

# Lesson 1: Solving Systems of Linear Equations by Graphing

Friday, August 31, 2018 2:33 AM

## ***Systems of Linear Equations Lesson #1: Solving Systems of Linear Equations by Graphing***

### ***Overview of Unit***

pg. 451

In this unit, we solve problems that involve systems of linear equations in two variables. We do this by inspection, by graphing, and algebraically using the method of substitution and the method of elimination.

### ***Exploring a Meaning of Two Intersecting Lines***

The Smith family and the Harper family are going to a book fair which is raising money for charity. Mr. Smith pays an entry fee of \$11 for three adults and one child. Mrs. Harper pays an entry fee of \$12 for two adults and three children.

We can determine the cost of an adult ticket and the cost of a child ticket by forming two linear equations and graphing them.

Let  $x$  be the entry fee for an adult ticket and let  $y$  be the entry fee for a child. The information about the Smith family can be modelled by the equation  $3x + y = 11$ , and information about the Harper family can be modelled by the equation  $2x + 3y = 12$ .

- a) Draw the graphs of the equations  $3x + y = 11$  and  $2x + 3y = 12$  on the grid without using technology.

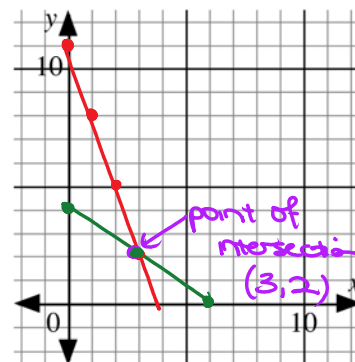
$y = mx + b$

$$3x + y = 11$$
$$y = -3x + 11$$

slope  $-3$  y-int  $11$

$$2x + 3y = 12$$
$$y = -\frac{2}{3}x + 4$$

slope  $-\frac{2}{3}$  y-int  $4$



- b) The graphs of the equations intersect at a point. State the coordinates of this point and explain what the coordinates represent in the context of the question.

$(3, 2)$

$x$  adult  $y$  child

The adult fee is \$3  
The child fee is \$2

### Systems of Equations

In the exploration on the previous page, we worked with the equation  $3x + y = 11$ . There are many values for  $x$  and  $y$  which satisfy this equation, e.g.  $x = 1$  and  $y = 8$ , or  $x = 2$  and  $y = 5$ , or  $x = 3$  and  $y = 2$ , etc.

We also worked with the equation  $2x + 3y = 12$ . There are also many values for  $x$  and  $y$  which satisfy this equation, e.g.  $x = 0$  and  $y = 4$ , or  $x = 3$  and  $y = 2$ , or  $x = 4.5$  and  $y = 1$ , etc.

If we consider both of these equations simultaneously, there is only one solution,  $x = 3$  and  $y = 2$ .

The equations  $3x + y = 11$  and  $2x + 3y = 12$ , considered at the same time, are called a **system of equations**.

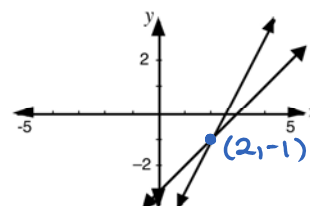
The **solution** to this system of equations is  $x = 3$  and  $y = 2$ . This is because  $x = 3$  and  $y = 2$  satisfy each equation in the system.

Graphically, the solution to the system is the **point of intersection** of the two lines.



A system of equations has been represented on the grid. The system has an integral solution.

- a) State the solution  $x = \underline{2}$ ,  $y = \underline{-1}$
- b) Write the solution as an ordered pair.  
 $(2, -1)$



Consider the system of equations  $2x + y = 2$ ,  $x - 3y = 15$ .

