

Arithmetic Sequences Lesson #1:

Investigating Patterns and Sequences

Overview

In this unit we investigate patterns used to define types of sequences. We then focus on arithmetic sequences and apply formal language to increasing and decreasing linear patterns. We derive a rule for determining the general term of an arithmetic sequence and explore problems relating to arithmetic growth and decay. As an extension we explore arithmetic series.

Investigation 1

Jesse is making a tower using playing cards. The top three rows of the tower are shown.

The top row (Row 1) requires three playing cards.

The second row (Row 2) requires six additional playing cards.

Continue the pattern for two more rows, and complete Tables A and B below.

Row 1

Row 2

Row 3

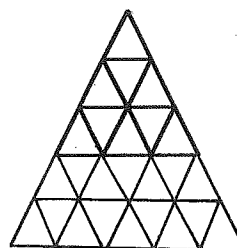


Table A

Row Number	1	2	3	4	5
Number of Additional Cards in the Row	3	6	9	12	15

Table B

Row Number	1	2	3	4	5
Number of Triangles in the Row	1	3	5	7	9

In grades 8 and 9 you learned how to determine a formula from a specific pattern. In this lesson, we focus on investigating patterns to identify different types of sequences, a skill which is useful in patterns of numbers which are more difficult to identify. In the next lesson we develop a formula and use this formula for patterns of numbers which are arithmetic.

Investigation 2

Each row in the triangle, named after French mathematician Blaise Pascal, begins and ends with the number 1.

Apart from the ones, every other number in a particular row can be determined by adding the two numbers diagonally above it.

Continue the pattern for rows 5, 6, and 7, and complete Table C.

Table C

Row Number	1	2	3	4	5	6	7
Sum of the Numbers in the Row	1	2	4	8	16	32	64

Row	1	1					
2	1	1					
3	1	2	1				
4	1	3	3	1			
5	1	4	6	4	1		
6	1	5	10	10	5	1	
7	1	6	15	20	15	6	1

Investigation 3

Triangle 1, shown in Diagram 1, is an equilateral triangle with sides of length 32 cm. A smaller triangle, Triangle 2, is placed inside Triangle 1 by joining the midpoints of Triangle 1 (as illustrated in Diagram 2). The pattern is continued as illustrated in Diagram 3.

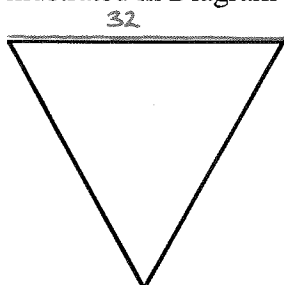


Diagram 1

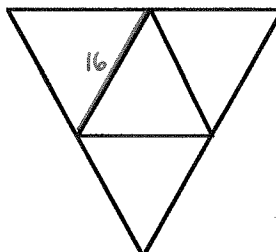


Diagram 2

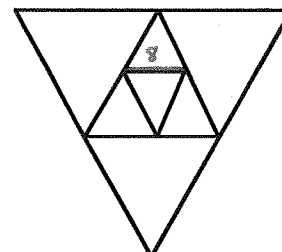


Diagram 3

a) Diagrams 4 and 5 in the sequence have not been completed. Complete each diagram.

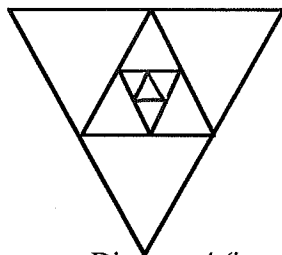


Diagram 4 (incomplete)

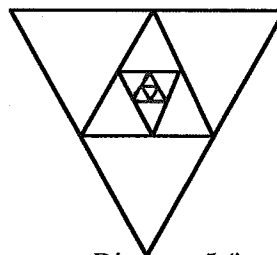


Diagram 5 (incomplete)

b) Complete Tables D and E below.

Table D	Triangle Number	1	2	3	4	5
	Length of Side of Triangle (cm)	32	16	8	4	2

Table E	Diagram Number	1	2	3	4	5
	Number of Triangles of any Size in the Diagram	1	5	9	13	17

Investigation 4

The mean daily temperature in Prince George on Dec 1 was 8°C . For the next six days, the mean daily temperature decreased by 4°C each day.

Complete Table F below.

Table F	Day Number	1	2	3	4	5
	Temperature ($^{\circ}\text{C}$)	8	4	0	-4	-8

In each of the tables A - F, the top row consists of the **natural numbers** 1, 2, 3, 4, etc., and the bottom row consists of a **sequence** of numbers related to the natural numbers in a specific order.

The first table you completed was

Table A

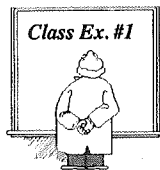
Row Number	1	2	3	4	5
Number of Cards in the Row	3	6	9	12	15

The sequence formed is 3, 6, 9, 12, 15. The sequence consists of **terms**.

$$t_1 = 3 \quad t_2 = 6 \quad t_3 = 9 \quad t_4 = 12 \dots$$

The first term, written t_1 , is equal to 3. The second term, t_2 , is equal to 6; $t_3 = 9$ etc.

