1.1 The Real Number System

This section will focus on different number systems that are part of a large collection of numbers. The systems will be listed from simple to complex. One which encompasses all the other number systems is the Real Number System.

Natural Numbers: $\{1, 2, 3...\}$

Whole Numbers: $\{0, 1, 2, 3...\}$

Integers: $\{... -3, -2, -1, 0, 1, 2, 3...\}$

Rational Numbers: All the numbers that **can** be written as a fraction with the denominator not equal to zero.

Examples: $-3,\ 0,\ 5,\ -\frac{2}{3},\ \frac{10}{7},\ 2.35,\ -2.\overline{35},\ \sqrt{4},\ -\sqrt{9},\ \sqrt{\frac{9}{16}}$

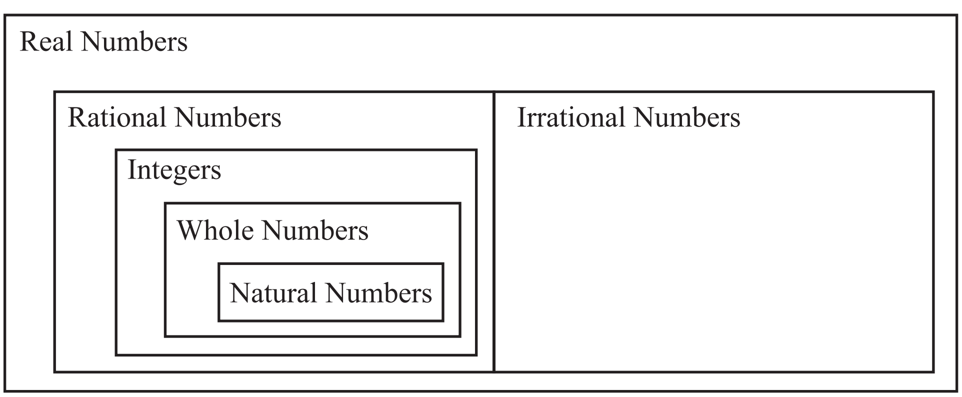
Note: Every terminating and repeating decimal can be written as a fraction.

Irrational Numbers: All the numbers that **cannot** be written as a fraction, a terminating decimal, or a repeating decimal.

Examples: $\sqrt{2},\ \pi,\ 1.62789...$

Note: There are an infinite number of irrationals of the form $\sqrt{n}$*,*
such as: $\sqrt{2},\ \sqrt{3},\ \sqrt{5}$*. However numbers such as:* $\sqrt{-1},\ \sqrt{-2},\ \sqrt{-3}$
are ***not*** *irrational numbers.*

Real Numbers: The rational numbers and irrational numbers combined.



1.1 Exercise Set

1. Determine whether each statement is true or false.

a) Every whole number is an integer. T / F

b) Every integer is a natural number. T / F

c) Every integer is a whole number. T / F

d) Irrational numbers are non-repeating, non-terminating decimals. T / F

e) Irrational numbers are real numbers. T / F

f) Every rational number can be written as a fraction. T / F

g) Every integer can be written as a fraction, therefore every integer is a rational number. T / F

h) Every rational number is a whole number. T / F

i) Every natural number is a whole number. T / F

j) There are more rational numbers than irrational numbers. T / F

k) There are an infinite number of real numbers between any two different real numbers. T / F

l) A rational number plus a irrational number is always an irrational number. T / F

m) A irrational number times an irrational number is always an irrational number. T / F

2. State if the following are Natural, Whole, Integer, Rational, Irrational or Real numbers.
(Some may belong to as many as five of these sets)

a) 5 ______________________

b) -5 ______________________

c) $-\frac{2}{3}$ ______________________

d) $\sqrt{10}$ ______________________

e) $\sqrt{-2}$ ______________________

f) π ______________________

g) $0.\overline{1738}$ ______________________

3. List the numbers in the following set that belong to each set of numbers.

$\left\{-4,\ -\frac{3}{4},\ 0,\ \sqrt{3},\ \frac{1}{4},\ 5,\ 7.3,\ 1.2583...\right\}$

a) Natural numbers ______________________

b) Whole numbers ______________________

c) Integers ______________________

d) Rational numbers ______________________

e) Irrational numbers ______________________

f) Real numbers ______________________

1.2 Square Roots

When a number is multiplied by itself, the number is squared. For example $3 \times 3 = 3^2$. This section focuses on which number needs to be squared in order to produce a given number. The process of undoing, or reversing the squaring process to find the original number used in squaring is called finding the square root.

The symbol $\sqrt{}$ is called a **radical** sign, and is used to indicate the **square root** of a number.

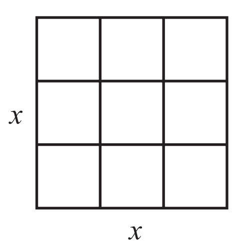
Perfect Square: A number with a rational square root.

Being familiar with the first 25 perfect square numbers is very helpful in determining square roots. Here is a list of the first 25 perfect square numbers:

$1 = 1^2$	$36 = 6^2$	$121 = 11^2$	$256 = 16^2$	$441 = 21^2$
$4 = 2^2$	$49 = 7^2$	$144 = 12^2$	$289 = 17^2$	$484 = 22^2$
$9 = 3^2$	$64 = 8^2$	$169 = 13^2$	$324 = 18^2$	$529 = 23^2$
$16 = 4^2$	$81 = 9^2$	$196 = 14^2$	$361 = 19^2$	$576 = 24^2$
$25 = 5^2$	$100 = 10^2$	$225 = 15^2$	$400 = 20^2$	$625 = 25^2$

Perfect square numbers are used to find square roots. For instance, if $64 = 8^2$ then $\sqrt{64} = \sqrt{8 \times 8} = 8$.

Example 1 A square has an area of nine square units. What must be the length of each equal side?



► *Solution*: Think of a number that when multiplied by itself gives 9. This will be the square root number.

$x^2 = 9 \rightarrow x = \sqrt{9} = \sqrt{3 \times 3} = 3.$

Therefore each side of the square is 3 units long.

Example 2 Determine the exact square root, if possible.

a) $\sqrt{25}$

b) $-\sqrt{16}$

c) $\sqrt{-4}$

d) $\sqrt{0}$

e) $\sqrt{\frac{4}{9}}$

f) $\sqrt{\frac{2}{18}}$

g) $\sqrt{5}$

h) $\sqrt{9+16}$

i) $\sqrt{9}+\sqrt{16}$

j) $\sqrt{4^2}$

k) $\sqrt{0.81}$

►*Solution*: **a)** $\sqrt{25} = \sqrt{5\times5} = 5$ because $5^2 = 25$

b) $-\sqrt{16} = -\sqrt{4\times4} = -4$ because $-(4)^2 = -16$

c) $\sqrt{-4}$ has no solution.

d) $\sqrt{0} = 0$ because $0^2 = 0$

e) $\sqrt{\frac{4}{9}} = \sqrt{\frac{2}{3}\times\frac{2}{3}} = \frac{2}{3}$ because $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

f) $\sqrt{\frac{2}{18}} = \sqrt{\frac{1}{9}} = \sqrt{\frac{1}{3}\times\frac{1}{3}} = \frac{1}{3}$ because $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

g) This square root cannot be reduced because 5 is not a perfect square.

h) $\sqrt{9+16} = \sqrt{25} = \sqrt{5\times5} = 5$ because $5^2 = 25$

i) $\sqrt{9}+\sqrt{16} = \sqrt{3\times3}+\sqrt{4\times4} = 3+4 = 7$ because $3^2 = 9$ and $4^2 = 16$

j) $\sqrt{4^2} = \sqrt{4\times4} = 4$

k) $\sqrt{0.81} = \sqrt{\frac{81}{100}} = \frac{9}{10} = 0.9$ because $0.9^2 = 0.81$

