Equations of Linear Relations Lesson #1:
The Equation of a Line in
Slope y-intercept Form → $y = mx + b$

Overview of Unit

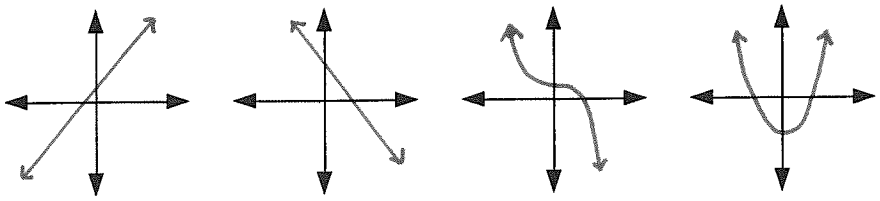
In this unit we express the equation of a linear relation in three different forms: slope y-intercept form, point-slope form, and general form. We relate linear relations expressed in these forms to their graphs.

We also determine the linear relation given: a graph, a table of data points, a point and the slope, two points, a point and the equation of a parallel or perpendicular line.

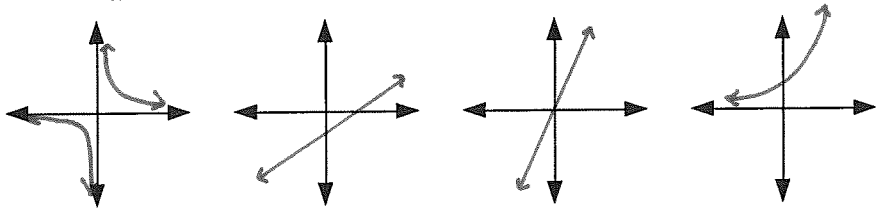
Investigating the Graphs of Linear and Non-Linear Relations

a) The equations of the graphs of some relations are given. In each case, use a graphing calculator to sketch the graph of the relation and make a rough sketch of the graph on the grid provided. Do not list any x- or y-intercepts.

$y = 3x + 1$ $y = -2x + 3$ $y = -x^3 + 1$ $y = x^2 - 3$



$y = \frac{1}{x}$ $y = \frac{1}{2}x - 2$ $y = 4x$ $y = 2^x$



b) List the equations of the graphs as linear or non-linear.

LINEAR: $y = 3x^1 + 1$, $y = -2x^1 + 3$, $y = \frac{1}{2}x^1 - 2$, $y = 4x^1$

NON-LINEAR: $y = -x^3 + 1$, $y = x^2 - 3$, $y = \frac{1}{x}$, $y = 2^x$

c) Compare the lists. Write a rule from the equation which can be used to determine whether the graph is a straight line or not.

If the exponents of the term in x and the term in y are both equal to one, then the graph is a straight line.

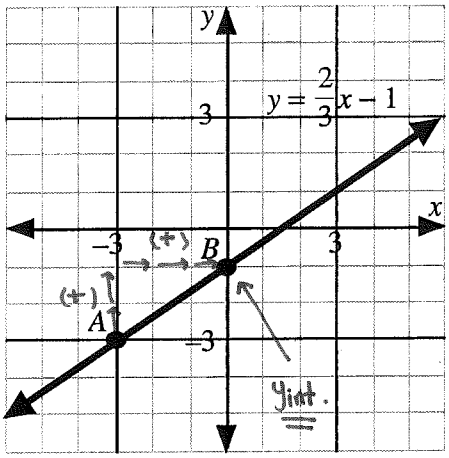
Linear Equation

A **linear equation** is an equation of the form $y = mx + b$, where $m, b \in R$.
The graph of a linear equation is a straight line.

Investigating m and b in the equation $y = mx + b$

Part One

Jenine used a graphing calculator to sketch the graph of the linear equation $y = \frac{2}{3}x - 1$.
Her sketch is shown on the grid.



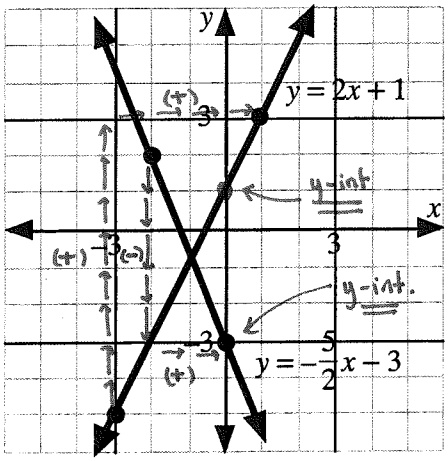
a) Use the sketch and points A and B to find the slope and y-intercept of the graph of $y = \frac{2}{3}x - 1$.

