Arithmetic Sequences Lesson #1:
Investigating Patterns and Sequences

Overview

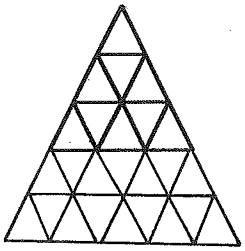
In this unit we investigate patterns used to define types of sequences. We then focus on arithmetic sequences and apply formal language to increasing and decreasing linear patterns. We derive a rule for determining the general term of an arithmetic sequence and explore problems relating to arithmetic growth and decay.
As an extension we explore arithmetic series.

Investigation 1

Jesse is making a tower using playing cards.
The top three rows of the tower are shown.

The top row (Row 1) requires three playing cards.

The second row (Row 2) requires six additional playing cards.



Continue the pattern for two more rows, and complete Tables A and B below.

In grades 8 and 9 you learned how to determine a formula from a specific pattern. In this lesson, we focus on investigatin patterns to identify different types of sequences, a skill which is useful in patterns of numbers which are more difficult to identify In the next lesson we develope a formula and use this formula for patterns of numbers which are arithmetic.

Table A

Row Number	1	2	3	4	5
Number of Additional Cards in the Row	3	6	9	12	15

Table B

Row Number	1	2	3	4	5
Number of Triangles in the Row	1	3	5	7	9

Investigation 2

Each row in the triangle, named after French mathematician Blaise Pascal, begins and ends with the number 1.

Apart from the ones, every other number in a particular row can be determined by adding the two numbers diagonally above it.

	Row
1	1
1 1	2
① ② 1	3
1 3 3 1	4
1 4 6 4 1	5
1 5 10 10 5 1	6
1 6 15 20 15 6 1	7

Continue the pattern for rows 5, 6, and 7, and complete Table C.

Table C

Row Number	1	2	3	4	5	6	7
Sum of the Numbers in the Row	1	2	4	8	16	32	64

Investigation 3

Triangle 1, shown in Diagram 1, is an equilateral triangle with sides of length 32 cm. A smaller triangle, Triangle 2, is placed inside Triangle 1 by joining the midpoints of Triangle 1 (as illustrated in Diagram 2). The pattern is continued as illustrated in Diagram 3.

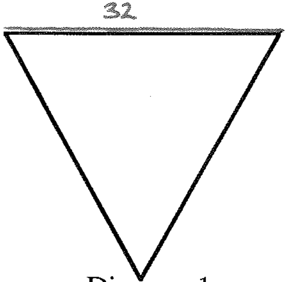


Diagram 1

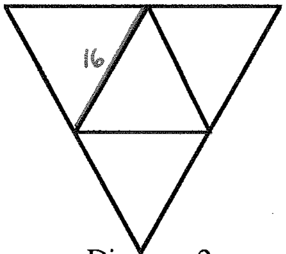


Diagram 2

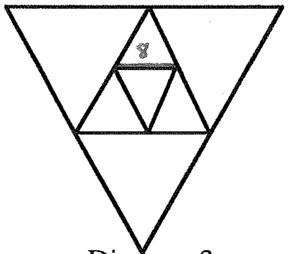


Diagram 3

a) Diagrams 4 and 5 in the sequence have not been completed. Complete each diagram.

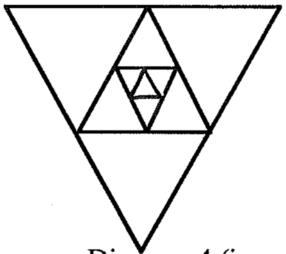


Diagram 4 (incomplete)

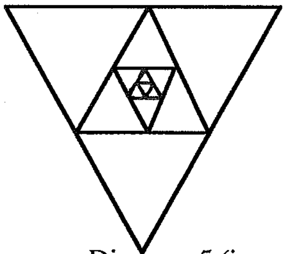


Diagram 5 (incomplete)

b) Complete Tables D and E below.

Table D

Triangle Number	1	2	3	4	5
Length of Side of Triangle (cm)	32	16	8	4	2

Table E

Diagram Number	1	2	3	4	5
Number of Triangles of any Size in the Diagram	1	5	9	13	17

Investigation 4

The mean daily temperature in Prince George on Dec 1 was 8°C. For the next six days, the mean daily temperature decreased by 4°C each day.

Complete Table F below.

Table F

Day Number	1	2	3	4	5
Temperature (°C)	8	4	0	-4	-8

In each of the tables A - F, the top row consists of the **natural numbers** 1, 2, 3, 4, etc., and the bottom row consists of a **sequence** of numbers related to the natural numbers in a specific order.

The first table you completed was **Table A**

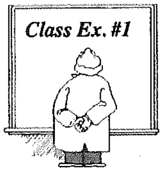
Row Number	1	2	3	4	5
Number of Cards in the Row	3	6	9	12	15

$t_1 = 3$ $t_3 = 9$
$t_2 = 6$ $t_4 = 12$...

The sequence formed is 3, 6, 9, 12, 15. The sequence consists of **terms.**

The first term, written t_1, is equal to 3. The second term, t_2, is equal to 6; $t_3 = 9$ etc.

The term is represented by the variabl "t", and the Subscript represents the term number
ex = t_5 "Term" Term#
(term 5)


Class Ex. #1

In each of the following:

- **a)** Complete the table using the information from Tables A - F.
- **b)** Complete the statement, explaining how to find the next term in the sequence from the previous term using only addition or multiplication.
- **c)** State the next two terms of the sequence.

Table A (pg 505)

n	1	2	3	4	5
t_n	3	6	9	12	15

+3 +3 +3

The next term can be calculated by adding 3 to the previous term.

$t_6 =$ 18 and $t_7 =$ 21 .

Table B (pg 505)

n	1	2	3	4	5
t_n	1	3	5	7	9

The next term can be calculated by adding 2 to the previous term.

$t_6 =$ 11 and $t_7 =$ 13 .

Table C (pg 505)

n	1	2	3	4	5	6	7
t_n	1	2	4	8	16	32	64

The next term can be calculated by multiplying the previous term by 2

$t_8 =$ 128 and $t_9 =$ 256 .

Table D (pg 506)

n	1	2	3	4	5
t_n	32	16	8	4	2

The next term can be calculated by multiplying the previous term by ½

$t_6 =$ 1 and $t_7 =$ ½ .

find out the nstant you are ultiplying by, take e term and vide it by the evious term. If e # repeats, it the constant

table D

$_2 \div t_1 \Rightarrow 16 \div 32 = \frac{1}{2}$
$_3 \div t_2 \Rightarrow 8 \div 16 = \frac{1}{2}$
$_4 \div t_3 \Rightarrow 4 \div 8 = \frac{1}{2}$

continues to repeat the term you are multiplying by is $\frac{1}{2}$

the # does not peat, then the sequence not geometric t can be arithmetic neither. more on e next page)

e will be nething similar this in lesson 2

Table E (pg 506)

n	1	2	3	4	5
t_n	1	5	9	13	17

The next term can be calculated by adding 4 to the previous term.

$t_6 =$ 21 and $t_7 =$ 25 .

Table F (pg 506)

n	1	2	3	4	5
t_n	8	4	0	-4	-8

The next term can be calculated by adding -4 to the previous term.

$t_6 =$ -12 and $t_7 =$ -16 .

Types of Sequences

There are different types of sequences.

A sequence in which the next term is formed by adding a constant (positive or negative) to the previous term is called an **arithmetic sequence**.

A sequence in which the next term is formed by multiplying the previous term by a constant (positive or negative) is called a **geometric sequence**.

There are other sequences, which are not arithmetic or geometric.

For example: Sequence of Prime Numbers 2, 3, 5, 7, 11, ...

A Fibonacci Sequence, 1, 1, 2, 3, 5, 8, 13, ... , is a special type of sequence which occurs in nature in such things as seed growth, leaves on stems, petals on flowers, etc.

In the remaining lessons in this unit, we focus on arithmetic sequences.
Geometric sequences will be studied in a later course.

Sequences

Arithmetic Sequences ***Geometric Sequences*** ***Other Sequences***

Adding a constant

Multiplying by a constant

Class Ex. #2

Classify the sequences in Tables A - F as arithmetic sequences or geometric sequences.

Arithmetic = A, B, E, F

Geometric = C, D

Finite and Infinite Sequences

Finite Sequence - a sequence that has a specific number of terms.

eg. 4, 10, 16, 22, 28 or 2, 4, 8, 16, ... 256.

↳ Ends at a specific #

Infinite Sequence - a sequence that has an unlimited number of terms.

eg. 4, 10, 16, 22, 28 ... or 2, 4, 8, 16, ...

↳ Does not end, is represented by "..."

A Sequence as a Function

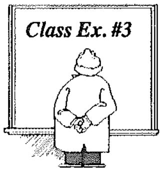
A sequence can be regarded as a **function** relating the set of natural numbers to the terms of the sequence.

The **domain** of the function is the set of natural numbers.

natural numbers are whole numbers, greater than zero.
∴ when drawing a graph, do NOT connect the points

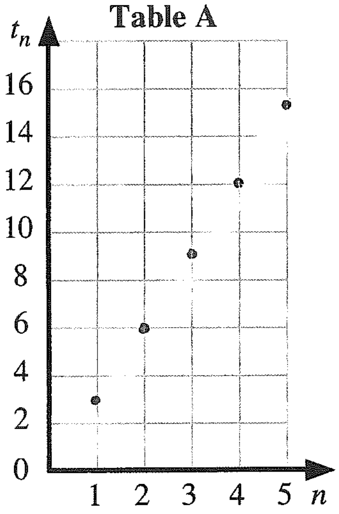
The **range** of the function is the set of terms of the sequence.

Some sequences can be represented by linear functions, and some can be represented by non-linear functions.



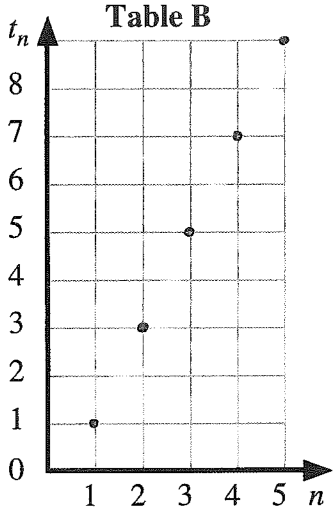
Using the information in **Tables A - F,** plot the points (n, t_n) on the grids. In each case, state whether the function represented by the sequence is linear or non-linear.

Table A



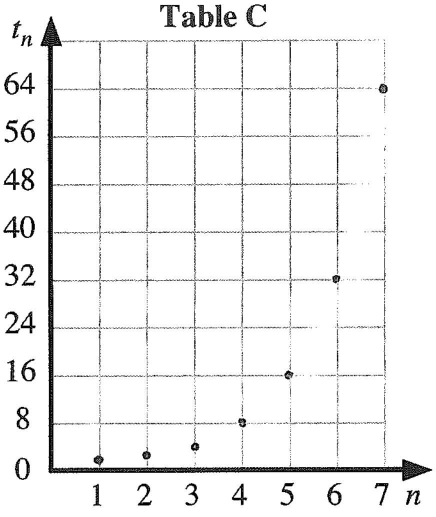
Linear

Table B



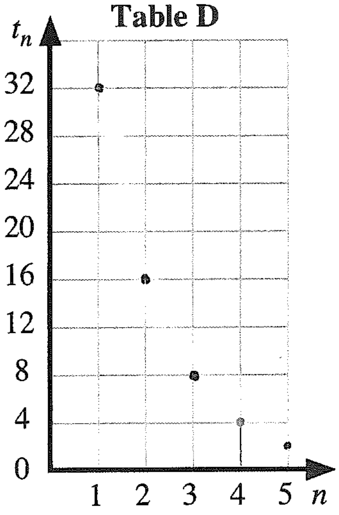
Linear

Table C



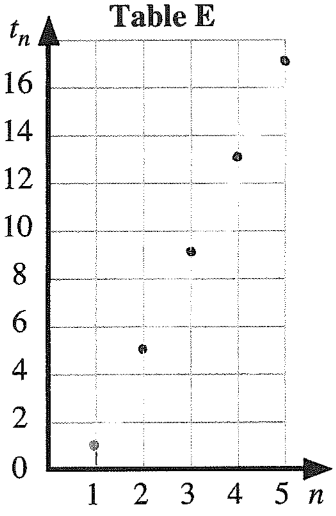
Non-linear

Table D



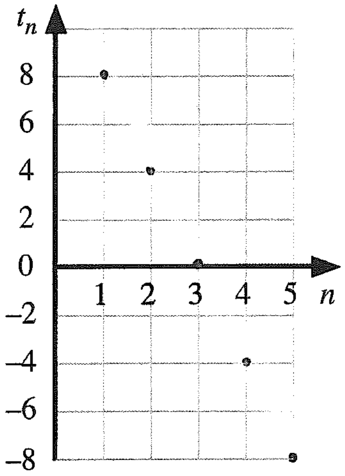
Non-linear

Table E

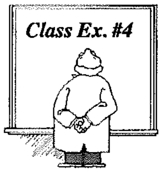


Linear

Table F



Linear



Circle the correct alternative in the following statements.

a) A sequence in which the next term is determined by adding a constant to the previous term is an [✓ arithmetic] / geometric sequence.

The sequence can be represented by a [✓ linear] / non-linear function.

b) A sequence in which the next term is determined by multiplying the previous term by a constant is an arithmetic / [✓ geometric] sequence.

The sequence can be represented by a linear / [✓ non-linear] function.

Complete Assignment Questions #1 - #12

Assignment

1. Consider the pattern of squares within squares shown below.

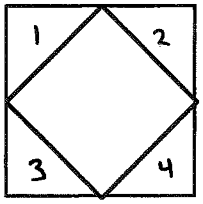


Diagram 1

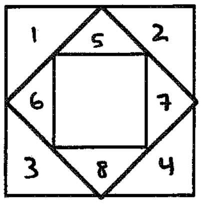


Diagram 2

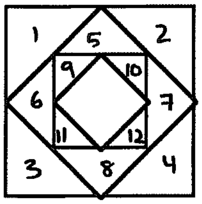


Diagram 3

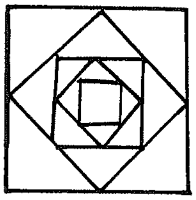


Diagram 4

a) Draw the next diagram in the pattern in the space above.

b) Complete the table.

Diagram Number	1	2	3	4
Number of Triangles in the Diagram	4	8	12	16

c) Consider the sequence of triangles in the table above.

(i) Complete the statement explaining how to find the next term in the sequence from the previous term.