1.2 Exercise Set

1. For each number, find its square root. If there is no square root, write ∅ (empty set).

a) 49 ________ **b)** 81 ________

c) 225 ________ **d)** 2500 ________

e) -9 ________ **f)** $\frac{4}{25}$ ________

g) $\frac{121}{9}$ ________ **h)** $\frac{225}{49}$ ________

i) 0.09 ________ **j)** 0.0625 ________

2. Simplify, if possible.

a) $\sqrt{25}$ ________ **b)** $\sqrt{144}$ ________

c) $-\sqrt{\frac{16}{9}}$ ________ **d)** $\sqrt{0.36}$ ________

e) $-\sqrt{\frac{36}{81}}$ ________ **f)** $\sqrt{-16}$ ________

g) $\sqrt{1}$ ________ **h)** $\sqrt{0.0064}$ ________

i) $\sqrt{0}$ ________ **j)** $\sqrt{6.25}$ ________

3. Simplify.

a) $\sqrt{9}+\sqrt{16}$ ________ **b)** $\sqrt{9+16}$ ________

c) $\sqrt{25}-\sqrt{36}$ ________ **d)** $\sqrt{1}+\sqrt{9}$ ________

e) $-\sqrt{16}-\sqrt{4}$ ________ **f)** $-\sqrt{25-9}$ ________

g) $-\sqrt{8^2}+\sqrt{15^2}$ ________ **h)** $-\sqrt{8^2+15^2}$ ________

i) $\sqrt{5^2+12^2}$ ________ **j)** $\sqrt{5^2}+\sqrt{12^2}$ ________

4. Determine the length of each side of the square.

a) 64 cm² **b)** 625 m²

c) 0.0049 mm² **d)** 0.01 ft²

5. Determine the rational square root of each number, if it exists. (Example: $\sqrt{9} = 3$)

a) 10 ________ **b)** 100 ________

c) 1000 ________ **d)** 36 ________

e) 360 ________ **f)** 3600 ________

g) 0.04 ________ **h)** 0.004 ________

i) 0.0004 ________ **j)** 0.81 ________

k) 0.081 ________ **l)** 0.0081 ________

6. A square piece of land has a total area of 2.89 miles². Determine:

a) The length of each side of the piece of land. **b)** The perimeter of the piece of land.

7. The amount a child consumes in calories each day is based on the formula $C = 600\sqrt{A}$, where C = number of calories and A = age.

a) Determine the calories needed for a

i) one year old

ii) nine year old

b) Determine the age of a person who consumes

i) 1200 calories per day

ii) 2400 calories per day

8. A rectangle has a length twice as long as it is wide, and an area of 242 m^2. Determine the length and width of the rectangle. (Area = length $\times$ width)

9. A rectangle has a length 25% longer than the width, and an area of 80 cm^2. Determine the length and width of the rectangle.

10. A right triangle made by cutting a diagonal on a square has an area of 242 cm^2. What is the perimeter of the original square?

11. A triangle is made by cutting a diagonal on a rectangle $33\frac{1}{3}$% longer than it is wide. If the area of the triangle is 54 cm^2, determine the length and width of the original triangle.

12. The area of a circle is determined by the equation $A = \pi r^2$. What is the radius of a circle that has an area of 49π cm^2?

13. A semi-circle has an area of 32π cm^2. What is the diameter of the semi-circle?

14. The volume of a rectangular solid with a square base is 245 cm^3. If the height of the rectangular solid is 5 cm, what is the length of the square base?

15. A rectangular solid has a base twice as long as it is wide, and a height of 8 cm. If the volume of the rectangular solid is 1936 cm^3, what is its length and width?

1.3 Square Roots of Non-Perfect Squares

As we saw from the previous section, the square roots of **perfect squares** are rational numbers. For example $\sqrt{16} = \sqrt{4 \times 4} = 4$ because $4^2 = 16$. In this section we will look at **non-perfect squares**, which are irrational numbers. For example $\sqrt{5}$ is irrational, because no two equal rational numbers multiply out to 5. We can, however, use a calculator to find a decimal approximation of these values.

Approximating Square Roots with a Calculator

Example 1 Find the decimal approximation of $\sqrt{12}$.

► *Solution*: No two rational numbers multiply out to $\sqrt{12}$. So, by calculator, $\sqrt{12} = 3.464...$ which is an irrational number. To two decimal places, $\sqrt{12} \simeq 3.46$.

Approximating Square Roots without a Calculator

Not every square root is a whole number. In fact most square roots are non-perfect squares. However we can approximate square roots that are not whole numbers.

Example 2 Find the decimal approximation of $\sqrt{11}$ without a calculator.

► *Solution*: $\sqrt{11}$ is not a whole number. Its value must lie between $\sqrt{9}$ and $\sqrt{16}$, that is $\sqrt{11}$ is between 3 and 4. Since $3^2 = 9$ and $4^2 = 16$, 11 is 2 units from 9 and 5 units from 16. Therefore $\sqrt{11}$ must be closer to 3 than to 4, say 3.3. Check $3.3^2 = 10.89$

Example 3 Find the decimal approximation of $\sqrt{110}$ without a calculator.

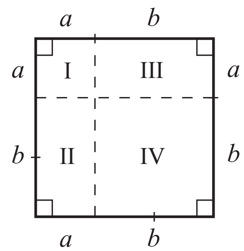
► *Solution*: $\sqrt{110}$ is not a whole number. Its value must lie between $\sqrt{100}$ and $\sqrt{121}$, that is $\sqrt{110}$ is between 10 and 11. Since $10^2 = 100$ and $11^2 = 121$, 110 is 10 units from 100 and 11 units from 121. Therefore $\sqrt{110}$ must be very close to 10.5. Check $10.5^2 = 110.25$

The Pythagorean Theorem: $a^2 + b^2 = c^2$

The Pythagorean Theorem describes the relationship between the sides of any right triangle. Side c is the **hypotenuse** (longest side) and sides a and b are the **legs** of the right triangle. There are over 300 ways of proving the Pythagorean Theorem. Below are two proofs.

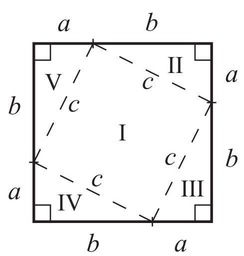
Proof 1: Consider a square with sides $a + b$.

Drawing two inner squares and two inner rectangles:



$$\begin{aligned}\text{Area of big square} &= \text{Area I} + \text{Area II} + \text{Area III} + \text{Area IV}\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

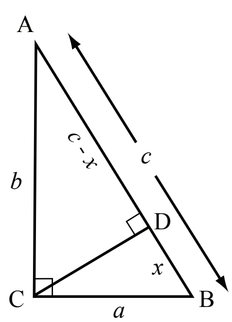
In the same square, draw four right triangles and an inner square:



$$\begin{aligned}\text{Area of big square} &= \text{Area I} + \text{Area II} + \text{Area III} + \text{Area IV} + \text{Area V}\\ &= c^2 + \tfrac{1}{2}ab + \tfrac{1}{2}ab + \tfrac{1}{2}ab + \tfrac{1}{2}ab\\ &= c^2 + 4(\tfrac{1}{2}ab)\\ &= c^2 + 2ab\end{aligned}$$

Since the area of both squares are equal: $a^2 + 2ab + b^2 = c^2 + 2ab \rightarrow a^2 + b^2 = c^2$

Proof 2: Objects of the same shape have sides that are proportional.



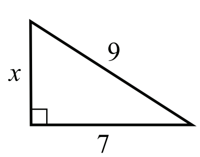
ΔABC, ΔACD and ΔCBD are similar to each other.

So $\dfrac{c}{a} = \dfrac{a}{x} \rightarrow a^2 = cx$

$\dfrac{c}{b} = \dfrac{b}{c-x} \rightarrow b^2 = c(c-x) = c^2 - cx$

Therefore $a^2 + b^2 = cx + c^2 - cx = c^2$

Example 4 Find x.



► *Solution*:

$x^2 + 7^2 = 9^2$

$x^2 = 9^2 - 7^2$

$x^2 = 32$

$x = \sqrt{32} \approx 5.6$

Non-Calculator Methods of Approximating Square Roots

Heron's Method

This method is named after the first-century Greek mathematician Heron of Alexandria who gave the first explicit description of iteratively computing the square root.

1. Find the closest perfect square a to non-perfect square x.
2. Use formula $\frac{1}{2}\left(a + \frac{x}{a}\right)$.
3. Repeat this formula to improve calculation.

Example 5 Find $\sqrt{8}$.

►*Solution*: The closest perfect square to $\sqrt{8}$ is $\sqrt{9} = 3^2$ therefore $a = 3$

$$\frac{1}{2}\left(a + \frac{x}{a}\right) = \frac{1}{2}\left(3 + \frac{8}{3}\right) = 2.83$$

Improve: $\frac{1}{2}\left(2.83 + \frac{8}{2.83}\right) = 2.828...$

Finding Square Roots Using an Algorithm

This method of computing square roots was taught in schools before the invention of calculators. While learning this algorithm may not be necessary in today's world with calculators, working out some examples could be a challenging exercise still.

1. Starting at the decimal point, separate the number into pairs of digits, to the left and right of the decimal point. No pair should straddle a decimal point.