Slope = $\frac{\text{rise}}{\text{run}}$ = $\frac{2}{3}$ y_{int} = -1

b) Compare the values found in **a)** with the coefficient of x and the constant term in the equation $y = \frac{2}{3}x - 1$.

The slope is the coefficient of x. (SAME)
The y-intercept is the constant term. (SAME)

c) Jenine sketched the graphs of two more linear equations. Use the grid to determine the slope and y-intercept of each graph.



equation	slope	y-intercept
$y = 2x + 1$	$\frac{8}{4} = 2$	1
$y = -\frac{5}{2}x - 3$	$\frac{-5}{2}$	-3

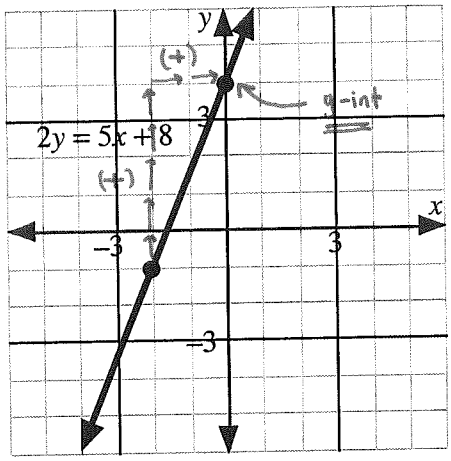
d) Make a conjecture about the slope and y-intercept of the graph of the linear equation $y = mx + b$.

Slope = m

y-int = b

Part Two

Hashib used a graphing calculator to graph the linear equation $2y = 5x + 8$.
The graph is shown on the grid.



a) Use the sketch to determine the slope and y-intercept of the graph of $2y = 5x + 8$.

Slope = $m = \frac{5}{2}$ y-intercept = $b = 4$

b) Explain why, in this case, the slope is not 5 (the coefficient of x) and the y-intercept is not 8 (the constant term).

The coefficient of y is 2.

The equation needs to be in the form $y = \frac{5}{2}x + 4$.

$\frac{2y}{2} = \frac{5x + 8}{2}$

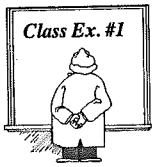
$y = \frac{5}{2}x + 4$

Slope y-intercept Form of the Equation of a Line → $y = mx + b$

The graph of an equation in the form $y = mx + b$ (or a function in the form $f(x) = mx + b$) is a straight line with **slope** m and **y-intercept** b.

The equation $y = mx + b$ is known as the **slope y-intercept form** of the equation of a line.

The graph of an equation in this form can be drawn without making a table of values.



Determine the slope and y-intercept of the graph of each linear equation listed below:

a) $y = 3x + 2$

Slope: 3

y_{int}: 2

b) $y = 7 - \frac{2}{3}x$

Slope: $-\frac{2}{3}$

$y_{int} = 7$

c) $6y = 8x + 1$

slope: 8

y_{int}: 1

Graphing an Equation of the Form $y = mx + b$

In this section, we will look at two ways of sketching the graph of a linear equation without using a graphing calculator or a table of values.

Consider the equation $y = 2x - 5$.

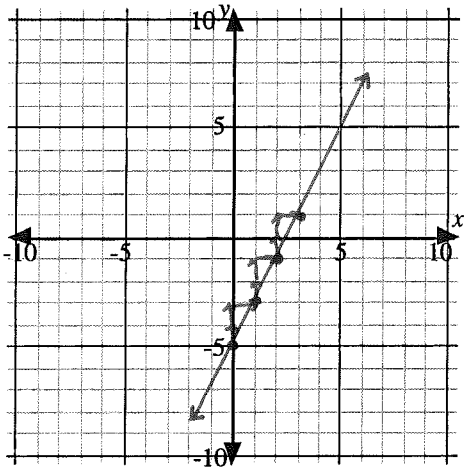
a) State the slope and y-intercept.

slope: $2 = \frac{2}{1}$ (2 - rise, 1 - run) $y_{int} = -5$

b) Mark the y-intercept on the grid.

c) Use the y-intercept and the formula $\text{slope} = \frac{\text{rise}}{\text{run}}$ to mark three other points on the grid. Join the points together, and extend the line.

d) Verify the graph using a graphing calculator.