- a) Graph the system of equations without using technology.

$$y = mx + b$$

$$2x + y = 2$$

$$y = -2x + 2$$

slope  $-\frac{2}{1}$  y-int  $2$

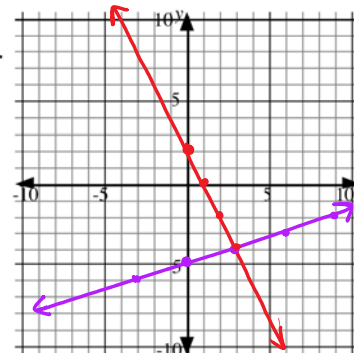
$$x - 3y = 15$$

$$-3y = -x + 15$$

$$y = \frac{1}{3}x - 5$$

slope  $\frac{1}{3}$  y-int  $-5$

- b) State the solution to the system of equations.  
 $(3, -4)$



- c) Algebraically verify the solution by replacing the values in the original equations.

$$2x + y = 2$$

$$2(3) + (-4) = 2$$

$$6 - 4 = 2$$

$$2 = 2 \checkmark$$

$$x - 3y = 15$$

$$3 - 3(-4) = 15$$

$$3 + 12 = 15$$

$$15 = 15 \checkmark$$

### Complete Assignment Questions #1 and #2

***Solving a System of Equations using a TI Graphing Calculator***

Check that the calculator is in “Function” mode.

Use the following procedure to find the solution to a system of equations.

1. Write each equation in terms of  $y$ .
2. Access the “Y= editor” by pressing the **Y=** key.
3. Enter one equation in **Y<sub>1</sub>**.
4. Enter the other equation in **Y<sub>2</sub>**.
5. Press the **GRAPH** key to display the graphs.
6. Access the intersect command by pressing **2nd** then **TRACE** and scroll down to “intersect”.  
The calculator will return to the display window with the graphs.
7. The calculator will display “First curve?”. Use the cursor key, if necessary, to select the first graph and then press **ENTER**.
8. The calculator will display “Second curve?”. Use the cursor key, if necessary, to select the second graph and then press **ENTER**.
9. The calculator will display “Guess?”. Press **ENTER**.



Consider the system of equations from the exploration at the beginning of this lesson.

$$\begin{aligned} 3x + y &= 11 \\ 2x + 3y &= 12. \end{aligned}$$

- a) Rewrite each equation in slope  $y$ -intercept form.
- b) Use a graphing calculator to graph each equation.
- c) State a suitable window which shows both sets of  $x$ - and  $y$ -intercepts and the point of intersection.
- d) Solve the system of equations using the features of the graphing calculator.  
Confirm the amount of the entry fees established in the exploration.



If a decimal value appears for the  $x$  and/or  $y$  coordinates, then the  $x$  and/or  $y$  value can be converted to an exact value (as long as it is not irrational and is within the limitations of the calculator) by using the following steps.

#### For the $x$ -coordinate

1. Exit the graphing screen by pressing **CLEAR** twice.
2. Press **X,T, θ, n** key, then press **ENTER** to import the  $x$ -coordinate.
3. To display the exact value,  
Press **MATH**, select “Frac ”, then press **ENTER**.

#### For the $y$ -coordinate

Except for step 2, the instructions to import the  $y$ -coordinate are the same as above.

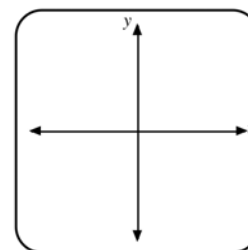
For step 2, press **ALPHA** **1** **ENTER** to import the  $y$ -coordinate value.  
Then proceed to step 3 above.



Class Ex. #4

- a) Solve the following system of equations using a graphing calculator.

$$\begin{aligned} 6a + 7b &= 5 \\ 3a &= 14b \end{aligned}$$



- b) List the answers as exact values using the technique above.
- c) Algebraically verify the solution.

#### Complete Assignment Questions #3 - #7

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## Assignment

1. Consider the system of equations  $x - 2y = 3$ ,  $x + y = 0$ .

a) Write each equation in slope y-intercept form.

b) Complete the table of values for each equation.

