Systems of Linear Equations Lesson #1:
Solving Systems of Linear Equations by Graphing

Overview of Unit

In this unit, we solve problems that involve systems of linear equations in two variables. We do this by inspection, by graphing, and algebraically using the method of substitution and the method of elimination.

Exploring a Meaning of Two Intersecting Lines

The Smith family and the Harper family are going to a book fair which is raising money for charity. Mr. Smith pays an entry fee of \$11 for three adults and one child. Mrs. Harper pays an entry fee of \$12 for two adults and three children.

We can determine the cost of an adult ticket and the cost of a child ticket by forming two linear equations and graphing them.

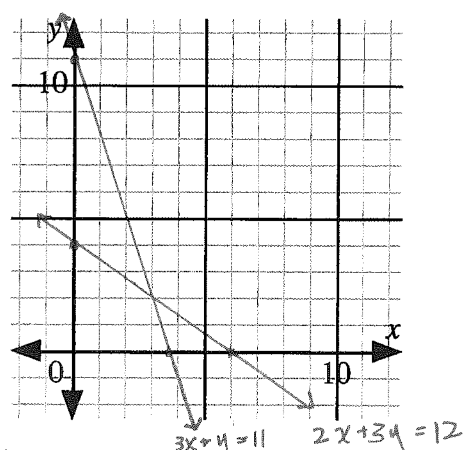
Let $\$x$ be the entry fee for an adult ticket and let $\$y$ be the entry fee for a child. The information about the Smith family can be modelled by the equation $3x + y = 11$, and information about the Harper family can be modelled by the equation $2x + 3y = 12$.

a) Draw the graphs of the equations $3x + y = 11$ and $2x + 3y = 12$ on the grid without using technology.

$3x + y = 11$
find x and y intercepts
x int ⇒ $y = 0$
$3x + (0) = 11$
$3x = 11$
$x = 11/3$
$\therefore (\frac{11}{3}, 0)$
y int ⇒ $x = 0$
$3(0) + y = 11$
$y = 11$
$\therefore (0, 11)$

plug in zero and solve for variable

$2x + 3y = 12$
find x and y intercepts
x int ⇒ $y = 0$
$2x + 3(0) = 12$
$2x = 12$
$x = 6$
$\therefore (6, 0)$
y int ⇒ $x = 0$
$2(0) + 3y = 12$
$3y = 12$
$y = 4$
$\therefore (0, 4)$



Plot points found and connect each graph.

b) The graphs of the equations intersect at a point. State the coordinates of this point and explain what the coordinates represent in the context of the question.

graphs intersect at point (3,2)

The cost of an adult ticket is \$3

The cost of a child ticket is \$2

Systems of Equations

In the exploration on the previous page, we worked with the equation $3x + y = 11$.
There are many values for x and y which satisfy this equation, e.g. $x = 1$ and $y = 8$, or $x = 2$ and $y = 5$, or $x = 3$ and $y = 2$, etc.

We also worked with the equation $2x + 3y = 12$.
There are also many values for x and y which satisfy this equation, e.g. $x = 0$ and $y = 4$, or $x = 3$ and $y = 2$, or $x = 4.5$ and $y = 1$, etc.

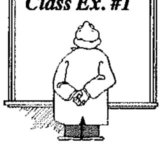
If we consider both of these equations simultaneously, there is only one solution, $x = 3$ and $y = 2$.

The equations $3x + y = 11$ and $2x + 3y = 12$, considered at the same time, are called a **system of equations**.

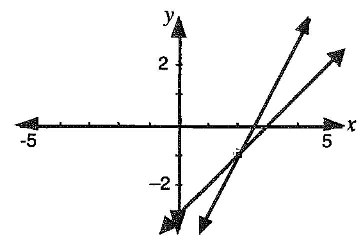
The **solution** to this system of equations is $x = 3$ and $y = 2$.
This is because $x = 3$ and $y = 2$ **satisfy** each equation in the system.

Graphically, the solution to the system is the point of intersection of the two lines.

Class Ex. #1



A system of equations has been represented on the grid.
The system has an integral solution.

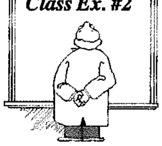


a) State the solution $x =$ <u>2</u> , $y =$ <u>-1</u>

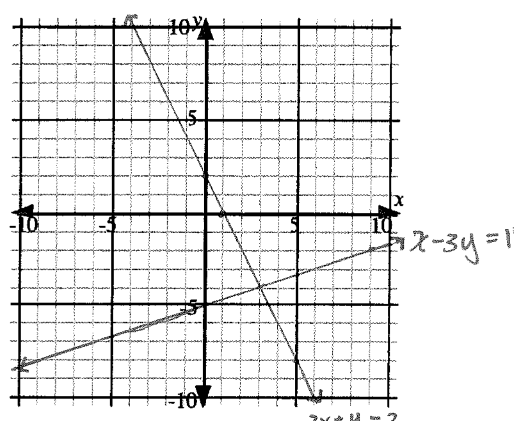
b) Write the solution as an ordered pair.

(2, -1)

Class Ex. #2



Consider the system of equations $2x + y = 2$, $x - 3y = 15$.



a) Graph the system of equations without using technology.

Find coordinates on each graph.
ex) find x and y intercepts

$2x + y = 2$
$2x = 2$ $\quad 2(0) + y = 2$
$x = 1$ $\quad y = 2$
$(1, 0)$ $\quad (0, 2)$

$x - 3y = 15$
$x - 3(0) = 15$ $\quad (0) - 3y = 15$
$x = 15$ $\quad y = -5$
$(15, 0)$ $\quad (0, -5)$

also point $(9, -2)$

b) State the solution to the system of equations.

$(3, -4)$

c) Algebraically verify the solution by replacing the values in the original equations.

$2x + y = 2$
LS: $2(3) + (-4) = 2$ ✓
RS: 2 ✓

$x - 3y = 15$
LS: $(3) - 3(-4) = 15$ ✓
RS: 15 ✓

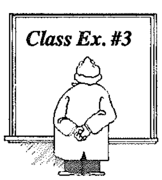
LS = RS
∴ verified ✓

Conplete Assignement Questions #1 and #2

Solving a System of Equations using a TI Graphing Calculator

Check that the calculator is in "Function" mode.
Use the following procedure to find the solution to a system of equations.

1. Write each equation in terms of y.
2. Access the "Y= editor" by pressing the $\boxed{\text{Y=}}$ key.
3. Enter one equation in $\boxed{Y_1}$.
4. Enter the other equation in $\boxed{Y_2}$.
5. Press the $\boxed{\text{GRAPH}}$ key to display the graphs.
6. Access the intersect command by pressing $\boxed{\text{2nd}}$ then $\boxed{\text{TRACE}}$ and scroll down to "**intersect**". The calculator will return to the display window with the graphs.
7. The calculator will display "**First curve?**". Use the cursor key, if necessary, to select the first graph and then press $\boxed{\text{ENTER}}$.
8. The calculator will display "**Second curve?**". Use the cursor key, if necessary, to select the second graph and then press $\boxed{\text{ENTER}}$.
9. The calculator will display "**Guess?**". Press $\boxed{\text{ENTER}}$.



Consider the system of equations from the exploration at the beginning of this lesson.

$$3x + y = 11$$
$$2x + 3y = 12.$$

a) Rewrite each equation in slope y-intercept form. $(y = mx + b)$

$3x + y = 11$
$y = -3x + 11$

$2x + 3y = 12$
$3y = -2x + 12$
$y = -\frac{2}{3}x + 4$

b) Use a graphing calculator to graph each equation.

c) State a suitable window which shows both sets of x- and y-intercepts and the point of intersection.

$x: [-5, 10, 1]$ $\quad$ $y: [-10, 15, 2]$

d) Solve the system of equations using the features of the graphing calculator. Confirm the amount of the entry fees established in the exploration.

$(3, 2)$ confirms the amount of the entry fees in the exploration.

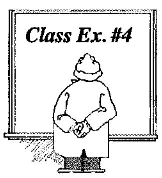
If a decimal value appears for the x and/or y coordinates, then the x and/or y value can be converted to an exact value (as long as it is not irrational and is within the limitations of the calculator) by using the following steps.

For the x-coordinate

1. Exit the graphing screen by pressing **CLEAR** twice.
2. Press **X,T, θ, n** key, then press **ENTER** to import the x-coordinate.
3. To display the exact value,
   Press **MATH**, select "**Frac** ", then press **ENTER**.

For the y-coordinate

Except for step 2, the instructions to import the y-coordinate are the same as above.
For step 2, press **ALPHA** **1** **ENTER** to import the y-coordinate value.
Then proceed to step 3 above.



a) Solve the following system of equations using a graphing calculator.

$$6a + 7b = 5$$
$$3a = 14b$$

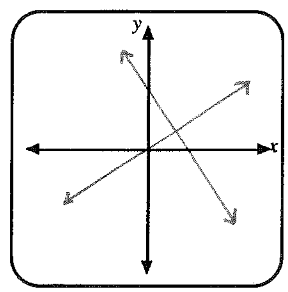
solve for "b"

$7b = -6a + 5$ $\quad$ $14b = 3a$

$b = -\frac{6}{7}a + \frac{5}{7}$ $\quad$ $b = \frac{3}{14}a$

Graph: $y_1 = -\frac{6}{7}x + \frac{5}{7}$

$y_2 = \frac{3}{14}x$



b) List the answers as exact values using the technique above.

$a = \frac{2}{3}$ $\quad$ $b = \frac{1}{7}$

c) Algebraically verify the solution.

$6a + 7b = 5$

LS: $6(\frac{2}{3}) + 7(\frac{1}{7}) = 4 + 1 = 5$

RS: 5

LS = RS

$3a = 14b$

LS: $3(\frac{2}{3}) = 2$

RS: $14(\frac{1}{7}) = 2$

LS = RS

Verified ✓

Complete Assignment Questions #3 - #7

Assignment

1. Consider the system of equations $x - 2y = 3$, $x + y = 0$.

a) Write each equation in slope y-intercept form.

$x - 3 = 2y$

$y = \frac{1}{2}x - \frac{3}{2}$

$y = -x$

b) Complete the table of values for each equation.

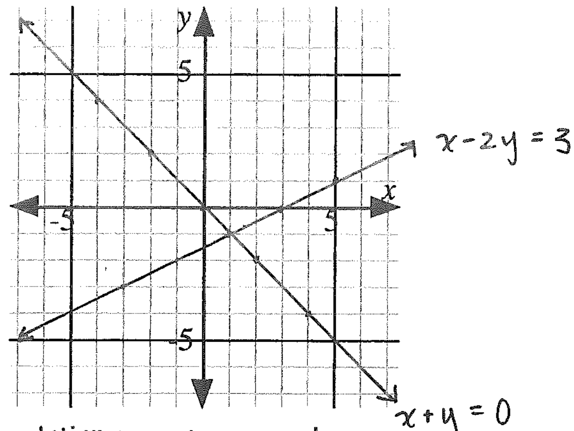
$x - 2y = 3$

x	y
–3	-3
–1	-2
1	-1
3	0
5	1

$x + y = 0$

x	y
–4	4
–2	2
0	0
2	-2
4	-4

c) Draw the lines on the grid and state the solution to the system.



solution: $x = 1$, $y = -1$

d) Verify the solution.

$x - 2y = 3$

LS: $(1) - 2(-1) = 3$

RS: 3

LS = RS

$x + y = 0$

LS: $(1) + (-1) = 0$

RS: 0

LS = RS

verified.

2. The following system of equations is given: $x - y = 7$, $x + 5y = -5$

a) Without using technology, graph each equation and hence solve the system.

$x - y = 7$

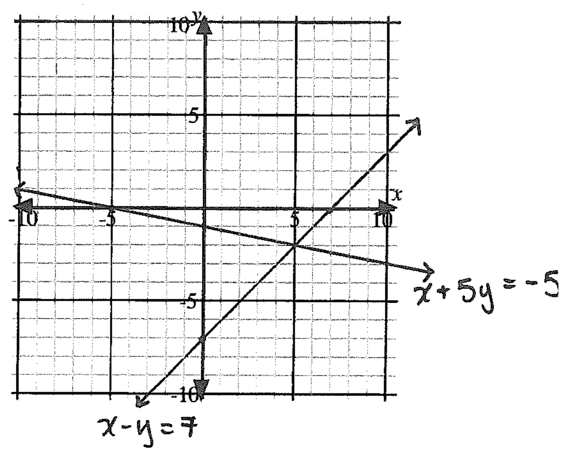
x int. = 7
$\therefore (7, 0)$

y int. = -7
$\therefore (0, -7)$

$x + 5y = -5$

x int = -5
$\therefore (-5, 0)$

y int. = -1
$\therefore (0, -1)$



solution: $x = 5$, $y = -2$

b) Verify the solution.

$x - y = 7$

LS = $(5) - (-2) = 7$

RS = 7

LS = RS

$x + 5y = -5$

LS = $(5) + 5(-2) = -5$

RS = -5

LS = RS

verified

3. In each case, solve the system of equations using technology. Verify the solution by replacing the values in the original equations.

a) $y = 3x - 7$
$y = -x + 9$

$x = 4$, $y = 5$

$y = 3x - 7$
RS $= 3(4) - 7 = 5$
LS $= 5$
LS = RS ✓

$y = -x + 9$
LS $= 5$
RS $= -(4) + 9 = 5$
LS = RS ✓

b) $y = -x$
$y = -\frac{1}{3}x + 3$

$x = -4.5$ $y = 4.5$

$y = -x$
LS $= 4.5$
RS $= -(-4.5) = 4.5$
LS = RS

$y = -\frac{1}{3}x + 3$
LS $= 4.5$
RS $= -\frac{1}{3}(-4.5) + 3 = 4.5$
LS = RS

c) $y = x - 2$
$y = \frac{3}{4}x - 4$

$x = -8$ $y = -10$

$y = x - 2$
LS $= -10$
RS $= (-8) - 2 = -10$
LS = RS

$y = \frac{3}{4}x - 4$
LS $= -10$
RS $= \frac{3}{4}(-8) - 4 = -10$
LS = RS

d) $3x + 2y = 5$
$x - y = 1$

$x = 1.4$ $y = 0.4$

$3x + 2y = 5$
LS $= 3(1.4) + 2(0.4) = 5$
RS $= 5$
LS = RS

$x - y = 1$
LS: $(1.4) - (0.4) = 1$
RS: 1
LS = RS

e) $4a - b = 6$
$3a + b = 1$

$a = 1$ $b = -2$

$4a - b = 6$
LS $= 4(1) - (-2) = 6$
RS $= 6$
LS = RS

$3a + b = 1$
LS $= 3(1) + (-2) = 1$
RS $= 1$
LS = RS

f) $0.6p - 0.8q = 2.6$
$5p + 6q = 9$

$p = 3$ $q = -1$

$0.6p - 0.8q = 2.6$
LS $= 0.6(3) - 0.8(-1) = 2.6$
RS $= 2.6$
LS = RS

$5p + 6q = 9$
LS $= 5(3) + 6(-1) = 9$
RS $= 9$
LS = RS

4. Solve the following systems of equations using technology. List the answers as exact values.

a) $4x - y + 6 = 0$, $\quad y = x + 2$

$y = 4x + 6$

$x = \frac{-4}{3}$ $y = \frac{2}{3}$

b) $8x - 3y = 5$, $\quad 5x + 3y = 2$

$8x - 5 = 3y$ $\quad 3y = -5x + 2$

$y = \frac{8}{3}x - \frac{5}{3}$ $\quad y = \frac{-5}{3}x + \frac{2}{3}$

$x = \frac{7}{13}$ $y = \frac{-3}{13}$

Multiple Choice

5. The ordered pair (x, y) which satisfies the system of equations $x - 2y = 6$, $x + 6y = 22$ is

A. $(-10, 2)$

B. $(2, 10)$

C. $(10, -2)$

D. $(10, 2)$

$x - 6 = 2y$ $\quad$ $6y = -x + 22$

$y = \frac{1}{2}x - 3$ $\quad$ $y = -\frac{1}{6}x + \frac{11}{3}$

graph: intersect at $x = 10$, $y = 2$

Numerical Response

6. If $7x - 5y = 19$ and $2x + 3y = 17$, then the value of x, to the nearest tenth, is ______ .

(Record your answer in the numerical response box from left to right)

4	.	6	

$7x - 19 = 5y$ $\quad$ $3y = -2x + 17$

$y = \frac{7}{5}x - \frac{19}{5}$ $\quad$ $y = -\frac{2}{3}x + \frac{17}{3}$

graph: intersect at $x = 4.58\ldots$

7. A pear costs 24 cents less than two apples. Four apples cost the same as three pears. In order to determine the cost of each piece of fruit, Courtney graphs the equations $y = 2x - 24$ and $4x = 3y$. The cost of a pear, in cents, is ______ .