Consider the equation $y = \frac{2}{3}x - 6$.

a) State the y-intercept.

$y_{int} = -6$

b) Determine the x-intercept algebraically.

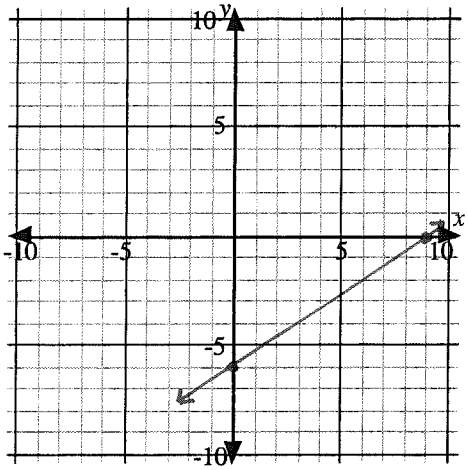
x_{int}: Let $y = 0$ $\quad (0) = \frac{2}{3}x - 6$

$6 = \frac{2}{3}x$ ($18 = 2x$, $x = 9$)

$x_{int} = 9$

c) Mark the x- and y-intercepts on the grid. Join the points together, and extend the line.

d) Verify the graph and the intercepts using a graphing calculator.



Complete Assignment Questions #1 - #14

Assignment

1. Each equation represents a relation.

a) $y = 6x + 1$ **b)** $y = x^2$ **c)** $y = 3x^4 + 5$ **d)** $y = -\frac{1}{4}x - 8$

e) $y = 1 - x$ **f)** $y = \frac{2}{1-x}$ **g)** $y = 4x$ **h)** $y = 4^x$

Without sketching the graph of the relation, list the letters a) through h) as linear or non-linear

LINEAR: a, d, e, g ← x variable has an exponent of 1. Recall: polynomial is of degree 1.

NON-LINEAR: b, c, f, h

2. State the slope and y-intercept of the graph of each linear equation.

a) $y = 7x - 2$ **b)** $y = \frac{4}{3}x + 3$ **c)** $y = 6 - \frac{1}{6}x$ **d)** $\frac{4y}{4} = \frac{6x+8}{4}$ **e)** $y = ax + b$

$y = \frac{3}{2} + 2$

slope: 7 slope: $\frac{4}{3}$ slope: $-\frac{1}{6}$ slope: $\frac{3}{2}$ slope: a

y_{int}: -2 y_{int}: 3 y_{int}: 6 y_{int}: 2 y_{int}: b

3. Write the equation of each line with the given slope and y-intercept.

a) slope $= 4$ **b)** slope $= \frac{1}{5}$ **c)** slope $= -3$ **d)** slope $= m$

y-intercept $= -9$ y-intercept $= \frac{1}{2}$ y-intercept $= 0$ y-intercept $= b$

$y = 4x - 9$ $y = \frac{1}{5}x + \frac{1}{2}$ $y = -3x$ $y = mx + b$

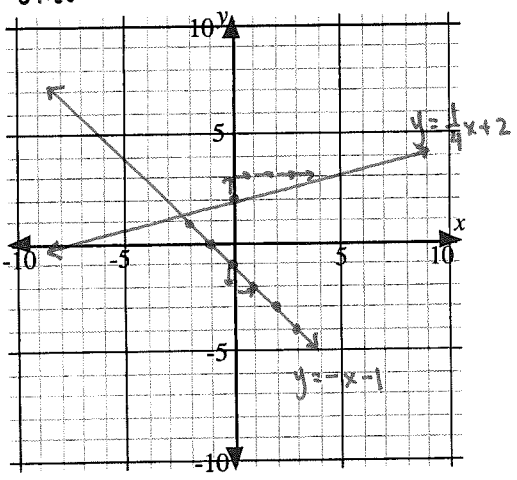
4. For each line, state the slope and the y-intercept. Graph the equation without using a graphing calculator.