The term is represented by the variable t , and the subscript represents the term number
ex = t_5 Term# (term 5)



In each of the following:

- Complete the table using the information from Tables A - F.
- Complete the statement, explaining how to find the next term in the sequence from the previous term using only addition or multiplication.
- State the next two terms of the sequence.

Table A (pg 505)

n	1	2	3	4	5
t_n	3	6	9	12	15

The next term can be calculated by adding 3 to the previous term.

$$t_6 = 18 \quad \text{and} \quad t_7 = 21$$

Table B (pg 505)

n	1	2	3	4	5
t_n	1	3	5	7	9

The next term can be calculated by adding 2 to the previous term.

$$t_6 = 11 \quad \text{and} \quad t_7 = 13$$

Table C (pg 505)

n	1	2	3	4	5	6	7
t_n	1	2	4	8	16	32	64

The next term can be calculated by multiplying the previous term by 2

$$t_8 = 128 \quad \text{and} \quad t_9 = 256$$

Table D (pg 506)

n	1	2	3	4	5
t_n	32	16	8	4	2

The next term can be calculated by multiplying the previous term by $\frac{1}{2}$

$$t_6 = 1 \quad \text{and} \quad t_7 = \frac{1}{2}$$

Table E (pg 506)

n	1	2	3	4	5
t_n	1	5	9	13	17

The next term can be calculated by adding 4 to the previous term.

$$t_6 = 21 \quad \text{and} \quad t_7 = 25$$

Table F (pg 506)

n	1	2	3	4	5
t_n	8	4	0	-4	-8

The next term can be calculated by adding -4 to the previous term.

$$t_6 = -12 \quad \text{and} \quad t_7 = -16$$

Types of Sequences

There are different types of sequences.

A sequence in which the next term is formed by adding a constant (positive or negative) to the previous term is called an **arithmetic sequence**.

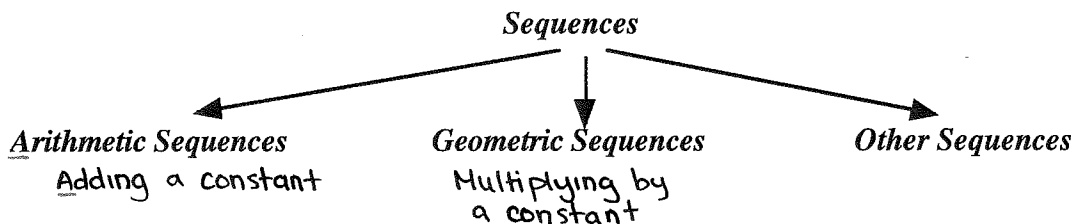
A sequence in which the next term is formed by multiplying the previous term by a constant (positive or negative) is called a **geometric sequence**.

There are other sequences, which are not arithmetic or geometric.

For example: Sequence of Prime Numbers 2, 3, 5, 7, 11, ...

A Fibonacci Sequence, 1, 1, 2, 3, 5, 8, 13, ... , is a special type of sequence which occurs in nature in such things as seed growth, leaves on stems, petals on flowers, etc.

In the remaining lessons in this unit, we focus on arithmetic sequences. Geometric sequences will be studied in a later course.



Classify the sequences in Tables A - F as arithmetic sequences or geometric sequences.

Arithmetic = A, B, E, F

Geometric = C, D

Finite and Infinite Sequences

Finite Sequence - a sequence that has a specific number of terms.

eg. 4, 10, 16, 22, 28 or 2, 4, 8, 16, ... 256.
 ↳ Ends at a specific #

Infinite Sequence - a sequence that has an unlimited number of terms.

eg. 4, 10, 16, 22, 28 ... or 2, 4, 8, 16, ...
 ↳ does not end, is represented by "..."

A Sequence as a Function

A sequence can be regarded as a **function** relating the set of natural numbers to the terms of the sequence.

The **domain** of the function is the set of natural numbers.

The **range** of the function is the set of terms of the sequence.

