%. A meal costs $7.25 to produce. How much will the customer be charged, after the markup, and then 5% GST are applied?

DISCOUNTS

When you go shopping, you have seen signs that say things like “Up to 50% Off” and “Discounted Prices.” These are **promotions**, activities that increase the awareness of a product or attract customers.
When working with sales or promotions, it is possible to find the ***discount*** – the amount saved – as well as the final sale price.

Example: Samantha is buying a new TV. She sees one marked $675.95, on sale for 20% off. How much will Samantha save (the discount) on her new TV?

Solution: The saving is 20% so find 20% of $675.95.

20% ÷ 100 = 0.20

0.20 × $675.95 = $135.19

Samantha will save $135.19 on the TV. The discount is $135.19

SALE PRICES

Once the discount is found for an item, the final price paid can be found easily by subtracting the discount from the original price. This is called the **sale price**.

Example: Samantha is buying a new TV. She sees one marked $675.95, on sale for 20% off. What is the sale price of the TV?

Solution: The saving is 20% so find 20% of $675.95. Then subtract this amount from the original price.

20% ÷ 100 = 0.20

0.20 × $675.95 = $135.19

The discount is $135.19.

The sale price of the TV is $675.95 – $135.19 = $540.76

Samantha will pay $540.76 for the TV.

SPECIAL SALE PRICES

Often, consumers wish to know the discount rate as a percent. To calculate this, take *the discount (in dollars) and divide it by the original cost (in dollars) and multiply by 100* to make the answer a percent.

Example 1: What is the percentage markdown of a sweater that regularly sells for $45.95, but is on sale for $24.99?

Solution: First, calculate the discount in dollars.

$45.95 - $24.99 = $20.96

The savings is the cost of one sweater - $20.96. This is the discount.

Divide that discount by the original price, and multiply by 100.

$20.96 ÷ $45.95 × 100 = 45.614

The percentage markdown is about 46%.

Example 2: Bamboo baskets are regularly priced at $19.98. They are on sale, advertised as “Buy two, get one at free.” What is the markdown (or discount rate), expressed as a percent?

Solution: First, calculate the regular cost of ***all*** the items – 3 baskets.

$19.98 × 3 = $59.94

The savings is the cost of one basket - $19.98. This is the discount.

Divide the savings by the regular price, and multiply by 100.

$19.98 ÷ 59.94 × 100 = 33.333 The discount rate/percentage markdown is 33%.

Notice that for both examples the part (discount or mark-up) is divided by the original amount (wholesale or retail price), times 100 to get a percentage.

Up until this point, taxes have been ignored. But taxes usually must be added to the sale price to calculate the final cost.

Remember that taxes are added only to the selling price, not the original price.

The example on the next page shows how to add the taxes with these special sale price types of questions.

Example 3: Samantha is buying a new TV. She sees one marked $675.95, on sale for 20% off. What is the final price of the TV if she is charged 12% HST?

Solution: First, find 20% of $675.95.

20% ÷ 100 = 0.20
0.20 × $675.95 = $135.19
The discount is $135.19.

Subtract this amount from the original price to get the sale price.
$675.95 – $135.19 = $540.76
The sale price of the TV is $540.76

The taxes are calculated on the sale price.
12% ÷ 100 = 0.12
$540.76 × 0.12 = $64.89

The final price, including taxes, is $540.76 + $64.89 = $605.65

CURRENCY EXCHANGE RATES

Different countries use different currencies and/or monetary units. For example, both Canada and the United States use a currency called the dollar, but they are different dollars. It is important to consider exchange rates when travelling to different countries. An exchange rate is simply the price of one country's currency in terms of another country's currency. The exchange rate will always be given to you, either in a chart or as a value.

Example 1: Lucas needs to convert $500 Canadian dollars (CAD) into American dollars (USD). If one Canadian dollar is worth 0.94192 of an American dollar, how many American dollars will Lucas receive?

Solution: To calculate this value, a proportion can be set up. Use the countries to help set-up the question properly.

$$\frac{\text{CAD}}{\text{USD}} \quad \frac{1}{0.94192} = \frac{500}{x}$$

$x = 500 \times 0.94192 \div 1 = \$470.96$ USD

Example 2: One Thai baht is worth 0.023541 of a Canadian dollar. How many bahts would a tourist in Thailand receive for $200 CAD?

Solution: To calculate this value, a proportion can be set up.

$$\frac{\text{Thai}}{\text{CAD}} \quad \frac{1}{0.023541} = \frac{x}{\$200}$$

$x = 200 \times 1 \div 0.023541 = 8495.92$ bahts

It is very important to use the English words as part of the proportion when setting up these problems. ***IT IS REALLY IMPORTANT!*** This will make sure that you put the

numbers in the correct spots for solving the problem. **DO NOT** just think you can multiply or divide. You will make mistakes if you do that.

MORE CURRENCY EXCHANGE RATES

Exchange rates change from day to day and from one currency to another. Exchanges set a **buying rate** and a **selling rate** for each currency, and these rates are different from each other. The buying rate is the rate that a bank will buy a currency from you while the selling rate is the rate at which the bank will sell a currency to you. It is important to always think about what the bank is doing in currency exchanges, not what the customer is doing!

Example 1: On a given day, the bank selling rate of the Swiss franc compared to the Canadian dollar is 1.0501, and the buying rate is 1.0213.

a) How many Swiss francs would Anne receive for $400 CAD?
b) If Anne sold the Swiss francs back to the bank on the same day, how much would she receive?
c) Why is there a difference, and how much is the difference?

Solutions: a) The bank is selling Swiss francs to Anne so use the selling rate of 1 Swiss franc costs 1.0501 CAD dollar.

$$\frac{\text{SFr}}{\text{CAD}} \quad \frac{1}{1.0501} = \frac{x}{\$400}$$

$x = 400 \times 1 \div 1.0501 = 380.92$ francs

b) The bank buys the Swiss francs back at a buying rate of 1 Swiss franc paying out $1.0213 CAD dollar.

$$\frac{\text{SFr}}{\text{CAD}} \quad \frac{1}{1.0213} = \frac{380.92\ \text{SFr}}{x}$$

$x = 380.92 \times 1.0213 \div 1 = 389.03$ CAD

c) There is a difference because the selling rate is always higher than the buying rate in order for the bank to make a profit!

The difference is $400 - $389.03 = $10.97
It cost Anne $10.97 to make these transactions.

When converting between currencies, if you are not told which number goes with which currency (ex: $1 CAD = $0.94192 US), then the “funky number” always goes with the Canadian currency and the other currency gets a 1.

Example 2: If the exchange rate is 0.366262 between Bahraini dinar and the Canadian dollar, how many dinar would you get for $550 CAD?

Solution: The “funky number” **0.366262** always goes with the Canadian currency, so the proportion is set up as shown.

$$\frac{\text{BAH}}{\textbf{CAD}} \quad \frac{1}{\textbf{0.366262}} = \frac{x}{550}$$

This would now be solved in the usual way by cross-multiply and divide.