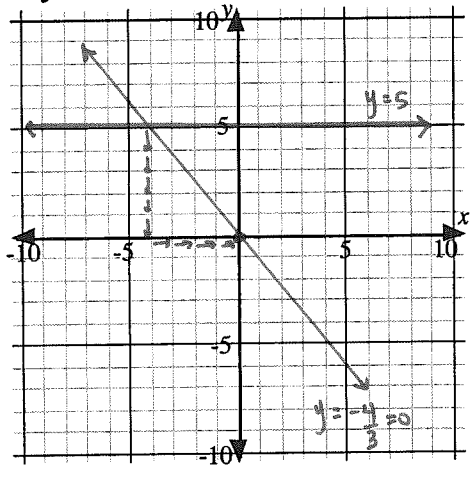
a) $y = \frac{1}{4}x + 2$ **b)** $y = -x - 1$ **c)** $y = -\frac{4}{3}x - 0$ **d)** $y = 5$

$y = 0x + 5$

m = slope = $\frac{1}{4}$ m = slope = -1 m = slope = $-\frac{4}{3}$ m = slope = 0

b = y_{int} = 2 b = y_{int} = -1 b = y_{int} = 0 b = y_{int} = 5





Recall: Only two points used to graph linear equation. Any other points can be use to check your work! In this case our two points are the x- and y-intercepts!

5. For each line, state the y-intercept. Determine the x-intercept algebraically, and graph the equation without using a graphing calculator.

a) $y = 2x + 6$

$y_{int} = 6$

x_{int}: Let $y = 0$

$0 = 2x + 6$

$-6 = 2x$

$x = -3$

$x_{int} = -3$

b) $y = -x - 4$

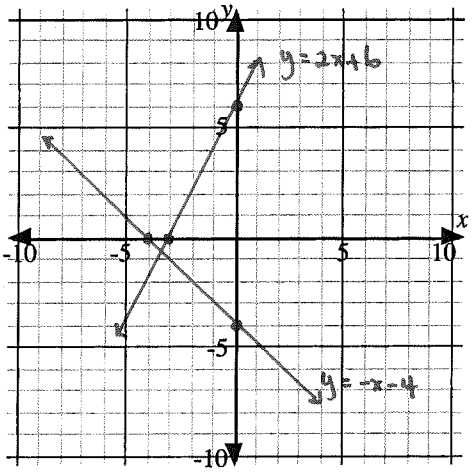
$y_{int} = -4$

x_{int}: Let $y = 0$

$0 = -x - 4$

$x = -4$

$x_{int} = -4$



c) $y = \frac{6}{7}x - 6$

$y_{int} = -6$

x_{int}: Let $y = 0$

$0 = \frac{6}{7}x - 6$

$6 = \frac{6}{7}x$

$42 = 6x$

$x = 7$

$x_{int} = 7$

d) $y = -\frac{1}{2}x + 1$

$y_{int} = 1$

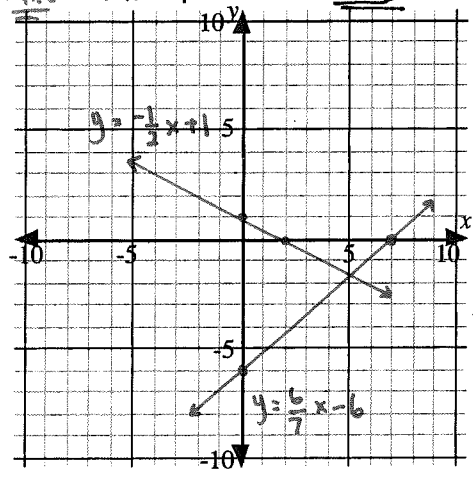
x_{int}: Let $y = 0$

$0 = -\frac{1}{2}x + 1$

$\frac{1}{2}x = 1$

$x = 2$

$x_{int} = 2$



6. Explain why the linear equation $y = 5x$ can be graphed using the method in question 4 but not by the method in question 5.

The method in question 4 needs a point and a slope. We have point (0,0) and slope 5. The method in question 5 needs two points to be joined. Since the x- and y-intercepts are the same points, the line cannot be drawn using the x-and y-intercepts.