The next term can be calculated by Adding 4 to the previous term.

(ii) State the value of the following terms. t_1 = 4 t_5 = 20 t_6 = 24

(iii) Circle the correct alternatives.
The sequence is [✓ arithmetic] / geometric and can be represented by a [✓ linear] / non-linear function.

2. Consider the following pattern of squares.

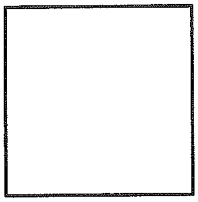


Diagram 1

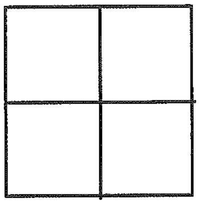


Diagram 2

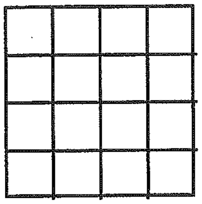


Diagram 3

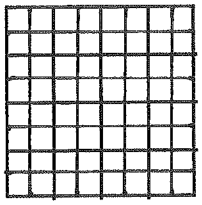


Diagram 4 (incomplete)

a) Complete diagram 4 in the pattern.

b) Complete the table.

Diagram Number	1	2	3	4
Number of Congruent Squares in the Diagram	1	4	16	64

c) Consider the sequence of squares in the table above.

(i) Complete the statement explaining how to find the next term in the sequence from the previous term.

The next term can be calculated by multiplying the previous term by 4.

(ii) State the value of the following terms. t_4 = 64 t_5 = 256 t_6 = 1024

(iii) Circle the correct alternatives.
The sequence is an arithmetic / [✓ geometric] sequence and can be represented by a linear / [✓ non-linear] function.

3. For each of the following sequences, determine if the sequence is arithmetic, geometric, or neither.

a) $\overset{+1\ \ +1\ \ldots}{1, 2, 3}$
Arithmetic

b) $\overset{\times -1\ \ \times -1\ \ \times -1\ \ldots}{-1, 1, -1, \ldots}$
Geometric

c) $\overset{\times 5\ \ \times 5\ \ldots}{12, 60, 300, \ldots .}$
Geometric

d) $\overset{\times\frac{1}{3}\ \ \times\frac{1}{3}\ \ \times\frac{1}{3}\ \ldots}{\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{2187}}$
Geometric

e) $\overset{+0\ \ +1\ \ +1}{1, 1, 2, 3, \ldots}$
Neither b/c you are not adding or multiplying by a constant #

f) $\overset{+(-50)\ \ +(-50)\ \ldots}{250, 200, 150, \ldots}$
Arithmetic

4. Classify each sequence in question **3.** as a finite sequence or an infinite sequence.

a) finite **b)** infinite **c)** infinite

d) finite **e)** infinite **f)** infinite

5. Write the next three terms of each of the infinite sequences in question **3.**

b) 1, -1, 1 c) 1500, 7500, 37 500

e) 5, 8, 13 f) 100, 50, 0

→ e) 1, 1, 2, 3, 5, 8, 13 ... These #'s are from the Fibonacci Sequence where you take the 2 previous terms and add them together to get the next term

6. For each of the following sequences of numbers:

I) • Identify the type of sequence as arithmetic or geometric.
II) • Write the next two terms of the sequence.
III) • Describe a rule which can be used to form the sequence using addition or multiplication.

+5 +5
a) 5, 10, 15, ...

I) Arithmetic

II) 20, 25

III) Add 5 to the previous term

•2 •2
b) 5, 10, 20, ...

I) Geometric

II) 40, 80

III) Multiply the previous term by 2

+3 +3
c) –8, –5, –2, ...

I) Arithmetic

II) 1, 4

III) Add 3 to the previous term

d) 80, 40, 20, ...
$\cdot\frac{1}{2}$ $\cdot\frac{1}{2}$

I) Geometric

II) 10, 5

III) Multiply the previous term by $\frac{1}{2}$

e) 25, 18, 11, ...
+(-7) +(-7)

I) Arithmetic

II) 4, -3

III) Add -7 to the previous term

f) –8, –11, –14, ...
+(-3) +(-3)

I) Arithmetic

II) -17, -20

III) Add -3 to the previous term

7. A fractal is a fragmented geometric shape that can be subdivided into parts, each of which is a scaled down copy of the whole. There is increased complexity of the shape at each step. An example is shown below.

Diagram 1

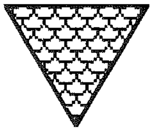


Diagram 2

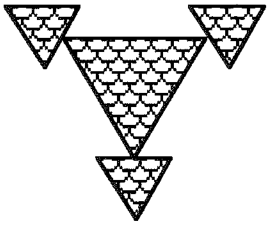
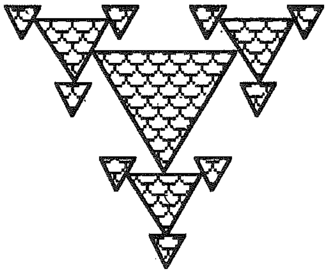
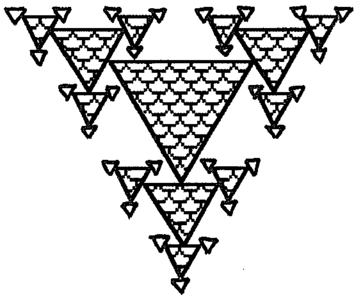


Diagram 3



a) Diagram 4 has been started below. Complete the diagram.

Diagram 4



b) Complete the table below.

Diagram Number	1	2	3	4
Number of New Triangles in the Diagram	1	3	9	27

c) Classify the sequence as arithmetic, [✓ geometric] or neither.

Multiply the previous term by 3

d) Determine the eighth term of the sequence.

$t_5 = 81$ $)\times 3$

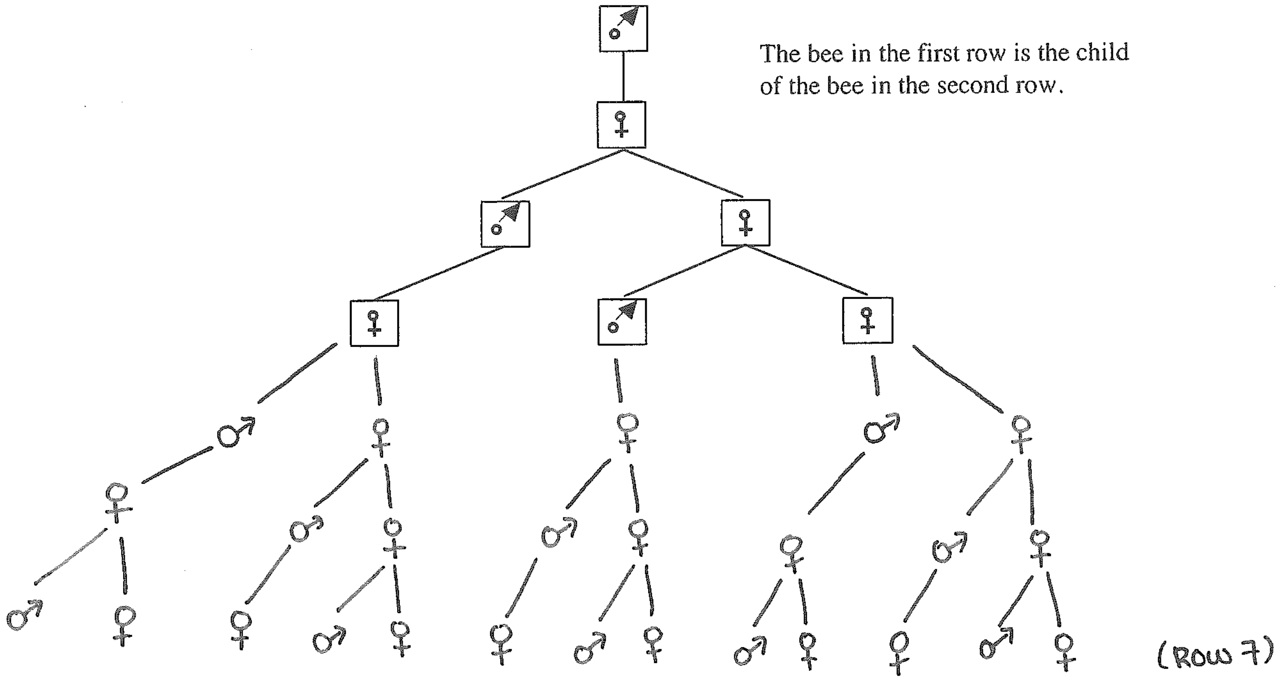
$t_6 = 243$

$t_7 = 729$ $)\times 3$

$\boxed{t_8 = 2187}$

8. The reproduction of bees in nature follows an interesting sequence. Whereas the female bee has two parents (a mother and a father), the male bee has only one parent (a mother).

a) Complete three more generations of the family tree for the male bee below.



b) Complete the following table.

Row Number	1	2	3	4	5	6	7
Number of Bees in the Row	1	1	2	3	5	8	13

c) Is the sequence represented in the table arithmetic, geometric or neither?

Neither

d) How many female bees are in row 7?

8

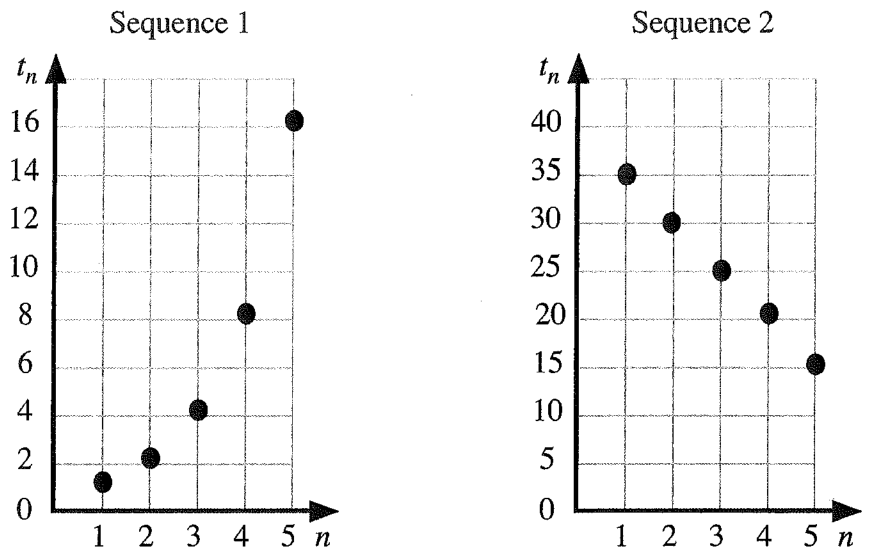
e) How many bees are in the eighth row?

Fibonacci Sequence

8+13 = 21

Use the following information to answer the next four questions.

The graphical representations of two sequences are shown in the grids below.



9. Explain how you can determine which one of the sequences is arithmetic.

If the graph is linear, the sequence is arithmetic
(Sequence 2 is arithmetic)

10. For each sequence, describe a rule for determining the next term from the previous term.

Sequence 1 = Multiply the previous term by 2

Sequence 2 = Add -5 to the previous term

Multiple Choice **11.** The eighth term of Sequence 2 is

A. 5

[✓ **B.**] 0

C. –5

D. –10

$t_5 = 15$) + -5
$t_6 = 10$) + -5
$t_7 = 5$) + -5
$t_8 = 0$

Numerical Response **12.** The eight term of Sequence 1 is 128 .

(Record your answer in the numerical response box from left to right.)

1	2	8	

$t_5 = 16$) ·2
$t_6 = 32$) ·2
$t_7 = 64$) ·2
$t_8 = 128$

Group Investigation

The following investigation can be used as a lead-in to the next lesson. The solution can be obtained by drawing a grid and using counters or coins; however, greater understanding may be obtained by having students act out the situation.

	A	B	C
1	S	S	S
2	S	S	S
3	S	S	

Eight students are arranged in a 3 × 3 grid. Students can move either left/right or up/down into an empty cell. A "move" consists of any one student moving into an adjacent cell.

a) Determine the smallest number of "moves" required for the student in cell A1 to end up in cell C3. 13 moves

b) Extend your thinking by developing a strategy for determining the smallest number of "moves" required to move a student from cell A1 to cell Z26 in a 26 × 26 grid.

Grid	# of Moves
2x2	5
3x3	13
4x4	21

} looks like an Arithmetic Sequence (add 8 to the previous term)

→ Extend the pattern for a 26x26 grid. You will get $\boxed{\text{197 moves}}$

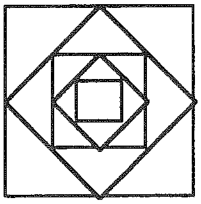
☆ There is also a rule to get from grid size to # of moves.

[(grid size)(8)] − 11 = # of moves

$(26 \times 8) - 11 = \boxed{197}$

Answer Key

1. a)



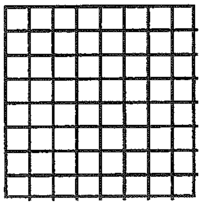
b) 8, 12, 16

c) i) adding four to the previous term

ii) $t_1 = 4, t_5 = 20, t_6 = 24$

iii) arithmetic, linear

2. a)



b) 1, 4, 16, 64

c) i) multiplying the previous term by 4

ii) $t_4 = 64, t_5 = 256, t_6 = 1024$

iii) geometric, non-linear

3. **a)** arithmetic **b)** geometric **c)** geometric **d)** geometric **e)** neither **f)** arithmetic

4. **a)** finite **b)** infinite **c)** infinite **d)** finite **e)** infinite **f)** infinite

5. **b)** 1, −1, 1 **c)** 1 500, 7 500, 37 500 **e)** 5, 8, 13 **f)** 100, 50, 0

6. **a)** Arithmetic. 20, 25. Add 5 to the previous term.

b) Geometric. 40, 80. Multiply the previous term by 2.

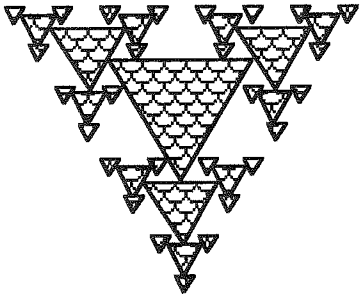
c) Arithmetic. 1, 4. Add 3 to the previous term.

d) Geometric. 10, 5. Multiply the previous term by $\frac{1}{2}$.

e) Arithmetic. 4, −3. Add −7 to the previous term.