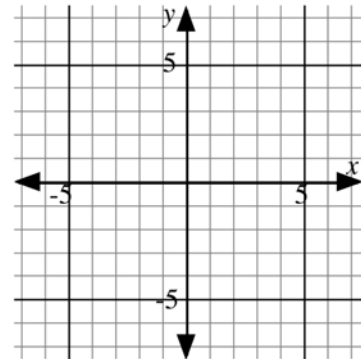
$$x - 2y = 3$$

$x$	$y$
-3	
-1	
1	
3	
5	

$$x + y = 0$$

$x$	$y$
-4	
-2	
0	
2	
4	

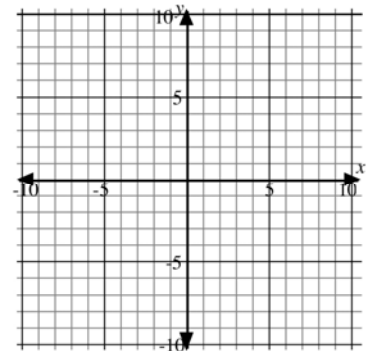
c) Draw the lines on the grid and state the solution to the system.



d) Verify the solution.

2. The following system of equations is given:  $x - y = 7$ ,  $x + 5y = -5$

a) Without using technology, graph each equation and hence solve the system.



b) Verify the solution.

3. In each case, solve the system of equations using technology. Verify the solution by replacing the values in the original equations.

a)  $y = 3x - 7$   
 $y = -x + 9$

b)  $y = -x$   
 $y = -\frac{1}{3}x + 3$

c)  $y = x - 2$   
 $y = \frac{3}{4}x - 4$

d)  $3x + 2y = 5$   
 $x - y = 1$

e)  $4a - b = 6$   
 $3a + b = 1$

f)  $0.6p - 0.8q = 2.6$   
 $5p + 6q = 9$

4. Solve the following systems of equations using technology. List the answers as exact values.

a)  $4x - y + 6 = 0$ ,  $y = x + 2$

b)  $8x - 3y = 5$ ,  $5x + 3y = 2$

**Multiple Choice**

5. The ordered pair  $(x, y)$  which satisfies the system of equations  $x - 2y = 6$ ,  $x + 6y = 22$  is

- A.  $(-10, 2)$
- B.  $(2, 10)$
- C.  $(10, -2)$
- D.  $(10, 2)$

**Numerical Response**

6. If  $7x - 5y = 19$  and  $2x + 3y = 17$ , then the value of  $x$ , to the nearest tenth, is \_\_\_\_\_.

(Record your answer in the numerical response box from left to right)

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7. A pear costs 24 cents less than two apples. Four apples cost the same as three pears. In order to determine the cost of each piece of fruit, Courtney graphs the equations  $y = 2x - 24$  and  $4x = 3y$ . The cost of a pear, in cents, is \_\_\_\_\_.

(Record your answer in the numerical response box from left to right)

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**Answer Key**

1. a)  $y = \frac{1}{2}x - \frac{3}{2}$ ,  $y = -x$     b) 

$x$	$y$
-3	-3
-1	-2
1	-1
3	0
5	1

$x$	$y$
-4	4
-2	2
0	0
2	-2
4	-4
- c)  $x = 1$ ,  $y = -1$
2. a)  $x = 5$ ,  $y = -2$
3. a)  $x = 4$ ,  $y = 5$     b)  $x = -4.5$ ,  $y = 4.5$     c)  $x = -8$ ,  $y = -10$   
d)  $x = 1.4$ ,  $y = 0.4$     e)  $a = 1$ ,  $b = -2$     f)  $p = 3$ ,  $q = -1$
4. a)  $x = -\frac{4}{3}$ ,  $y = \frac{2}{3}$     b)  $x = \frac{7}{13}$ ,  $y = -\frac{3}{13}$

5. D

6.

4	.	6	
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7.

4	8		
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