7. Consider the graph of the function with equation $y = x + 0$

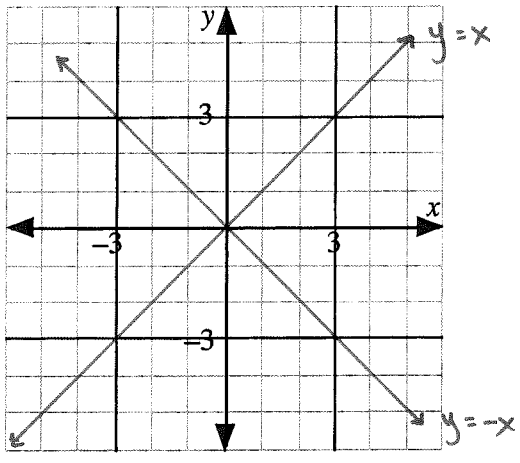
a) State the values of m and b.

m: slope $= 1$ $\quad b$: y-intercept $= 0$

b) Determine the x- and y-intercepts.

$x_{int} = 0$ $\quad y_{int} = 0$

c) Sketch the graph on the grid provided without using a graphing calculator.



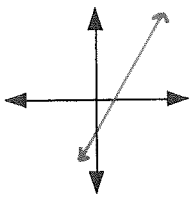
d) Determine the domain and range of the function.

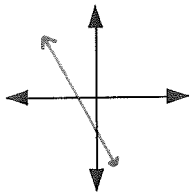
$D: \{x \mid x \in \mathbb{R}\}$ $\quad R: \{y \mid y \in \mathbb{R}\}$

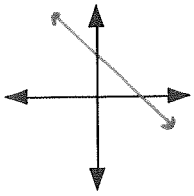
e) Use a graphing calculator to graph the line $y = -x$, and sketch the graph on the grid.

8. Use a graphing calculator to sketch the graph of each of the following linear equations. Complete the table giving the x-intercept to the nearest hundredth.

i) $y = 7x - 8$ **ii)** $y = -\frac{31}{2}x - 25$ **iii)** $y = 75 - \frac{5}{3}x$







2nd [TRACE] [2] LeftBound [◁] RightBound [▷] [ENTER]

slope	7
x-intercept	1.14
y-intercept	−8

slope	−31/2
x-intercept	−1.61
y-intercept	−25

slope	−5/3
x-intercept	45
y-intercept	75

once on correct side of x-intercept [ENTER] in each case.

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin= -4
 Xmax= 4
 Xscl= 1
 Ymin= -10
 Ymax= 5
 Yscl= 2
 Xres=1
```

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin= -4
 Xmax= 4
 Xscl= 1
 Ymin= -40
 Ymax= 10
 Yscl= 10
 Xres=1
```