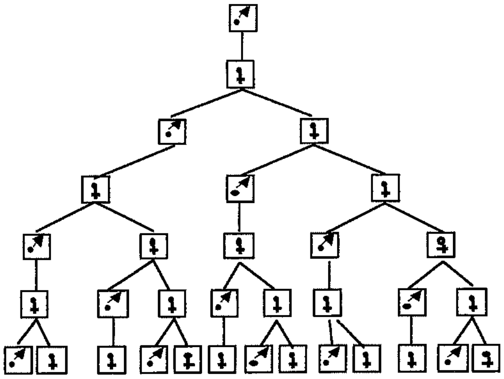
f) Arithmetic. −17, −20. Add −3 to the previous term.

7. a)



b) 3, 9, 27 **c)** Geometric **d)** 2187

8. a)



b) 2, 3, 5, 8, 13 **c)** Neither **d)** 8 **e)** 21

9. If the graph is linear, the sequence is arithmetic. Sequence 2 is arithmetic.

10. For sequence 1, multiply the previous term by 2. For sequence 2, add −5 to the previous term.

11. B **12.**

1	2	8	

Group Investigation

a) 13

b) 2 x 2 grid → 5 moves. 3 x 3 grid → 13 moves. 4 x 4 grid → 21 moves.
Extend the pattern: 26 x 26 grid → 197 moves.

Arithmetic Sequences Lesson #2:
Arithmetic Sequences

Arithmetic Sequence

An **arithmetic sequence** is a sequence in which each term is formed from the preceding term by ***adding*** a constant (positive or negative).

Complete the following for the sequence 7, 10, 13, 16, ...

- Each term is determined by adding 3 to the previous term.
- Calculate the differences: $t_2 - t_1 = 3$ $t_3 - t_2 = 3$ $t_4 - t_3 = 3$

10 - 7 13 - 10 16 - 3

Notice that there is a **common difference** between successive terms.

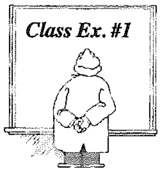
The common difference in this example is 3.

Finding a Common Difference

To find a common difference in an arithmetic sequence, we can subtract any term from the term after it.

For example $t_2 - t_1$ = common difference, or
$t_5 - t_4$ = common difference, etc.

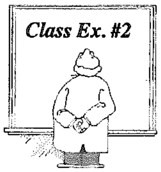
$$\boxed{\text{common difference} = t_n - t_{n-1}}$$



Consider the sequence 16, 13, 10, 7, ...

The **common difference** in the sequence is -3.

$t_2 - t_1 \rightarrow 13 - 16 = -3$
$t_3 - t_2 \rightarrow 10 - 13 = -3$
$t_4 - t_3 \rightarrow 7 - 10 = -3$



For each of the following :
- Determine which sequences are arithmetic.
- Find the common difference for those sequences which are arithmetic .

a) 2, 4, 6, 8, ...
Arithmetic
common difference = 2

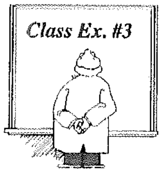
b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \ldots$
Not Arithmetic

c) −10, −4, 2, 8, ...
Arithmetic
common difference = 6

d) 4, 8, 16, 32, ...
Not Arithmetic

In an arithmetic sequence we often use the following terminology.

The **first term** in an arithmetic sequence is represented by t_1, or a, and the **common difference** is represented by d.



→ t_1 → common difference

State the values of a and d in the following sequences:

i) $-8, 2, 12, 22, ...$

$a = -8$
$d = 10$

ii) $15, 10, 5, 0 . . . .$

$a = 15$
$d = -5$

Investigation

Investigating the Formula for the General Term of an Arithmetic Sequence

Consider the sequence $2, 12, 22, 32, 42 ....$

a) State the following

$t_1 = 2$ $t_2 = 12$ $t_3 = 22$ $t_4 = 32$ $t_5 = 42$ $a = 2$ $d = 10$

b) Complete the following pattern which describes each term in the sequence in terms of the first term, a, and the common difference, d.

$t_1 = 2$	$t_1 = a$
$t_2 = 2 + 1(10) = 12$	$t_2 = a + (1)d$
$t_3 = 2 + 2(10) = 22$	$t_3 = a + 2d$
$t_4 = 2 + 3(10) = 32$	$t_4 = a + 3(d)$
$t_5 = 2 + 4(10) = 42$	$t_5 = a + 4(d)$
$t_{30} = 2 + 29(10) = 292$	$t_{30} = a + 29(d)$
$t_n = 2 + (n-1)(10)$	$t_n = a + (n-1)(d)$

↳ "n" represents any natural #

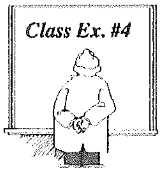
The Formula for the General Term of an Arithmetic Sequence

The formula for the general term of an arithmetic sequence is

☆ $\boxed{t_n = t_1 + (n-1)d}$ or ☆ $\boxed{t_n = a + (n-1)d}$

where, t_n is the general term of the arithmetic sequence
$a = t_1$, is the first term
d is the common difference
n is the position of the term in the sequence

The general arithmetic sequence is $a, a + d, a + 2d, a + 3d, ... a + (n - 1)d$.


Class Ex. #4

Consider the arithmetic sequence $-6, -1, 4, 9, \ldots$.

a) Determine the formula for the general term of the sequence.

$t_n = a + (n-1)d$

$\{$if $a = -6$ and $d = 5$ then...$\}$

$t_n = -6 + (n-1)(5)$ ⇒ expand and simplify

$t_n = -6 + 5n - 5$

$\boxed{t_n = 5n - 11}$

↳ $-11 + 5n$ is not wrong but formula's are normally written in descending powers.

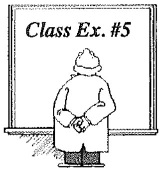
b) Determine the value of the twelfth term of the sequence.

↳ $t_{12} \Rightarrow n = 12$

$t_n = 5n - 11$

$t_{12} = 5(12) - 11$

$\boxed{t_{12} = 49}$


Class Ex. #5

Find the number of terms in the arithmetic sequence $\underbrace{3, -1, -5, \ldots, -117}$.

↳ find "n"

Write down what you know.

$a = 3 \quad d = -4 \quad \underbrace{t_n = -117}$

-117 is the last term, so if we solve for "n" that will tell us how many terms there are

$t_n = a + (n-1)d$

$-117 = 3 + (n-1)(-4)$

$-117 = 3 - 4n + 4$

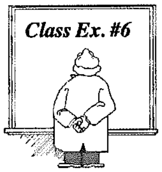
$\frac{-124}{-4} = \frac{\cancel{-4}n}{\cancel{-4}}$

$\boxed{31 = n}$ → There are 31 terms in this sequence

Complete Assignment Questions #1 - #9

Arithmetic Means

The terms placed between two non-consecutive terms of an arithmetic sequence are called **arithmetic means**. For example, in the sequence 5, 10, 15, 20, the numbers 10 and 15 are arithmetic means between 5 and 20. In order to determine arithmetic means between two given terms, it is helpful to think of the two given terms as the **first** and **last** terms of a sequence.


Class Ex. #6

Place three arithmetic means between -4 and 8.

$\underset{t_1}{-4}\ \underset{t_2}{\_\_}\ \underset{t_3}{\_\_}\ \underset{t_4}{\_\_}\ \underset{t_5}{8}$

$a = -4$

$n = 5$

$t_5 = 8$

$d = ?$ → solve for "d" then you can find the Arithmetic Means

$t_n = a + (n-1)d$

$t_5 = -4 + (5-1)d$

$8 = -4 + (4)d$

$\frac{12}{4} = \frac{\cancel{4}d}{\cancel{4}}$

$3 = d$ → Add 3 to the previous term or use the formula

$t_2 = -4 + (2-1)(3) = -1$

$t_3 = -4 + (3-1)(3) = 2$

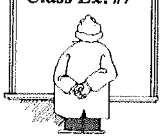
$t_4 = -4 + (4-1)(3) = 5$

$\boxed{\text{Arithmetic Means} = -1, 2, 5}$

$-4, -1, 2, 5, 8$

Solving Sequence Problems Where Both "a" and "d" are Unknown

Class Ex. #7



Consider the sequence $x+2, 3x-1, 2x+1$.

a) Determine the value of x such that $x+2$, $3x-1$, and $2x+1$ form an arithmetic sequence.

↳ use a "common difference" to solve problems like these → $d = t_n - t_{n-1}$

$t_1 = x+2$

$t_2 = 3x-1$

$t_3 = 2x+1$

→ B/c we are using a "common difference" we can say:

$t_2 - t_1 = t_3 - t_2$

$\Rightarrow (3x-1)-(x+2) = (2x+1)-(3x-1)$

$3x-1-x-2 = 2x+1-3x+1$ → $2x-3 = -x+2$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{5}{3}$

$\boxed{x = \frac{5}{3}}$

b) Determine the numerical value of the three terms.

$t_1 = x+2 \rightarrow (\frac{5}{3})+2 = \frac{11}{3}$

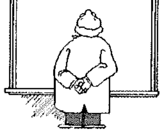
$t_2 = 3x-1 \rightarrow 3(\frac{5}{3})-1 = 4$

$t_3 = 2x+1 \rightarrow 2(\frac{5}{3})+1 = \frac{13}{3}$

$\boxed{\frac{11}{3}, 4, \frac{13}{3}}$

The next two class examples show two different ways of solving the same problem. Class Example #8 uses arithmetic means, and Class Example #9 uses a system of linear equations.

Class Ex. #8



The third and eighth terms of an arithmetic sequence are 12 and −18, respectively.

a) Use arithmetic means to determine the fifth term of the sequence.

original

$\underline{\ \ }\ \underline{\ \ }\ 12\ \underline{?}\ \underline{\ \ }\ \underline{\ \ }\ -18$

$t_1\ t_2\ t_3\ t_4\ t_5\ t_6\ t_7\ t_8$

→ For cases like these, it's easier to make 12 term 1 and −18 term 6

NEW

$\underline{\ \ }\ \underline{\ \ }\ \boxed{12\ \underline{\ \ }\ \underline{?}\ \underline{\ \ }\ \underline{\ \ }\ -18}$

$t_1\ t_2\ t_3\ t_4\ t_5\ t_6$

→ Now it looks similar to Class Ex. 6, solve for t_3

$a = 12$

$d = ?$

$t_n = -18$

$n = 6$

$t_n = a+(n-1)d$

$-18 = 12+(6-1)d$

$-18 = 12+5d$

$\frac{-30}{5} = \frac{\cancel{5}d}{\cancel{5}}$

$-6 = d$

Add −6 to the previous term

$\underline{\ \ }\ \underline{\ \ }\ 12\ \underline{6}\ \underline{0}\ \underline{\ \ }\ \underline{\ \ }\ -18$

$\boxed{t_5 = 0}$

when using the formula & solve, be careful of the "n" value

↳ $t_3 = 12+(3-1)(-6)$

$\boxed{t_3 = 0}$

↳ Again, the question asked for t_5 but since we manipulated the question, we are really looking for t_3 of the NEW sequence. t_5 of original = t_3 of new

b) State the first term, a, and the common difference, d, of the sequence.

$d = -6$

$a = ?$

$24\ 18\ 12\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ -18$ (+6, +6)

$\Rightarrow \boxed{t_1 = 24}$

c) Complete the following: $\frac{t_8 - t_3}{8-3} = \frac{-18-12}{8-3} = \frac{-30}{5} = -6$

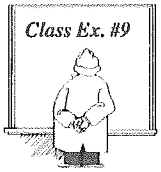
d) Write t_3 and t_8 in terms of a and d and prove that $\frac{t_8 - t_3}{8-3} = d$.

if $t_n = a+(n-1)d$ then...

$$\frac{[a+(8-1)d]-[a+(3-1)d]}{8-3} \rightarrow \frac{a+7d-a-2d}{5} \rightarrow \frac{\cancel{5}d}{\cancel{5}} = \boxed{d}$$

e) Suggest a formula for finding the common difference of a sequence if you are given the value of the p^{th} term and the q^{th} term.

$d = \frac{t_q - t_p}{q-p}$, if $q > p$



The third and eighth terms of an arithmetic sequence are 12 and −18, respectively. Use a system of linear equations to determine the values of the first term and the common difference. Hence, determine the fifth term of the sequence.

→ using elimination or substitution, find "d" to find the first term (a)

① ind
rmulas

$t_3 = 12$

$t_3 = a + (3-1)d$

$\therefore a + 2d = 12$

$t_8 = -18$

$t_8 = a + (8-1)d$

$\therefore a + 7d = -18$

② using elimination:

$$\begin{aligned} a + 2d &= 12 \\ -(a + 7d &= -18) \\ \hline \frac{-5d}{-5} &= \frac{30}{-5} \\ \boxed{d = -6} \end{aligned}$$

Now that "d" is found substitute it back into one of the equations

③ $a + 2d = 12$

$a + 2(-6) = 12$

$a - 12 = 12$

$\boxed{a = 24}$

④ $t_n = a + (n-1)d$

$t_5 = 24 + (5-1)(-6)$

$\boxed{t_5 = 0}$

Complete Assignment Questions #10 - #16

Assignment

ECALL:
mon difference
$n - t_{n-1}$
it 2 or 3 times
double check
rk.