2. Find the largest number whose square is equal to or less than the leading digit pair. Put that number to the left of the square root, and above the first digit pair.

3. Square that number, and subtract this square from the leading digit pair.

4. Bring down the next digit pair, and put it to the right of the difference you just calculated.

5. Multiply the number on the top by two, and put it to the left of the difference you calculated in step 3. Leave an empty place next to it, eg. 4 __.

6. Find the largest single-digit number to put in this blank decimal place such that this combined number times the single-digit number itself will be less than the current difference, eg. $4\underline{N} \times \underline{N}$. Put this number in the blank space, and in the next decimal place on the top row.

7. Now subtract the product you just found.

8. Repeat for the next digit, starting at step 4.

Example 6 Find $\sqrt{536}$ using an algorithm.

► *Solution*:

$$\begin{array}{rl} & \overline{)\,536.00} \end{array}$$

Step 1: Separate the number into pairs of digits, counting from the decimal point.

$$\begin{array}{rl} & \ \ 2 \\ 2 & \overline{)\,536.00} \\ & \ \ \underline{4} \\ & \ \ 1 \end{array}$$

Steps 2,3: 2 is the largest number whose square is less than 5. Subtract 2 × 2 and the difference is 1.

$$\begin{array}{rl} & \ \ 2\ \ . \\ 2 & \overline{)\,536.00} \\ & \ \ \underline{4} \downarrow \\ 4\underline{\ \ } & \ \ 136 \end{array}$$

Steps 4,5: Bring down the next pair of digits, 36. Double the number from the top row is 4. Place the number to the left of the 136 and leave a space.

$$\begin{array}{rl} & \ \ 2\ 3. \\ 2 & \overline{)\,536.00} \\ & \ \ \underline{4} \downarrow \\ 4\underline{3} & \ \ 136 \\ & \ \ \underline{129} \\ & \ \ \ \ \ 7 \end{array}$$

Steps 6,7: 3 is the largest number to fill in the space since $43 \times 3 = 129 < 136$ and $44 \times 4 = 176$. Subtract 129; the difference is 7.

$$\begin{array}{rl} & \ \ 2\ 3. \\ 2 & \overline{)\,536.00} \\ & \ \ \underline{4} \downarrow\ \downarrow \\ 4\underline{3} & \ \ 136 \\ & \ \ \underline{129} \\ 46\underline{\ \ } & \ \ \ \ \ 700 \end{array}$$

Repeat Steps 4,5: Bring down next pair of digits, 00. Double the number on the top row is 46. Place the number to the left of the difference and leave a space.

$$\begin{array}{rl} & \ \ 2\ 3.\ 1 \\ 2 & \overline{)\,536.00} \\ & \ \ \underline{4} \downarrow\ \downarrow \\ 4\underline{3} & \ \ 136 \\ & \ \ \underline{129} \\ 46\underline{1} & \ \ \ \ \ 700 \\ & \ \ \ \ \ \underline{461} \\ & \ \ \ \ \ 239 \end{array}$$

Repeat Steps 6,7: 1 is the largest number to fill in the space since $461 \times 1 = 461 < 700$ and $462 \times 2 = 924 > 700$. Subtract, and the difference is 239. Repeat.

1.3 Exercise Set

1. State if the square root of the number is rational or irrational.

a) 121 ____________ **b)** 60 ____________

c) 729 ____________ **d)** 750 ____________

e) $\frac{81}{5}$ ____________ **f)** $\frac{8}{18}$ ____________

g) 1.6 ____________ **h)** $\frac{24}{54}$ ____________

i) 0.9 ____________ **j)** 0.09 ____________

2. Between what two integers does the irrational number fall?

a) $\sqrt{75}$ ____________ **b)** $\sqrt{110}$ ____________

c) $\sqrt{90}$ ____________ **d)** $-\sqrt{300}$ ____________

e) $-\sqrt{160}$ ____________ **f)** $-\sqrt{125}$ ____________

g) $\sqrt{204}$ ____________ **h)** $-\sqrt{470}$ ____________

i) $\sqrt{300}$ ____________ **j)** $-\sqrt{500}$ ____________

3. Determine the closest integer to the irrational number.

a) $\sqrt{5}$ ____________ **b)** $\sqrt{19}$ ____________

c) $\sqrt{180}$ ____________ **d)** $\sqrt{450}$ ____________

e) $\sqrt{590}$ ____________ **f)** $-\sqrt{5}$ ____________

g) $-\sqrt{19}$ ____________ **h)** $-\sqrt{180}$ ____________

i) $-\sqrt{450}$ ____________ **j)** $-\sqrt{590}$ ____________

4. Without using a calculator, determine the square roots of the irrational numbers to one decimal place.

a) $\sqrt{75}$ ________ **b)** $\sqrt{110}$ ________

c) $\sqrt{90}$ ________ **d)** $-\sqrt{300}$ ________

e) $-\sqrt{160}$ ________ **f)** $-\sqrt{125}$ ________

g) $\sqrt{204}$ ________ **h)** $-\sqrt{470}$ ________

i) $\sqrt{300}$ ________ **j)** $-\sqrt{500}$ ________

5. Without using a calculator determine the square roots of the irrational numbers to two decimal places. (Hint: Start with your answers to question 4, then apply Heron's Method)

a) $\sqrt{75}$ ________ **b)** $\sqrt{110}$ ________

c) $\sqrt{90}$ ________ **d)** $-\sqrt{300}$ ________

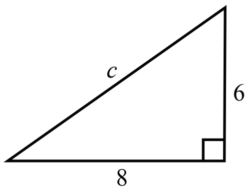
e) $-\sqrt{160}$ ________ **f)** $-\sqrt{125}$ ________

g) $\sqrt{204}$ ________ **h)** $-\sqrt{470}$ ________

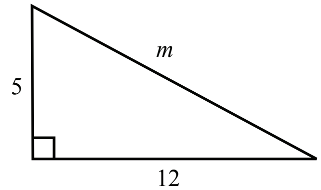
i) $\sqrt{300}$ ________ **j)** $-\sqrt{500}$ ________

6. Using the Pythagorean Theorem, solve to one decimal place.

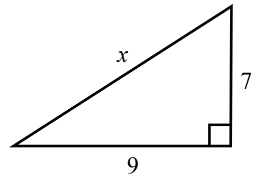
a)



b)

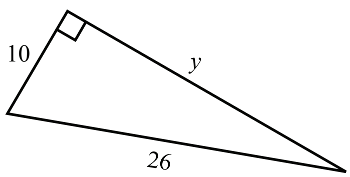


c)

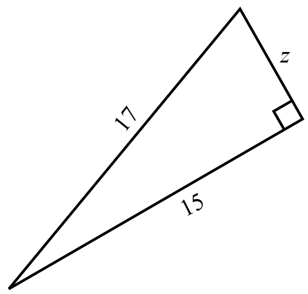


$c =$ ________ $m =$ ________ $x =$ ________

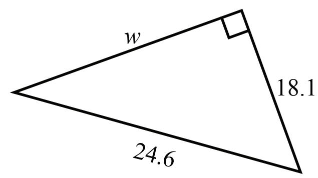
d)



e)

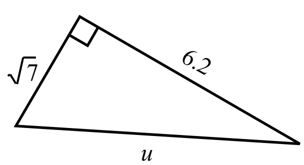


f)

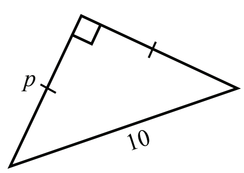


$y =$ ________ $z =$ ________ $w =$ ________

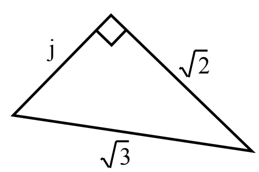
g)



h)



i)



$u =$ ________ $p =$ ________ $j =$ _______

7. Pythagorean Triples are sets of numbers that satisfy the property $a^2 + b^2 = c^2$. For example, the set of numbers 5, 12, 13 is a Pythagorean Triple since $5^2 + 12^2 = 13^2$ $(25 + 144 = 169)$. Which of the following sets of numbers are Pythagorean Triples? Explain your choices.