(Record your answer in the numerical response box from left to right)

4	8		

graph: $y_1 = 2x - 24$

$y_2 = \frac{4}{3}x$

x = apples

y = pears

solution: $x = 36$ $y = 48$

$\therefore$ pear costs 48 cents

Answer Key

1. a) $y = \frac{1}{2}x - \frac{3}{2}$, $y = -x$ b)

x	y
-3	-3
-1	-2
1	-1
3	0
5	1

x	y
-4	4
-2	2
0	0
2	-2
4	-4

c) $x = 1$, $y = -1$

2. a) $x = 5, y = -2$

3. a) $x = 4, y = 5$ b) $x = -4.5, y = 4.5$ c) $x = -8, y = -10$
d) $x = 1.4, y = 0.4$ e) $a = 1, b = -2$ f) $p = 3, q = -1$

4. a) $x = -\frac{4}{3}, y = \frac{2}{3}$ b) $x = \frac{7}{13}, y = -\frac{3}{13}$

5. D

6.

4	.	6	

7.

4	8		

Systems of Linear Equations Lesson #2:
Determining the Number of Solutions to a System of Linear Equations

Exploration | *Determining the Number of Solutions to a System of Linear Equations*

As part of a high school work experience course, three students have been placed in three different burger restaurants. Deja has been placed at Burger Shack, Shelly has been placed at Big's Burgers, and John has been placed at The Burger Haven.

Detailed below are the first two orders taken by each student and the total cost calculated by the student for each order.

Burger Shack For the first order Deja charges \$28 for 4 burgers and 2 salads. For the second order she charges \$34 for 6 burgers and a salad.

Big's Burgers Shelly charges \$18 for a burger and 3 salads, and \$54 for 3 burgers and 9 salads.

The Burger Haven John charges \$32 for 2 burgers and 4 salads, and \$42 for 3 burgers and 6 salads.

a) Let x dollars be the cost of a burger and y dollars be the cost of a salad. Write a system of equations for each scenario.

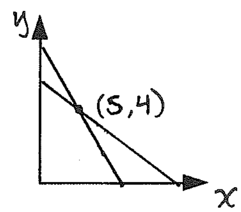
Burger Shack	Big's Burgers	The Burger Haven
$4x + 2y = 28$	$x + 3y = 18$	$2x + 4y = 32$
$6x + y = 34$	$3x + 9y = 54$	$3x + 6y = 42$

b) Consider the equations for the Burger Shack. Write each equation in terms of y and use a graphing calculator to determine the cost of a burger and the cost of a salad.

$4x + 2y = 28$
$2y = -4x + 28$
$y = -2x + 14$

$6x + y = 34$
$y = -6x + 34$

burger \$5
salad \$4

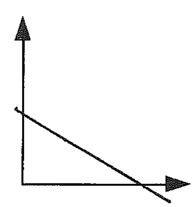


c) Repeat part **b)** for Big's Burgers. Can you determine the cost of a burger and the cost of a salad? Explain.

$x + 3y = 18$
$3y = -x + 18$
$y = -\frac{1}{3}x + 6$

$3x + 9y = 54$
$9y = -3x + 54$
$y = -\frac{1}{3}x + 6$

Both equations are the same.
↳ cannot determine the cost.



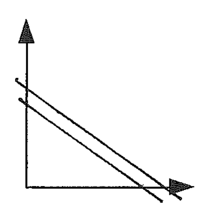
d) Repeat part **b)** for The Burger Haven. Can you determine the cost of a burger and the cost of a salad? Explain how you can tell that the student must have made an error in at least one of the calculations.

$2x + 4y = 32$
$4y = -2x + 32$
$y = -\frac{1}{2}x + 8$

$3x + 6y = 42$
$6y = -3x + 42$
$y = -\frac{1}{2}x + 7$

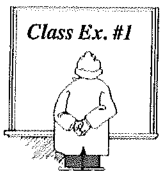
(1 burger and 2 salads cannot cost \$16 and \$14)

Both equations have the same slope ∴ lines are parallel



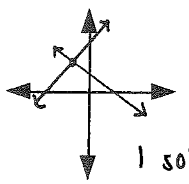
Number of Solutions to a System of Equations

In all of the examples in Lesson #1, each system of equations had one unique solution. However, we have seen on the previous page that a system of two linear equations may have no solution, only one solution or an infinite number of solutions.



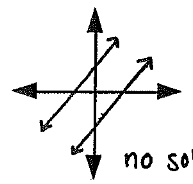
Graph each system of equations on the grid provided. State the number of solutions for each system.

a) $y = x + 5$
$y = -x + 1$



1 solution

b) $y = 2x - 8$
$y = 2x + 6$

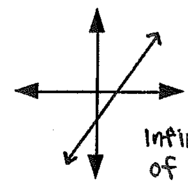


no solution

c) $y = 3x - 2$
$6x - 2y - 4 = 0$

same equation

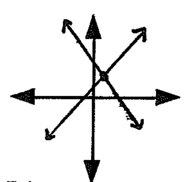
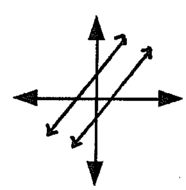
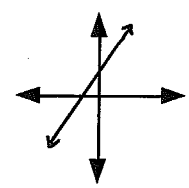
$6x - 4 = 2y$
$y = 3x - 2$

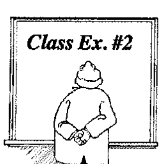


Infinite number of solutions

The number of solutions can be determined from the graph as above, or directly from the equations if they are expressed in slope y-intercept form.

d) Complete the following chart.

Number of Solutions	one solution	no solution	infinitely many
Graphical Example	 Lines intersect at one point	 Lines are parallel	 Lines are coincident
Slopes and Intercepts	slopes are different	Slopes are equal and intercepts are different	Slopes are equal and intercepts are equal



Without graphing, analyze each system to determine whether the system has one solution, no solution, or infinitely many solutions.

a) $3x + 5y = 15,\ y = -\frac{3}{5}x$

$5y = -3x + 15$
$y = -\frac{3}{5}x + 3$

slopes are the same!

no solution

b) $x - 4y + 8 = 0,\ y = \frac{1}{4}x + 2$

$x + 8 = 4y$
$y = \frac{1}{4}x + 2$

equations are the same!

infinitely many solutions

c) $7x + y = 12,\ x - 6y = 5$

$y = -7x + 12$
$-6y = -x + 5$
$y = \frac{1}{6}x - \frac{5}{6}$

slopes are different!

one solution.

Complete Assignment Questions #1 - #5

Solving a System Graphically by Changing the Calculator Window

Often, the solution to a system of equations will not be visible using the default window of the graphing calculator. When this occurs, we must change the window settings.

We use the following graphing calculator window format:

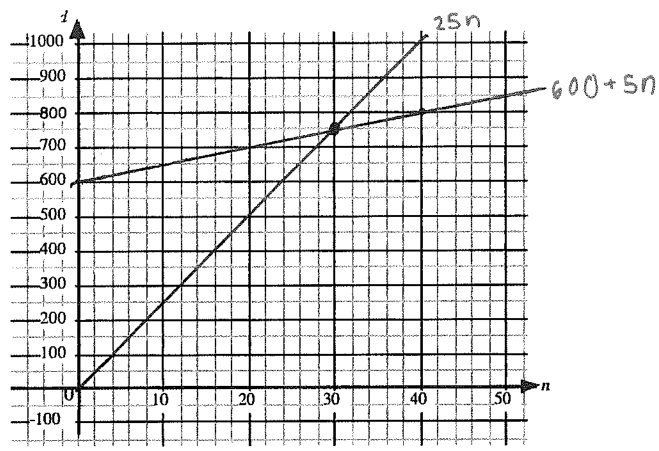
x:[x_{min}, x_{max}, x_{scl}] y:[y_{min}, y_{max}, y_{scl}]

Class Ex. #3

At a local High School, the Students' Council decided to sell sweaters to students. The cost of designing the sweaters included a fixed cost of \$600 plus \$5 per sweater.

The Students' Council planned to sell the sweaters for \$25 each. The cost and revenue can be represented by the following system of equations where d represents the dollar cost and n represents the number of sweaters sold.

Cost of sweaters in dollars $d = 600 + 5n$
Revenue of sweaters in dollars $d = 25n$



a) Use a graphing calculator to graph each equation and sketch each graph on the grid provided.

b) How much profit or loss is made if

i) twenty sweaters are sold?

cost = 600 + 5(20) = \$700
revenue = 25(20) = \$500
Profit = revenue − cost
= 500 − 700 = −200 → b/c (−) a loss is made and not a profit
∴ loss = \$200

ii) fifty sweaters are sold?

cost = 600 + 5(50) = \$850
revenue = 25(50) = \$1250
Profit = revenue − cost
= 1250 − 850 = 400 → (+) number ∴ profit is made and not a loss
∴ Profit = \$400

c) The break even point is the point where no profit or loss is made. Mark the break-even point on the grid.

d) Use a graphing calculator to determine the number of sweaters which must be sold in order to break even.

30 sweaters

e) If all 850 students in the school purchased a sweater, how much profit would the Students' Council make?

cost = 600 + 5(850) = \$4850
revenue = 25(850) = \$21250
Profit = \$21250 − \$4850
Profit = \$16400

Complete Assignment Questions #6 - #11

Assignment

1. How can you tell by graphing a system of linear equations whether the system has no solution, one solution, or infinitely many solutions?

if the lines are parallel, there is no solution

if the lines intersect, there is one solution.

if the lines are coincident, there are infinitely many solutions.

2. Graph each system and determine whether the system has no solution, one solution, or infinitely many solutions.

a) $x = 2y - 5$

$y = \frac{1}{2}(x + 5)$

$x + 5 = 2y$ $\quad y = \frac{1}{2}x + \frac{5}{2}$

$y = \frac{1}{2}x + \frac{5}{2}$

infinitely many solutions.

b) $6x - y = 5$ $\quad y = 6x - 5$

$y = 6x + 7$

no solution.

c) $2x - 5y = 10$

$3x - 4y = 24$

$2x - 10 = 5y$ $\quad 3x - 24 = 4y$

$y = \frac{2}{5}x - 2$ $\quad y = \frac{3}{4}x - 6$

one solution.

3. How can you tell by writing a system of linear equations in the form $y = mx + b$ whether the system has no solution, one solution, or infinitely many solutions?

m = slope

- if the values of m are identical but the values of b are different, there is no solution.
- if the values of m are different, there is one solution
- if the values of m are identical and the values of b are identical, there are infinitely many solutions.

4. Rearrange each equation into the form $y = mx + b$ and state whether the system has no solution, one solution, or infinitely many solutions.

a) $6x - y = 1$ $\quad y = 6x - 1$

$y = 6x + 1$

no solution.

b) $8x - y = 13$ $\quad y = 8x - 13$

$x - 8y = 13$

$x - 13 = 8y$

$y = \frac{1}{8}x - \frac{13}{8}$

one solution.

c) $5y + x - 10 = 0$ $\quad 5y = -x + 10$

$y = -\frac{1}{5}x + 2$ $\quad y = \frac{-1}{5}x + 2$

infinitely many solutions.

5. Write an equation which forms a system with the equation $3x - y = 9$ so that the system has

$y = 3x - 9$

a) no solution

↳ same slope, $m = 3$

$y = 3x - 4$

or

$3x - y = 4$

b) one solution

(different slope)

$7x - 3y = 6$

c) an infinite number of solutions

$2(3x - y = 9)$

$6x - 2y = 18$

↳ identical m and b values

6. All 480 tickets for a school concert were sold. Seats in the front part of the hall cost \$6 each, and seats in the back part of the hall cost \$4 each. The receipts totalled \$2 530.

Information from the number of tickets can be represented by $f + b = 480$.

a) State an equation which can be formed from the costs of the tickets.

$6f + 4b = 2530$

b) Graph the system to determine the number of tickets sold for each part of the hall.

$f + b = 480$ $\quad$ $6f + 4b = 2530$

graph $\rightarrow$ y_1: $b = 480 - f$ $\quad$ $4b = 2530 - 6f$

y_2: $b = -\frac{3}{2}f + \frac{1265}{2}$

intersect at $(305, 175)$
$f \quad b$
$\therefore$ front: 305 tickets
back: 175 tickets

c) State the graphing window used.

$x: [-100, 500, 100]$ $\quad$ $y: [-100, 700, 100]$

d) Verify the solution.

$\underline{f + b = 480}$
LS: $(305) + (175) = 480$
RS: 480
LS = RS

$\underline{6f + 4b = 2530}$
LS: $6(305) + 4(175) = 2530$
RS: 2530
LS = RS

verified ✓

7. Six pencils and four crayons cost \$3.40. Three similar pencils and ten similar crayons cost \$4.90. **Describe** a method to determine how much you would expect to pay for a set of eight pencils and twelve crayons and then calculate the cost.

Let \$$x$ be the cost of a pencil and \$$y$ be the cost of a crayon

- form two equations from the given information
  ↳ $6x + 4y = 3.4$ $\quad$ ↳ $3x + 10y = 4.9$
- Write each equation in slope y-intercept form. $(y = mx + b)$
  ↳ $4y = -6x + 3.4$ $\quad$ ↳ $10y = -3x + 4.9$
  $y = -\frac{3}{2}x + \frac{17}{20}$ $\quad$ $y = -\frac{3}{10}x + \frac{49}{100}$
- Graph the system of equations and determine coordinates (x, y) of the intersection point.
  ↳ the point is $(0.3, 0.4)$

The answer is $8x + 12y = 8(0.3) + 12(0.4) = 7.2$

Answer = \$7.20

Multiple Choice **8.** The solution to the system $\begin{cases} 4x - 3y = 9 \\ 8x - 6y = 81 \end{cases}$ has

A. no solution

B. one solution

C. two solutions

D. infinitely many solutions

$y = \frac{4}{3}x - 3$

$y = \frac{4}{3}x - \frac{27}{2}$

Numerical Response

9. If $3x + 2y = 48$ and $2x + 3y = 12$, then the value of $x - 2y$, to the nearest tenth, is ______ .

$2y = -3x + 48$ $\quad$ $3y = -2x + 12$

$y = -\frac{3}{2}x + 24$ $\quad$ $y = -\frac{2}{3}x + 4$

(Record your answer in the numerical response box from left to right)

4	8	.	0

graph $y_1 = -\frac{3}{2}x + 24$

$y_2 = -\frac{2}{3}x + 4$

intersection point $(24, -12)$

$x - 2y$

$= (24) - 2(-12)$

$= 48$

10. The value of k, $k \in N$, for which the system of equations $10x + ky = -8$ and $-15x - 6y = 12$ has an infinite number of solutions, is ______ .

↳ ∴ identical m and b values

(Record your answer in the numerical response box from left to right)

4			

$10x + ky = -8$

$ky = -10x - 8$

$y = -\frac{10}{k}x - \frac{8}{k}$ ↳ "b" value

$-15x - 6y = 12$

$6y = -15x - 12$

$y = -\frac{5}{2}x - 2$ ↳ "b" value

$-\frac{8}{k} = -2$

$-8 = -2k$

$k = 4$

b/c both equations should have identical "b" values, you are able to set these val equal to one another a solve for k!