1. For the following arithmetic sequences:
   i) Determine the common difference. **ii)** Find the next three terms of the sequence.

a) 8, 14, 20, ...

i) $20 - 14 = 6$

$14 - 8 = 6$

$\boxed{d = 6}$

ii) 26, 32, 38

b) −5, 7, 19, ...

i) $19 - 7 = 12$

$7 - (-5) = 12$

$\boxed{d = 12}$

ii) 31, 43, 55

c) 70, 53, 36, 19, ...

i) $53 - 70 = -17$

$36 - 53 = -17$

$\boxed{d = -17}$

ii) 2, −15, −32

d) 7.1, 4.2, 1.3, ...

i) $1.3 - 4.2 = -2.9$

$4.2 - 7.1 = -2.9$

$\boxed{d = -2.9}$

ii) −1.6, −4.5, −7.4

e) $\frac{2}{3}, \frac{1}{15}, -\frac{8}{15}, \ldots$

i) $\frac{1}{15} - \frac{2}{3} = -\frac{3}{5}$

$\frac{-8}{15} - \frac{1}{15} = \frac{-3}{5}$

$\boxed{d = -\frac{3}{5}}$

ii) $\boxed{\frac{-17}{15}, \frac{-26}{15}, \frac{-7}{3}}$

f) $-2x + 3y, -5x + y, -8x - y, \ldots$

i) $(-8x - y) - (-5x + y) =$

$-8x - y + 5x - y = -3x - 2y$

$\boxed{d = -3x - 2y}$

ii) $-8x - y + (-3x - 2y) = -11x - 3y$

$-11x - 3y + (-3x - 2y) = -14x - 5y$

$-14x - 5y + (-3x - 2y) = -17x - 7y$

$\boxed{-11x - 3y, -14x - 5y, -17x - 7y}$

2. In each of the following sequences, the value of one term is given. Write the missing terms of the sequence if the common difference is as indicated.

a) −6, −3, 0, 3, 6: $d = 3$ **b)** 18, 11, 4, −3, −10: $d = -7$

c) 5, 3, 1, −1, −3: $d = -2$ **d)** 5, 7.5, 10, 12.5, 15: $d = 2.5$

3. Calculate the first four terms of the arithmetic sequences with given term and common difference, d.

a) $t_1 = 5, d = 6$

$t_1 = 5$

$t_2 = 5+6 = 11$

$t_3 = 11+6 = 17$

$t_4 = 17+6 = 23$

$\boxed{5, 11, 17, 23}$

b) $t_3 = 15, d = -2$

$\underline{19} \overset{-2}{\curvearrowright} \underline{17} \overset{-2}{\curvearrowright} \underset{t_3}{\underline{15}} \overset{-2}{\curvearrowright} \underline{13}$

$\boxed{19, 17, 15, 13}$

c) $t_5 = 20, d = -1$

$\underline{24} \overset{-1}{\curvearrowright} \underline{23} \overset{-1}{\curvearrowright} \underline{22} \overset{-1}{\curvearrowright} \underline{21} \; \underset{t_5}{\underline{20}}$

$+1 \quad +1 \quad +1$

$\boxed{24, 23, 22, 21}$

4. Consider the sequence 12, 5, −2, −9, ...

a) Determine the formula for the general term of the sequence.

$a = 12$

$d = -7$

$n = ?$

$t_n = ?$

$t_n = a + (n-1)d$

$t_n = 12 + (n-1)(-7)$

$t_n = 12 - 7n + 7$

$\boxed{t_n = -7n + 19}$ → normally this would be written in descending powers, but since the leading coefficient is negative and the constant is positive, it is sometimes written like:

$\boxed{t_n = 19 - 7n}$ but both answers are correct.

b) Determine the nineteenth term of the sequence.

$t_n = -7n + 19$

$t_{19} = -7(19) + 19 = \boxed{-114}$

c) Which of the numbers −268 and −350 are terms of the sequence?

$t_n = -7n + 19$

$-268 = -7n + 19$

$\frac{-287}{-7} = \frac{\cancel{-7}n}{\cancel{-7}} \rightarrow \underline{41 = n}$

$-350 = -7n + 19$

$\frac{-369}{-7} = \frac{\cancel{-7}n}{\cancel{-7}} \rightarrow \underbrace{52.714... = n}$

Remember "n" represents all natural numbers, b/c this answer is a decimal, you can exclude it.

∴ $\boxed{\text{-268 is the 41st term}}$

5. Determine the indicated terms in each arithmetic sequence.

Find the general solution (t_n) first then solve for the specific term.

a) −1, −4, −7, −10, ... t_5, t_{24}, t_n

$a = -1$

$d = -3$

$t_n = ?$

$n = ?$

$t_n = a + (n-1)d$

$t_n = -1 + (n-1)(-3)$

$t_n = -1 - 3n + 3$

$\boxed{t_n = -3n + 2}$

$\boxed{t_n = 2 - 3n}$

$t_5 = -3(5) + 2$

$\boxed{t_5 = -13}$

$t_{24} = -3(24) + 2$

$\boxed{t_{24} = -70}$

b) −21, −6, 9, 24, ... t_{10}, t_{90}, t_n

$a = -21$

$d = 15$

$t_n = ?$

$n = ?$

$t_n = a + (n-1)d$

$t_n = -21 + (n-1)(15)$

$t_n = -21 + 15n - 15$

$\boxed{t_n = 15n - 36}$

$t_{10} = 15(10) - 36$

$\boxed{t_{10} = 114}$

$t_{90} = 15(90) - 36$

$\boxed{t_{90} = 1314}$

<u>Class Ex 7</u> **c)** $-b, 2a-b, 4a-b, 6a-b, \ldots . t_{12}, t_n$

Common difference = $t_n - t_{n-1}$

$t_2 - t_1 \rightarrow (2a-b) - (-b) = 2a - b + b = 2a$

$t_3 - t_2 \rightarrow (4a-b) - (2a-b) = 4a - b - 2a + b = 2a$

ecall the "a" in
e formula
n = a+(n-1)d" is
e first term.
nce "a" is a
ariable in this
uestion "t_1" is
ritten instead
avoid confusion →

$d = 2a$

$t_n = ?$

$n = ?$

$t_1 = -b$

$t_n = t_1 + (n-1)d$

$t_n = -b + (n-1)(2a)$

$t_n = -b + 2an - 2a$

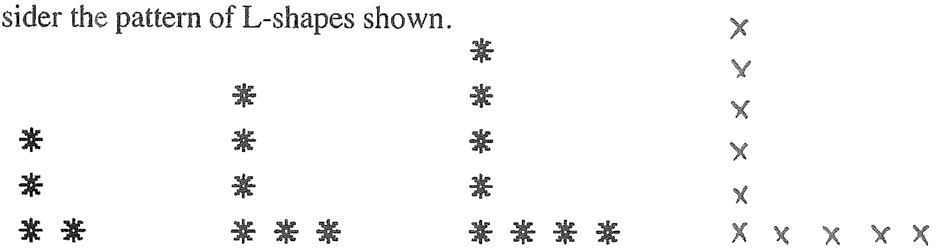
$\boxed{t_n = 2an - 2a - b}$

$t_{12} = 2a(12) - 2a - b$

$t_{12} = 24a - 2a - b$

$\boxed{t_{12} = 22a - b}$

6. Consider the pattern of L-shapes shown.



a) State the number of stars in each of the first four patterns.

Diagram ① = 4 ② = 6 ③ = 8 ④ = 10

b) How many stars are in the 34th pattern?

$a = 4$ $t_n = a + (n-1)d$ → $t_{34} = 2(34) + 2$

$d = 2$ $t_n = 4 + (n-1)(2)$ $\boxed{t_{34} = 70}$

$t_n = ?$ $t_n = 4 + 2n - 2$

$n = ?$ $t_n = 2n + 2$

7. Determine the number of terms in each sequence. (solve for "n")

a) 4, 7, 10, ..., 49

$a = 4$ $t_n = a + (n-1)d$

$d = 3$ $49 = 4 + (n-1)(3)$

$t_n = 49$ $49 = 4 + 3n - 3$

$n = ?$ $49 = 1 + 3n$

$\frac{48}{3} = \frac{\cancel{3}n}{\cancel{3}}$

$16 = n$ → $\boxed{16 \text{ terms}}$

b) –52, –56, –60, ..., –148

$a = -52$ $t_n = a + (n-1)d$

$d = -4$ $-148 = -52 + (n-1)(-4)$

$t_n = -148$ $-148 = -52 - 4n + 4$

$n = ?$ $-148 = -48 - 4n$

$\frac{-100}{-4} = \frac{\cancel{-4}n}{\cancel{-4}}$

$25 = n$ → $\boxed{25 \text{ terms}}$

8. How many multiples of 5 are there from 25 to 315, inclusive? (including 25 and 315)

Find "n" number of terms

$\underbrace{25, 30, 35, 40 \ldots 315}$

These are all multiples of 5. Notice they go up by 5 each time, therefore 5 is the common difference

$a = 25$ $t_n = a + (n-1)d$

$d = 5$ $315 = 25 + (n-1)(5)$

$t_n = 315$ $315 = 25 + 5n - 5$

$n = ?$ $315 = 20 + 5n$

$\frac{295}{5} = \frac{\cancel{5}n}{\cancel{5}}$ → $59 = n$ → $\boxed{59 \text{ multiples}}$

9. Consider the sequence of multiples of 7 between 51 and 275.

↳ That does not necessarily mean these are the first/last terms of the sequence. You are looking for multiples of 7 between 51 and 275

Method to find this on pg 544 question 9. →

a) State the first and last terms of the sequence.

$t_1 = 56$ $t_n = 273$

b) How many multiples of 7 are there between 51 and 275?

$a = 56$ $t_n = a + (n-1)d$

$d = 7$ $273 = 56 + (n-1)(7)$

$t_n = 273$ $273 = 56 + 7n - 7$

$n = ?$ $273 = 49 + 7n$

$\frac{224}{7} = \frac{\cancel{7}n}{\cancel{7}}$ → $32 = n$

$\boxed{32 \text{ multiples}}$

Class Ex 8 **10.** Determine the 4^{th}, 5^{th}, 6^{th}, and 7^{th} terms of the arithmetic sequence in which $t_3 = -24$ and $t_8 = -94$.

original sequence $\_\ \_\ -24\ \_\ \_\ \_\ \_\ -94$

$t_1\ t_2\ t_3\ t_4\ t_5\ t_6\ t_7\ t_8$

New sequence $\_\ \_\ \boxed{-24\ \_\ \_\ \_\ \_\ -94}$

$t_1\ t_2\ t_3\ t_4\ t_5\ t_6$

$a = -24$
$d = ?$
$t_n = -94$
$n = 6$

$t_n = a + (n-1)d$
$-94 = -24 + (6-1)d$
$-94 = -24 + 5d$
$-70 = 5d$
$-14 = d$

now that "d" is found, plug it back into the formula or continuously add to term 3 of the original sequence

$t_4 = -24 + (-14) = -38$
$t_5 = -38 + (-14) = -52$
$t_6 = -52 + (-14) = -66$
$t_7 = -66 + (-14) = -80$

$\boxed{-38, -52, -66, -80}$

Class Ex 7 **11.** The terms $2x + 3$, $3x + 1$, and $8x - 1$ are consecutive terms in an arithmetic sequence. Calculate the value of x, and state the three terms.

$t_1 = 2x+3$
$t_2 = 3x+1$
$t_3 = 8x-1$

① calculate the value of x use common difference

$t_2 - t_1 = t_3 - t_2$
$(3x+1) - (2x+3) = (8x-1) - (3x+1)$
$3x + 1 - 2x - 3 = 8x - 1 - 3x - 1$
$x - 2 = 5x - 2$
$0 = 4x$
$\boxed{0 = x}$

② state the three terms (plug in "x")

$t_1 = 2x+3$
$= 2(0)+3 \rightarrow 3$

$t_2 = 3x+1$
$= 3(0)+1 \rightarrow 1$

$t_3 = 8x-1$
$8(0) - 1 \rightarrow -1$

$\boxed{3, 1, -1}$

12. The terms $x + 3$, $3x - 1$, and $7x - 2$ are consecutive terms in an arithmetic sequence. Calculate the value of x, and determine the general term of the sequence.

$t_1 = x+3$
$t_2 = 3x-1$
$t_3 = 7x-2$

① value of x

$t_2 - t_1 = t_3 - t_2$
$(3x-1) - (x+3) = (7x-2) - (3x-1)$
$3x - 1 - x - 3 = 7x - 2 - 3x + 1$
$2x - 4 = 4x - 1$
$-3 = 2x$
$\boxed{\frac{-3}{2} = x}$

② General term.
-you only need to solve for t_1 because you need t_1 in the formula

$t_1 = x + 3 \rightarrow (\frac{-3}{2}) + 3 = \frac{3}{2} = a$

"d" $= t_2 - t_1 \rightarrow (3x-1) - (x+3) \rightarrow 2x - 4 \rightarrow 2(\frac{-3}{2}) - 4 =$

$t_n = a + (n-1)d$
$t_n = \frac{3}{2} + (n-1)(-7)$
$t_n = \frac{3}{2} - 7n + 7$

$\rightarrow \boxed{t_n = -7n + \frac{17}{2}}$

13. In an arithmetic sequence, the seventh term is 3 and the sixteenth term is 9.

a) Use arithmetic means to determine the common difference and the first term of the sequence.

$\_\ \_\ \ldots \boxed{3\ \_\ \_\ \ldots\ 9}$

$t_1 \qquad t_{10}$

$a = 3$
$d = ?$
$t_n = 9$
$n = 10$

$t_n = a + (n-1)d$
$9 = 3 + (10-1)d$
$\frac{6}{9} = \frac{\cancel{9}d}{\cancel{9}}$
$\boxed{\frac{2}{3} = d}$

Remember not to confuse the first term of the original sequence with the new sequence

$t_n = a + (n-1)d$
$t_{16} = a + (16-1)(\frac{2}{3})$
$9 = a + (15)(\frac{2}{8})$
$9 = a + 10$

$\rightarrow -1 = a$
$\therefore \boxed{t_1 = -1}$

b) Use a system of equations to determine the common difference and the first term of the sequence. (Elimination and Substitution)

$t_n = a + (n-1)d$

$t_7 = a + (7-1)d$

$3 = a + 6d$

$t_{16} = a + (16-1)d$

$9 = a + 15d$

① $9 = a + 15d$

$-(3 = a + 6d)$

$\frac{6}{9} = \frac{\cancel{9}d}{\cancel{9}}$

$\boxed{\frac{2}{3} = d}$

② $3 = a + 6(\frac{2}{3})$

$3 = a + 4$

$-1 = a$

$\boxed{t_1 = -1}$

you can use either formula ($3 = a + 6d$ or $9 = a + 15d$) to find the first term

c) Calculate t_{19} and determine the general term of the sequence.

$a = -1$

$d = \frac{2}{3}$

$t_{19} = ?$

$n = 19$

$t_n = a + (n-1)d$

$t_{19} = -1 + (19-1)(\frac{2}{3})$

$t_{19} = -1 + 12$

$\boxed{t_{19} = 11}$

$t_n = a + (n-1)d$

$t_n = -1 + (n-1)(\frac{2}{3})$

$t_n = -1 + \frac{2}{3}n - \frac{2}{3}$

$\boxed{t_n = \frac{2}{3}n - \frac{5}{3}}$

14. Use linear systems to determine a, d, and t_n for the sequences in which the following two terms are given.

a) $t_5 = 21, t_{10} = 41$

① $t_5 = a + (5-1)d$

$21 = a + 4d$

$t_{10} = a + (10-1)d$

$41 = a + 9d$

② $41 = a + 9d$

$-(21 = a + 4d)$

$\frac{20}{5} = \frac{\cancel{5}d}{\cancel{5}}$

$\boxed{4 = d}$

③ $21 = a + 4d$

$21 = a + 4(4)$

$21 = a + 16$

$\boxed{5 = a}$

④ $t_n = a + (n-1)d$

$t_n = 5 + (n-1)(4)$

$t_n = 5 + 4n - 4$

$\boxed{t_n = 4n + 1}$

b) $t_4 = -9, t_{15} = -31$

① $t_4 = a + (4-1)d$

$-9 = a + 3d$

$t_{15} = a + (15-1)d$

$-31 = a + 14d$

② $-31 = a + 14d$

$-(-9 = a + 3d)$

$\frac{-22}{11} = \frac{\cancel{11}d}{\cancel{11}}$

$\boxed{-2 = d}$

③ $-9 = a + 3d$

$-9 = a + 3(-2)$

$-9 = a - 6$

$\boxed{-3 = a}$

④ $t_n = a + (n-1)d$

$t_n = -3 + (n-1)(-2)$

$t_n = -3 - 2n + 2$

$\boxed{t_n = -2n - 1}$

Multiple Choice

15. Which of the following represents an arithmetic sequence with a common difference of –4?

A. 8, 4, 2, 1 . . . → No common difference because it's Geometric

B. 20, 24, 28, 32 . . . → $d = +4$

C. 32, –8, 2, –0.5 . . . → Geometric

[✓ **D.**] 20, 16, 12, 8 . . .