Example: 1, 2, 3 ___No___ $1^2 + 2^2 \neq 3^2$, $(1 + 4 \neq 9)$

a) 7, 24, 25 __________ ______________________________

b) 8, 15, 17 __________ ______________________________

c) 9, 14, 16 __________ ______________________________

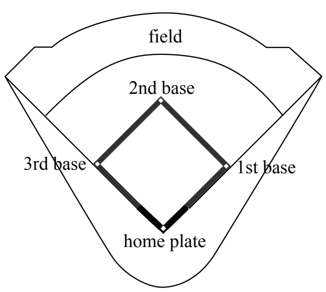
d) 9, 40, 41 __________ ______________________________

e) 15, 36, 39 __________ ______________________________

f) $\sqrt{2}, \sqrt{3}, \sqrt{5}$ __________ ______________________________

g) $\sqrt{10}$, 4, 5 __________ ______________________________

Use the diagram for questions 8 - 11.



Baseball Diamond

8. A baseball diamond is 90 feet on each side. How far is it from second base to home plate?

9. The short stop plays exactly halfway between second base and third base. How far is the throw to first base from short stop?

10. The second base player backs up 30 ft from second base directly in line with second and third base to field a ball. How far is a throw to home plate?

11. A fielder caught a fly ball on the first base line 126 feet from first base, how far would he have to throw to get the ball to third base?

1.4 Defining a Power

Many mathematical situations require multiplying a number by itself repeatedly. Writing these expressions in **exponential form** provides an efficient method for representing the repeated multiplication of the same factor.

The expression $10 \times 10 \times 10 \times 10 = 10^4$ is read "ten to the fourth power". We call the number 4 an **exponent** and we say that the number 10 is the **base**. An expression for a power is called **exponential notation**.

Exponential Notation

An **exponent** (or power) is a number that indicates how many times another number (called the base) is used as a factor.

For any natural number n greater than or equal to two, $b^n = \underbrace{b \times b \times b \times b \times \ldots\ldots \times b}_{n \text{ factors}}$

The expression $2^5 = \underbrace{2 \times 2 \times 2 \times 2 \times 2}_{5 \text{ factors}}$ is read "two to the fifth power"; it tells us to use two as a factor five times.

The Product of Negative Numbers

The product of an even number of negative numbers is positive. The product of an odd number of negative numbers is odd.

$(-1)^{30}$ is positive since 30 is even. $(-1)^{19}$ is negative since 19 is odd.

Another important concept in determining if a product of negative numbers is even or odd is where the negative sign lies in relation to the brackets.

i) $(-2)^4 = (-2)\times(-2)\times(-2)\times(-2) = 16$ — *The negative sign is inside the brackets therefore -2 is the base and an even number of negative numbers is positive.*

ii) $-(2)^4 = -(2)\times(2)\times(2)\times(2) = -16$ — *The negative is outside the bracket therefore only the two is the base and there is only one negative sign. Therefore the answer is negative.*

iii) $-2^4 = -2 \times 2 \times 2 \times 2 = -16$ — *Without the brackets, the base is only two. The negative sign does not belong to the exponent, and the answer is negative again.*

One and Zero as Exponents

Look for a pattern in the following:

$$\begin{aligned} 2\times2\times2\times2\times2 &= 2^5 \\ 2\times2\times2\times2 &= 2^4 \\ 2\times2\times2 &= 2^3 \\ 2\times2 &= 2^2 \end{aligned}$$

On the left side of the equation each step is being divided by 2. On the right side of the equation the exponent decreases by 1 on each step.

To continue the pattern we say:

$$\begin{aligned} 2 &= 2^1 \\ 1 &= 2^0 \end{aligned}$$

Exponents of 0 and 1

$a^1 = a$, for any number a.
$a^0 = 1$, for any non-zero number a.

note: 0^0 is not defined.

Examples: $5^0 = 1$, $17^1 = 17$, $\left(\frac{2}{3}\right)^0 = 1$, $-5^0 = -1$, $(-5)^0 = 1$

Example 1 Simplify.

a) $(a+b)^0$ **b)** $a+b^0$ **c)** a^0+b^0 **d)** $(a\cdot b)^0$ **e)** $a\cdot b^0$ **f)** $(-a)^0$ **g)** $-a^0$

► *Solution*: **a)** 1

b) $a+1$

c) $1+1=2$

d) 1

e) $a\cdot 1 = a$

f) 1

g) -1

1.4 Exercise Set

1. Complete the table.

	Base 2	Base 3	Base 4	Base 5
Second Power	$2^2 = 2 \times 2 = 4$			
Third Power	$2^3 = 2 \times 2 \times 2 = 8$			
Fourth Power				

2. For each equation, find the integer that can be used as the exponent to make the equation correct.

a) $8 = 2^?$ ________ **b)** $81 = 3^?$ ________

c) $625 = 5^?$ ________ **d)** $64 = 8^?$ ________

e) $64 = 4^?$ ________ **f)** $64 = 2^?$ ________

g) $216 = 6^?$ ________ **h)** $1024 = 2^?$ ________

i) $2401 = 7^?$ ________ **j)** $2187 = 3^?$ ________

3. Circle positive or negative for the values, $a > 0$.

a) $-a^{30}$ + / − **b)** $-(a)^{30}$ + / −

c) $(-a)^{30}$ + / − **d)** $-(-a)^{30}$ + / −

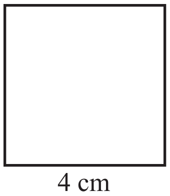
e) $-a^{25}$ + / − **f)** $-(a)^{25}$ + / −

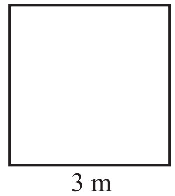
g) $(-a)^{25}$ + / − **h)** $-(-a)^{25}$ + / −

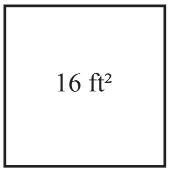
i) $(-a)^{\text{even}}$ + / − **j)** $(-a)^{\text{odd}}$ + / −

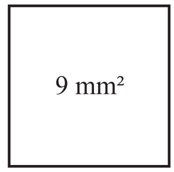
4. Assume that $a > 1$. In order from least to greatest arrange the following: $-(-a)^3,\ -a^3,\ (-a)^4,\ -a^4$.

5. Write the area of each square as a power.

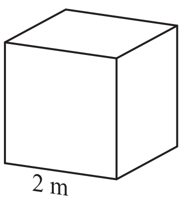
a) 

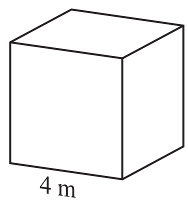
b) 

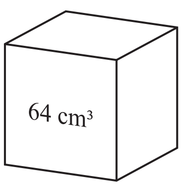
c) 

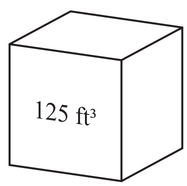
d) 

6. Write the volume of each cube as a power.

a) 

b) 

c) 

d) 

7. Write in exponential notation. All values are positive.

a) $4 \times 4 \times 4 \times 4 \times 4$ ________

b) $5 \times 5 \times 5$ ________

c) $7 \times 7 \times 7 \times 7 \times 7$ ________

d) $10 \times 10 \times 10 \times 10$ ________

e) $a \times a \times a \times a$ ________

f) $(-a) \times (-a) \times (-a) \times (-a)$ ________

g) $\underbrace{b \times b \times b \times \ldots. \times b}_{\text{20 times}}$ ________

h) $\underbrace{b \times b \times b \times \ldots. \times b}_{n \text{ times}}$ ________