11. The value of a, $a \in N$, for which the system of equations $ax + 5y = 10$ and $6x + 2y = 7$ has no solution, is ______ .

(Record your answer in the numerical response box from left to right)

↳ identical m value

1	5		

$5y = -ax + 10$ $\quad$ $2y = -6x + 7$

$y = -\frac{a}{5}x + 2$ $\quad$ $y = -3x + \frac{7}{2}$

$-\frac{a}{5} = -3$

$-a = -15$

$a = 15$

Answer Key

1. If the lines are parallel, there is no solution.
   If the lines intersect, there is one solution.
   If the lines are coincident, there are infinitely many solutions.

2. **a)** infinitely many solutions **b)** no solution **c)** one solution

3. If the values of *m* are identical but the values of *b* are different, there is no solution.
   If the values of *m* are different, there is one solution.
   If the values of *m* are identical and the values of *b* are identical, there are infinitely many solutions.

4. **a)** no solution **b)** one solution **c)** infinitely many solutions

5. **a)** e.g. $3x - y = 4$ **b)** e.g. $7x - 3y = 6$ **c)** e.g. $6x - 2y = 18$

6. **a)** $6f + 4b = 2530$ **b)** Front 305 tickets, Back 175 tickets **c)** x:[−100, 500, 100] y:[−100, 700, 100]

7. Let $\$x$ be the cost of a pencil and $\$y$ be the cost of a crayon. Form two equations from the given information. These are $6x + 4y = 3.4$ and $3x + 10y = 4.9$ Write each equation in slope y-intercept form. Graph the system of equations and determine the coordinates (x, y) of the intersection point. The answer is $8x + 12y = 8(0.3) + 12(0.4) = 7.2$ *Answer* = $7.20

8. A

9.

4	8	.	0

10.

4			

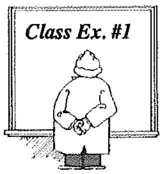
11.

1	5		

Systems of Linear Equations Lesson #3:
Solving Systems of Linear Equations by Inspection and by Substitution

Method of Inspection

In some simple cases, a system of linear equations can be solved by mentally trying different values for the variables until a correct solution is reached. This is called the method of inspection and is really only practical if the equations are very simple.



The sum of two numbers is 14 and the difference between the numbers is 2. Form two equations in two variables and determine the numbers by inspection.

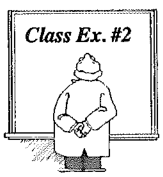
Let x = the larger number , Let y = the smaller number

$x+y=14$
$x-y=2$

inspection method
while adding two numbers which equal to 14, inspect the same two numbers and see if they subtract to 2.

$13+1=14$ $13-1=12$ NOT 2
$10+4=14$ $10-4=6$ CLOSER, BUT NOT 2.
$8+6=14$ $8-6=2$

by inspection, $x=8$ and $y=6$



Solve the system $x+2y=12$ and $x+3y=17$ by inspection.

Interpret the two equations in words

i) $x+2y=12$
↳ "a number plus twice another number equals 12"

ii) $x+3y=17$
↳ "a number plus three times another number equals 17"

iii) Guess and check numbers in $x+2y=12$, then try them in $x+3y=17$

* numbers should be whole because inspection is only pratical when equations are simple.

$x+2y=12$
Try $x=2$
$(2)+2y=12$
$2y=10$
$y=5$

↳ try $x=2$ and $y=5$ in other equation.

$x+3y=17$
$(2)+3(5)=17$
$17=17$
Works!

$\therefore$ $x=2$ and $y=5$

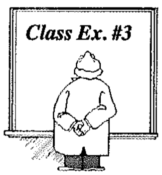
Complete Assignment Questions #1 - #3

Method of Substitution

If the equations are too complex to be solved by inspection, then algebraic procedures such as the method of substitution and the method of elimination (next lesson) may be used.

When using the method of substitution, there are four steps which are shown in the flowchart below.

Step 1		Step 2		Step 3		Step 4
Choose the simpler equation and express one variable in terms of the other.	→	Substitute the expression from step 1 into the other equation.	→	Solve the single variable equation.	→	Substitute the solution from step 3 into the equation in step 1 to find the value of the other variable.



Consider the following system of equations: $x + 4y = 17$
$2x - y = 7$

a) Solve the system using the method of substitution by rewriting the first equation in the form $x =$ .....

$x + 4y = 17$
$x = -4y + 17$

Plug into other equation

$2x - y = 7$
$2(-4y+17) - y = 7$
$-8y + 34 - y = 7$
$-9 = -27$
$\underline{y = 3}$

solve for 'y'

$x + 4y = 17$
$x + 4(3) = 17$
$x + 12 = 17$
$\underline{x = 5}$

Plug y value found into the other equation & solve for x.

$\boxed{x = 5 \quad y = 3}$

b) Solve the system using the method of substitution by rewriting the first equation in the form $y =$ .....

$x + 4y = 17$
$4y = -x + 17$
$y = -\frac{1}{4}x + \frac{17}{4}$

$2x - y = 7$
$2x - (-\frac{1}{4}x + \frac{17}{4}) = 7$
$4\times(2x + \frac{1}{4}x - \frac{17}{4}) = (7)\times 4$
$8x + x - 17 = 28$
$9x = 45$
$\underline{x = 5}$

multiplying by 4 makes the numbers easier to work with! No fractions!

$y = -\frac{1}{4}x + \frac{17}{4}$
$y = -\frac{1}{4}(5) + \frac{17}{4}$
$\underline{y = 3}$

$\boxed{x = 5 \quad y = 3}$

c) Which method, a) or b), was simpler?

d) Verify that the solution satisfies both equations.

$\underline{x + 4y = 17}$
LS: $(5) + 4(3) = 17$
RS: 17
LS = RS

$\underline{2x - y = 7}$
LS: $2(5) - (3) = 7$
RS: 7
LS = RS

verified ✓

e) Check the solution using a graphing calculator.

$x = 5 \quad y = 3$

Class Ex. #4

Consider the following system of equations: $4x + 3y = 0,\ 8x - 9y = 5.$

a) Solve and verify the system using the method of substitution.

$4x + 3y = 0$
$3y = -4x$
$y = -\frac{4}{3}x$

$8x - 9y = 5$
$8x - 9(-\frac{4}{3}x) = 5$
$8x + 12x = 5$
$20x = 5$
$x = \frac{1}{4}$

$y = -\frac{4}{3}x$
$y = -\frac{4}{3}(\frac{1}{4})$
$y = -\frac{1}{3}$

$\boxed{x = \frac{1}{4} \quad y = -\frac{1}{3}}$

verify

$4x + 3y = 0$
LS: $4(\frac{1}{4}) + 3(-\frac{1}{3}) = 0$
RS: 0
LS = RS

$8x - 9y = 5$
LS: $8(\frac{1}{4}) - 9(-\frac{1}{3}) = 5$
RS: 5
LS = RS

verified ✓

b) Check the solution using a graphing calculator.

Class Ex. #5

Consider the following system of equations: $5(2a - 3) + b = 5, \quad 6a - 2(b - 4) = 20.$

a) Solve the system using the method of substitution.

$5(2a - 3) + b = 5$
$10a - 15 + b = 5$
$10a + b = 20$
$b = -10a + 20$

$6a - 2(b - 4) = 20$
$6a - 2(-10a + 20 - 4) = 20$
$6a + 20a - 40 + 8 = 20$
$26a = 52$
$a = 2$

$b = -10a + 20$
$b = -10(2) + 20$
$b = 0$

$\boxed{a = 2 \quad b = 0}$

b) Verify algebraically that the solution satisfies both equations.

$5(2a - 3) + b = 5$
LS: $5(2(2) - 3) + (0) = 5$
RS: 5
LS = RS

$6a - 2(b - 4) = 20$
LS: $6(2) - 2((0) - 4) = 12 + 8 = 20$
RS: 20
LS = RS

verified ✓

Complete Assignment Questions #4 - #10

Assignment

1. Solve the following linear systems by method of inspection.

 a) $x+y=9,\ x-y=1$ b) $x+y=12,\ x-y=0$ c) $x+y=4,\ x-y=6$

 * x must be larger

 $x=5,\ y=4$

 ↳ #'s are equal

 $\therefore x=6,\ y=6$

 $x=5,\ y=-1$

2. At the Little River Pow Wow, a vendor sells a salmon burger and two cans of cola for \$8. If two salmon burgers and two cans of cola sell for \$14, then determine the cost of

 a) a salmon burger $14-8=6$

 \$6

 b) a can of cola

 $s+2c=8$
 $(6)+2c=8$
 $c=1$

 \$1

3. Tickets are on sale for a music concert. Three adult tickets and two child tickets cost \$90. Three adult tickets and four child tickets cost \$120.

 a) Write a system of equations in two variables to represent the above information.

 x = adult ticket
 y = child ticket

 $3x+2y=90$, $3x+4y=120$

 b) Determine the total cost of two adult tickets and three child tickets.

 $120-90=30$

 The difference between the 2 equations is \$30 because of 2 child tickets ∴ 1 child ticket = \$15 = y

 $3x+2(15)=90$
 $3x=60$
 $x=20$ = adult ticket

 $2(20)+3(15)$
 = \$85

4. In each of the following systems:
 - solve the system using the method of substitution
 - verify the solution satisfies both equations
 - check the solution by graphing

 a) $y=x+2,\quad 3x+4y=1$

 $3x+4y=1$
 $3x+4(x+2)=1$
 $3x+4x+8=1$
 $7x=-7$
 $x=-1$

 $y=x+2$
 $y=(-1)+2$
 $y=1$

 $y=x+2$
 LS: $y=1$
 RS: $x+2=(-1)+2=1$
 LS = RS

 $3x+4y=1$
 LS: $3(-1)+4(1)=1$
 RS: 1
 LS = RS

 verified ✓

 b) $x-2y=10,\quad x+5y+4=0$

 $x=2y+10$ $(2y+10)+5y+4=0$
 $7y=-14$
 $y=-2$

 $x-2y=10$
 $x-2(-2)=10$
 $x+4=10$
 $x=6$

 $x-2y=10$
 LS: $(6)-2(-2)=10$
 RS: 10
 LS = RS

 $x+5y+4=0$
 LS: $(6)+5(-2)+4=0$
 RS: 0
 LS = RS

 verified ✓

5. Solve each of the following systems by substitution. Check each solution.

a) $4p + q = 0, \quad 7p + 4q = 3$

$q = -4p$

$7p + 4(-4p) = 3$

$7p - 16p = 3$

$-9p = 3$

$p = -\frac{1}{3}$

$q = -4(-\frac{1}{3})$

$q = \frac{4}{3}$

$p = -\frac{1}{3}, \ q = \frac{4}{3}$

$4p + q = 0$

LS: $4(-\frac{1}{3}) + \frac{4}{3} = 0$

RS: 0

LS = RS

$7p + 4q = 3$

LS: $7(-\frac{1}{3}) + 4(\frac{4}{3}) = 3$

RS: 3

LS = RS

verified ✓

b) $6u - 3v + 4 = 0, \quad 3u = 3v - 5$

$\frac{3u}{3} = \frac{3v - 5}{3}$

$u = v - \frac{5}{3}$

$6u - 3v + 4 = 0$

$6(v - \frac{5}{3}) - 3v + 4 = 0$

$6v - 10 - 3v + 4 = 0$

$3v = 6$

$v = 2$

$u = (2) - \frac{5}{3}$

$u = \frac{1}{3}$

$v = 2, \ u = \frac{1}{3}$

$6u - 3v + 4 = 0$

LS: $6(\frac{1}{3}) - 3(2) + 4 = 0$

RS: 0

LS = RS

$3u = 3v - 5$

LS: $3(\frac{1}{3}) = 1$

RS: $3(2) - 5 = 1$

LS = RS

verified ✓

c) $2x - 5y = -7$

$\frac{1}{2}x - y = 3$

$y = \frac{1}{2}x - 3$

$2x - 5y = -7$

$2x - 5(\frac{1}{2}x - 3) = -7$

$2x - \frac{5}{2}x + 15 = -7$

$-\frac{1}{2}x = -22$

$x = 44$

$y = \frac{1}{2}(44) - 3$

$y = 19$

$x = 44, \ y = 19$

$2x - 5y = -7$

LS: $2(44) - 5(19) = -7$

RS: -7

LS = RS

$\frac{1}{2}x - y = 3$

LS: $\frac{1}{2}(44) - (19) = 3$

RS: 3

LS = RS

verified ✓

d) $2(x + 2) + y = 8$

$7x - 2(y - 3) + 24 = 0$

$2(x+2) + y = 8$

$2x + 4 + y = 8$

$y = -2x + 4$

$7x - 2(y - 3) + 24 = 0$

$7x - 2((-2x+4) - 3) + 24 = 0$

$7x + 4x - 8 + 6 + 24 = 0$

$11x = -22$

$x = -2$

$y = -2(-2) + 4$

$y = 8$

$x = -2, \ y = 8$

$2(x+2) + y = 8$

LS: $2((-2)+2) + (8) = 8$

RS: 8

LS = RS

$7x - 2(y - 3) + 24 = 0$

LS: $7(-2) - 2((8) - 3) + 24 = 0$

RS: 0

LS = RS

verified ✓

6. The straight line $px + qy + 14 = 0$ passes through the points $(-3, 1)$ and $(-4, 6)$.

a) Substitute the x and y-coordinates of the two points into the equation of the line to form two equations in p and q.

$-3p + q + 14 = 0$

$-4p + 6q + 14 = 0$

b) Solve this system of equations by substitution to determine the values of p and q and write the equation of the line.

$-3p + q + 14 = 0$
$q = 3p - 14$

$-4p + 6q + 14 = 0$
$-4p + 6(3p - 14) + 14 = 0$
$-4p + 18p - 84 + 14 = 0$
$14p = 70$
$p = 5$

$q = 3p - 14$
$q = 3(5) - 14$
$q = 1$

$\underline{p = 5,\ q = 1}$

$px + qy + 14 = 0$
$\underline{5x + y + 14 = 0}$

c) Verify the equation in **b)** using the slope formula and the point-slope equation of a line formula.

(slope formula)
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
$m = \dfrac{6 - 1}{-4 + 3}$
$m = -5$

(point-slope formula)
$y - y_1 = m(x - x_1)$
$y - (1) = (-5)(x - (-3))$
$y - 1 = -5(x + 3)$
$y - 1 = -5x - 15$
$\underline{5x + y + 14 = 0}$ OR $\underline{y = -5x - 14}$

7. Solve the following systems by substitution. Explain the results.

a) $y = 3x - 7$
$6x - 2y = 14$

$6x - 2(3x - 7) = 14$
$6x - 6x + 14 = 14$
$0 = 0$

The two equations are identical.
There are an infinite number of solutions.

b) $x = 3y + 2$
$2x - 6y = 5$

$2(3y + 2) - 6y = 5$
$6y + 4 - 6y = 5$
$4 = 5$

the two lines are parallel
There are no solutions.

Multiple Choice

8. If $x + 2y = 10$ and $x - 2y = 2$, then $x + y$ is equal to

A. 8
B. 12
C. 13
D. -2

$x + 2y = 10$
$x = -2y + 10$
$x = 10 - 2y$

$x - 2y = 2$
$(10 - 2y) - 2y = 2$
$10 - 4y = 2$
$8 = 4y$
$\underline{y = 2}$

$x = 10 - 2y$
$x = 10 - 2(2)$
$\underline{x = 6}$

$x + y$
$= (6) + (2)$
$= \underline{8}$

9. When solving a system of equations, one of which is $\frac{x}{2} - \frac{y}{3} = 1$, a substitution which can be made is

A. $x = \frac{1}{3}(2y + 1)$

B. $y = \frac{1}{2}(3x - 1)$

C. $x = \frac{1}{2}(3y + 6)$

D. $y = \frac{1}{2}(3x - 6)$

→ multiply by 6 to make equation easier to work with, no fractions!