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin= -10
 Xmax= 60
 Xscl= 10
 Ymin= -20
 Ymax= 100
 Yscl= 10
 Xres=1
```

Multiple Choice

9. Which of the following does not represent the equation of a straight line?

A. $y = 3x^1$

B. $y = 11 - 3x^1$

C. $y = \frac{x^1}{3}$ ... Also can write as $y = \frac{1}{3}x^1$

(D.) All of the above represent the equation of a straight line.

All are of degree 1.

10. Which of the following statements is false for the line $y = -\frac{1}{2}x + 1$?

Slope = $-\frac{1}{2}$ ✓✓

y-int = 1 ✓✓

x_{int}: $0 = -\frac{1}{2}x + 1$

$\frac{1}{2}x = 1$

$x_{int} \rightarrow x = 2$ ✓✓

A. The graph of the line falls from left to right. ✓ slope is (−)

B. The x-intercept is 2. ✓

C. The graph passes through the point $(8, -3)$. ✓

(D.) The line is perpendicular to the line $y = -2x + 4$.

Problem: The ⊥ slope to $-\frac{1}{2}$ must be 2 not −2.

Check Point $(8, -3)$: Let $x = 8$

$y = -\frac{1}{2}(8) + 1$

$= -4 + 1$

$y = -3$ ✓✓

11. Which of the following statements is true for the line $2y = \frac{1}{4}x + 6$?

x_{int}:

Problem: not in $y = mx + b$ form. Must isolate for y.

$y = \frac{1}{8}x + 3$ (slope, y-int.)

A. The x-intercept is 24. ✗

B. The y-intercept is 6. ✗

C. The slope is $\frac{1}{8}$. ✓

D. The graph passes through the point $(-4, 5)$. ✗

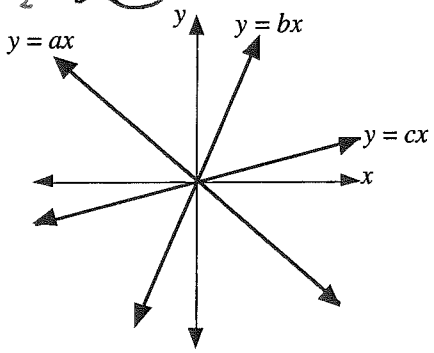
$0 = \frac{1}{8}x + 3$

$\frac{1}{8}x = -3$

$x = -24$

Let $x = -4$: $2y = \frac{1}{4}(-4) + 6$ $\quad 2y = -1 + 6$ $\quad \frac{2y}{2} = \frac{5}{2}$ $\quad y = 2.5$

12. The lines $y = ax$, $y = bx$, and $y = cx$ are shown. Which of the following statements is true?



A. $a < b < c$

B. $a < c < b$

C. $c < a < b$

D. $c < b < a$

c: small positive (rising)

b: large positive

a: negative (falling)

Use the following information to answer questions 13 and 14.

Consider the line with equation $y = 3x + 5$. The line intersects the x-axis at P and the y-axis at Q. Triangle POQ is formed where O is the origin.

slope = 3 $\quad$ $y_{int} = 5$

Numerical Response

13. The area of $\triangle POQ$, in square units, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

4	.	2	

x_{int}: Let $y = 0$

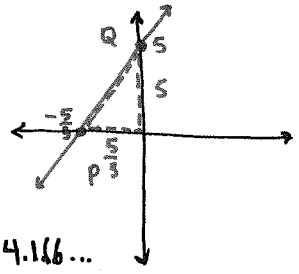
$0 = 3x + 5$

$3x = -5$

$x = -\frac{5}{3}$

$x_{int} = -\frac{5}{3}$

Now: We have two points need to plot and graph the linear function, both the x- and y-intercepts.



Area $= \frac{1}{2}bh = \frac{1}{2}\left(\frac{5}{3}\right)(5) = \frac{25}{6} = 4.166\ldots$

14. To the nearest tenth, the perimeter of $\triangle POQ$ is _____.

(Record your answer in the numerical response box from left to right)

1	1	.	9

Goal: We just need to determine the length of the hypotenuse since we already have the base and the height based on the x- and y-intercepts.

length of PQ $= \sqrt{\left(\frac{5}{3}\right)^2 + 5^2} = 5.270\ldots$

Perimeter $= \frac{5}{3} + 5 + 5.270\ldots$

$= 11.937\ldots$

Answer Key

1. LINEAR a), d), e), g). NON-LINEAR b), c), f), h).

2. **a)** slope = 7, y-int = –2 **b)** slope = $\frac{4}{3}$, y-int = 3

c) slope = $-\frac{1}{6}$, y-int = 6 **d)** slope = $\frac{3}{2}$, y-int = 2 **e)** slope = a, y-int = b