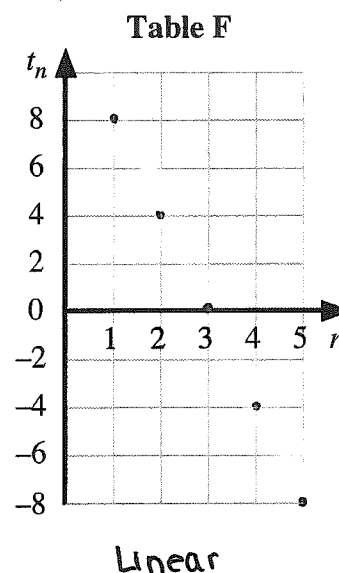
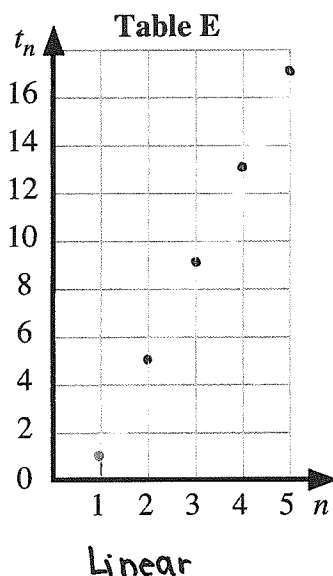
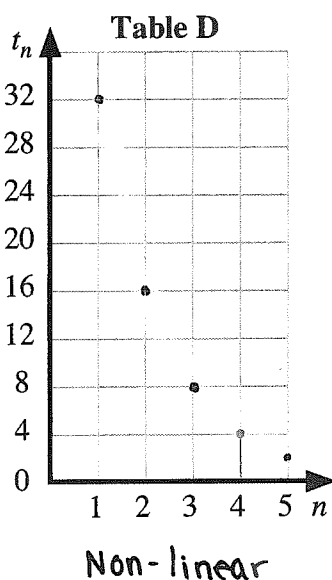
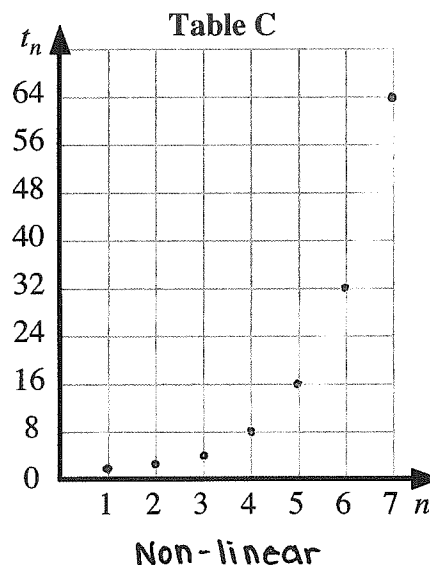
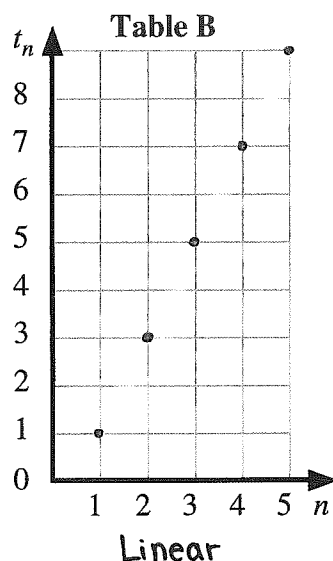
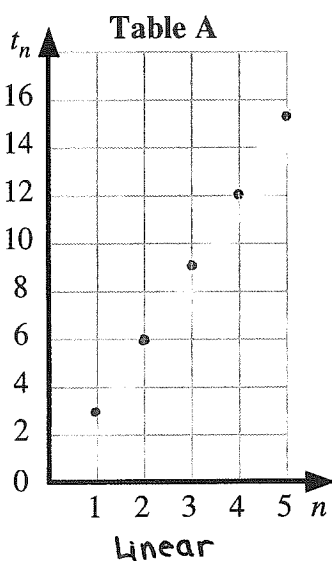
natural numbers are whole numbers, greater than zero.
 \therefore when drawing a graph, do NOT connect the points

Some sequences can be represented by linear functions, and some can be represented by non-linear functions.

Class Ex. #3



Using the information in **Tables A - F**, plot the points (n, t_n) on the grids. In each case, state whether the function represented by the sequence is linear or non-linear.





Circle the correct alternative in the following statements.

- a) A sequence in which the next term is determined by adding a constant to the previous term is an arithmetic / geometric sequence.

The sequence can be represented by a linear / non-linear function.

- b) A sequence in which the next term is determined by multiplying the previous term by a constant is an arithmetic / geometric sequence.

The sequence can be represented by a linear / non-linear function.

Complete Assignment Questions #1 - #12

Assignment

1. Consider the pattern of squares within squares shown below.

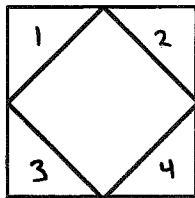


Diagram 1

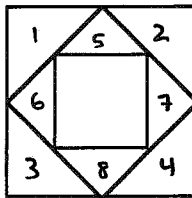


Diagram 2

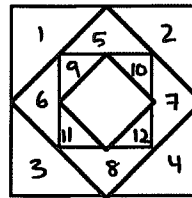


Diagram 3

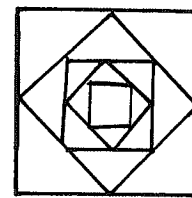


Diagram 4

- a) Draw the next diagram in the pattern in the space above.
b) Complete the table.

Diagram Number	1	2	3	4
Number of Triangles in the Diagram	4	8	12	16

- c) Consider the sequence of triangles in the table above.

- (i) Complete the statement explaining how to find the next term in the sequence from the previous term.

The next term can be calculated by Adding 4 to the previous term.

- (ii) State the value of the following terms. $t_1 = \underline{4}$ $t_5 = \underline{20}$ $t_6 = \underline{24}$

- (iii) Circle the correct alternatives.

The sequence is arithmetic / geometric and can be represented by a linear / non-linear function.