$6\left(\frac{x}{2}\right) - 6\left(\frac{y}{3}\right) = 6(1)$

$3x - 2y = 6$ OR $3x - 6 = 2y$

$3x = 2y + 6$ $\qquad y = \frac{3}{2}x - 3$

$x = \frac{1}{3}(2y + 6)$ $\qquad y = \frac{1}{2}(3x - 6)$

Numerical Response

10. If $s - 8t + 20 = 5s - 7t + 1 = 0$, then the value of $s + t$, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right)

7	.	0	

$s = 8t - 20$

$5s - 7t + 1 = 0$

$5(8t - 20) - 7t + 1 = 0$

$40t - 100 - 7t + 1 = 0$

$33t = 99$

$t = 3$

$s = 8(3) - 20$

$s = 24 - 20$

$s = 4$

$s + t$

$= (4) + (3)$

$= 7.0$

Answer Key

1. **a)** $x = 5, y = 4$ **b)** $x = 6, y = 6$ **c)** $x = 5, y = -1$

2. **a)** \$6 **b)** \$1

3. **a)** $3x + 2y = 90, 3x + 4y = 120$ **b)** \$85

4. **a)** $x = -1, y = 1$ **b)** $x = 6, y = -2$

5. **a)** $p = -\frac{1}{3}, q = \frac{4}{3}$ **b)** $u = \frac{1}{3}, v = 2$ **c)** $x = 44, y = 19$ **d)** $x = -2, y = 8$

6. **a)** $-3p + q + 14 = 0,\ -4p + 6q + 14 = 0$ **b)** $p = 5, q = 1$ $\quad 5x + y + 14 = 0$

7. **a)** There are an infinite number of solutions of the form $x = a, y = 3a - 7, a \in R$ because the equations are identical, (the resulting equation reduces to $0 = 0$).
b) There are no solutions since the graphs of the equations are parallel lines, (the resulting equation reduces to $4 = 5$).

8. A **9.** D **10.**

7	.	0	

Systems of Linear Equations Lesson #4: Solving Systems of Linear Equations by Elimination

So far we have used three methods to solve systems of equations: graphing, inspection, and substitution. In this lesson we will learn another algebraic technique: the method of elimination. This method is particularly useful when the equations involve fractions.

Method of Elimination

In using the method of elimination, there are four steps which are shown below.

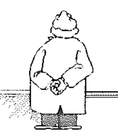
Step 1: If necessary, multiply each equation by a constant to obtain coefficients for x (or y) that are identical (except perhaps for the sign).

→ **Step 2:** Add or subtract the two equations to eliminate one of the variables.

→ **Step 3:** Solve the resulting equation to determine the value of one of the variables.

→ **Step 4:** Substitute the solution into either of the original equations to determine the value of the other variable.

Class Ex. #1



Consider the system of equations:

$$\begin{aligned} &2x + 7y = 13 \\ + \; &3x - 7y = 2 \\ \hline &5x = 15 \\ &\frac{5x}{5} = \frac{15}{5} \\ &x = 3 \end{aligned}$$

a) Add the two equations. This will eliminate the variable y.

b) Use the equation in **a)** to determine the value of x and hence solve the system.

$2x + 7y = 13$ $\quad 7y = 7$
$2(3) + 7y = 13$ $\quad y = 1$
$6 + 7y = 13$

$x = 3, \ y = 1$

c) Verify the solution satisfies both equations.

$2x + 7y = 13$

LS	RS
$2(3) + 7(1)$ $= 13$	13

$3x - 7y = 2$

LS	RS
$3(3) - 7(1)$ $= 2$	2

Class Ex. #2

Consider the system of equations:

$$\begin{aligned} &2x + 6y = 6 \\ - \; &2x + 3y = 4.5 \\ \hline &3y = 1.5 \\ &y = 0.5 \end{aligned}$$

a) Subtract the two equations. This will eliminate the variable x.

b) Use the equation in **a)** to determine the value of y and hence solve the system.

$2x + 6y = 6$ $\quad 2x = 3$
$2x + 6(0.5) = 6$ $\quad x = 3/2$
$2x + 3 = 6$ $\quad x = 1.5$

$x = 1.5, \ y = 0.5$

c) Verify the solution satisfies both equations.

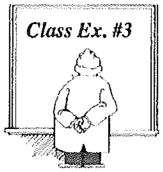
$2x + 6y = 6$

LS	RS
$2(1.5) + 6(0.5)$ $= 6$	6

$2x + 3y = 4.5$

LS	RS
$2(1.5) + 3(0.5)$ $= 4.5$	4.5

Complete Assignment Questions #1 - #3



Consider the system of equations: $2x + 3y = 4$
$4x - y = 22$

a) Does adding or subtracting the equations eliminate either of the variables? No.

b) Multiply the second equation by 3 and then add the two equations.

$3(4x - y = 22)$
$12x - 3y = 66$

$2x + 3y = 4$
$+\ 12x - 3y = 66$
$14x = 70$

c) Solve and verify the system.

$14x = 70$
$x = 5$

$2x + 3y = 4$
$2(5) + 3y = 4$
$10 + 3y = 4$
$3y = -6$
$y = -2$

$x = 5,\ y = -2$

$2x + 3y = 4$
LS: $2(5) + 3(-2) = 4$
RS: 4
LS = RS

$4x - y = 22$
LS: $4(5) - (-2) = 22$
RS: 22
LS = RS

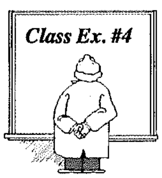
d) Consider the original system. Multiply the first equation by an appropriate number which will eliminate x by addition or subtraction. Solve the system.

$2(2x + 3y = 4)$
$4x + 6y = 8$

$4x + 6y = 8$
$-\ 4x - y = 22$
$7y = -14$
$y = -2$

$2x + 3y = 4$
$2x + 3(-2) = 4$
$2x - 6 = 4$
$2x = 10$
$x = 5$

$x = 5,\ y = -2$



Consider the system of equations: $5a + 3b = 3$
$3a - 7b = 81$

a) Choose appropriate whole numbers to multiply each equation so that the system can be solved by eliminating b. equation 1 $(\times 7)$ equation 2 $(\times 3)$

b) Solve and verify the system by eliminating b.

side work
$7(5a + 3b = 3)$
$3(3a - 7b = 81)$

$35a + 21b = 21$
$+\ 9a - 21b = 243$
$44a = 264$
$a = 6$

$5a + 3b = 3$
$5(6) + 3b = 3$
$30 + 3b = 3$
$3b = -27$
$b = -9$

$a = 6,\ b = -9$

$5a + 3b = 3$
LS: $5(6) + 3(-9) = 3$
RS $= 3$
LS = RS

$3a - 7b = 81$
LS: $3(6) - 7(-9) = 81$
RS: 81
LS = RS

verified ✓

c) Choose appropriate whole numbers to multiply each equation so that the system can be solved by eliminating a. equation 1 $(\times 3)$, equation 2 $(\times 5)$

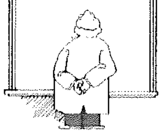
d) Solve the system by eliminating a.

side work
$3(5a + 3b = 3)$
$5(3a - 7b = 81)$

$15a + 9b = 9$
$-\ 15a - 35b = 405$
$44b = -396$
$b = -9$

$5a + 3b = 3$
$5a + 3(-9) = 3$
$5a - 27 = 3$
$5a = 30$
$a = 6$

$a = 6,\ b = -9$



Class Ex. #5

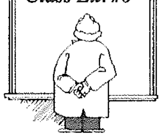
Solve the following system using elimination. $4x + 2y - 13 = 0,\quad 3x = 5y + 26$

$4x + 2y = 13 \ (\times 5)$
$3x - 5y = 26 \ (\times 2)$

(add) + $20x + 10y = 65$
$6x - 10y = 52$
$26x = 117$
$x = \frac{9}{2}$

$4x + 2y = 13$
$4(\frac{9}{2}) + 2y = 13$
$18 + 2y = 13$
$2y = -5$
$y = -\frac{5}{2}$

$x = \frac{9}{2}, \frac{-5}{2}$



Class Ex. #6

Solve the following system using elimination.

* multiplying by 15 and 10 removes the fraction so it is easier to solve

$\frac{x-2}{3} - \frac{y+2}{5} = 2, \ (\times 15)$ | $\frac{3}{5}(x+1) - \frac{4}{5}(y-3) = \frac{21}{2} \ (\times 10)$

$15(\frac{x-2}{3}) - 15(\frac{y+2}{5}) = 15(2)$
$5(x-2) - 3(y+2) = 30$
$5x - 10 - 3y - 6 = 30$
$5x - 3y = 46$

$10[\frac{3}{5}(x+1)] - 10[\frac{4}{5}(y-3)] = 10(\frac{21}{2})$
$6(x+1) - 8(y-3) = 105$
$6x + 6 - 8y + 24 = 105$
$6x - 8y = 75$

$5x - 3y = 46 \ (\times 8)$
$6x - 8y = 75 \ (\times -3)$

(add) + $40x - 24y = 368$
$-18x + 24y = -225$
$22x = 143 \rightarrow x = \frac{13}{2}$

$6x - 8y = 75$
$6(\frac{13}{2}) - 8y = 75$
$39 - 8y = 75$
$-8y = 36$
$y = \frac{-9}{2}$

$x = \frac{13}{2}, y = \frac{-9}{2}$

Complete Assignment Questions #4 - #12

Assignment

1. In each of the following systems:
   - solve the system using the method of elimination by adding the equations.
   - verify the solution satisfies both equations.

a) + $8x - y = 10$ (add)
$4x + y = 14$
$12x = 24$
$x = 2$

$4x + y = 14$
$4(2) + y = 14$
$y = 6$

$x = 2, y = 6$

$8x - y = 10$
LS: $8(2) - (6) = 10$
RS: 10
LS = RS

$4x + y = 14$
LS: $4(2) + (6) = 14$
RS: 14
LS = RS

verified ✓

b) + $x + 2y = 3$ (add)
$-x + 3y = 2$
$5y = 5$
$y = 1$

$x + 2y = 3$
$x + 2(1) = 3$
$x = 1$

$x = 1, y = 1$

$x + 2y = 3$
LS: $(1) + 2(1) = 3$
RS: 3
LS = RS

$-x + 3y = 2$
LS: $-(1) + 3(1) = 2$
RS: 2
LS = RS

verified ✓

c) + $4a - 3b = 2$ (add)
$-4a - b = 6$
$-4b = 8$
$b = -2$

$4a - 3(-2) = 2$
$4a + 6 = 2$
$4a = -4$
$a = -1$

$a = -1, b = -2$

$4a - 3b = 2$
LS: $4(-1) - 3(-2) = 2$
RS: 2
LS = RS

$-4a - b = 6$
LS: $-4(-1) - (-2) = 6$
RS: 6
LS = RS

verified ✓

2. In each of the following systems:
   - solve the system using the method of elimination by subtracting the equations.
   - verify the solution satisfies both equations.

a) $7x + y = 15$ (subtract)
$3x + y = 3$

$4x = 12$
$x = 3$

$3x + y = 3$
$3(3) + y = 3$
$9 + y = 3$
$y = -6$

$x = 3,\ y = -6$

$7x + y = 15$
LS: $7(3) + (-6) = 15$
RS: 15
LS = RS

$3x + y = 3$
LS: $3(3) + (-6)$
$= 3$
RS: 3
LS = RS

verified ✓

b) $5m + 3n = 10$ (subtract)
$5m - 2n = -15$

$5n = 25$
$n = 5$

$5m + 3n = 10$
$5m + 3(5) = 10$
$5m = -5$
$m = -1$

$m = -1,\ n = 5$

$5m + 3n = 10$
LS: $5(-1) + 3(5)$
$= 10$
RS: 10
LS = RS

$5m - 2n = -15$
LS: $5(-1) - 2(5)$
$= -15$
RS: -15
LS = RS

verified ✓

c) $4a - 3b = -18$ (subtract)
$-2a - 3b = -9$

$6a = -9$
$a = -\frac{3}{2}$

$4a - 3b = -18$
$4(-\frac{3}{2}) - 3b = -18$
$-6 - 3b = -18$
$12 = 3b$
$b = 4$

$a = -\frac{3}{2},\ b = 4$

$4a - 3b = -18$
LS: $4(-\frac{3}{2}) - 3(4)$
$= -6 - 12 = -18$
RS: -18
LS = RS

$-2a - 3b = -9$
LS: $-2(-\frac{3}{2}) - 3(4)$
$= -9$
RS: -9
LS = RS

verified ✓

3. Solve and verify each of the following systems using the method of elimination.

a) $-10p + 10q = 3$ (add)
$10p + 5q = 6$

$15q = 9$
$q = \frac{3}{5}$

$10p + 5q = 6$
$10p + 5(3/5) = 6$
$10p + 3 = 6$
$10p = 3$
$p = \frac{3}{10}$

$p = \frac{3}{10},\ q = \frac{3}{5}$

$-10p + 10q = 3$
LS: $-10(\frac{3}{10}) + 10(\frac{3}{5})$
$= -3 + 6$
$= 3$
RS: 3
LS = RS

$10p + 5q = 6$
LS: $10(\frac{3}{10}) + 5(\frac{3}{5})$
$= 3 + 3$
$= 6$
RS: 6
LS = RS

verified ✓

b) $x + 4y = -0.5$ (subtract)
$5x + 4y = 2.3$

$-4x = -2.8$
$x = 0.7$

$x + 4y = -0.5$
$(0.7) + 4y = -0.5$
$4y = -1.2$
$y = -0.3$

$x = 0.7,\ y = -0.3$

$x + 4y = -0.5$
LS: $(0.7) + 4(-0.3)$
$= -0.5$
RS: -0.5
LS = RS

$5x + 4y = 2.3$
LS: $5(0.7) + 4(-0.3)$
$= 2.3$
RS: 2.3
LS = RS

verified ✓

c) $4x + 2y - 31 = 0$ (add)
$-4x + 6y - 13 = 0$

$8y - 44 = 0$
$8y = 44$
$y = \frac{11}{2}$

$4x + 2y - 31 = 0$
$4x + 2(\frac{11}{2}) - 31 = 0$
$4x = 20$
$x = 5$

$x = 5,\ y = \frac{11}{2}$

$4x + 2y - 31 = 0$
LS: $4(5) + 2(\frac{11}{2}) - 31 = 0$
RS: 0
LS = RS

$-4x + 6y - 13 = 0$
LS: $-4(5) + 6(\frac{11}{2}) - 13 = 0$
RS: 0
LS = RS

verified ✓

4. Solve each of the following systems by elimination. Check each solution.

a) $2a + 5b = 16$
$2(a - b = 1)$

* multiply by 2 to have the equations match and be able to do elimination.

$-$ $2a + 5b = 16$
$2a - 2b = 2$ (subtract) *cancels out the "2a"

$7b = 14$
$b = 2$

$a - b = 1$
$a - (2) = 1$
$a = 3$

$a = 3,\ b = 2$

$2a + 5b = 16$
LS: $2(3) + 5(2) = 16$
RS: 16
LS = RS

$a - b = 1$
LS: $(3) - (2) = 1$
RS: 1
LS = RS

verified ✓

b) $4x - 3y = 9$
$2(2x - 5y = 1)$

$-$ $4x - 3y = 9$
$4x - 10y = 2$ (subtract)

$7y = 7$
$y = 1$

$2x - 5y = 1$
$2x - 5(1) = 1$
$2x = 6$
$x = 3$

$x = 3,\ y = 1$

$4x - 3y = 9$
LS: $4(3) - 3(1) = 9$
RS: 9
LS = RS

$2x - 5y = 1$
LS: $2(3) - 5(1) = 1$
RS: 1
LS = RS

verified ✓

c) $5x - 2y = 0.6$
$2(2x + y = 1.5)$

$+$ $5x - 2y = 0.6$
$4x + 2y = 3$ (add)

$9x = 3.6$
$x = 0.4$

$2x + y = 1.5$
$2(0.4) + y = 1.5$
$0.8 + y = 1.5$
$y = 0.7$

$x = 0.4,\ y = 0.7$

$5x - 2y = 0.6$
LS: $5(0.4) - 2(0.7) = 0.6$
RS: 0.6
LS = RS

$2x + y = 1.5$
LS: $2(0.4) + (0.7) = 1.5$
RS: 1.5
LS = RS

verified ✓

5. Solve each of the following systems by elimination. Check each solution.

a) $2x + 4y = 7$, $4x - 3y = 3$ (×2)

$-$ $4x + 8y = 14$
$4x - 3y = 3$ (subtract)

$11y = 11$
$y = 1$

$2x + 4y = 7$
$2x + 4(1) = 7$
$2x = 3$
$x = 3/2$

$x = \frac{3}{2},\ y = 1$

$2x + 4y = 7$
LS: $2(3/2) + 4(1) = 7$
RS: 7
LS = RS

$4x - 3y = 3$
LS: $4(3/2) - 3(1) = 3$
RS = 3
LS = RS

verified ✓

b) $5x = 8y$, $4x - 3y + 17 = 0$

* rearrange equation to match the other, it is easier to work this way.