Numerical Response

16. Twenty-seven arithmetic means are inserted between the first and last terms of a sequence. The number of terms in the sequence is 29 .

(Record your answer in the numerical response box from left to right.)

2	9		

$27 + 2 = 29$

27 arithmetic means

First + last term

Answer Key

1. a) i) 6 ii) $t_4 = 26, t_5 = 32, t_6 = 38$ **b)** i) 12 ii) $t_4 = 31, t_5 = 43, t_6 = 55$
c) i) -17 ii) $t_5 = 2, t_6 = -15, t_7 = -32$ **d)** i) -2.9 ii) $t_4 = -1.6, t_5 = -4.5, t_6 = -7.4$
e) i) $-3/5$ ii) $t_4 = -17/15, t_5 = -26/15, t_6 = -7/3$
f) i) $-3x - 2y$, ii) $t_4 = -11x - 3y, t_5 = -14x - 5y, t_6 = -17x - 7y$

2. a) $-6, -3, 0, 3, 6$ **b)** $18, 11, 4, -3, -10$ **c)** $5, 3, 1, -1, -3$ **d)** $5, 7.5, 10, 12.5, 15$

3. a) 5, 11, 17, 23 **b)** 19, 17, 15, 13 **c)** 24, 23, 22, 21

4. a) $t_n = 19 - 7n$ **b)** -114 **c)** -268 is the 41st term

5. a) $-13, -70, t_n = 2 - 3n$ **b)** $114, 1314, t_n = 15n - 36$ **c)** $22a - b, t_n = 2an - 2a - b$

6. a) 4, 6, 8, 10 **b)** 70

7. a) 16 **b)** 25 **8.** 59 **9. a)** 56 and 273 **b)** 32

10. $-38, -52, -66, -80$

11. $x = 0$; $3, 1, -1$ **12.** $x = -\frac{3}{2}$; $t_n = \frac{17}{2} - 7n$

13. a) $d = \frac{2}{3}, t_1 = -1$; **b)** $d = \frac{2}{3}, a = -1$; **c)** $t_{19} = 11$, $t_n = \frac{2}{3}n - \frac{5}{3}$

14. a) $a = 5, d = 4$, $t_n = 4n + 1$ **b)** $a = -3, d = -2$, $t_n = -2n - 1$

15. D

16.

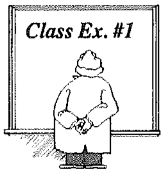
2	9		

Arithmetic Sequences Lesson #3:
Arithmetic Growth and Decay

Generating Number Patterns Exhibiting Arithmetic Growth

Many real-life scenarios can be represented by a pattern of numbers which exhibit arithmetic growth.

Class Ex. #1



BP Birthplace Forest program in Calgary enables parents to honour their children by planting a tree when their child is born. At the same time, this shows concern for the urban environment by encouraging the growth of city forests. Some of the trees which have been planted are evergreen trees which grow an average of 12 to 18 inches per year. The program was launched in the year 2000.

a) An evergreen tree, 6 inches tall was planted in June 2007 and has a growth rate of 15 inches per year. Two students were asked to determine the height of the tree in June 2018. Joel formed an arithmetic sequence beginning 6, 21, 36, ... and Jenna formed an arithmetic sequence beginning 21, 36, 51 ... . Use the formulas in the previous lesson to determine each student's answer to the problem.

JOEL
$t_{12} = ?$
$a = 6$
$d = 15$
$n = 12$
$t_{12} = 6 + (12-1)(15)$
$\boxed{t_{12} = 171 \text{ inches}}$

JENNA
$t_{11} = ?$
$a = 21$
$d = 15$
$n = 11$
$t_{11} = 21 + (11-1)(15)$
$\boxed{t_{11} = 171 \text{ inches}}$

b) Determine the formula for the general term of Joel's arithmetic sequence.

$t_n = a + (n-1)d$
$t_n = 6 + (n-1)(15)$
$t_n = 6 + 15n - 15$
$\boxed{t_n = 15n - 9}$

c) Determine the formula for the general term of Jenna's arithmetic sequence.

$t_n = a + (n-1)d$
$t_n = 21 + (n-1)(15)$
$t_n = 21 + 15n - 15$
$\boxed{t_n = 15n + 6}$

d) Explain why the formulas in **b)** and **c)** are different.

B/c Joel's sequence started at 6, he is including the year 2007, therefore, there are 12 terms, where as Jenna did not include the year of 2007 so she only has 11 terms making the general term different

e) If the tree continues to grow at the same rate, in which year will it reach a height of 28 ft?

Beware of units! The question is asking for the height in ft but the general formula(s) created are in inches. (Convert ft to inches)

12 inches = 1 ft
28 ft × 12 inches
= 336 inches

JOEL = including 2007
$t_n = 15n - 9$
$336 = 15n - 9$
$\frac{345}{15} = \frac{\cancel{15}n}{\cancel{15}}$
$23 = n$
$\boxed{\text{year} = 2029}$

JENNA from year 2008
$t_n = 15n + 6$
$336 = 15n + 6$
$\frac{330}{15} = \frac{\cancel{15}n}{\cancel{15}}$
$22 = n$
$\boxed{\text{year} = 2029}$

$\underbrace{2007 \_\_ \_\_ \_\_ \_\_ \dots t_n}_{n=22}$

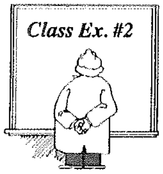
2007 is already included in "n" so to find what year it will reach 336" add 2007 w 22
2007 + 22 = 2029

$\underbrace{2007}_{} + \underbrace{\_\_ \_\_ \_\_ \dots t_n}_{n=22}$
2007 + 22 = 2029

when "d" is greater than zero ($d > 0$) there is Growth. when "d" is less than zero ($d < 0$) there is Decay.

Generating Number Patterns Exhibiting Arithmetic Decay

Many real-life scenarios can be represented by a pattern of numbers which exhibit arithmetic decay or arithmetic depreciation.



A printing press was bought in the year 1999. It depreciates in value by the same amount each year. Five years after its purchase, the printing press had a value of $311 000. It had a scrap value of $2900 in the year 2017.

a) Use an arithmetic sequence to determine the annual depreciation.

↳ Find "d" the common difference...

Method 1 including year 1999

$t_n = a + (n-1)d$

$t_6 = 311\,000 \quad t_{19} = 2900$

① $t_6 = a + (6-1)d$

$a + 5d = 311000$

② $t_{19} = a + (19-1)d$

$a + 18d = 2900$

③ ellimination:

$a + 18d = 2900$

$-(a + 5d = 311\,000)$

$\frac{\cancel{13}d}{\cancel{13}} = \frac{-308\,100}{13}$

$\boxed{d = -23\,700}$

$t_6 = 311\,000$ b/c this is the value 5 years including 1999 (5+1=6)

Method 2 starting in year 2000

$t_n = a + (n-1)d$

$t_5 = 311\,000 \quad t_{18} = 2900$

① $t_5 = a + (5-1)d$

$311000 = a + 4d$

② $t_{18} = a + (18-1)d$

$2900 = a + 17d$

③ ellimination:

$a + 17d = 2900$

$-(a + 4d = 311\,000)$

$\frac{\cancel{13}d}{\cancel{13}} = \frac{-308\,100}{13}$

$\boxed{d = -23\,700}$

b) Determine the purchase price of the printing press.

↳ Look for the first term (a or t_1)

$t_n = a + (n-1)d$

$t_6 = a + (6-1)(-23\,700)$

$311\,000 = a - 118\,500$

$\boxed{429\,500 = a}$

This answer is correct b/c it includes the year 1999

$t_n = a + (n-1)d$

$t_5 = a + (5-1)(-23\,700)$

$311000 = a - 94800$

$\underbrace{405800 = a}$

This answer is not necessarily correct... You need to go back one year b/c it does not include 1999

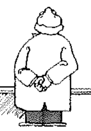
$405800 + 23\,700 = \boxed{429\,500 = a}$

Relating Arithmetic Sequences to Linear Functions

We can relate arithmetic sequences to linear functions over the natural numbers. Consider the following example:

A pile of bricks is arranged in rows. There are 28 bricks in the first row, and the number of bricks in each successive row is two more than in the previous row.

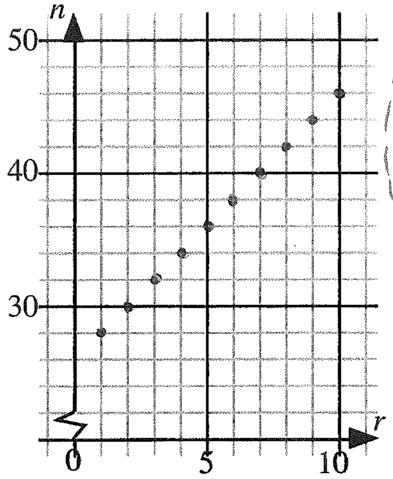
Class Ex. #3



a) Complete the table of values showing the number of bricks in each of the first 10 rows.

Row Number, r	Number of Bricks, n
1	28
2	30
3	32
4	34
5	36
6	38
7	40
8	42
9	44
10	46

b) Plot the ordered pairs on the grid. and classify the relationship as [✓ linear] or non-linear.



RECALL:
Do NOT connect the dots b/c this is discrete data. The points plotted are natural numbers (which are whole numbers, greater than zero)

c) Determine if the range is an arithmetic sequence.

Range $\{n \mid 28 \le n \le 46, n \in N\}$

Range is arithmetic b/c it is linear and b/c each data point increases by a common difference of +2.

d) State the domain of the relationship.

Domain $\{r \mid 1 \le r \le 10, r \in N\}$

"natural numbers"

e) Explain why the graph does not have an intercept on the vertical axis?

There is no such thing as row zero.

f) Write the equation for the number of bricks in a row, *n*, as a function of the row number, *r*.

$t_n = a + (n-1)d$

$t_n = 28 + (n-1)(2)$

$t_n = 28 + 2n - 2$

$\underline{t_n = 2n + 26}$

writing this formula in terms of n+r looks like:

$t_n = 2n + 26$

$\downarrow \quad \downarrow$

$\boxed{n = 2r + 26}$

"y-axis" # of bricks (n)

"x-axis" Row number (r)

Complete Assignment Questions #1 - #8

Assignment

1. A contractor charges \$68 for the first hour of work. This includes a rate for one hour of work and a fixed travel fee. For a two hour job, the contractor charges \$110, and for a three hour job, the contractor charges \$152.

   a) Using these three numbers as the first three terms of an arithmetic sequence, determine the values of *a* and *d*.

   $\underset{t_1}{68},\ \underset{t_2}{110},\ \underset{t_3}{152}$ $\boxed{a=68}$

   $d = t_2 - t_1$

   $= 110 - 68 = \boxed{42}$

   b) State the rate for each additional hour of work.

   $\boxed{\$42}$

   c) Determine the amount of the travel fee.

   $68 - 42 = \boxed{\$26}$

   d) Determine the total cost for a six hour job.

   $t_n = a + (n-1)d$

   $t_6 = 68 + (6-1)(42) \rightarrow \boxed{\$278}$

2. For a forthcoming horticultural exhibition, bulbs were planted in rows. The number of bulbs in each row forms an arithmetic sequence. There are 58 bulbs in the eighth row and 107 bulbs in the fifteenth row.

   How many bulbs in total are in the first three rows?

   $\underset{t_1}{9} \xrightarrow{+7} \underset{t_2}{16} \xrightarrow{+7} 23 \xrightarrow{+7} 30\ \ 37\ \ 44\ \ 51\ \ \underset{t_8}{58}$ } Side wor double che

   ① Use a proccess of elimination to find "d"

   $t_8 = 58 \quad t_{15} = 107$

   I) $t_n = a + (n-1)d$

   $t_8 = a + (8-1)d$

   $58 = a + 7d$

   II) $t_{15} = a + (15-1)d$

   $107 = a + 14d$

   III) $107 = a + 14d$

   $-(58 = a + 7d)$

   $\frac{49}{7} = \frac{\cancel{7}d}{\cancel{7}}$

   $7 = d$

   ② Once "d" is found, find t_1, t_2, t_3, then add them together to get the total # of bulbs in the first 3 rows

   $t_1 = a \rightarrow 58 - 7(7) = a \rightarrow 9$

   $t_2 = 9 + 7 = 16$

   $t_3 = 16 + 7 = 23$

   $9 + 16 + 23 = \boxed{48 \text{ bulbs}}$

3. Consider the linear function with equation $y = 3x + 5$.

   a) Sketch the graph of the linear function on the grid.

   b) Restrict the domain to the set of natural numbers. Mark with dots points on the graph which represent the function on the restricted domain.

   c) Write the first five elements of the range in numerical order.

   8, 11, 14, 17, 20

   d) Show that the elements of the range form an arithmetic sequence and state the common difference.

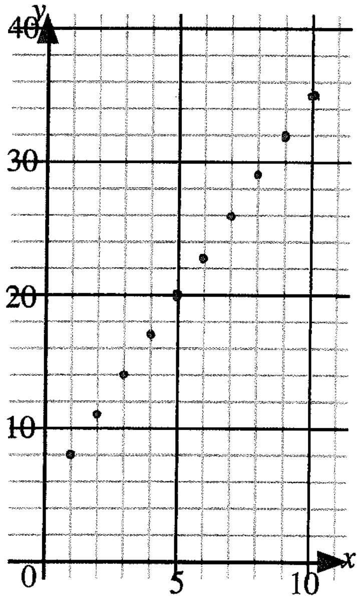
   8, 11, 14, 17....

   common difference

   $t_2 - t_1 \rightarrow 11 - 8 = \boxed{3}$

x	y
1	8
2	11
3	14
4	17
5	20
6	23
7	26
8	29
9	32
10	35

plug in the value for x in the equation $y = 3x + 5$ to get the y value :)



4. Charity starts a new job as a geologist in the mining sector. Her rate of pay for the first year is $36 000, with an increase of $2 750 per year thereafter.

a) Calculate her rate of pay in the seventh year.