8. Evaluate.

a) 5^3 ________ **b)** 3^5 ________

c) 6^2 ________ **d)** 2^6 ________

e) $(-3)^4$ ________ **f)** -3^4 ________

g) -2^5 ________ **h)** $-(-2)^5$ ________

i) -2^6 ________ **j)** $-(-2)^6$ ________

9. Evaluate. $a \neq 0, \quad b \neq 0, \quad a \neq b, \quad a + b \neq 0$

a) 6^0 ________ **b)** $(-6)^0$ ________

c) $-(6)^0$ ________ **d)** -6^0 ________

e) $2^0 + 3^0$ ________ **f)** $2^0 - 3^0$ ________

g) $-2^0 - 3^0$ ________ **h)** $(2^0 + 3^0)^0$ ________

i) $-(-2^0 - 3^0)$ ________ **j)** $3^0 \times 4^0$ ________

k) $(a + b)^0$ ________ **l)** $a^0 - b^0$ ________

m) $-a^0 - b^0$ ________ **n)** $-(a + b)^0$ ________

10. Write as a repeated factor.

a) 2^4 ________________ **b)** $(-2)^4$ ________________

c) -2^4 ________________ **d)** a^5 ________________

e) $(-a)^5$ ________________ **f)** a^n ________________

11. Use $<$, $>$ or $=$ to write a true sentence.

a) 2^3 ____ 3^2

b) 2^4 ____ 4^2

c) (-2^4) ____ -2^4

d) $-(-2)^3$ ____ 2^3

e) $(-2)^6$ ____ 6^2

f) $(-5)^3$ ____ $(-3)^5$

g) $(-5)^4$ ____ $(-4)^5$

h) $(-2)^5$ ____ 5^2

i) $\left(\frac{2}{3}\right)^2$ ____ $\left(\frac{2}{3}\right)^3$

j) $\left(\frac{5}{2}\right)^3$ ____ $\left(\frac{5}{2}\right)^4$

k) $\left(\frac{3}{5}\right)^5$ ____ $\left(\frac{2}{5}\right)^6$

l) $\left(\frac{8}{7}\right)^{100}$ ____ $\left(\frac{8}{7}\right)^{101}$

m) $\left(-\frac{2}{3}\right)^3$ ____ $\left(-\frac{2}{3}\right)^5$

n) $\left(-\frac{3}{2}\right)^2$ ____ $\left(-\frac{3}{2}\right)^5$

o) $\left(-\frac{2}{3}\right)^2$ ____ $\left(-\frac{2}{3}\right)^4$

p) $\left(-\frac{3}{2}\right)^2$ ____ $\left(-\frac{3}{2}\right)^4$

12. Write in exponential form.

a) $2 + 2 + 2 + 2$ ________

b) $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$ ________

c) $5 + 5 + 5 + 5 + 5$ ________

d) $4 + 4 + 4 + 4$ ________

e) $6 + 6 + 6 + 6 + 6 + 6$ ________

f) $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$ ________

g) $8 + 8$ ________

h) $9 + 9 + 9$ ________

13. Suppose the width of a square is four times the width of another square. How do the areas of the squares compare?

14. Suppose the volume of a cube is 343 times the volume of another cube. How do the lengths of of the sides compare?

1.5 Orders of Operations

If we want to solve $4 + 2 \times 3$, do we add first, then multiply, or do we multiply first, then add? Does it make a difference? If we add first, the answer is 18; if we multiply first, the answer is 10. What if we write $2(3 + 4)$? What do the brackets mean? What about $6^2 \div 2$? To come to an answer that everyone can agree on, we must make rules to standardize the order in which we perform mathematical operations.

Rules for Order of Operations

1. Do all calculations within brackets or parentheses first. When more than one kind of grouping of symbols occurs, do the innermost one first, then work from the inside out.
2. Evaluate all exponential expressions.
3. Do all multiplication and division in order from left to right.
4. Do all addition and subtraction in order from left to right.

To remember the order of operations, the acronym **BEDMAS** is used.

B Brackets
E Exponents
D Division
M Multiplication
A Addition
S Subtraction

Example 1 Simplify.

a) $5 + 3 \times 4$

b) $2^3 + 2 \times 3$

c) $6 - (2 + 3)^2$

d) $(3 - 2 \times 4)^2 - \left(3 + \dfrac{6^2}{2}\right)$

► *Solution*:

a) $5 + 3 \times 4 = 5 + 12 = 17$

b) $2^3 + 2 \times 3 = 8 + 2 \times 3 = 8 + 6 = 14$

c) $6 - (2 + 3)^2 = 6 - 5^2 = 6 - 25 = -19$

d) $(3 - 2 \times 4)^2 - \left(3 + \dfrac{6^2}{2}\right) = (3 - 8)^2 - \left(3 + \dfrac{36}{2}\right) = (-5)^2 - (3 + 18) = 25 - 21 = 4$

1.5 Exercise Set

1. Calculate.

a) $6 + 2 \times 3$ ________

b) $2 \times 3 + 2 \times 4$ ________

c) $4 \times 6 - 5 \times 3$ ________

d) $16 - 8 \div 4 - 2$ ________

e) $12 \div 3 - 16 \div 8$ ________

f) $25 - 18 \div 6 - 10$ ________

g) $7 - 3 - 10 \div 2$ ________

h) $-6 \times 2 - 4 - 2$ ________

i) $6 - 3 \times 4 - 5$ ________

j) $63 \div 7 \div 3 \times 2$ ________

2. Simplify.

a) $6 - (2 \times 3)$ ________

b) $(6 - 2) + 3$ ________

c) $-8 - (5 - 3)$ ________

d) $(-8 - 5) - 3$ ________

e) $-(8 - 3) + (3 - 7)$ ________

f) $100 \div (10 \div 5)$ ________

g) $(100 \div 10) \div 5$ ________

h) $128 \div (32 \div 2)$ ________

i) $(128 \div 32) \div 2$ ________

j) $5 \times 10 - (7 + 3) \div 5 - 24$ ________

3. Simplify.

a) 3×2^3 ________

b) $(3 \times 2)^3$ ________

c) $-5 - 3^2$ ________

d) $(-5 - 3)^2$ ________

e) $2^4 \div 2^2 \times 2^5 \div 2^3$ ________

f) $(2^4 \div 2^2)(2^5 \div 2^3)$ ________

g) $\dfrac{6 + 3 \times 4}{6 + 3 \times 4}$ ________

h) $\dfrac{(6 + 3)(4)}{6 + 3 \times 4}$ ________

i) $\dfrac{15 + 2 \times 5}{15 - 2 \times 5}$ ________

j) $\dfrac{(15 + 2)(5)}{15 - 2 \times 5}$ ________

4. Simplify.

a) $12 + 2[(20 - 8) - (1 + 3^2)]$

b) $\dfrac{3^3 - (1^3 + 2^3)}{2}$

c) $4 + 3(2^2 - 1)^3$

d) $4^2[(8 + 4) \div 6]$

e) $\dfrac{(-5)^2 - 3 \times 5}{3^2 + 3 \times 2(-1)^5}$

f) $\dfrac{(-2)^3 + 4^2}{3 - 5^2 + 3 \times 6}$

g) $4^2 \times 3 \div 8 - \dfrac{(4)(6 - 10)}{2} - 24 \div 2^3$

h) $-5^2 + \dfrac{(3)(4 - 8)}{2} + 10 \div 5$

i) $2^3 \div 4 \times 2 + 3(5 - 2) - 3 \times 2$

j) $\dfrac{(6 - 5)^4 + 21}{27 - 4^2}$

k) $\dfrac{40 - 1^3 - 2^4}{3(2 + 5) + 2}$

l) $20 \div 4 + \{2 \times 3^2 - [3 + (6 - 2)]\}$

m) $3 + 2\{3[(4 - 2)^2 + 1]\}$

n) $-6 - 3^2\{-2(2 - 3)^3 + (4 - 2)^2\}$

5. Insert parentheses to make the expression true.

a) $3 + 3 \times 4^2 = 96$

b) $4 \times 2 \times 3^2 = 576$

c) $6 + 2 \times 3^2 = 42$

d) $12 \div 4 + 2 \times 3 - 2 = 4$

e) $12 \div 4 + 2 \times 3 - 2 = 2$

f) $4 \times 2 \times 9 - 7 - 7 = 9$

g) $8 - 9 - 12 + 5 = 16$

h) $4 \times 2 + 3 - 7 + 4 = 0$

i) $12 \div 4 + 2 \times 3 - 2 = 5$

j) $4 - 2^3 \times 5 \div 24 - 4 = 2$

6. Insert parentheses in the expression $3 + 5 \times 4 - 6 \div 2$ to produce the following values:

a) 29

b) -2

c) -8

d) 20

7. Insert any operation signs (parentheses, $+$, $-$, $\times$, $\div$) so that the given numbers make the statement true.

a) 2 3 5 6 = 23

b) 5 6 7 9 = 64

c) 7 5 4 2 = 2

d) 6 3 2 1 = 1

8. Bill enters $24 \div 2 \times 3$ into his calculator and expects to get 4. What mistake is he making?

9. Sue enters $8 + 4 \div 2$ into her calculator and expects to get 6. What mistake is she making?

1.6 Exponent Laws

Multiplying with Exponents

We can simplify the expression $2^3 \times 2^4$ using the definition of exponents.

$$2^3 \times 2^4 = \underbrace{\overbrace{(2\times2\times2)}^{\text{three 2's}}\overbrace{(2\times2\times2\times2)}^{\text{four 2's}}}_{\text{seven 2's}} = 2^7$$

Notice that the exponent 7 is the sum of the exponents 3 and 4. This is the basis for the product rule of exponents.