$5x - 8y = 0$ (×4) ⇒ $20x - 32y = 0$
$4x - 3y = -17$ (×5) ⇒ $20x - 15y = -85$ $-$ (subtract)

$-17y = 85$
$y = -5$

$5x = 8y$
$5x = 8(-5)$
$5x = -40$
$x = -8$

$x = -8,\ y = -5$

$5x = 8y$
LS: $5(-8) = -40$
RS: $8(-5) = -40$
LS = RS

$4x - 3y + 17 = 0$
LS: $4(-8) - 3(-5) + 17 = 0$
RS: 0
LS = RS

verified ✓

c) $7e + 4f - 1 = 0$, $5e + 3f + 1 = 0$

$7e + 4f = 1$ (×3) ⇒ $21e + 12f = 3$ $-$ (subtract)
$5e + 3f = -1$ (×4) ⇒ $20e + 12f = -4$

$e = 7$

$7e + 4f = 1$
$7(7) + 4f = 1$
$49 + 4f = 1$
$4f = -48$
$f = -12$

$e = 7,\ f = -12$

$7e + 4f - 1 = 0$
LS: $7(7) + 4(-12) - 1 = 0$
RS: 0
LS = RS

$5e + 3f + 1 = 0$
LS: $5(7) + 3(-12) + 1 = 0$
RS: 0
LS = RS

verified ✓

d) $3x + 2y - 6 = 0$, $9x = 5y + 18$

$3x + 2y = 6$ (×3) ⇒ $9x + 6y = 18$ $-$ (subtract)
$9x - 5y = 18$ $\quad$ $9x - 5y = 18$

$11y = 0$
$y = 0$

$3x + 2y = 6$
$3x + 2(0) = 6$
$3x = 6$
$x = 2$

$x = 2,\ y = 0$

$3x + 2y - 6 = 0$
LS: $3(2) + 2(0) - 6 = 0$
RS: 0
LS = RS

$9x = 5y + 18$
LS: $9(2) = 18$
RS: $5(0) + 18 = 18$
LS = RS

verified ✓

6. Consider the system of equations $x - 2y + 1 = 0$, $2x + 3y = 12$. Solve the system by

a) elimination

$2(x - 2y = -1)$ $\Rightarrow$ $2x - 4y = -2$ (subtract)
$2x + 3y = 12$ $\quad$ $2x + 3y = 12$
$-7y = -14$
$y = 2$

$2x + 3y = 12$
$2x + 3(2) = 12$
$2x + 6 = 12$
$2x = 6$
$x = 3$

$x = 3,\ y = 2$

b) substitution

$x - 2y + 1 = 0$
$x = 2y - 1$

$2x + 3y = 12$
$2(2y - 1) + 3y = 12$
$4y - 2 + 3y = 12$
$7y = 14$
$y = 2$

$x = 2y - 1$
$x = 2(2) - 1$
$x = 3$

$x = 3,\ y = 2$

Which method do you prefer?

Personal choice.

7. Consider the system of equations: $11x + 3y + 2 = 0$, $11x - 5y - 62 = 0$.

Solve the system by

a) elimination

$11x + 3y = -2$ (subtract)
$11x - 5y = 62$
$8y = -64$
$y = -8$

$11x + 3y = -2$
$11x + 3(-8) = -2$
$11x - 24 = -2$
$11x = 22$
$x = 2$

$x = 2,\ y = -8$

b) substitution

$11x + 3y + 2 = 0$
$3y = -11x - 2$
$y = -\frac{11}{3}x - \frac{2}{3}$

$11x - 5y - 62 = 0$
$11x - 5\left(-\frac{11}{3}x - \frac{2}{3}\right) - 62 = 0$
$11x + \frac{55}{3}x + \frac{10}{3} - 62 = 0$
$\frac{88}{3}x = \frac{176}{3}$
$x = 2$

$y = -\frac{11}{3}x - \frac{2}{3}$
$y = -\frac{11}{3}(2) - \frac{2}{3}$
$y = -8$

$x = 2,\ y = -8$

Which method do you prefer?

Personal choice.

8. Solve each of the following systems by elimination. Explain the results.

a) $-2x + 6y - 1 = 0,\ \ 5x - 15y + 2.5 = 0$

$-2x + 6y = 1$ (×5)
$5x - 15y = -2.5$ (×2)

$-10x + 30y = 5$
$+\ 10x - 30y = -5$ (add)
$0 = 0$

$-2x + 6y = 1$
$6y = 2x + 1$
$y = \frac{1}{6}(2x+1)$

$5x + 2.5 = 15y$
$y = \frac{5x + 2.5}{15}$
$y = \frac{1}{3}x + \frac{1}{6}$
$y = \frac{1}{6}(2x+1)$

There are an infinite number of solutions of the form
$x = a \quad y = \frac{1}{6}(2a+1)$
The graph of the equations are identical

b) $2x - 4y = 7,\ \ -7x + 14y = -21$

$2x - 4y = 7$ (×7)
$-7x + 14y = -21$ (×2)

$14x - 28y = 49$
$+\ -14x + 28y = -42$
$0 = 7$

There are no solutions since the graphs of the equations are parallel lines.

9. Solve each of the following systems by elimination.

a) $3x - \frac{1}{2}y = 5$ (×2)
$\frac{1}{3}x + \frac{1}{4}y = 3$ (×4)

$6x - y = 10$
$+\ \frac{4}{3}x + y = 12$
$\frac{22}{3}x = 22$
$x = 3$

$6x - y = 10$
$6(3) - y = 10$
$18 - y = 10$
$8 = y$

$\underline{\underline{x = 3,\ y = 8}}$

b) $\frac{m}{2} - \frac{n-4}{4} = 2$ (×8)
$\frac{3m}{4} - \frac{n}{5} = 5$ (×20)

$4m - 2(n-4) = 16$
$15m - 4n = 100$
$\Rightarrow$
$4m - 2n + 8 = 16$
$15m - 4n = 100$

$4m - 2n = 8$ (× -2)
$15m - 4n = 100$
$\Rightarrow$
$-8m + 4n = -16$
$+\ 15m - 4n = 100$ (add)
$7m = 84$
$m = 12$

$4m - 2n = 8$
$4(12) - 2n = 8$
$48 - 2n = 8$
$-2n = -40$
$n = 20$

$\underline{\underline{m = 12,\ n = 20}}$

Multiple Choice

10. When b is eliminated from the equations $2x + b = 8$ and $5x + 2b = 2$, we obtain

A. $7x = 10$
B. $9x = 18$
C. $x = -14$
D. $3x = -6$

$2x + b = 8 \; (\times 2) \Rightarrow 4x + 2b = 16$ (subtract)
$5x + 2b = 2 \qquad\qquad 5x + 2b = 2$
$-x = 14$
$x = -14$

11. The solution to the systems of equations $x + y = 0$, $\frac{1}{2}x + \frac{1}{3}y = 1$ is
$(\times 2) \qquad (\times 6)$

A. $x = 6, y = -6$
B. $x = 1, y = -1$
C. $x = 0, y = -0$
D. $x = -6, y = 6$

$2x + 2y = 0$ (subtract)
$3x + 2y = 6$
$-x = -6$
$x = 6$

$x + y = 0$
$(6) + y = 0$
$y = -6$

Numerical Response

12. If $\frac{1}{3}x + 5 = \frac{2}{3}y$ and $\frac{1}{2}x + \frac{1}{3}y = \frac{1}{3}$, then the value of $y - \frac{1}{2}x$, to the nearest tenth, is ______ .
(Record your answer in the numerical response box from left to right)

7	.	5	

$\frac{1}{3}x + 5 = \frac{2}{3}y \; (\times 3) \Rightarrow x + 15 = 2y \Rightarrow x - 2y = -15$
$\frac{1}{2}x + \frac{1}{3}y = \frac{1}{3} \; (\times 6) \qquad 3x + 2y = 2 \qquad 3x + 2y = 2$ + (add)
$4x = -13$
$x = \frac{-13}{4}$

$x + 15 = 2y$
$\left(\frac{-13}{4}\right) + 15 = 2y$
$\frac{47}{4} = 2y$
$\frac{47}{8} = y$

$y - \frac{1}{2}x$
$= \left(\frac{47}{8}\right) - \frac{1}{2}\left(\frac{-13}{4}\right)$
$= 7.5$

Answer Key

1. a) $x = 2,\ y = 6$ **b)** $x = 1,\ y = 1$ **c)** $a = -1,\ b = -2$

2. a) $x = 3,\ y = -6$ **b)** $m = -1,\ n = 5$ **c)** $a = -\frac{3}{2},\ b = 4$

3. a) $p = \frac{3}{10},\ q = \frac{3}{5}$ **b)** $x = 0.7,\ y = -0.3$ **c)** $x = 5,\ y = \frac{11}{2}$

4. a) $a = 3,\ b = 2$ **b)** $x = 3,\ y = 1$ **c)** $x = 0.4,\ y = 0.7$

5. a) $x = \frac{3}{2},\ y = 1$ **b)** $x = -8,\ y = -5$ **c)** $e = 7,\ f = -12$ **d)** $x = 2,\ y = 0$

6. $x = 3,\ y = 2$ **7.** $x = 2,\ y = -8$

8. a) There are an infinite number of solutions of the form $x = a, y = \frac{1}{6}(2a + 1), a \in R$ because the equations are identical (the resulting equation reduces to $0 = 0$).
b) There are no solutions since the graphs of the equations are parallel lines (the resulting equation reduces to e.g. $0 = 7$).

9. a) $x = 3,\ y = 8$ **b)** $m = 12,\ n = 20$ **10.** C **11.** A **12.**

7	.	5	

Systems Of Linear Equations Lesson #5:
Number and Money Applications

We have discussed three different methods for solving systems of equations.

Methods for
Solving Systems of Equations

Graphing | **Substitution** | **Elimination**

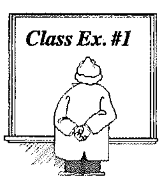
In this lesson we apply these methods in problem solving.

Problem Solving

We can solve a variety of types of problems using a system of equations. There are four general steps to problem solving which are shown in the flowchart below.

Step 1		Step 2		Step 3		Step 4
Introduce variables to represent the unknown values.	→	Form a system of equations involving the variables.	→	Solve the system.	→	Answer the problem and check the solution.

Number Applications



The difference between two numbers is 9. The larger number is 3 more than twice the smaller number. Find the numbers.

Let x be the larger number and y be the smaller number

$x - y = 9$ * "the difference between two numbers is 9"

$x = 2y + 3$ * "larger number is 3 more than twice the smaller number"

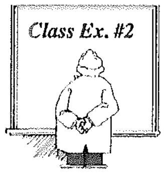
$x - y = 9$
$(2y+3) - y = 9$
$y = 6$

$x = 2y + 3$
$x = 2(6) + 3$
$x = 15$

<u>The numbers are 15 and 6</u>

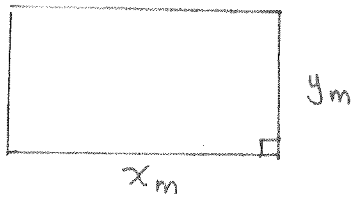
CHECK: $\underline{x - y = 9}$
LS: $15 - 6$
$= 9$
RS: 9
LS = RS

$\underline{x = 2y + 3}$
LS: 15
RS: $2(6) + 3$
$= 15$
LS = RS



The perimeter of a rectangle is 40 metres. The width is 4 metres less than the length. Find the dimensions of the rectangle.

y x



$2x + 2y = 40 \qquad y = x - 4$

sub into formula found.

$2x + 2y = 40$
$2x + 2(x-4) = 40$
$2x + 2x - 8 = 40$
$4x = 48$
$x = 12$

sub. into equation to solve "y"

$y = x - 4$
$y = (12) - 4$
$y = 8$

check

$2x + 2y = 40$

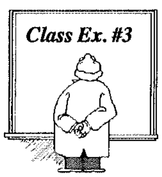
LS	RS
$2(12) + 2(8) = 40$	40

$y = x - 4$

LS	RS
(8)	$(12) - 4 = 8$

The length is 12m
The width is 8m

Money Applications



Gary had a total of $260 in five-dollar bills and ten-dollar bills.
If he has 33 bills in total, how many of each denomination does he have?

Let x be the number of five-dollar bills and y be the number of ten-dollar bills.

$5x + 10y = 260$
$x + y = 33 \quad (\times 5)$

$5x + 10y = 260$
$-\ 5x + 5y = 165$ (subtract)
$5y = 95$
$\underline{y = 19}$

sub into equation to solve for x.

$x + y = 33$
$x + (19) = 33$
$\underline{x = 14}$

check answers

$5x + 10y = 260$
LS: $5(14) + 10(19) = 260$
RS: 260
LS = RS

$x + y = 33$
LS: $(14) + (19) = 33$
RS: 33
LS = RS

There are 14 five-dollar bills and 19 ten-dollar bills

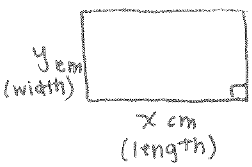
Complete Assignment questions #1 - #10

Assignment

In problems #1 - #7 use the following procedure:

a) Introduce variables to represent the unknown values.
b) Form a system of equations involving the variables.
c) Solve the system.
d) Answer the problem and check the solution.

1. A rectangle is to be drawn with perimeter 64 cm. If the length is to be 14 cm more than the width, determine the area of the rectangle.



$2x + 2y = 64$

$x = y + 14$

check: $\underline{2x + 2y = 64}$

LS: $2(23) + 2(9)$
$= 64$
RS: 64
LS = RS

$\underline{x = y + 14}$

LS = 23
RS = $(9) + 14$
$= 23$
LS = RS

$2x + 2y = 64$
$2(y + 14) + 2y = 64$
$2y + 28 + 2y = 64$
$4y = 36$
$y = 9$

$x = y + 14$
$x = (9) + 14$
$x = 23$

The length is 23 cm and the width is 9 cm

Area = L×W = 23×9
Area = 207 cm²

2. The sum of two numbers is 3, and twice the larger number is 36 more than three times the smaller number. Find the numbers.

Let the larger number be x and the smaller number be y.

$x + y = 3 \Rightarrow y = 3 - x$
$2x = 3y + 36$

$2x = 3(3 - x) + 36$
$2x = 9 - 3x + 36$
$5x = 45$
$x = 9$

$y = 3 - (9)$
$y = -6$

The numbers are 9 and −6

check: $\underline{x + y = 3}$

LS: $(9) + (-6)$
$= 3$
RS: 3
LS = RS

$\underline{2x = 3y + 36}$

LS: $2(9) = 18$
RS: $3(-6) + 36$
$= 18$
LS = RS

3. Five pencils and four pens cost \$6.15. Three similar pencils and eight similar pens cost \$9.85. How much would you expect to pay for a set of eight pencils and seven pens?

Let \$$x$ be the cost of a pencil and \$$y$ be the cost of a pen.