$t_1 = 36000$ $\quad$ $t_n = a + (n-1)d$

$d = 2750$ $\quad$ $t_7 = 36000 + (7-1)(2750)$

$\boxed{t_7 = 52500}$

b) In which year will she first earn more than $60 000?

↳ continue to add 2750, recording the term, until the value becomes greater than $60 000.

$t_7 = 52500$

$t_8 = 55\ 250$

$t_9 = 58\ 000$

$t_{\textcircled{10}} = \underline{\underline{60\ 750}}$

→ $\boxed{\text{year } 10}$

5. Chairs in an auditorium are arranged in rows in such a way that the first two rows each have the same number of chairs. The third and fourth rows each have three more chairs than the first and second row; the fifth and sixth rows each have three more chairs than the third and fourth row, etc. The sequence of number of chairs for every second row forms an arithmetic sequence. The first two rows each have 27 chairs, and the last two rows each have 114 chairs.

a) How many rows of chairs are there?

$\underset{\text{27 chairs in row 1+2}}{27},\ \underset{\text{row 3+4}}{30},\ \underset{\text{row 5+6}}{33}\ldots.\ \underset{\text{row n+m}}{114}$

$t_n = a + (n-1)d$

$a = 27$ $\quad$ $114 = 27 + (n-1)3$

$d = 3$ $\quad$ $114 = 27 + 3n - 3$

$t_n = 114$ $\quad$ $90 = 3n$

$n = ?$ $\quad$ $\underline{30 = n}$

→ There are 30 rows w̄ a different amount of chairs. B/c 2 rows share the same amount of chairs multiply 30 by 2 to get the total amount of rows.

$30 \times 2 = \boxed{60 \text{ rows}}$

b) How many chairs are in the **i)** thirteenth **ii)** thirtieth row?

i) The 13th row is the same as t_7 in the arithmetic sequence created

$t_7 = 27 + (7-1)(3)$

$t_7 = \boxed{45 \text{ chairs}}$

ii) The 30th row is the same as t_{15}

$t_{15} = 27 + (15-1)(3)$

$t_{15} = \boxed{69 \text{ chairs}}$

6. A sports utility vehicle sells for $35 000. The vehicle depreciates $5000 the first year and $2400 each year thereafter. Calculate the value of the vehicle at the end of the eleventh year.

$t_1 = 35\,000 - 5000 = \underbrace{30\,000}$ → value of the vehicle in the first year

$d = -2400$

↖ negative b/c it depreciates.

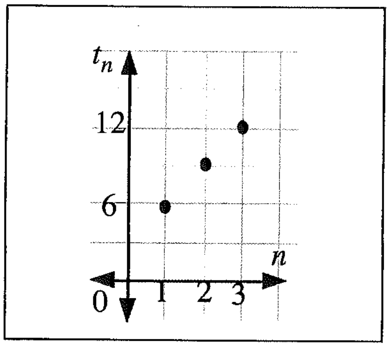
$t_n = a + (n-1)d$

$t_{11} = 30\,000 + (11-1)(-2400)$

$t_{11} = 30\,000 - 24\,000$

$t_{11} = \boxed{\$6000}$

Use the following graph to answer questions #7 and #8.



Sidework

$6 \xrightarrow{+3} 9 \xrightarrow{+3} 12 \ldots$

$t_1 \quad t_2 \quad t_3$

Multiple Choice **7.** The graph above represents arithmetic growth. If the graph continues indefinitely, then the fifteenth term of the sequence is

[✓ **A.**] 48
B. 51
C. 90
D. 153

$a = 6$
$d = 3$
$t_{15} = ?$
$n = 15$

$t_n = a + (n-1)d$

$t_{15} = 6 + (15-1)(3)$

$t_{15} = 6 + 42$

$\boxed{t_{15} = 48}$

Numerical Response **8.** The formula for this sequence is $t_n = mn + b$.
The value of b is 3 .

(Record your answer in the numerical response box from left to right.)

3			

$t_n = a + (n-1)d$

$t_n = 6 + (n-1)3$

$t_n = 6 + 3n - 3$ → $t_n = 3n + 3$

$\uparrow \quad \uparrow\uparrow\uparrow\uparrow$

$t_n = mn + b$

$\boxed{b = 3}$

Answer Key

1. a) $a = 68$, $d = 42$ **b)** $42 **c)** $26 **d)** $278 **2.** 48
3. c) 8, 11, 14, 17, 20 **d)** common difference = 3
4. a) $52500 **b)** year 10 **5. a)** 60 **b) i)** 45 **ii)** 69 **6.** $6000
7. A **8.**

3			

Arithmetic Sequences Lesson #4:
Extension: Arithmetic Series

Arithmetic Series

When the terms of an arithmetic sequence are added, the result is known as an **arithmetic series**.

For example $3, 5, 7, 9, 11$ → arithmetic sequence.

$3 + 5 + 7 + 9 + 11$ → arithmetic series.

The symbol, S_n is used to represent the sum of n terms of an arithmetic series.

In the example above $S_5 =$ 35 .

$3 + 5 + 7 + 9 + 11 = \underline{\underline{35}}$

Investigation

Investigating the Sum of an Arithmetic Series

To illustrate the method for determining a formula for the sum of n terms of an arithmetic series, the story of the great mathematician Karl Gauss (1777-1855) is frequently told.

> When Karl was about 10 years old, he was placed in Master Buttner's arithmetic class. Master Buttner often gave his class long arithmetic problems to keep them quiet for a time. On one particular day, Master Buttner asked his class to add the whole numbers from 1 to 100. While all the students began to work madly on this assignment, Karl laid his slate on the desk and informed Master Buttner he was finished. Master Buttner asked Karl what his answer was. To Master Buttner's astonishment, Karl gave the correct answer of 5050.

Here we apply a method similar to his to determine the answer.

$$S_{100} = 1 + 2 + 3 + 4 + 5 + \ldots + 96 + 97 + 98 + 99 + 100$$

$$S_{100} = 100 + 99 + 98 + 97 + 96 + \ldots + 5 + 4 + 3 + 2 + 1$$

Add the rows and complete the work to show that $S_{100} = 5050$.

$$2S_{100} = 101 + 101 + 101 + 101 + 101 \ldots\ldots + 101 + 101 + 101$$

$$101 \times 100 = 2S_{100} = 10100$$

$$\frac{10100}{2} = \frac{\cancel{2}S_{100}}{\cancel{2}} \rightarrow \boxed{S_{100} = 5050}$$

Formulas for the Sum of an Arithmetic Series

In an arithmetic series of n terms,

the first term is a, the second term is $a + d$, and the last term, $t_n = a + (n - 1)d$.

The sum of n terms of the series can be written as

$$S_n = a + (a + d) + (a + 2d) + \ \ldots \ + (a + (n - 3)d) + (a + (n - 2)d) + (a + (n - 1)d$$

OR

$$S_n = (a + (n - 1)d) + (a + (n - 2)d) + (a + (n - 3)d.+ \ldots + (a + 2d) + (a + d) + a$$

Adding these two lines together gives

$$2S_n = (2a + (n - 1)d) + (2a + (n - 1)d) + (2a + (n - 1)d) + \ldots + (2a + (n - 1)d) \\ + (2a + (n - 1)d) + (2a + (n - 1)d)$$

$$2S_n = n(2a + (n - 1)d)$$

Dividing by 2 gives the **formula for the sum of *n* terms of an arithmetic series.**

$$\boxed{S_n = \frac{n[2a + (n - 1)d]}{2}}$$

★use this formula when the common difference is known

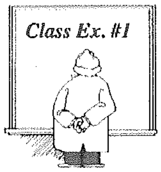
This formula connects a, d, n, and S_n.
If any three of these values are known, the fourth can be determined.

However, when the common difference of the series is not known, another **formula for the sum of *n* terms of an arithmetic series** can be formed by replacing $a + (n - 1)d$ by t_n to give

$$\boxed{S_n = \frac{n(a + t_n)}{2}}$$

★use this when the last term is known

The above formula can be thought of as the average of the first and last terms multiplied by the number of terms.



Determine the sum of the first fourteen terms of the arithmetic series 9 + 15 + 21 + ...

a = still the first term of the sequence.

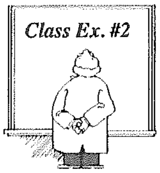
d = still the common difference.

n = still the number of terms.

$a=9$
$d=6$
$n=14$

$$S_n = \frac{n[2a+(n-1)d]}{2}$$

$$S_{14} = \frac{14[2(9)+(14-1)(6)]}{2} = \boxed{672}$$



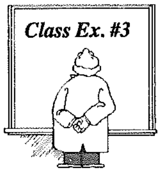
Determine the sum of $\underline{\text{22 terms}}$ of an arithmetic sequence with $t_1 = -18$ and $t_{22} = 45$.

↳ Total # of terms, use the formula w/ t_n in it.

$a = -18$
$d = ?$
$n = 22$
$t_n = 45$

$$S_n = \frac{n(a+t_n)}{2}$$

$$S_{22} = \frac{22(-18+45)}{2} = \boxed{297}$$



Find the sum of the terms in the sequence 17, 12, 7, ... −38.

$a = 17$
$d = -5$
$n = ?$
$t_n = -38$

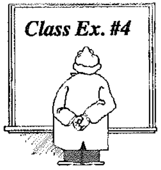
① First find "n"

$$t_n = a+(n-1)d$$
$$-38 = 17+(n-1)(-5)$$
$$-38 = 17-5n+5$$
$$\frac{-60}{-5} = \frac{\cancel{-5}n}{\cancel{-5}}$$
$$12 = n$$

② Then find the sum (use any formula)

$$S_n = \frac{n(a+t_n)}{2}$$
$$S_{12} = \frac{12(17+(-38))}{2}$$
$$\boxed{S_{12} = -126}$$

★ Important



Tarvarus starts work at a salary of \$16 000 per $\overset{\text{year}}{\text{annum}}$. He receives annual increases of \$850. He works for the firm for twelve years.

a) Calculate his salary $\underline{\text{in the twelfth year}}$. → t_n not S_n

$a = 16\,000$
$d = 850$
$n = 12$

$$t_n = a+(n-1)d$$
$$t_{12} = 16\,000+(12-1)(850)$$
$$\boxed{t_{12} = \$25\,350}$$

b) How much has he earned $\underline{\text{in total}}$ over the twelve years?

$$S_n = \frac{n(a+t_n)}{2}$$

$$S_{12} = \frac{12(16\,000+25\,350)}{2} = \boxed{\$248\,100}$$

Complete Assignment Questions #1 - #4

Investigation #2 *Investigating an Arithmetic Series Defined in Terms of S_n.*

Corrie was given two questions on a sequences and series assignment. The first question is listed below.

"Find the first four terms of the series defined by $S_n = 2n^2 - n$."

a) Complete her work below to find the first four terms.

$S_n = 2n^2 - n$

$S_1 = 2(1)^2 - 1 = 1 \Rightarrow t_1 = S_1 \quad \therefore t_1 = 1$

$S_2 = 2(2)^2 - 2 = 6 \Rightarrow S_2 = S_1 + t_2 \Rightarrow t_2 = S_2 - S_1 \Rightarrow t_2 = 6 - 1 \quad \therefore t_2 = 5$

$S_3 = 2(3)^2 - 3 = 15 \Rightarrow S_3 = \underline{S_2} + t_3 \Rightarrow t_3 = S_3 - \underline{S_2} \Rightarrow t_3 = 15 - 6 \quad \therefore t_3 = 9$

$S_4 = 2(4)^2 - 4 = 28 \Rightarrow S_4 = \underline{S_3} + \underline{t_4} \Rightarrow t_4 = S_4 - S_3 \Rightarrow t_4 = 28 - 15 \quad \therefore t_4 = 13$

b) Express t_{10} in terms of S.

$t_{10} = S_{10} - S_9$

c) Express t_n in terms of S.

$t_n = S_n - S_{n-1}$

d) The second question Corrie received was "Find t_n if $S_n = 2n^2 - n$."

i) Find t_n using the formula $t_n = a + (n - 1)d$.

$a = 1$
$d = 4$

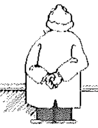
$t_n = 1 + (n-1)(4)$
$t_n = 1 + 4n - 4$
$\boxed{t_n = 4n - 3}$

ii) Find t_n using the formula in **c)**.

$t_n = S_n - S_{n-1}$
$t_n = [2n^2 - n] - [2(n-1)^2 - (n-1)]$
$t_n = 2n^2 - n - 2(n-1)^2 + (n-1)$
$t_n = 2n^2 - n - 2(n^2 - 2n + 1) + n - 1$
$t_n = \cancel{2n^2} \cancel{-n} \cancel{-2n^2} + 4n - 2 \cancel{+n} - 1$
$\boxed{t_n = 4n - 3}$

Remember the formula $\boxed{t_n = S_n - S_{n-1},\ n \geq 2, n \in N}$

Class Ex. #5



For a certain arithmetic series, $S_n = \frac{1}{2}n(11 - n)$.

Determine the first four terms of the corresponding arithmetic sequence.