The Product Rule

If a is a real number, and m and n are integers, then:

$a^m \times a^n = a^{m+n} \qquad (a \neq 0)$

(When multiplying, if the bases are the same, keep the base, and add the exponents)

Common Mistakes:

1. $2^3 \times 2^4 \neq 4^{3+4} = 4^7$
2. $2^3 \times 2^4 \neq 4^{3\times4} = 4^{12}$
3. $2^3 \times 2^4 \neq 2^{3\times4} = 2^{12}$

Example 1 Simplify.

a) $3^5 \times 3^4$

b) $2^3 \times 2^4 \times 2^6$

► *Solution*: **a)** $3^5 \times 3^4 = 3^{5+4} = 3^9$

b) $2^3 \times 2^4 \times 2^6 = 2^{3+4+6} = 2^{13}$

Dividing with Exponents

Next we simplify a quotient.

$$\begin{aligned}\frac{3^6}{3^2} &= \frac{3\times3\times3\times3\times3\times3}{3\times3}\\ &= \frac{\cancel{3}\times\cancel{3}}{\cancel{3}\times\cancel{3}}\times3\times3\times3\times3\\ &= 3\times3\times3\times3\\ &= 3^4\end{aligned}$$

Notice that the exponent 4 is the difference of the exponents 6 and 2. This is the basis for the quotient rule of exponents.

The Quotient Rule

If a is a real number, and m and n are integers, then:

$$\frac{a^m}{a^n} = a^{m-n},\ (a \neq 0)$$

(When dividing, if the bases are the same, keep the base, and subtract the exponents)

Common Mistakes:

1. $\frac{3^8}{3^2} \neq 3^4$

2. $\frac{3^8}{3^2} \neq 1^{8-2} = 1^6 = 1$

3. $\frac{3^8}{3^2} \neq 1^4$

Example 2 Simplify.

a) $\frac{5^8}{5^4}$ **b)** $\frac{3^4\times3^5}{3^6}$

► *Solution*: **a)** $\frac{5^8}{5^4} = 5^{8-4} = 5^4$

b) $\frac{3^4\times3^5}{3^6} = 3^{4+5-6} = 3^3$

Zero Exponent

Suppose the numerator and denominator have the same base, both raised to the same power. We know that any expression divided by itself is equal to one. For example $\frac{5\times5\times5}{5\times5\times5} = 1$. But $\frac{5\times5\times5}{5\times5\times5} = \frac{5^3}{5^3}$, if we apply the quotient rule then $\frac{5^3}{5^3} = 5^{3-3} = 5^0$. Therefore $5^0 = 1$.

> **The Zero Exponent Rule**
>
> For any non-zero real number a:
>
> $a^0 = 1,\ (a \neq 0)$
>
> (Any non-zero number real number raised to the zero power is one, 0^0 is undefined)

Examples: $3^0 = 1$, $\left(\frac{2}{3}\right)^0 = 1$

Combined Operations

Example 3 Simplify

a) $(-2)^5 \times 2^3$

b) $\frac{(-3)^8}{3^4}$

c) $2^7 - 2^2 \times 2^3$

d) $3 \times 3^4 + \frac{3^5}{3^3}$

► *Solution*:

a) $(-2)^5 = -2^5$ because an odd number of negatives is negative.
Therefore $(-2)^5 \times 2^3 = -2^5 \times 2^3 = -2^{5+3} = -2^8$

b) $(-3)^8 = 3^8$ because an even number of negatives is positive.
Therefore $\frac{(-3)^8}{3^4} = \frac{3^8}{3^4} = 3^{8-4} = 3^4$.

c) $2^7 - 2^2 \times 2^3 = 2^7 - 2^{2+3} = 2^7 - 2^5 = 128 - 32 = 96$

d) $3 \times 3^4 + \frac{3^5}{3^3} = 3^{1+4} + 3^{5-3} = 3^5 + 3^2 = 243 + 9 = 252$

1.6 Exercise Set

1. Fill in the blanks with the correct word.

a) When multiplying two numbers with the same base, the ________ rule says to keep the base and ________ the exponents.

b) In the number 3^4, the 3 is referred to as the __________ and the 4 is the __________ . The expression is read "three to the fourth ____________ ."

c) When dividing two numbers with the same base, the ________ rule says to keep the base and ________ the exponents.

2. Determine if the expression is true or false. If it is false, correct the expression.

a) $(2 \times 3)^2 = 2 \times 3^2$

b) $2^3 \times 2^6 = 2^9$

c) $\frac{2^8}{2^4} = 2^2$

d) $3^4 \times 3^5 = 9^9$

e) $\frac{2^7}{2^4} = 2^3$

f) $2 \times 3^0 = 1$

g) $\left(\frac{3}{4}\right)^3 = \frac{27}{12}$

h) $(-5)^0 = -1$

i) $-(-2)^4 = 2^4$

j) $-(-2)^3 = 2^3$

k) $\left(\frac{3}{5}\right)^3 = \frac{9}{15}$

l) $3^0 + 4^0 = (3 + 4)^0$

m) $-(-5)^3 = 5^3$

n) $-(-5)^2 = 5^2$

3. Which of the following is equal to one?

a) 3×2^0 **b)** -3×2^0 **c)** $(3 \times 2)^0$ **d)** $3(-2)^0$

4. Simplify.

a) 3^0 ________ **b)** -3^0 ________

c) $(-3)^0$ ________ **d)** 2×3^0 ________

e) $(2 \times 3)^0$ ________ **f)** $-(2 \times 3)^0$ ________

g) $(-2 \times 3)^0$ ________ **h)** $(2 + 3)^0$ ________

i) $2^0 + 3^0$ ________ **j)** $-2^0 - 3^0$ ________

k) $(-2)^0 + (-3)^0$ ________ **l)** $(-2)^0 - (-3)^0$ ________

m) $\dfrac{-2^0}{-3^0}$ ________ **n)** $-\left(\dfrac{2}{3}\right)^0$ ________

o) $\dfrac{0^2}{2^0}$ ________ **p)** $\left(\dfrac{0}{2}\right)^2$ ________

5. Multiply and simplify. Leave answer in exponential form.

a) $2^5 \times 2^4$ ________ **b)** $3^5 \times 3^3$ ________

c) $(-2)^4(-2)^3$ ________ **d)** $5^4 \times 5^3 \times 5^0$ ________

e) $(-3)^4(-3)^2(-3)^3$ ________ **f)** $4^3 \times 4^0 \times 4$ ________

g) $7 \times 7^2 \times 7^4$ ________ **h)** $(-4)(-4)^2(-4)^3$ ________

i) $2^3 \times 2^4 \times 3 \times 3^2$ ________ **j)** $(-3)(-3)^3(-5)^2(-5)^4$ ________

6. Divide and simplify. Leave answer in exponential form.

a) $\dfrac{2^6}{2^3}$ ________ **b)** $\dfrac{3^8}{3^4}$ ________

c) $\dfrac{(-3)^5}{(-3)^2}$ ________ **d)** $\dfrac{(-1)^8}{(-1)^3}$ ________

e) $\dfrac{4^9 \times 3^6}{4^5 \times 3^2}$ ________ **f)** $\dfrac{(-2)^5 \times 7^4}{(-2)^5 \times 7}$ ________

g) $\dfrac{8^{12}}{(-8)^6}$ ________ **h)** $\dfrac{(-2)^8}{2^3}$ ________

i) $\dfrac{3^5 \times 5^6}{(-3)^2 \times 5^3}$ ________ **j)** $\dfrac{(-5)^7 \times 7^4}{5^3 \times (-7)^2}$ ________

7. Simplify. Leave answer in exponential form.

a) $\dfrac{2^7 \times 2^5}{2^8}$ ________ **b)** $\dfrac{(-3)^4(-3)^3}{(-3)^5}$ ________

c) $\dfrac{5^7}{5^3 \times 5^4}$ ________ **d)** $\dfrac{(-7)^4(-7)^5}{(-7)^3(-7)^2}$ ________

e) $\dfrac{10 \times 10^7 \times 10^4}{10^6 \times 10^5}$ ________ **f)** $\dfrac{(-11)^3(-11)^5(-11)^0}{(-11)(-11)^2}$ ________

g) $\dfrac{(-2)^5 \times 2^3 \times (-2)^4}{2 \times (-2)^2 \times 2^2}$ ________ **h)** $\dfrac{5^4 \times (-5)^5 \times 5^5}{-5 \times 5^3}$ ________

i) $\dfrac{(-7)^0 \times 7^9}{(-7)^2(-7)^3(-7)}$ ________ **j)** $\dfrac{(-3)^{10}}{3^3 \times (-3)^3 \times 3}$ ________