$5x + 4y = 6.15$ (×2)
$3x + 8y = 9.85$

$10x + 8y = 12.30$
$-\ 3x + 8y = 9.85$ (subtract)
$7x = 2.45$
$x = 0.35$

$5x + 4y = 6.15$
$5(0.35) + 4y = 6.15$
$1.75 + 4y = 6.15$
$4y = 4.40$
$y = 1.10$

Pen cost \$1.10
Pencil cost \$0.35

$8(0.35) + 7(1.10) = 10.50$

$\boxed{\text{Eight pencils and seven pens costs \$10.50}}$

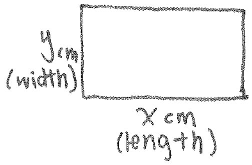
CHECK: $\underline{5x + 4y = 6.15}$

LS: $5(0.35) + 4(1.10)$
$= 6.15$
RS: 6.15
LS = RS

$\underline{3x + 8y = 9.85}$

LS: $3(0.35) + 8(1.10)$
$= 9.85$
RS: 9.85
LS = RS

4. The perimeter of a rectangle is 40 cm. If the length were doubled and the width halved, the perimeter would be increased by 16 cm. Find the dimensions of the original rectangle.



$2x + 2y = 40$

$2(2x) + 2(\frac{1}{2}y) = 40 + 16$

$2x + 2y = 40$

$4x + y = 56 \rightarrow y = 56 - 4x$

$2x + 2y = 40$

$2x + 2(56 - 4x) = 40$

$2x + 112 - 8x = 40$

$-6x = -72$

$x = 12$

$y = 56 - 4x$

$y = 56 - 4(12)$

$y = 8$

$x = 12, \ y = 8$

The length is 12 cm and the width is 8 cm

CHECK: $2x + 2y = 40$

LS: $2(12) + 2(8) = 40$

RS: 40

LS = RS

$4x + y = 56$

LS: $4(12) + (8) = 56$

RS: 56

LS = RS

5. A small engineering company has an old machine which produces 30 components per hour, and has recently installed a new machine which produces 40 components per hour. Yesterday, both machines were in operation for different periods of time. If 545 components were produced when the total number of hours of operation was 15 hours, determine for how many hours each machine was operating.

Let x = # hours old machine was in operation

and y = # hours new machine was in operation.

$x + y = 15 \rightarrow y = 15 - x$

$30x + 40y = 545$

$30x + 40(15 - x) = 545$

$30x + 600 - 40x = 545$

$-10x = -55$

$x = 5.5$

$y = 15 - x$

$y = 15 - (5.5)$

$y = 9.5$

CHECK: $x + y = 15$

LS: $(5.5) + (9.5) = 15$

RS: 15

LS = RS

$30x + 40y = 545$

LS: $30(5.5) + 40(9.5) = 545$

RS: 545

LS = RS

The old machine operated for $5\frac{1}{2}$ hours and the new machine operated for $9\frac{1}{2}$ hours

6. In a hockey arena, a seat at rink level costs three times as much as a seat in the upper level. If five seats at rink level cost $112 more than eight seats in the upper level, find the cost of a seat at rink level.

Let $x be the cost of a rink level seat and $y be the cost of an upper level se

$x = 3y$

$5x = 8y + 112$

$5(3y) = 8y + 112$

$15y = 8y + 112$

$7y = 112$

$y = 16$

$x = 3y$

$x = 3(16)$

$x = 48$

CHECK: $x = 3y$

LS = (48)

RS = $3(16) = 48$

LS = RS

$5x = 8y + 112$

LS: $5(48) = 240$

RS: $8(16) + 112 = 240$

LS = RS

A seat at rink level cost $48

7. Rachel had been saving quarters and dimes to buy a new toy. She had 103 coins and had saved \$21.40. How many coins of each type had she saved?

Let x = # quarters and y = # dimes

$x+y=103 \rightarrow y=103-x$

$0.25x+0.1y=21.4$

$0.25x+0.1(103-x)=21.4$

$0.25x+10.3-0.1x=21.4$

$0.15x=11.1$

$x=74$

$y=103-x$

$y=103-(74)$

$y=29$

CHECK: $\underline{x+y=103}$

LS: $(74)+(29)$

$=103$

RS: 103

LS = RS

$\underline{0.25x+0.1y=21.4}$

LS: $0.25(74)+0.1(29)$

$=21.4$

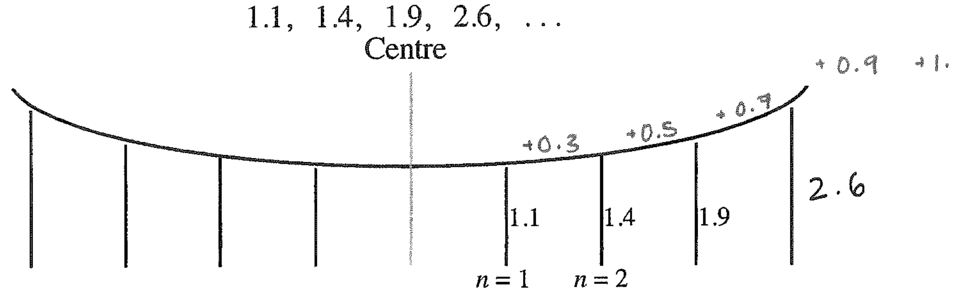
RS: 21.4

LS = RS

She saved 74 quarters and 29 dimes

8. The heights, in metres, of the vertical rods of a suspension bridge, as you move out from the centre of the bridge, form the sequence,

1.1, 1.4, 1.9, 2.6, ...



a) Without a calculator determine the next two terms in the sequence.

$2.6+0.9=\underline{3.5}$ $\qquad 3.5+1.1=\underline{4.6}$

b) The height, h metres, of the n^{th} rod is given by the formula $h=a+bn^2$. Using the terms of the sequence given to form a system of equations, determine the values of a and b and state the formula.

$h=a+bn^2$

(term 1) $1.1=a+b(1)^2$

(term 2) $1.4=a+b(2)^2$

$\underline{a=1,\ b=0.1}$ $\qquad \underline{h=1+0.1n^2}$

$a+b=1.1$

$-\ a+4b=1.4$ (subtract)

$-3b=-0.3$

$b=0.1$

$a+b=1.1$

$a+(0.1)=1.1$

$a=1$

$h=a+bn^2$

$h=(1)+(0.1)n^2$

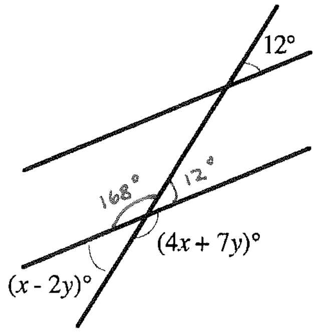
$h=1+0.1n^2$

c) Use this formula to verify the answers in **a**).

$n=5 \quad h=1+0.1(5)^2=3.5$

$n=6 \quad h=1+0.1(6)^2=4.6$

Numerical Response **9.** The diagram shows two parallel lines and a transversal.



$180° - 12°$
$= 168°$

$x - 2y = 12$ $(\times 4)$ $\Rightarrow$ $4x - 8y = 48$
$4x + 7y = 168$ $\quad$ $4x + 7y = 168$ (subtract)
$-15y = -120$
$\underline{y = 8}$

$x - 2y = 12$
$x - 2(8) = 12$
$\underline{x = 28}$

The value of $x + y$, to the nearest whole number, is _____ .

(Record your answer in the numerical response box from left to right)

3	6		

$x + y$
$= 28 + 8$
$= 36$

10. A number consists of two digits whose sum is 11. If the digits are reversed, the original number is increased by 27. The original number is _____ .

(Record your answer in the numerical response box from left to right)

4	7		

Let x = units digit and y = tens digit

yx = original number
xy = new number

original number $= 10y + x$
new number $= 10x + y$

* multiply by 10 b/c that variable is the tens digit.

$10x + y = 10y + x + 27$
$9x - 9y = 27$

$x + y = 11$ $(\times 9)$ $\Rightarrow$ $9x + 9y = 99$ (add)
$9x - 9y = 27$ $\quad$ $+\ 9x - 9y = 27$
$18x = 126$
$\underline{x = 7}$

$x + y = 11$
$(7) + y = 11$
$\underline{y = 4}$

$\boxed{\text{original number} = yx = 47}$

New number $= xy = 74$
$74 - 47 = 27$ ✓ verified.

Answer Key

1. 207 cm^2 **2.** 9, −6 **3.** \$10.50 **4.** 12 cm by 8 cm

5. $5\frac{1}{2}$ hours old and $9\frac{1}{2}$ hours new **6.** \$48 **7.** 74 quarters, 29 dimes

8. a) 3.5, 4.6, **b)** $a = 1$, $b = 0.1$, $h = 1 + 0.1n^2$

9.

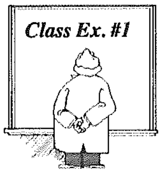
3	6		

10.

4	7		

Systems of Linear Equations Lesson #6:
Mixture and Percentage Applications

Mixture Applications



Cashew nuts costing \$22/kg are mixed with Brazil nuts costing \$16/kg. The mixture weighs 50 kg and sells for \$18/kg.

a) How much does it cost to buy the whole mixture? $50(18) = \underline{900}$

b) Form a system of equations and solve it to determine the number of kilograms of each type of nut used in the mixture.

Let x = # kg of cashew nuts and y = # kg of Brazil nuts.

$x + y = 50 \rightarrow y = 50 - x$

$22x + 16y = 900$

$22x + 16(50 - x) = 900$

$22x + 800 - 16x = 900$

$6x = 100$

$x = \frac{50}{3}$

$x = 16\frac{2}{3}$

$y = 50 - x$

$y = 50 - (16\frac{2}{3})$

$y = \frac{100}{3}$

$y = 33\frac{1}{3}$

CHECK: $\underline{x + y = 50}$

LS: $16\frac{2}{3} + 33\frac{1}{3}$

$= 50$

RS $= 50$

LS = RS

$\underline{22x + 16y = 900}$

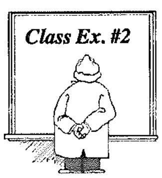
LS: $22(16\frac{2}{3}) + 16(33\frac{1}{3})$

$= 900$

RS $= 900$

LS = RS

The mixture had $16\frac{2}{3}$ kg of cashew nuts and $33\frac{1}{3}$ kg of Brazil nuts.



Lora invested her inheritance of \$48 000 in two different mutual funds. At the end of one year one fund had earned 10.5% interest and the other fund had earned 12% interest. If she received a total of \$5520 in interest, how much did she invest in each mutual fund?

Let x = amount invested in 10.5% interest fund

Let y = amount invested in 12% interest fund

$x + y = 48\,000 \rightarrow y = 48000 - x$

$\underbrace{1.105x}_{\text{"earned 10.5\%" } \therefore 110.5\% = 1.105} + \underbrace{1.12y}_{\text{"earned 12\%" } \therefore 112\% = 1.12} = \underbrace{53520}_{\text{initial + earned } = 48000 + 5520 = 53520}$

$1.105x + 1.12y = 53520$

$1.105x + 1.12(48000 - x) = 53520$

$1.105x + 53760 - 1.12x = 53520$

$-0.015x = -240$

$\underline{x = 16000}$

$y = 48000 - x$

$y = 48000 - (16000)$

$\underline{y = 32000}$

CHECK: $\underline{x + y = 48000}$

LS: $16000 + 32000$

$= 48000$

RS: 48000

LS = RS

$\underline{1.105x + 1.12y = 53520}$

LS: $1.105(16000) + 1.12(32000)$

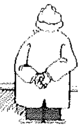
$= 53520$

RS: 53520

LS = RS

She invested \$16000 and \$32000 in each mutual fund.

Class Ex. #3



Earl the chemist has to make 180 mL of 60% hydrochloric acid (HCl) solution. He has available a one litre bottle of 45% HCl solution and a one litre bottle of 70% HCl solution by volume. How many mL of each solution are mixed to make the 60% HCl solution?

Let x = # mL of 45% HCl and y = # mL of 70% HCl

$x+y=180 \qquad y=180-x$

$0.45x+0.7y=0.60(180)$

$\hookrightarrow$ 60% = 0.6 ∴ 0.6(180) b/c that is the goal of the question.

$0.45x+0.7y=108$

$0.45x+0.7(180-x)=108$

$0.45x+126-0.7x=108$

$-0.25x=-18$

$x=72$

$y=180-x$

$y=180-(72)$

$y=108$

CHECK: $x+y=180$

LS: $(72)+(108)$
$=180$

RS: 180

LS = RS

$0.45x+0.7y=108$

LS: $0.45(72)+0.7(108)$
$=108$

RS $=108$

LS = RS.

72 mL of 45% HCl solution and 108 mL of 70% HCl solution are mixe

Complete Assignment questions #1 - #8

Assignment

In problems #1 - #8 use the following procedure.

a) Introduce variables to represent the unknown values.
b) Form a system of equations involving the variables.
c) Solve the system.
d) Answer the problem and check the solution.

1. Candy costing \$6 per kg is mixed with candy costing \$4.50 per kg to produce 112 kg of candy worth \$ 612. How many kg of each type of candy were used?

Let x = # kg at \$6 per kg and y = # kg at \$4.50 per kg

$x+y=112 \rightarrow y=112-x$

$6x+4.5y=612$

$6x+4.5(112-x)=612$

$6x+504-4.5x=612$

$1.5x=108$

$x=72$

$y=112-x$

$y=112-(72)$

$y=40$

CHECK: $x+y=112$

LS: $(72)+(40)$
$=112$

RS: 112

LS = RS

$6x+4.5y=612$

LS: $6(72)+4.5(40)=$
$=432+180$
$=612$

RS $=612$

LS = RS

72 kg of \$6 per kg candy and 40 kg of \$4.50 per kg candy.

2. Chad invested $\frac{3}{4}$ of his \$56 000 lottery winnings in two different mutual funds. At the end of the year the *Balanced Fund* had earned 6.5% interest, but the *Emerging Markets Fund* had lost 3%. If the value of Chad's funds increased by \$1 590, determine the amount invested in each fund.

Let x = amount invested in Balanced Fund and y = amount invested in Emerging Markets Fund.

$\frac{3}{4} \times 56000 = 42000 + 1590 = 43590$

"earned 6.5%" = 106.5% = 1.065

"lost 3%" = 97% = 0.97

$x + y = 42000$ $\quad y = 42000 - x$

$1.065x + 0.97y = 43590$

$1.065x + 0.97(42000 - x) = 43590$

$1.065x + 40740 - 0.97x = 43590$

$0.095x = 2850$

$x = 30000$

$y = 42000 - x$

$y = 42000 - (30000)$

$y = 12000$

CHECK: $\underline{x + y = 42000}$
LS: $(30000) + (12000) = 42000$
RS: 42000
LS = RS

$\underline{1.065x + 0.97y = 43590}$
LS: $1.065(30000) + 0.97(12000) = 43590$
RS: 43590
LS = RS

\$30000 invested in Balanced Fund and \$12000 invested in Emerging Markets Fund.

3. Shoji invested \$7 000, part at 9% interest and part at 6% interest. The interest obtained from the 6% investment was half of the interest obtained from the 9% investment. How much was invested at each rate?

Let \$$x$ = amount at 9% and \$$y$ = amount at 6%

$x + y = 7000 \rightarrow y = 7000 - x$

$0.06y = \frac{1}{2}(0.09x)$

$0.06(7000 - x) = \frac{1}{2}(0.09x)$

$420 - 0.06x = 0.045x$

$420 = 0.105x$

$x = 4000$

$y = 7000 - x$

$y = 7000 - (4000)$

$y = 3000$

CHECK: $\underline{x + y = 7000}$
LS: $4000 + 3000 = 7000$
RS: 7000
LS = RS

$\underline{0.06y = \frac{1}{2}(0.09x)}$
LS: $0.06(3000) = 180$
RS: $\frac{1}{2}(0.09)(4000) = 180$
LS = RS

Shoji invested \$4000 at 9% and \$3000 at 6%

4. 300 grams of Type A Raisin Bran is mixed with 500 grams of Type B Raisin Bran to produce a mixture which is 11% raisins. Type A Raisin Bran has twice as many raisins per kilogram as Type B. What percentage of raisins are in each type of Raisin Bran?