① $S_n = \frac{1}{2}n(11-n)$

$S_1 = \frac{1}{2}(1)(11-1) = 5$

$S_2 = \frac{1}{2}(2)(11-2) = 9$

$S_3 = \frac{1}{3}(3)(11-3) = 12$

$S_4 = \frac{1}{2}(4)(11-3) = 14$

5, 9, 12, 14
are not the answers.
These are the values
in the series not the
sequence

② $S_1 = t_1$

$S_2 = t_1 + t_2$

$S_3 = t_1 + t_2 + t_3$

$S_4 = t_1 + t_2 + t_3 + t_4$

③ $S_1 = t_1$

$S_2 - t_1 = t_2$

$S_3 - t_1 - t_2 = t_3$

$S_4 - t_1 - t_2 - t_3 = t_4$

④ $t_1 = 5$

$t_2 = (9-5) = 4$

$t_3 = (12-5-4) = 3$

$t_4 = (14-5-4-3) = 2$

$\boxed{\text{First 4 terms } 5,4,3,2}$

Complete Assignment Questions #5 - #10

Assignment

1. Find the sum of each series.

a) 2 + 3 + 4 + ... (first 30 terms)

$a = 2$
$d = 1$
$n = 30$

$\Rightarrow S_n = \dfrac{n[2a+(n-1)d]}{2}$

$S_{30} = \dfrac{30[2(2)+(30-1)(1)]}{2}$

$S_{30} = \boxed{495}$

b) (–8) + (–4) + 0 + ... (first 27 terms)

$a = -8$
$d = 4$
$n = 27$

$S_n = \dfrac{n[2a+(n-1)d]}{2}$

$S_{27} = \dfrac{27[2(-8)+(27-1)(4)]}{2}$

$S_{27} = \boxed{1188}$

c) 2.5 + 2.7 + 2.9 + ... to 16 terms

$a = 2.5$
$d = 0.2$
$n = 16$

$S_n = \dfrac{n[2a+(n-1)d]}{2}$

$S_{16} = \dfrac{16[2(2.5)+(16-1)(0.2)]}{2}$

$S_{16} = \boxed{64}$

d) $\frac{5}{2} + \frac{11}{6} + \frac{7}{6} + \frac{1}{2} + \dots$ to 12 terms

$a = \frac{5}{2}$
$d = \frac{-2}{3}$
$n = 12$

$S_n = \dfrac{n[2a+(n-1)d]}{2}$

$S_{12} = \dfrac{12[2(\frac{5}{2})+(12-1)(\frac{-2}{3})]}{2}$

$\boxed{S_{12} = -14}$

2. Find the sum of each arithmetic series given the first and last terms.

a) $a = 8, t_{15} = 120$

$$S_n = \frac{n(a + t_n)}{2}$$

$$S_{15} = \frac{15(8 + 120)}{2}$$

$$\boxed{S_{15} = 960}$$

b) $t_1 = -11, t_{23} = -253$

$$S_n = \frac{n(a + t_n)}{2}$$

$$S_{23} = \frac{23[(-11) + (-253)]}{2}$$

$$\boxed{S_{23} = -3036}$$

3. Find the sum of each series.

a) 11 + 23 + 35 + 47 + ... + 179

$a = 11$
$d = 12$
$n = ?$
$t_n = 179$

$t_n = a + (n-1)d$

$179 = 11 + (n-1)(12)$

$179 = 11 + 12n - 12$

$179 = 12n - 1$

$\frac{180}{12} = \frac{\cancel{12}n}{\cancel{12}}$

$15 = n$ →

$$S_n = \frac{n(a+t_n)}{2}$$

$$S_{15} = \frac{15(11 + 179)}{2}$$

$$\boxed{S_{15} = 1425}$$

b) 29 + 21 + 13 + 5 + ... −27

$a = 29$ $d = -8$ $n = ?$ $t_n = -27$

$t_n = a + (n-1)d$

$-27 = 29 + (n-1)(-8)$

$-27 = 29 - 8n + 8$

$-27 = -8n + 37$

$\frac{-64}{-8} = \frac{-\cancel{8}n}{\cancel{-8}}$

$8 = n$ →

$$S_n = \frac{n(a+t$$

$$S_8 = \frac{8[29+(-}{2}$$

$$\boxed{S_8 = 8}$$

4. As payment for her yard work, a father agrees to give his daughter an allowance of \$3.50 in the first week of the year with an increase of 50 cents each week until the last week of the year.

a) How much money did she receive for an allowance in the last week of the year?

$a = 3.50$
$d = 0.50$
$n = 52$
$t_n = ?$

There are about 52 weeks in a year → n = 52
365÷7 = 52.14...
↳ 52

$t_n = a + (n-1)d$

$t_{52} = 3.50 + (52-1)(0.50)$

$$\boxed{t_{52} = \$29}$$

b) What was the total amount of money her father gave her in allowances for the year?
↳ Sn

$$S_n = \frac{n(a+t_n)}{2}$$

$$S_{52} = \frac{52(3.50 + 29)}{2}$$

$$\boxed{S_{52} = \$845}$$

Sidework

$\underset{1}{5}\ \underbrace{\_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_}_{\text{8 Arithmetic means.}}\ \underset{t_{10}}{12}$

terms total

5. On a math assignment, Bob "Bubbles" Burble was asked to find the sum of eight arithmetic means placed between –15 and 12. Bubbles proceeded to find the eight arithmetic means and determine the sum. Along came Sally "Sequential" Sequence and asks Bubbles why he first found all the means. Sally told Bubbles he does not have to find the means at all! Is Sally correct in stating that Bubbles does not have to find all the means? If so, show clearly Sally's method, and determine the sum.

Bob doesn't have to find any Arithmetic Means because he already has enough info to use a formula. Once he has found the value using the formula, subtract the first + last term to get the total sum of the Arithmetic Means.

$a = -15$
$d = ?$
$n = 10$
$t = 12$

$S_n = \dfrac{n(a+t_n)}{2}$

$S_{10} = \dfrac{10(-15+12)}{2}$

$S_{10} = -15$

$\rightarrow S_{10} - t_1 - t_{10} = \text{Sum of AM}$

$-15 - (-15) - 12 = \boxed{-12}$

be careful of signs.

6. Consider the series defined by $S_n = 3n - 1$.

a) Find the first four terms of the series.

$S_n = 3n-1$
$S_1 = 3(1)-1 \rightarrow 2$
$S_2 = 3(2)-1 \rightarrow 5$
$S_3 = 3(3)-1 \rightarrow 8$
$S_4 = 3(4)-1 \rightarrow 11$

$t_1 = 2$
$t_2 = 5-2 = 3$
$t_3 = 8-2-3 = 3$
$t_4 = 11-2-3-3 = 3$

$\boxed{\text{First four terms } 2,3,3,3}$

b) Is the sequence arithmetic? Explain.

Not arithmetic b/c there is no common difference between the terms.

7. Consider the series defined by $S_n = 3n^2 - n$.

a) Find the first four terms of the series.

$S_n = 3n^2-n$
$S_1 = 3(1)^2-1 \rightarrow 2$
$S_2 = 3(2)^2-2 \rightarrow 10$
$S_3 = 3(3)^2-3 \rightarrow 24$
$S_4 = 3(4)^2-4 \rightarrow 44$

$t_1 = 2$
$t_2 = 10-2 = 8$
$t_3 = 24-2-8 = 14$
$t_4 = 44-2-8-14 = 20$

$\boxed{\text{First four terms } 2,8,14,20}$

b) Determine the eighth term of the corresponding sequence.

$a = 2$
$d = 6$
$n = 8$

$t_n = a+(n-1)d$
$t_8 = 2+(8-1)(6)$
$\boxed{t_8 = 44}$

Multiple Choice **8.** Which of the following is an arithmetic series?

A. $1 + 4 + 9 + 16 + 25$ no common difference, needs a common difference to be "arithmetic"
B. $1, 3, 5, 7, \ldots$ Sequence
[✓ **C.**] $6 + 2 + (-2) + (-6)$
D. $8, -8, 8, -8$ Sequence

9. The sum of the first 100 terms of the arithmetic series $3 + 1 + (-1) + (-3) + \ldots$ is

A. -1020
B. -1005
C. -9705
[✓ **D.**] -9600

$a = 3$
$d = -2$
$n = 100$
$t_n = ?$

$$S_n = \frac{n[2a + (n-1)d]}{2}$$

$$S_{100} = \frac{100[2(3) + (100-1)(-2)]}{2}$$

$$S_{100} = \underline{\underline{-9600}}$$

Numerical Response **10.** If the *n*th term of an arithmetic series is $3n - 7$, then the sum of the first 18 terms is _387_.

(Record your answer in the numerical response box from left to right.)

3	8	7	

→ need to find the first term, the last term or the common difference

$a = ?$
$d = ?$
$t_n = ?$
$n = 18$

$t_n = 3n - 7$
$t_1 = 3(1) - 7 \rightarrow -4$ = first term
$t_{18} = 3(18) - 7 \rightarrow 47$ = last term

$$S_n = \frac{n(a + t_n)}{2}$$

$$S_{18} = \frac{18(-4 + 47)}{2} = \boxed{387}$$

Answer Key

1. **a)** 495 **b)** 1188 **c)** 64 **d)** –14
2. **a)** 960 **b)** –3036 **3.** **a)** 1425 **b)** 8
4. **a)** \$29, **b)** \$845
5. Sally is correct. We are given: i) the first and last terms, ii) the number of terms, 10 (eight arithmetic means plus the first and last term) and iii) the series is arithmetic. ∴ given this information, all Bubbles has to do is substitute the appropriate numbers in the general arithmetic series formula $S_n = \dfrac{n(a + t_n)}{2}$ to get the answer S_{10}. Subtract the first and last terms to get the answer of –12.
6. **a)** 2, 3, 3, 3 **b)** no, because there is no common difference between successive terms.
7. **a)** 2, 8, 14, 20 **b)** 44 **8.** C **9.** D **10.**

3	8	7	

Arithmetic Sequences Lesson #5:
Practice Test

1. An example of a finite sequence is
   ↳ Does not continue

   [✓ **A.**] $3, 8, 13, 18, \dots 5n-2, \dots 498,\ n \in N$ ↳ end of sequence
   B. $3, 8, 13, 18, \dots 5n-2, \dots\ n \in N$ ↳ continues
   C. $3 + 8 + 13 + 18 + \dots$ series
   D. $3 + 8 + 13 + 18$ series

2. Which of the following statements regarding the sequence $\underbrace{2, 4, 7, 11, 16, \dots}$ is correct?

   Not Arithmatic = No common difference
   Not Geometric = No common multiple

   A. The sequence is arithmetic.
   B. The sequence is geometric.
   C. The sequence is both arithmetic and geometric.
   [✓ **D.**] The sequence is neither arithmetic nor geometric.

Numerical Response 1. On a particular street, the house numbers on one side of the street form an arithmetic sequence. If the first two houses are numbered 8991 and 8995, and the last house is numbered 10039 then the number of houses on this side of the street is 263.

(Record your answer in the numerical response box from left to right.)

2	6	3	

$t_n = 10039$
$n = ?$
$d = 4$
$a = 8951$

$t_n = a + (n-1)d$
$10039 = 8991 + (n-1)(4)$
$10039 = 8991 + 4n - 4$
$10039 = 8987 + 4n$
$\frac{1052}{4} = \frac{\cancel{4}n}{\cancel{4}}$
$\boxed{263 = n}$

3. The common difference of the arithmetic sequence defined by $t_n = \frac{1}{3}(7 - 2n),\ n \in N,$ is

   A. $\frac{7}{3}$ **B.** $-\frac{7}{3}$
   [✓ **C.**] $-\frac{2}{3}$ **D.** $\frac{2}{3}$

   $t_n = \frac{1}{3}(7-2n)$
   $t_1 = \frac{1}{3}(7-2(1)) \rightarrow \frac{5}{3}$
   $t_2 = \frac{1}{3}(7-2(2)) \rightarrow 1$
   common difference = $t_2 - t_1$
   $1 - (\frac{5}{3}) = \boxed{\frac{-2}{3}}$

4. As part of a new training routine, Jesse does some burpees and push-ups. On the first day he does eight burpees and ten push-ups. Each day he increases the number of burpees by three and the number of push-ups by four. How many burpees and push-ups does he do on the nineteenth day?

   A. 57 and 76
   [✓ **B.**] 62 and 82
   C. 65 and 86
   D. 68 and 90

   Burpees
   $t_{19} = ?$
   $n = 19$
   $a = 8$
   $d = 3$
   $t_n = a + (n-1)d$
   $t_{19} = 8 + (19-1)(3)$
   $t_{19} = \boxed{62}$

   Push-Ups
   $t_{19} = ?$
   $n = 19$
   $a = 10$
   $d = 4$
   $t_n = a + (n-1)d$
   $t_{19} = 10 + (19-1)(4)$
   $t_{19} = \boxed{82}$

5. If the 15^{th} and 16^{th} terms of an arithmetic sequence are 99 and 92 respectively, the 5^{th} term is

A. 29

B. 36

C. 162

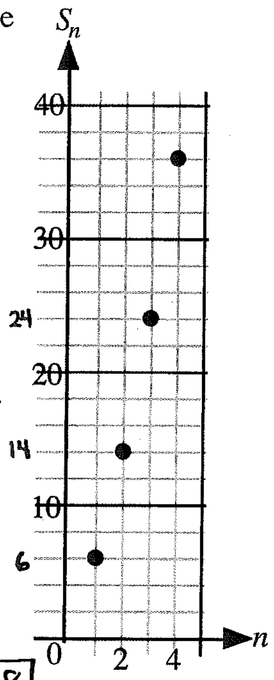
[✓ **D.**] 169

$t_{16} = 92$
$n = 16$
$d = t_{16} - t_{15} \rightarrow 92 - 99 = -7$
$a = ?$

① $t_n = a + (n-1)d$
$t_{16} = a + (16-1)(-7)$
$92 = a - 105$
$197 = a$

② $t_5 = 197 + (5-1)(-7)$
$\boxed{t_5 = 169}$

Numerical Response 2. The graph shown plots the sum of n terms of an arithmetic sequence as a function of $n \in N$. Each point on the graph is of the form (n, S_n), where S_n is the sum of n terms of the sequence.



Lesson 4
Class Ex 5

The sum of the second and third terms of the sequence is 18.

Method 1:
Subtraction
and theory

$S_1 = 6$
$S_2 = 14$
$S_3 = 24$
$S_4 = 36$

$t_1 = S_1 \rightarrow 6$
$t_2 = S_2 - t_1 \rightarrow 14 - 6 = \underline{8}$
$t_3 = S_3 - t_1 - t_2 \rightarrow 24 - 6 - 8 = \underline{\underline{10}}$
$t_2 + t_3 = 10 + 8 = \boxed{18}$

Method 2:
Formula

→ Solve for tn then plug in "n"

$$S_n = \frac{n(a + t_n)}{2}$$
$$S_n(2) = n(a + t_n)$$
$$\frac{S_n(2)}{n} = a + t_n$$
$$\frac{S_n(2)}{n} - a = t_n$$

$$\frac{S_2(2)}{2} - 6 = t_2$$
$$\frac{(14)(2)}{2} - 6 = t_2 = \underline{\underline{8}}$$
$$\frac{S_3(2)}{3} - 6 = t_3$$
$$\frac{(24)(2)}{3} - 6 = \underline{\underline{10}}$$

$t_2 + t_3 =$
$8 + 10 = \boxed{18}$

(Record your answer in the numerical response box from left to right.)

1	8		

6. $p-1$, $p+3$, $3p-1$, in that order, form an arithmetic sequence. Which of the following is/are true about p?