8. Simplify, then evaluate.

a) $2^3 \times 2^4 - 2^2 \times 2^3$ ________

b) $2^5 \div 2 + 2^4 \times 2^2$ ________

c) $(-3)^4 + (-3)^6 \div (-3)^5 - (-3)^2$ ________

d) $3^7 \div (3^6 \div 3^3) \div 3^2$ ________

e) $5^9 \div 5^5 \times 5 \div 5^3$ ________

f) $5^9 \div (5^5 \times 5) \div 5^3$ ________

g) $(-2)^2 \times 2^4 + (-2)^3 \times 2$ ________

h) $(-3)^6 \div 3^4 + (-3)^3 \div 3$ ________

i) $(-5)^3 \div 5^2 + (-3)^3 \div 3^2$ ________

j) $(-8)^2 \times 8 + (-8)^5 \div 8^2$ ________

k) $\dfrac{(-2)^5 + (-2)^2}{(-2)^4}$ ________

l) $\dfrac{(-3)^4 - 3^2}{(-3)^3}$ ________

9. Simplify.

a) $2^{a+3} \times 2^{a-1}$ ________

b) $\dfrac{3^{2m}}{3^{m-1}}$ ________

c) $\dfrac{5^{4a-3} \times 5^{3-a}}{5^{2a+1}}$ ________

d) $\dfrac{7^{6-2a}}{7^{a+3} \times 7^{2-3a}}$ ________

1.7 Power Rules

Consider an expression like $(5^3)^2$.

We know that to square a number means to multiply it by itself.

$$(5^3)^2 = 5^3 \times 5^3$$

$$= (5 \times 5 \times 5)(5 \times 5 \times 5)$$

$$= 5 \times 5 \times 5 \times 5 \times 5 \times 5$$

$$= 5^6$$

Notice that the exponent 6 is the product of the exponents 3 and 2. This is the basis for the power rule of exponents.

The Power Rule

For any real number a, and any integers m and n:

$(a^m)^n = a^{m \times n}$

(To raise a power to a power, multiply the exponents)

Common Mistakes:

1. $(3 + 5)^2 \neq 3^2 + 5^2 = 34$. The correct answer is $(3 + 5)^2 = 8^2 = 64$. An exponent can belong to more than one base within a set of parentheses **only** when the bases are related by multiplication or division, **not** addition or subtraction.

2. $2^4 \times 2^3 = 2^{4+3} = 2^7$ but $(2^4)^3 = 2^{4 \times 3} = 2^{12}$. Be careful not to mix up the product rule and exponent rule.

Example 1 Simplify.

a) $(2^3)^4$

b) $(3^2)^5$

► *Solution*: **a)** $(2^3)^4 = 2^{3 \times 4} = 2^{12}$

b) $(3^2)^5 = 3^{2 \times 5} = 3^{10}$

Raising a Product or Quotient to a Power

The power rule can be applied to both products and quotients.

$$\begin{aligned}(3\times4)^2 &= (3\times4)(3\times4)\\ &= (3\times3)(4\times4)\\ &= 3^2\times4^2\end{aligned}
\qquad\qquad
\begin{aligned}\left(\tfrac{2}{5}\right)^3 &= \left(\tfrac{2}{5}\right)\left(\tfrac{2}{5}\right)\left(\tfrac{2}{5}\right)\\ &= \frac{2\times2\times2}{5\times5\times5}\\ &= \frac{2^3}{5^3}\end{aligned}$$

> **Rule for Raising a Product to a Power**
>
> For any integer n, and any real numbers a and b:
>
> $(ab)^n = a^n b^n$
>
> (To raise a product to a power, raise each factor to that power)

Example 2 Remove brackets and simplify; evaluate if possible.

a) $(2\times3)^4$ **b)** $(a^2b^3)^n$

► *Solution*: **a)** $(2\times3)^4 = 2^4\times3^4 = 16\times81 = 1296$

b) $(a^2b^3)^n = a^{2n}b^{3n}$

> **Rule for Raising a Quotient to a Power**
>
> For any integer n, and any real numbers a and b, $b\neq0$:
>
> $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
>
> (To raise a quotient to a power, raise the numerator and the denominator to that power)

Example 3 Remove brackets and simplify; evaluate if possible.

a) $\left(\frac{2}{5}\right)^3$ **b)** $\left(\frac{a^3}{b^2}\right)^n$

► *Solution*: **a)** $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125}$

b) $\left(\frac{a^3}{b^2}\right)^n = \frac{a^{3n}}{b^{2n}}$

1.7 Exercise Set

1. State whether the equation is an example of the product rule, quotient rule, power rule, raising a product to a power, or raising a quotient to a power.

a) $(2^5)^3 = 2^{15}$ _______________ **b)** $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$ _______________

c) $2^5 \times 2^3 = 2^8$ _______________ **d)** $\frac{2^6}{2^2} = 2^4$ _______________

e) $(2 \times 3)^3 = 2^3 \times 3^3$ _______________ **f)** $\left[\left(\frac{4}{3}\right)^2\right]^3 = \frac{4^6}{3^6}$ _______________

g) $[(3^4)^5]^2 = 3^{40}$ _______________ **h)** $3^2 \times 3^4 \times 3^3 = 3^9$ _______________

i) $\frac{5^7}{5^3} = 5^4$ _______________ **j)** $(2 \times 3 \times 4)^2 = 2^2 \times 3^2 \times 4^2$ _______________

2. Fill in the blanks to make the statement true.

a) $2^3 =$ ____ $\times$ ____ $\times$ ____ **b)** $(-3)\times(-3)\times(-3) =$ ____

c) $2^m \times 2^n =$ ____ **d)** $(2 \times 3)^m =$ ____ $\times$ ____

e) $\left(\frac{2}{3}\right)^n =$ ____ **f)** $(2^m)^n =$ ____

g) $\frac{2^m}{2^n} =$ ____ **h)** $(2^3)^2 =$ ____

i) $3^1 =$ ____ **j)** $(2 \times 3)^4 =$ ____

3. Write in exponential form.

a) $(2^3)^4$ __________ **b)** $(4^3)^3$ __________

c) $(5^3)^6$ __________ **d)** $(3^7)^5$ __________

e) $(7^0)^5$ __________ **f)** $[(-2)^2]^3$ __________

g) $-(2^2)^3$ __________ **h)** $[(-2)^3]^5$ __________

i) $-(2^3)^5$ __________ **j)** $[(2^3)^4]^2$ __________

4. Write in exponential form without parentheses.

a) $(2 \times 3)^2$ ________ **b)** $(2 + 3)^2$ ________

c) $(5 \times 7)^3$ ________ **d)** $(5 + 7)^3$ ________

e) $(2^3 \times 5^2)^4$ ________ **f)** $(2^3 + 5^2)^4$ ________

g) $(6^0 \times 3^2)^5$ ________ **h)** $\left(\frac{7}{11}\right)^3$ ________

i) $\left(\frac{7^0}{11^5}\right)^3$ ________ **j)** $[(2 \times 3^4)^2]^3$ ________

5. Determine if the values are positive or negative.

a) $[(-2)^2]^3$ ________ **b)** $[(-2)^3]^3$ ________

c) $(-2^2)^3$ ________ **d)** $(-2^2)^4$ ________

e) $(-2^3)^2$ ________ **f)** $[(-2^2)^3]^3$ ________

6. Evaluate.

a) $(-2 \times 3)^2$ ________ **b)** $-(2 \times 3)^2$ ________

c) $(-2 \times 3)^3$ ________ **d)** $(-2 + 3)^4$ ________

e) $(-3 + 2)^4$ ________ **f)** $(-2 + 3)^5$ ________

g) $(-3 + 2)^5$ ________ **h)** $\left(-\frac{6}{2}\right)^3$ ________

i) $\left(-\frac{6}{2}\right)^4$ ________ **j)** $\left(\frac{(-2)^3}{(-2)^2}\right)^3$ ________