Let x = percentage of raisins in Type A

Let y = percentage of raisins in Type B

$x = 2y$

$300x + 500y = 800(11)$

$300x + 500y = 8800$

$300(2y) + 500y = 8800$

$600y + 500y = 8800$

$1100y = 8800$

$y = 8$

$x = 2(8)$

$x = 16$

CHECK: $\underline{300x + 500y = 8800}$
LS: $300(16) + 500(8) = 8800$
RS: 8800

$\underline{x = 2y}$
LS: 16
RS: $2(8) = 16$
LS = RS

Type A has 16% raisins and

Type B has 8% raisins.

5. A scientist has to make 800 ml of 61% sulfuric acid solution. He has available a one litre bottle of 40% sulfuric acid solution and a one litre bottle of 75% sulfuric acid solution by volume.

a) How many ml of each solution are mixed to make the 61% sulfuric acid solution?

Let x = # mL of 40% sulfuric acid and y = # mL of 75% sulfuric acid.

$x+y=800$

$0.4x+0.75y=0.61(800)$

$y=800-x$

$0.4x+0.75y=488$

CHECK: $x+y=800$

LS: $(320)+(480)$
$=800$

RS: 800

LS = RS

$0.4x+0.75y=488$

LS: $0.4(320)+0.75(480)$
$=488$

RS: 488

LS = RS

$0.4x+0.75(800-x)=488$

$0.4x+600-0.75x=488$

$-0.35x=-112$

$x=320$

$y=800-x$

$y=800-(320)$

$y=480$

320 mL of 40% sulfuric acid and 480 mL of 75% sulfuric acid

b) What is the maximum volume, rounded down to the nearest ml, of 61% sulfuric acid solution which the scientist could mix with the original bottles of sulfuric acid?

use 1 litre of 75% sulfuric acid
∴ $y=1000$ mL

$\frac{x}{320}=\frac{y}{480} \Rightarrow \frac{x}{320}=\frac{1000}{480}$

$x=666.6\ldots$

1000 mL of 75% sulfuric acid solution

666 mL of 40% sulfuric acid solution

maximum volume = 1666 mL

6. One year a man saved \$5000. The next year his income increased by 10% and his expenditure decreased by 16%. He was able to save \$14 600. Calculate his income in the second year.

Let \$$x$ = income in year 1 and \$$y$ = expenditure in year 1

$x-y=5000 \rightarrow x=y+5000$

$1.1x-0.84y=14600$

$1.1(y+5000)-0.84y=14600$

$1.1y+5500-0.84y=14600$

$0.26y=9100$

$y=35000$

$x=y+5000$

$x=35000+5000$

$x=40000$

CHECK: $x-y=5000$

LS: $40000-35000$
$=5000$

RS: 5000

LS = RS

$1.1x-0.84y=14600$

LS: $1.1(40000)-0.84(35000)$
$=14600$

RS: 14600

LS = RS

$40000+10\%=44000$

Income in year 2 = \$44000

7. Pure gold (24-carat) is often mixed with other metals to produce jewellery. 12-carat gold is $^{12}/_{24}$ or 50% gold, 6-carat gold is $^{6}/_{24}$ or 25% gold, etc.
A jeweller has some 12- carat gold and some 21-carat gold and wants to produce 90 grams of 75% gold.

a) What percentage of gold is 21-carat?

$\frac{21}{24} = 87.5\%$ gold

b) How many grams of 12-carat gold and of 21 carat gold are needed to produce the mixture?

Let x = # grams of 12 carat gold and y = # grams of 21 carat gold.

$x + y = 90 \qquad y = 90 - x$

$0.5x + 0.875y = 0.75(90)$
$0.5x + 0.875y = 67.5$
$0.5x + 0.875(90 - x) = 0.75(90)$
$0.5x + 78.75 - 0.875x = 67.5$
$-0.375x = -11.25$
$\underline{x = 30}$

$y = 90 - x$
$y = 90 - 30$
$\underline{y = 60}$

CHECK: $\underline{x + y = 90}$
LS: $30 + 60 = 90$
RS: 90
LS = RS

$\underline{0.5x + 0.875y = 67.5}$
LS: $0.5(30) + 0.875(60) = 67.5$
RS: 67.5
LS = RS

<u>30g of 12 carat gold and 60g of 21 carat gold</u>

Multiple Choice **8.** A shopkeeper wishes to mix two types of tea together. One type sells at \$8 per kg and the second type sells at \$12 per kg. He wishes to make 100 kg of the mixture to sell at \$11 per kg. The number of kg of the first type of tea in this mixture should be

A. 25
B. $33\frac{1}{3}$
C. 50
D. 75

Let x = # kg of first type
Let y = # kg of second type

$x + y = 100 \Rightarrow y = 100 - x$
$8x + 12y = 11(100) \Rightarrow 8x + 12y = 1100$

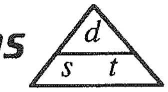
$8x + 12(100 - x) = 1100$
$8x + 1200 - 12x = 1100$
$-4x = -100$
$\underline{\underline{x = 25}}$

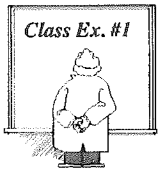
Answer Key

1. 72 kg of \$6/kg candy, 40 kg of \$4.50/kg candy
2. \$30 000, \$12 000
3. \$3000 at 6% and \$4000 at 9%
4. 16% in type A, 8% in type B
5. **a)** 320 ml of 40% solution, 480 ml of 75% solution **b)** 1666 ml
6. \$44 000
7. **a)** 87.5% **b)** 30 g of 12-carat gold, 60 g of 21-carat gold
8. A

Systems of Linear Equations Lesson #7:

Distance, Speed, and Time Applications





Class Ex. #1

A student drove the 1245 km from Edmonton to Vancouver in $16\frac{1}{2}$ hours. This included a one hour stop in Golden and a 30 minute stop in Kamloops. She averaged 100 km/h on the divided highways and 75 km/h on the non-divided mountainous roads. How much time did she spend on the divided highways?

driving time $= 16\frac{1}{2} - 1 - \frac{1}{2} = 15$ hours

	Distance (km)	Speed (km/h)	Time (h)
Highway	$100x$	100	x
Mountainous Roads	$75y$	75	y

$x + y = 15 \rightarrow y = 15 - x$

$100x + 75y = 1245$

$100x + 75(15 - x) = 1245$

$100x + 1125 - 75x = 1245$

$25x = 120$

$\underline{x = 4.8}$

$y = 15 - x$

$y = 15 - 4.8$

$\underline{y = 10.2}$

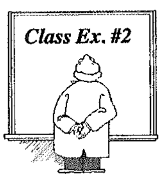
CHECK: $\underline{x + y = 15}$

LS: $(4.8) + (10.2)$
$= 15$
RS: 15
LS = RS

$\underline{100x + 75y = 1245}$

LS: $100(4.8) + 75(10.2)$
$= 1245$
RS: 1245
LS = RS

<u>She spent 4.8 hours on the divided highways</u>



Class Ex. #2

A small cruise boat took 3 hours to travel 36 km down a river with the current. On the return trip it took 4 hours against the current. Find the speed of the current and the speed of the small cruise boat in still water.

Let x km/h be the speed of the boat in still water
Let y km/h be the speed of the current

	Distance (km)	Speed (km/h)	Time (h)
Downstream	$3(x+y)$	$x + y$	3
Upstream	$4(x+y)$	$x - y$	4

subtracted b/c when going against the current (upstream) the speed will be lower than going downstream

$3(x+y) = 36$
$4(x-y) = 36$

$\Rightarrow$

$3x + 3y = 36 \ (\times 4)$
$4x - 4y = 36 \ (\times 3)$

$12x + 12y = 144$
$+\ 12x - 12y = 108$
$24x = 252$
$\underline{x = 10.5}$

$3x + 3y = 36$
$3(10.5) + 3y = 36$
$3y = 4.5$
$\underline{y = 1.5}$

CHECK: $\underline{3(x+y) = 36}$

LS: $3(10.5 + 1.5)$
$= 3(12) = 36$
RS: 36
LS = RS

$\underline{4(x-y) = 36}$

LS: $4(10.5 - 1.5)$
$= 4(9) = 36$
RS: 36
LS = RS

<u>Still water speed = 10.5 km/h</u>
<u>Current speed = 1.5 km/h</u>

Complete Assignment questions #1 - #6

Assignment

In problems #1 - #6 use the following procedure.

a) Introduce variables to represent the unknown values.
b) Form a system of equations involving the variables.
c) Solve the system.
d) Answer the problem and check the solution.

1. A cycle road test consists of a series of uphill and downhill sections.
Li Na averaged 20 km/hr on the uphill sections and 40 km/hr on the downhill sections.
If she completed the 35 km course in 1.3 hours, determine the length of the downhill sections.

	Distance (km)	Speed (km/h)	Time (h)
uphill	$20x$	20	x
downhill	$40y$	40	y

$x+y=1.3 \rightarrow y=1.3-x$

$20x+40y=35$

$20x+40(1.3-x)=35$
$20x+52-40x=35$
$-20x=-17$
$\underline{x=0.85}$

$y=1.3-x$
$y=1.3-0.85$
$\underline{y=0.45}$

These are times! Question is asking for distance!

CHECK: $\underline{x+y=1.3}$
LS: $0.85+0.45=1.3$
RS: 1.3
LS = RS

$\underline{20x+40y=35}$
LS: $20(0.85)+40(0.45)$
$=35$
RS: 35
LS = RS

Distance downhill $=40y$
$=40(0.45)$
$=18$ km

★ do not forget this step!

<u>Length of downhill sections = 18 km</u>

2. A train travels 315 km in the same time that a car travels 265 km. If the train travels, on average, 20 km/h faster than the car, find the average speed of the car and the time taken to travel 265 km.

	Distance (km)	Speed (km/h)	Time (h)
Train	315	x	$\frac{315}{x}$
Car	265	y	$\frac{265}{y}$

$x=y+20$

$\frac{315}{x}=\frac{265}{y}$
$315y=265x$

$315y=265(y+20)$
$315y=265y+5300$
$50y=5300$
$y=106$
(speed of car)

$x=y+20$
$x=(106)+20$
$x=126$

Time taken $=\frac{265}{y}=\frac{265}{106}=2.5$

CHECK: $\underline{\frac{315}{x}=\frac{265}{y}}$
LS: $\frac{315}{(126)}=2.5$
RS: $\frac{265}{(106)}=2.5$
LS = RS

$\underline{x=y+20}$
LS: 126
RS: $(106)+20=126$
LS = RS

<u>Average speed of car = 106 km/h</u> <u>Time taken = 2.5 hours</u>

3. A small plane flying into a wind takes 3hr to travel the 780 km journey from Victoria to Prince Rupert. At the same time, a similar plane leaves Prince Rupert and reaches Victoria in $2\frac{1}{2}$ hr. If the planes have the same cruising speed in windless conditions, determine the speed of the wind.

	Distance (km)	Speed (km/h)	Time (h)
V → PR	$3(x-y)$	$x-y$	3
PR → V	$2.5(x+y)$	$x+y$	2.5

Let x km/h = plane speed
Let y km/h = wind speed

$$3(x-y)=780 \Rightarrow 3x-3y=780 \ (\times 5)$$
$$2.5(x+y)=780 \qquad 2.5x+2.5y=780 \ (\times 6)$$

$$\begin{aligned} &15x-15y=3900 \quad \text{(add)}\\ +\ &15x+15y=4680\\ \hline &30x=8580\\ &\underline{x=286} \end{aligned}$$

CHECK: $3x-3y=780$
LS: $3(286)-3(26)$
$=780$
RS: 780
LS = RS

$2.5x+2.5y=780$
LS: $2.5(286)+2.5(26)$
$=780$
RS: 780
LS = RS

$$\begin{aligned} 3x-3y&=780\\ 3(286)-3y&=780\\ 858-3y&=780\\ -3y&=-78\\ \underline{y}&\underline{=26} \end{aligned}$$

Wind speed is 26 km/h

4. A cyclist leaves home at 7:30 am to cycle to school 7 km away. He cycles at 10 km/h until he has a puncture; then he has to push his bicycle the rest of the way at 3 km/h. He arrives at school at 8:40 am. How far did he have to push his bicycle?

	Distance (km)	Speed (km/h)	Time (h)
Cycle	x	10	$x/10$
Push	y	3	$y/3$

7:30AM → 8:40AM = 1hr. 10min. = $\frac{7}{6}$ hr.

$$x+y=7 \longrightarrow y=7-x$$
$$\left(\frac{x}{10}+\frac{y}{3}=\frac{7}{6}\right)(\times 30) \rightarrow 3x+10y=35$$

$$\begin{aligned} 3x+10y&=35\\ 3x+10(7-x)&=35\\ 3x+70-10x&=35\\ -7x&=-35\\ \underline{x}&\underline{=5} \end{aligned}$$

$$\begin{aligned} y&=7-x\\ y&=7-(5)\\ \underline{y}&\underline{=2} \end{aligned}$$

CHECK: $x+y=7$
LS: $(5)+(2)$
$=7$
RS: 7
LS = RS

$\frac{x}{10}+\frac{y}{3}=\frac{7}{6}$
LS: $\frac{5}{10}+\frac{2}{3}=\frac{7}{6}$
RS = $\frac{7}{6}$
LS = RS

He had to push his bicycle 2 km

Multiple Choice 5. Chris walks at 8 km/h and runs at 12 km/h. One day he walks and runs on the way from his house to the library. It takes him 20 minutes.
On his way back from the library he again walks and runs, but he runs twice as far as he did on the way to the library. The journey home takes $17\frac{1}{2}$ minutes.
The distance between his house and the library is

Distance = $x+y$
$= 2+1$
$= 3$ km

A. 3 km
B. 4 km
C. 5 km
D. 6 km

House → Library

	D (km)	S (km/h)	T (h)
walk	x	8	$x/8$
run	y	12	$y/12$

Library → House

	D (km)	S (km/h)	T (h)
walk	$x-y$	8	$\frac{x-y}{8}$
run	$2y$	12	$\frac{2y}{12} = \frac{y}{6}$

Let x = distance walked, Let y = distance ran.

$\frac{x}{8} + \frac{y}{12} = \frac{20}{60}$
$120\left(\frac{x}{8}\right) + 120\left(\frac{y}{12}\right) = 120\left(\frac{20}{60}\right)$
$(15x + 10y = 40) \div 5$ to simplify
$3x + 2y = 8$

$\frac{x-y}{8} + \frac{y}{6} = \frac{17.5}{60}$
$120\left(\frac{x-y}{8}\right) + 120\left(\frac{y}{6}\right) = 120\left(\frac{17.5}{60}\right)$
$15(x-y) + 20y = 35$
$15x - 15y + 20y = 35$
$(15x + 5y = 35) \div 5$ to simplify
$3x + y = 7$

$3x + 2y = 8$
$-\ 3x + y = 7$ (subtra
$y = 1$

$3x + y = 7$
$3x + (1) = 7$
$3x = 6$
$x = 2$

Numerical Response 6. Raj left home at 1 pm to travel 675 km to visit his sister. He averaged 110km/h for the first part of the trip during which he had a 1 hour rest, and 90 km/h for the second part of the trip during which he had a 30 minute rest. He reached his destination at 9 pm. The number of minutes taken for the first part of the trip, to the nearest minute, was ______ .