✓**1.** p is even **2.** p is odd ✓**3.** p is a perfect square

A. 1 only

[✓ **B.**] 1 and 3 only

C. 2 only

D. 2 and 3 only

common difference:
set equal to each other

$t_2 - t_1 = t_3 - t_2$
$(p+3) - (p-1) = (3p-1) - (p+3)$
$p + 3 - p + 1 = 3p - 1 - p - 3$
$2p + 4 = 4p - 4$
$8 = 2p$
$\boxed{4 = p}$

7. Two students are asked to write the first four terms of an arithmetic sequence.
   Rob writes the sequence $-14, -6, 2, 10 \ldots.$
   Jason writes the sequence $166, 162, 158, 154 \ldots..$

   Which statement is true about the fifteenth term of these sequences?

   A. t_{15} is the same in each sequence
   [✓ B.] t_{15} is smaller in Rob's sequence
   C. t_{15} is smaller in Jason's sequence
   D. there is not enough information to answer the question

ROB

-14
8
15
= ?

$t_n = a + (n-1)d$
$t_{15} = -14 + (15-1)(8)$
$t_{15} = \underline{\underline{98}}$

JASON

$a = 166$
$d = -4$
$n = 15$
$t_n = ?$

$t_n = a + (n-1)d$
$t_{15} = 166 + (15-1)(-4)$
$t_{15} = \underline{\underline{110}}$

Use the following information to answer questions #3 - #4.

A child arranges animal blocks in rows on a floor. There are 64 animal blocks in the fifth row and 92 animal blocks in the ninth and last row. Assume that the number of animal blocks from row to row form an arithmetic sequence.

Numerical Response 3. The number of animal blocks in the first row is 36.
(Record your answer in the numerical response box from left to right.)

3	6		

First find d, then find a (first row)

① $t_5 = 64$
$t_5 = a + (5-1)d$
$64 = a + 4d$

$t_9 = 92$
$t_9 = a + (9-1)d$
$92 = a + 8d$

② Elimination:
$$\begin{array}{r} 92 = a + 8d \\ -(64 = a + 4d) \\ \hline \frac{28}{4} = \frac{\cancel{4}d}{\cancel{4}} \\ 7 = d \end{array}$$

③ $64 = a + 4d$ ← formula from t_5
$64 - 4d = a$
$64 - 4(7) = a$
$64 - 28 = a$
$\boxed{36 = a}$

Numerical Response 4. The total number of blocks used in the arrangement is 576.
(Record your answer in the numerical response box from left to right.)

5	7	6	

$$S_n = \frac{n(a + t_n)}{2}$$
$$S_9 = \frac{9(36 + 92)}{2}$$
$$\boxed{S_9 = 576}$$

$$S_n = \frac{n[2a + (n-1)d]}{2}$$
$$S_9 = \frac{9[2(36) + (9-1)(7)]}{2}$$
$$\boxed{S_9 = 576}$$

8. If $x+2$, $3x-4$, and $7x-6$ are the first three terms of an arithmetic sequence, then the first term of the sequence has a numerical value of

↳ solve for x then replace it in the first term.

[✓ **A.**] 0
B. −2
C. 2
D. 4

① common difference

$$t_2 - t_1 = t_3 - t_2$$
$$(3x-4)-(x+2) = (7x-6)-(3x-4)$$
$$3x-4-x-2 = 7x-6-3x+4$$
$$2x-6 = 4x-2$$
$$-4 = 2x$$
$$-2 = x$$

② term 1 = $x+2$

$$t_1 = (-2)+2$$
$$\boxed{t_1 = 0}$$

9. The number of multiples of 12 between 179 and 892 is

To know the first multiple, take 179 and divide it by 12: $179 \div 12 = 14.91\overline{6}$, take this value then round to the nearest whole #, then take the number and multiply it by 12: $15 \times 12 = \underline{180}$

A. 59
[✓ **B.**] 60
C. 61
D. 62

The first multiple of 12 that is between 179 and 892 is 180 and the last term is 888

$t_1 = 180$
$t_n = 888$
$n = ?$
$d = 12$

$$t_n = a+(n-1)d$$
$$888 = 180 + (n-1)(12)$$
$$888 = 180 + 12n - 12$$
$$888 = 168 + 12n$$
$$720 = 12n$$
$$\boxed{60 = n}$$

This works with fin
the last term also
$892 \div 12 = 74.\overline{3} \rightarrow 74$
$12 \times 74 = 888$
Double check:
$12 \times 75 = 900$, is not between 179 and 89

10. The first term of an arithmetic sequence is 20 and the last term is −76. If there are seven terms in the sequence, then the middle term

A. −12
B. −20
[✓ **C.**] −28
D. −44

$\underline{20}$ ___ ___ $\underset{\uparrow\, t_4}{\underline{?}}$ ___ ___ $\underline{-76}$

$a = 20$
$d = ?$
$t_n = -76$
$n = 7$

① Find "d"

$$t_n = a+(n-1)d$$
$$-76 = 20+(7-1)d$$
$$-76 = 20+6d$$
$$-96 = 6d$$
$$-16 = d$$

② Find t_4 (the middle term)

$$t_4 = 20+(4-1)(-16)$$
$$\boxed{t_4 = -28}$$

11. As part of a fitness program, Melissa walks for 35 minutes on day 1 and increases the walking time by 8 minutes each day. She completes the fitness program on the day she first spends more than 3 hours walking. The program is completed on day ______ .

↳ look for "n"

A. 17
B. 18
C. 19
[✓ **D.**] 20

$a = 35$
$d = 8$
$n = ?$
$t_n = \underbrace{180}$

3 hours, but to keep the same units, convert it into minutes.

$$t_n = a+(n-1)d$$
$$180 = 35+(n-1)8$$
$$180 = 35+8n-8$$
$$180 = 27+8n$$
$$\frac{153}{8} = \frac{\cancel{8}n}{\cancel{8}}$$
$$\rightarrow \underbrace{19.125} = n$$

"n" represents the # of days, the answer must be greate than this value, anything lower than this would be less than 180 minutes (3 hours) and Melissa onl completes the program whe she walks for more than 3 hours.

Use the following information to answer questions #12 and #13.

Mary Ann was a statistician. She was paid according to a salary grid with annual increases from year one to year ten. The sequence of salaries from years one to ten formed an arithmetic sequence. After year ten she reached her maximum salary. She earned $65 328 in the fifth year and $81 276 in the ninth year. She worked as a statistician for twelve years.

12. Mary Ann's pay in her first year of work was

[✓ **A**] $49 380

B. $53 367

C. $57 354

D. none of the above

$t_1 = ?$
$t_5 = 65\ 328$
$t_9 = 81\ 276$
$d = ?$
$n = ?$

① $t_n = a + (n-1)d$
$t_5 = a + (5-1)d$
$65\ 328 = a + 4d$
$t_9 = a + (9-1)d$
$81\ 276 = a + 8d$

② $81\ 276 = a + 8d$
$-(65\ 328 = a + 4d)$ ⇒ Elimination (subtract)
$\frac{15\ 948}{4} = \frac{\cancel{4}d}{\cancel{4}}$
$3987 = d$

③ $65\ 328 = a + 4d$
$65\ 328 - 4d = a$
$65\ 328 - 4(3987) = \boxed{\$49\ 380}$

13. The total amount that Mary Ann earned as a statistician was

A. $85 263

B. $673 215

[✓ **C**] $843 741

D. $1 346 430

↳ The question is asking for the total amount of money made in the 12 years, but her salary stays constant after Mary Ann reaches her tenth year of work, so we are not looking for S_{12}, we are looking for $S_{10} + t_{10} + t_{10}$ or $S_{10} + 2(t_{10})$.

Amount: $\underbrace{\overset{\text{year 1}}{t_1}, \overset{\text{year 2}}{t_2}, \overset{\text{year 3}}{t_3}, t_4, t_5, t_6, t_7, t_8, t_9, \overset{\text{year 10}}{t_{10}}}_{S_{10}} + \underbrace{\overset{\text{year 11}}{t_{10}}}_{t_{10}} + \underbrace{\overset{\text{year 12}}{t_{10}}}_{t_{10}}$

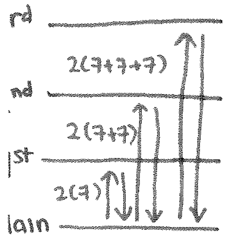
$d = 3987$
$a = 49\ 380$
$n = 10$
$t_n = ?$

① $S_n = \frac{n[2a + (n-1)d]}{2}$
$S_{10} = \frac{10[2(49\ 380 + (10-1)(3987))]}{2}$
$S_{10} = \$673\ 215$

② $t_n = a + (n-1)d$
$t_{10} = 49\ 380 + (10-1)(3987)$
$t_{10} = \$85\ 263$

$\Rightarrow S_{10} + t_{10} + t_{10} = 673\ 215 + 85\ 263 + 85\ 263$
$= \boxed{\$843\ 741}$

14. Joe, from Perfection Millworks, has fourteen counter tops to deliver to fourteen floors in an office building. Because of the size of the counter tops, he can only get one counter top into the elevator at a time. He starts at the main floor and takes the first counter top to the first floor, returns to the main floor, picks up the second counter top and takes it to the second floor, and so on. He continues in this way until all fourteen countertops have been delivered. If the distance between floors is exactly 7 m, how far has the elevator travelled when Joe has delivered all the counter tops and returned to the main floor?



↳ The total distance including up and down, what is Sn?

A. 637 m

B. 735 m

C. 1274 m

[✓ **D.** 1470 m]

$a = 14$

$d = 14$

$n = 14$

$t_n = ?$

not 7 b/c you are moving up and down so its 7x2 = 14

$$S_n = \frac{n[2a+(n-1)d]}{2}$$

$$S_{14} = \frac{14[2(14)+(14-1)(14)]}{2}$$

$$S_{14} = \boxed{1470\text{m}}$$

The sequence looks like this:

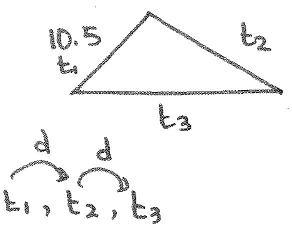
al tance: $\overset{\text{Floor 1}}{14}, \overset{\text{Floor 2}}{28}, \overset{\text{Floor 3}}{42}, 56 \ldots 95$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_{14}$

Numerical Response **5.** The side lengths of a triangle form an arithmetic sequence. The shortest side has a length of 10.5 cm and the perimeter of the triangle is 45 cm. The length, in cm, of the longest side is 19.5 .

(Record your answer in the numerical response box from left to right.)

1	9	.	5



longest side = t_3

$t_1 = a = 10.5$

$S_n = 45$ (perimeter, $t_1+t_2+t_3$)

$n = 3$

$t_n = ?$

$$S_n = \frac{n(a+t_n)}{2}$$

$$S_n(2) = n(a+t_n)$$

$$\frac{S_n(2)}{n} = a + t_n$$

$$\frac{S_n(2)}{n} - a = t_n$$

$$\frac{(45)(2)}{3} - 10.5 = t_3$$

$$\boxed{19.5 = t_3}$$

19.5 cm

15. All the terms in a particular arithmetic sequence are whole numbers. If the first three terms can be represented by $2x + 10$, $4x + 30$, and $8x + 60$, then the sum of the first four terms of the corresponding series is

A. 340

B. 170

[✓ **C.** 60]

D. 30

$t_1 = 2x + 10$

$t_2 = 4x + 30$

$t_3 = 8x + 60$

① Solve for x

$t_2 - t_1 = t_3 - t_2$

$(4x+30)-(2x+10) = (8x+60)-(4x+30)$

$4x+30-2x-10 = 8x+60-4x-30$

$2x+20 = 4x+30$

$-10 = 2x$

$\underline{-5 = x}$

② solve for the terms

$t_1 = 2(-5)+10 = 0$ +10

$t_2 = 4(-5)+30 = 10$ +10

$t_3 = 8(-5)+60 = 20$

$t_4 = 20+10 = 30$

The Common difference is 10

③ Add each term together or use Sn formu

$0 + 10 + 20 + 30 = \boxed{60}$

Written Response - 5 marks

1. The manager of a condo development receives a base salary of \$15 000 per year plus \$800 for every condo unit sold.

↳ $d = 800$

- Write the first four terms of the arithmetic sequence for the manager's earnings if 1, 2, 3, ... units are sold.

The base salary is 15 000 plus 800 for every condo SOLD, if one condo is sold, $t_1 = 15\,000 + 800 = 15\,800$.

$\boxed{15\,800,\ 16\,600,\ 17\,400,\ 18\,200}$

- Determine the formula for the general term of the arithmetic sequence in the form $t_n = a + (n-1)d$.

$a = 15\,800$

$d = 800$

$t_n = t_n$

$n = n$

$t_n = a + (n-1)d$

$\boxed{t_n = 15\,800 + (n-1)(800)}$

- Use the general term formula to calculate his earnings if

i) 23 units are sold

$n = 23$

$t_n = ?$

$t_{23} = 15\,800 + (23-1)(800)$

$t_{23} = \boxed{\$33\,400}$

ii) 54 units are sold

$n = 54$

$t_n = ?$

$t_{54} = 15\,800 + (54-1)(800)$

$t_{54} = \boxed{\$58\,200}$

- Write the equation for the manager's earnings, E, as a linear function of the number, n, of units sold. Write the equation in the form $E = mn + b$.

↳ This is the simplified version of the formula $t_n = a + (n-1)d$

$t_n = 15\,800 + (n-1)(800)$

$t_n = 15\,800 + 800n - 800$

$t_n = 15\,000 + 800n$

$\boxed{E = 800n + 15\,000}$

$E = 800n + 15000$

$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \swarrow$

$E = mn + b$ ←

- Use the linear function from the previous bullet to calculate his earnings if

i) 23 units are sold

$n = 23$

$E = ?$

$E = 800(23) + 15\,000$

$\boxed{E = \$33\,400}$

ii) 54 units are sold

$n = 54$

$E = ?$

$E = 800(54) + 15000$

$\boxed{E = \$58\,200}$

Answer Key

1. A 2. D 3. C 4. B 5. D 6. B 7. B 8. A
9. B 10. C 11. D 12. A 13. C 14. D 15. C

Numerical Response

1.

2	6	3	

2.

1	8		

3.

3	6		

4.

5	7	6	

5.

1	9	.	5

Written Response

1.
- 15,800, 16,600, 17,400, 18,200,
- $t_n = 15800 + (n - 1)(800)$
- i) \$33,400 ii) \$58,200
- $E = 800n + 15000$
- i) \$33,400 ii) \$58,200