7. Write each expression as 2 raised to a power, then simplify.

a) 4×8 __________

b) 16×32 __________

c) 16 __________

d) $8 + 8$ __________

e) 4^{10} __________

f) 16^{4} __________

g) 64^{3} __________

h) 4^{a} __________

i) 16^{3a} __________

j) $(8^{a})^{2a}$ __________

8. Simplify, then evaluate.

a) $(2^8 \div 2^6) - (2^3 \div 2^2)^3$

b) $(2^3)^2 + 3^3 \times 3^2 - 4^3 \div 4$

c) $(2 + 3)^2 - (3 - 5)^3 - (2 \times 3)^2$

d) $(3^2 + 3)^2 + (2^2 + 2)^2$

e) $(3^2)^2 + 3^2 + (2^2)^2 + 2^2$

f) $(3^2)^2 + (3^2 + 2^2)^2 + 2^2$

g) $[(-2)^2]^3 + [(-2)^3]^3 - (-3)^5$

h) $[(-4)^2 - 3^2]^2 + [(-3)^2 - 2^4]^2$

i) $[(-2)^5 - (-3)^3]^2 + [(-2)^2 - (-3)^2]^2$

j) $[(-2)^8 - (3)^5]^2 - [-(-2)^9 + (-3)^5]$

k) $[(-5)(-4)]^2 + [(-3)^3]^2 - [(-2)^5 \div (-4)^2]^3$

l) $\left[\frac{(-2)^5}{2^2}\right]^3 - \left[\frac{(-2)^4}{2^3}\right]^3 + \left[\frac{(-2)^3}{2}\right]^3$

9. Use the approximation $2^{10} \simeq 1000$ to estimate the value of the following powers of 2.

(Example: $2^{15} = 2^{10} \times 2^5 \doteq 1000 \times 32 = 32\,000$ or 32×10^3)

a) 2^{12} ____________ **b)** 2^{25} ____________

c) 2^{33} ____________ **d)** 2^{104} ____________

e) 2^{200} ____________ **f)** 2^{197} ____________

10. Simplify.

a) $(2^{3a+1})^2$ ____________ **b)** $\left(\dfrac{3^{1-2a}}{3^{a+2}}\right)^4$ ____________

c) $(5^3)^{2-a}$ ____________ **d)** $\left[7^{2(3-a)}\right]^3$ ____________

11. Solve for x.

a) $\dfrac{2^{26}}{2^x} = 2^x$ ____________ **b)** $\dfrac{2^x}{2^9} = 2^{2x}$ ____________

c) $\dfrac{9^{2x}}{27^2} = 3^{2x}$ ____________ **d)** $\dfrac{4^{3x}}{8^x} = 2^7$ ____________

1.8 Chapter Review

Section 1.1

1. Determine whether each statement is always true, sometimes true, or never true.

a) A negative integer is a rational number. ________________

b) An irrational number times an irrational number is irrational. ________________

c) Fractions are irrational numbers. ________________

2. Which of the following are irrational numbers:
$-\frac{2}{3},\ \sqrt{2},\ \pi,\ 0,\ -3\frac{1}{2},\ 2.1111...,\ 1.121121112...$

Section 1.2

3. Simplify.

a) $\sqrt{144} + \sqrt{25}$ __________

b) $\sqrt{144 + 25}$ __________

c) $\sqrt{25} - \sqrt{9}$ __________

d) $\sqrt{25 - 9}$ __________

4. A rectangle half as wide as it is long has an area of 98 cm². Determine the perimeter of the rectangle.

5. A right triangle has one leg twice as long as the other with an area of 289 cm². What is the length of each leg?

Section 1.3

6. Between what two integers does the irrational number fall?

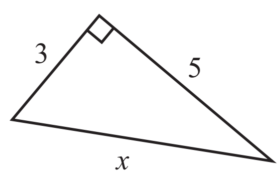
a) $\sqrt{327}$ __________ **b)** $-\sqrt{173}$ __________

7. Without a calculator determine the square roots of the irrational numbers to one decimal place.

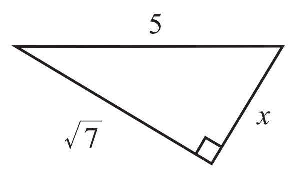
a) $\sqrt{327}$ __________ **b)** $-\sqrt{173}$ __________

8. Solve for x to one decimal place.

a)



b)



Section 1.4

9. Find the missing exponents to make the equation correct.

a) $64 = 8^? = 4^? = 2^?$ ____________ **b)** $81 = 9^? = 3^?$ ____________

c) $256 = 16^? = 4^? = 2^?$ ____________ **d)** $729 = 27^? = 9^?$ ____________

10. Assume that $0 < a < 1$. Arrange the following in order from least to greatest: $-(-a)^3$, $-a^3$, $(-a)^4$, $-a^4$

11. Evaluate.

a) -3^0 ________ **b)** $(-3)^0$ ________

c) $(2+3)^0$ ________ **d)** 2^0+3^0 ________

e) $(2-3)^{100}$ ________ **f)** $(2-3)^{199}$ ________

g) $-(-1)^{50}-(-1)^{51}$ ________ **h)** $-1^{100}-1^{101}$ ________

i) -1^0-1^0 ________ **j)** $-(1-1)^0$ ________

12. Use $<$, $>$ or $=$ to write a true sentence.

a) $(-3)^5$ ____ -3^5 **b)** $(-3)^8$ ____ -3^8

c) $\left(-\frac{3}{4}\right)^2$ ____ $\left(-\frac{3}{4}\right)^4$ **d)** $\left(-\frac{4}{3}\right)^2$ ____ $\left(-\frac{4}{3}\right)^4$

e) 2^5 ____ 5^2 **f)** $(-4)^2$ ____ $(-2)^4$

g) $\left(-\frac{7}{9}\right)^5$ ____ $\left(-\frac{7}{9}\right)^7$ **h)** $\left(-\frac{9}{7}\right)^5$ ____ $\left(-\frac{9}{7}\right)^7$

Section 1.5

13. Simplify.

a) $4 - 3 \times 2^2$ ________ **b)** $4 - (3 \times 2)^2$ ________

c) $(4 - 3) \times 2^2$ ________ **d)** $\dfrac{(8 - 2) \times 3}{8 - 2 \times 3}$ ________

e) $\dfrac{8 + 2 \times 3}{(8 - 2) \times 3}$ ________ **f)** $\dfrac{8 - 2 \times 3}{(8 + 2) \times 3}$ ________

g) $-2 \times 3^2 - 4 \times 2^2$ ________ **h)** $(-2 \times 3)^2 - (4 \times 2)^2$ ________

i) $-2 \times (3^2 - 4) \times 2^2$ ________ **j)** $(-2 \times 3^2 - 4) \times 2^2$ ________

14. Insert parentheses to make the expression true.

a) $-4 + 1^2 + 2 - 3^3 = 8$ **b)** $6 \div 2 + 1 \times 4 = 8$

c) $3 + 2 \times 3 - 1^2 = 11$ **d)** $8 \div 8 - 4 + 3 = 8$

Section 1.6

15. Simplify. Leave answer in exponential form.

a) $3^4 \times 2^3 \times 3^2 \times 2$ ____________ **b)** $3^0 \times 3^1 \times 3^2$ ____________

c) $(-3)^4 \times 3^2$ ____________ **d)** $(-5)^3 \times 5^2 \times (-5)^0$ ____________

e) $\dfrac{4^3 \times 4^5}{4^4}$ ____________ **f)** $\dfrac{(-3)^5}{(-3)^2 \times 3^2}$ ____________

g) $\dfrac{(-2)^5 \times 2^2}{(-2)^2}$ ____________ **h)** $\dfrac{2^4 \times 2^x}{2^{x-1}}$ ____________

16. Simplify, then evaluate.

a) $3^2 \times 3 - 3^3 \times 3^0$ ________ **b)** $2^4 \div 2 + 2^3 \times 2^2$ ________

c) $(-4)^5 \div 4^3 + (-4)^3 \div 4$ ________ **d)** $(-5)^4 \div 5^2 - (-5)^2$ ________

e) $\dfrac{(-2)^4 + (-2)^3}{(-2)^2}$ ________ **f)** $\dfrac{-3^0 - 3^3}{(-2)^5 - (-2)^2}$ ________

Section 1.7

17. Write in exponential form.

a) $(3^2)^4$ ________ **b)** $(2^5)^6$ ________

c) $[(3^2)^4]^3$ ________ **d)** $[(-2)^2]^3$ ________

e) $[(-5)^3]^3$ ________ **f)** $[(-4)^3]^6$ ________

18. Evaluate.

a) $(2^5 \div 2^3)^2$ ________ **b)** $(2^3 \times 2)^2$ ________

c) $(1 + 2)^3 - (5 - 2)^2$ ________ **d)** $(2^2)^3 - (2^2 + 1)^2$ ________

e) $[(-3)^2 - 2^2]^2 - [(-2)^2 - 3]^2$ ________ **f)** $\left[\dfrac{(-3)^5}{3^3}\right]^2 - \left[\dfrac{(-2)^5}{2^3}\right]^3$ ________