(Record your answer in the numerical response box from left to right)

2	7	0	

1pm → 9pm = 8h
Driving time: 8h − 1h − 0.5h = 6.5h

	Distance (km)	Speed (km/h)	Time (h)
First Part	$110x$	110	x
Second Part	$90y$	90	y

CHECK: $110x + 90y = 675$
LS: $110(4.5) + 90(2)$
$= 675$
RS: 675
LS = RS

$x + y = 6.$
LS: $(4.5)+(2$
$= 6.5$
RS: 6.5
LS = RS

$110x + 90y = 675$
$x + y = 6.5$ $(\times 90)$

$110x + 90y = 675$
$-\ 90x + 90y = 585$ (subtract)
$20x = 90$
$x = 4.5$

$y = 6.5 - x$
$y = 6.5 - (4.5)$
$y = 2$

$4.5\text{h} = 4.5(60\text{ min}) = 270$ min.

Answer Key

1. 18 km **2.** 106 km/h, $2\frac{1}{2}$ hr **3.** 26 km/h **4.** 2 km **5.** A **6.**

2	7	0	

Systems of Linear Equations Lesson #8:
Practice Test

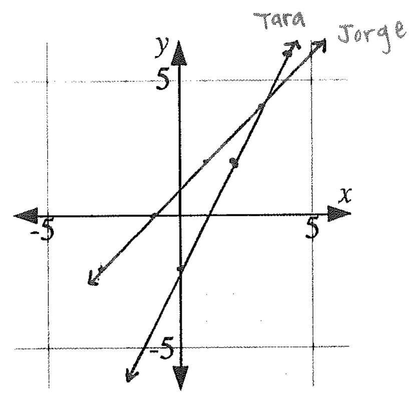
Numerical Response 1. Two students worked together to solve a system of equations which had integer solutions. Tara made a table of values for the first equation and Jorge made a table of values for the second equation.

Tara

x	y
–2	–6
0	–2
2	2
4	6

Jorge

x	y
–3	–2
–1	0
1	2
3	4
5	6



Using the students' results to determine the solution to the system, the value of $x + y$ is ___7___ .

(Record your answer in the numerical response box from left to right)

7			

$(3,4)$ $3+4=7$

1. The ordered pair (x, y) which satisfies the system of equations $x - 3y = 8,\ x + 4y = -13$, is

A. $(-1, 3)$

B. $(3, -1)$

C. $(-1, -3)$

D. $(3, 1)$

$x-3y=8$
$-\ x+4y=-13$ (subtract)
$-7y=21$
$\underline{y=-3}$

$x-3y=8$
$x-3(-3)=8$
$x+9=8$
$\underline{x=-1}$

$\therefore (-1,-3)$

2. The number of solutions to the system of equations $6x - 2y = 24,\ 5y = 15x - 64$, is

A. zero

B. one

C. two

D. infinite

$6x-2y=24$
$-2y=-6x+24$
$y=\underline{3}x-12$

$5y=15x-64$
$y=\underline{3}x-\frac{64}{5}$

parallel lines

Numerical Response 2. Alyssa graphs the equations $x - y = -4$ and $x + 2y = 4$. The y-coordinate, to the nearest hundredth, of the point of intersection is _____ .

(Record your answer in the numerical response box from left to right)

2	.	6	7

$x-y=-4$
$x+4=y$
$y=x+4$

$x+2y=4$
$2y=-x+4$
$y=-\frac{1}{2}x+2$

graph: $\left.\begin{array}{l} y_1 = x+4 \\ y_2 = -\frac{1}{2}x+2 \end{array}\right\}$ $x=-1.33\ldots$ $y=2.66\ldots$

3. Consider the following two systems of equations.

a) $y = \frac{2}{3}x + 1, \quad y = \frac{1}{2}x - 2$ b) $4x + 5y = 18, \quad 2x + 3y = 1$

$5y = -4x + 18$ $3y = -2x + 1$

$y = -4/5\,x + 18/5$ $y = -2/3\,x + 1/3$

Solve the systems of equations using a graphing calculator and determine which one of the following statements is true.

A. In system **a)**, $x + y = 29$. x a) $x = -18,\ y = -11$

B. In system **b)**, $x + y = 40.5$. x b) $x = 24.5,\ y = -16$

C. One of the values of x is 42.5 more than the other. ✓

D. One of the values of y is 42.5 more than the other. x

4. The system $y = 4x - 8, \quad x = by + c,$ has an infinite number of solutions if

A. $b = 0.25$ and $c = -2$ $by = x - c$

B. $b = 0.25$ and $c = 2$ $y = \frac{1}{b}x - \frac{c}{b}$

↳ for an infinite number of solutions the equations must be identical.

C. $b = 0$ and $c = 0$

D. $b = 4$ and $c = -8$

$\frac{1}{b} = 4$ $\frac{c}{b} = 8$

$b = \frac{1}{4}$ $c = 8b$

$b = 0.25$ $c = 8(0.25) = 2$

5. The graphs of $y = ax + b$ and $y = cx + d$ are parallel.
The number of solutions to the system $y = ax + b,\ y = cx + d$ is

A. zero **B.** one **C.** two **D.** infinite

6. For which system of equations graphed on a grid is the point $(-2, -1)$ a solution?

A. $x - 2y = 3, \quad x + 5y = -11$ x

B. $3x - 3y = -9, \quad x - 4y = -6$ x

C. $2x - 10y = 6, \quad x + 5y = -3$ x

D. $x + y = -3, \quad -2x + 5y = -1$ ✓

replace $x = -2$, $y = -1$ in the left side of each equation to see which equation is satisfied by the point $(-2, -1)$.

A. $(-2) - 2(-1) = 0$ x

B. $3(-2) - 3(-1) = -3$ x

C. $2(-2) - 10(-1) = 6$ ✓ $(-2) + 5(-1) = -7$ x

D. $(-2) + (-1) = -3$ ✓ $-2(-2) + 5(-1) = -1$ ✓

7. When solving a system of equations, one of which is $\frac{x}{4} - \frac{y}{3} = 2$, a substitution which can be made is

A. $x = \frac{1}{4}(3y + 2)$

B. $x = \frac{1}{3}(4y + 6)$

C. $x = \frac{1}{4}(3y + 24)$

D. $x = \frac{1}{3}(4y + 24)$

$\frac{x}{4} - \frac{y}{3} = 2$

$12\left(\frac{x}{4}\right) - 12\left(\frac{y}{3}\right) = 12(2)$

$3x - 4y = 24$

$3x = 4y + 24$

$x = \frac{1}{3}(4y + 24)$

8. In solving the system $3a - 2b = 14,\ 2a + b = 7$ by elimination, an equation which arises could be

A. $-7b = 49$

B. $-b = 7$

C. $5a = 21$

D. $7a = 28$

$3a - 2b = 14$

$2a + b = 7$ (×2) ⇒

$3a - 2b = 14$

$+\ 4a + 2b = 14$ (add)

$7a = 28$

$a = 4$

9. If $x + y = 12$ and $x - y = 2$, then $x + 2y$ is equal to

A. 10

B. 17

C. 19

D. 34

$x + y = 12$

$+\ x - y = 2$ (add)

$2x = 14$

$x = 7$

$x + y = 12$

$(7) + y = 12$

$y = 5$

$x + 2y$

$= (7) + 2(5)$

$= 17$

10. If $3(x - 2) + y = 7$ and $4x - 3(y - 1) = 16$, then y is equal to

A. $-\frac{15}{13}$

B. $-\frac{5}{13}$

C. 1

D. 4

$3x - 6 + y = 7$

$3x + y = 13$

$4x - 3y + 3 = 16$

$4x - 3y = 13$

$3x + y = 13$ (×4)

$4x - 3y = 13$ (×3)

$12x + 4y = 52$

$-\ 12x - 9y = 39$ (subtract)

$13y = 13$

$y = 1$

11. Solve the following system using elimination.

$$\frac{2p}{3} - \frac{3q}{4} = \frac{11}{2}, \quad \frac{5p}{9} + \frac{q}{6} = 3$$

The value of p is

A. 6

B. $\frac{75}{38}$

C. $\frac{48}{11}$

D. $\frac{7}{24}$

$12\left(\frac{2p}{3}\right) - 12\left(\frac{3q}{4}\right) = 12\left(\frac{11}{2}\right)$

$8p - 9q = 66$

$18\left(\frac{5p}{9}\right) + 18\left(\frac{q}{6}\right) = 18(3)$

$10p + 3q = 54$

$8p - 9q = 66$
$10p + 3q = 54$ (×3)

$8p - 9q = 66$ (add)
$+\ 30p + 9q = 162$

$38p = 228$
$p = 6$

Numerical Response 3. If $m - 2n - 30 = 2m - n - 39 = 0$, then the value of $m - n$, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right)

2	3		

$m - 2n - 30 = 0$ $\quad 2m - n - 39 = 0$
$m - 2n = 30$ $\quad 2m - n = 39$

$m - 2n = 30$ (×2)
$2m - n = 39$

$\Rightarrow$ $2m - 4n = 60$ (subtract)
$-\ 2m - n = 39$

$-3n = 21$
$n = -7$

$m - 2n = 30$
$m - 2(-7) = 30$
$m + 14 = 30$
$m = 16$

$m - n$
$= (16) - (-7)$
$= 23$

12. If $\frac{2x + y}{3} - 5 = 0$ and $\frac{3x - y}{5} = 1$, then the value of y is

A. 4

B. 7

C. $\frac{13}{5}$

D. $\frac{23}{25}$

$\frac{2x+y}{3} = 5$

$2x + y = 15$

$\frac{3x-y}{5} = 1$

$3x - y = 5$

$2x + y = 15$ (add)
$+\ 3x - y = 5$

$5x = 20$
$x = 4$

$2x + y = 15$
$2(4) + y = 15$
$8 + y = 15$
$y = 7$

Numerical Response 4. The straight line $ax + y = b$ passes through the points $(-1, 1)$ and $(-5, 4)$. The value of ab, to the nearest hundredth, is ______.

(Record your answer in the numerical response box from left to right)

0	.	1	9

$x = -1,\ y = 1$: $ax + y = b$, $a(-1) + (1) = b$, $-a + 1 = b$, $-a - b = -1$

$x = -5,\ y = 4$: $a(-5) + (4) = b$, $-5a + 4 = b$, $-5a - b = -4$

$-a - b = -1$
$-\ (-5a - b = -4)$ (subtract)
$4a = 3$
$a = \frac{3}{4}$

$-a - b = -1$
$-(\frac{3}{4}) - b = -1$
$-b = -\frac{1}{4}$
$b = \frac{1}{4}$

$ab = (\frac{3}{4})(\frac{1}{4}) = 0.1875$

Use the following information to answer the next two questions.

> Lisbeth cycled 100 km from Calgary to Canmore. On the uphill sections her average speed was 12 km/h, and on the rest of the trip her average speed was 28 km/h.
>
> time = $\frac{\text{distance}}{\text{speed}}$
>
> She cycled for a total of 5 hours on the journey.

13. If x km represents the distance travelled uphill and y km represents the distance travelled on the rest of the trip, which of the following systems could be used to determine the values of x and y?

A. $x + y = 100, \quad 12x + 28y = 5$

B. $x + y = 100, \quad \frac{x}{12} + \frac{y}{28} = 5$

C. $x + y = 5, \quad 12x + 28y = 100$

D. $x + y = 5, \quad \frac{x}{12} + \frac{y}{28} = 100$

distance: $x + y = 100$ time: $\frac{x}{12} + \frac{y}{28} = 5$

Numerical Response 5. The distance travelled uphill, to the nearest km, is ______.

(Record your answer in the numerical response box from left to right)

3	0		

$x + y = 100 \rightarrow y = 100 - x$

$\frac{x}{12} + \frac{y}{28} = 5 \rightarrow 84(\frac{x}{12}) + 84(\frac{y}{28}) = 84(5)$
$7x + 3y = 420$

$7x + 3y = 420$
$7x + 3(100 - x) = 420$
$7x + 300 - 3x = 420$
$4x = 120$
$\underline{x = 30}$

14. For a birthday gift Isabelle was given an electronic piggy bank containing loonies and toonies. The display showed that the bank contained 26 coins with a value of \$40. The value of the toonies in the piggy bank, in dollars, was

A. 28
B. 24
C. 14
D. 12

Let x = # of loonies and y = # of toonies

$x + y = 26 \rightarrow y = 26 - x$
$x + 2y = 40$

$x + 2y = 40$
$x + 2(26 - x) = 40$
$x + 52 - 2x = 40$
$-x = -12$
$\underline{x = 12}$

$y = 26 - x$
$y = 26 - (12)$
$\underline{y = 14}$

value of toonies $= 14(2) = \underline{28}$

15. Susan solves the system of equations $\frac{2}{x} + \frac{3}{y} = 2$, $\frac{8}{x} - \frac{9}{y} = 1$ by first substituting a for $\frac{1}{x}$ and b for $\frac{1}{y}$. The value of xy is

A. 6
B. $\frac{1}{6}$
C. $-\frac{49}{720}$
D. $-\frac{720}{49}$

$2a + 3b = 2$ $(\times 3)$
$8a - 9b = 1$

$6a + 9b = 6$ (add)
$+\ 8a - 9b = 1$
$14a = 7$
$a = \frac{1}{2}$

$a = \frac{1}{2} \Rightarrow x = 2$

$2a + 3b = 2$
$2(\frac{1}{2}) + 3b = 2$
$1 + 3b = 2$
$3b = 1$
$b = \frac{1}{3}$

$b = \frac{1}{3} \Rightarrow y = 3$

$xy = (2)(3) = 6$

Written Response - 5 marks

1. Erika plans to set up an internet connection with *Y2K Internet Company*. There are three plans to choose from.

- Plan 1 costs \$20 per month and includes a user fee of 40¢ per hour.
- Plan 2 costs \$15 per month and includes a user fee of 80¢ per hour.
- Plan 3 costs \$60 per month for unlimited use.

- What factor would determine which plan is most economical?

  The expected number of hours of internet use per month.

- Let y = total cost per month in dollars and x = number of hours of use per month. Write a linear equation for each of the three plans.

  Plan 1: $y = 20 + 0.4x$ Plan 2: $y = 15 + 0.8x$ Plan 3: $y = 60$

- Use a graphical method to determine when plans 1 and 2 are equally economical to use. State the graphing window used.

  $y_1: 20 + 0.4x$
  12.5 hours $x: [0, 50, 10]$ $y: [0, 50, 10]$ $y_2: 15 + 0.8x$

- Algebraically verify the solution in the bullet above algebraically.

  $y = 20 + 0.4x$
  $y = 15 + 0.8x$

  $$20 + 0.4x = 15 + 0.8x$$
  $$\frac{5}{0.4} = \frac{0.4x}{0.4}$$
  $$12.5 = x$$
  $$x = 12.5$$
  12.5 hours

- For each of plans 1 and 2, determine the number of hours of use which could be obtained for \$60.

  Plan 1: $y = 20 + 0.4x$
  $60 = 20 + 0.4x$
  $40 = 0.4x$
  $x = 100$ 100 hours

  Plan 2: $y = 15 + 0.8x$
  $60 = 15 + 0.8x$
  $45 = 0.8x$
  $x = 56.25$ $56\frac{1}{4}$ hours

- Devise a simple rule which would determine which plan is most economical depending on the expected number of hours of internet use per month.

  - Plan 2 for up to 12.5 hrs
  - Plan 1 for between 12.5 hrs and 100 hrs
  - Plan 3 for more than 100 hrs

Answer Key

1. C	2. A	3. C	4. B	5. A	6. D	7. D	8. D
9. B	10. C	11. A	12. B	13. B	14. A	15. A	

Numerical Response

1. | 7 | | | |

2. | 2 | . | 6 | 7 |

3. | 2 | 3 | | |

4. | 0 | . | 1 | 9 |

5. | 3 | 0 | | |

Written Response

1.
- The expected number of hours of internet use per month.
- $y = 20 + 0.4x$, $y = 15 + 0.8x$, $y = 60$
- 12.5 hours, eg. x:[0, 50, 10], y:[0, 50, 10]
- 12.5 hours
- 100 hours, $56\frac{1}{4}$ hours
- Plan 2 for up to 12.5 hours, Plan 1 for between 12.5 and 100 hours, Plan 3 for more than 100 hours