3. **a)** $y = 4x - 9$ **b)** $y = \frac{1}{5}x + \frac{1}{2}$ **c)** $y = -3x$ **d)** $y = mx + b$

4. **a)** slope = $\frac{1}{4}$, y-int = 2 **b)** slope = –1, y-int = –1

c) slope = $-\frac{4}{3}$, y-int = 0 **d)** slope = 0, y-int = 5

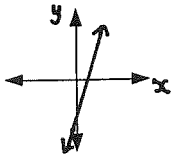
5. **a)** y-int = 6, x-int = –3 **b)** y-int = –4, x-int = –4 **c)** y-int = –6, x-int = 7 **d)** y-int = 1, x-int = 2

6. The method in #4 needs a point and a slope. We have point (0, 0) and slope = 5. The method in #5 needs two points to be joined. Since the x- and y-intercepts are the same point, the line cannot be drawn.

7. **a)** $m = 1$, $b = 0$ **b)** x-int = 0 and y-int = 0. **d)** $D = x \in R$ $R = y \in R$

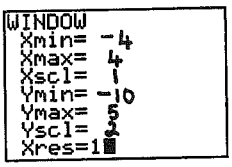
8.

i) $y = 7x - 8$

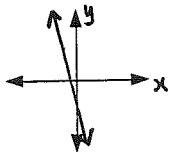


slope	7
x-intercept	1.14
y-intercept	–8

Graphing Window which includes both intercepts:

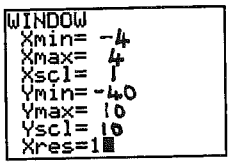


ii) $y = -\frac{31}{2}x - 25$

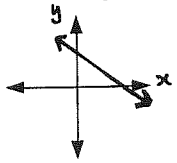


slope	$-\frac{31}{2}$
x-intercept	–1.61
y-intercept	–25

Graphing Window which includes both intercepts:

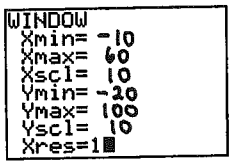


iii) $y = 75 - \frac{5}{3}x$



slope	$-\frac{5}{3}$
x-intercept	45
y-intercept	75

Graphing Window which includes both intercepts:



9. D **10.** D **11.** C **12.** B

13.

4	.	2	

14.

1	1	.	9

Equations of Linear Relations Lesson #2:
Writing Equations Using $y = mx + b$

Review

We have learned that the graph of an equation in the form $y = mx + b$ is a straight line with slope m and y-intercept b.

Using the Form $y = mx + b$ to Write the Equation of a Line

The form $y = mx + b$ can be used to determine the equation of a line when the following information is given:

- the slope of the line
- the y-intercept of the line.

Class Ex. #1

Write the equation of a line passing through the point $(0, 2)$ with slope $\frac{5}{2}$.

y_{int}.

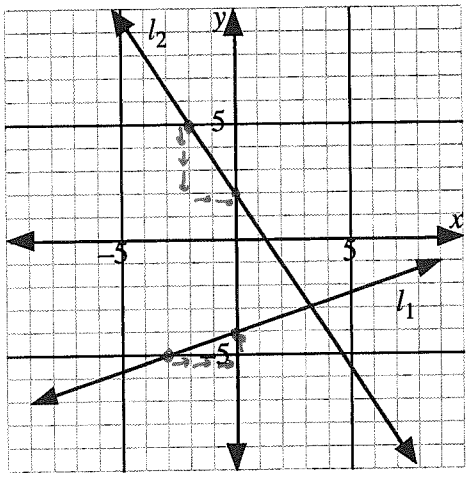
m = slope = $\frac{5}{2}$

$b = y_{int} = 2$

$y = \frac{5}{2}x + 2$

Class Ex. #2

Each line on the grid passes through points with integer coordinates. In each case, state the slope and y–intercept of the line, and determine the equation of the line.



line 1 (l_1):

m = Slope = $\frac{1}{3}$

$b = y_{int} = -4$

$y = \frac{1}{3}x - 4$

line 2 (l_2):

m = slope = $-\frac{3}{2}$

$b = y_{int} = 2$

$y = -\frac{3}{2}x + 2$

Class Ex. #3

Determine the equation of the following lines.

a) The line parallel to $y = \frac{1}{3}x + 4$, and with the same y-intercept as $y = 6x - 7$.

$m = \frac{1}{3}$ $\quad$ $y_{int} = -7$

equation $y = \frac{1}{3}x - 7$

b) The line passing through $(0, 9)$ and perpendicular to the line joining $(2, -6)$ and $(-5, 0)$.

$m = \frac{0 - (-6)}{-5 - 2} = -\frac{6}{7}$

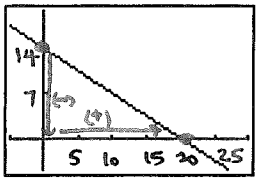
perpendicular slope = $\frac{7}{6}$

$y_{int} = 9$

equation $y = \frac{7}{6}x + 9$

The diagram shows the display from a graphing calculator screen. The intercepts are integers. Determine the equation of the line shown.

```
WINDOW
Xmin=-5
Xmax=30
Xscl=5
Ymin=-5
Ymax=20
Yscl=7
Xres=1
```



m = Slope = $\frac{\text{rise}}{\text{run}} = \frac{-14}{20} = \frac{-7}{10}$

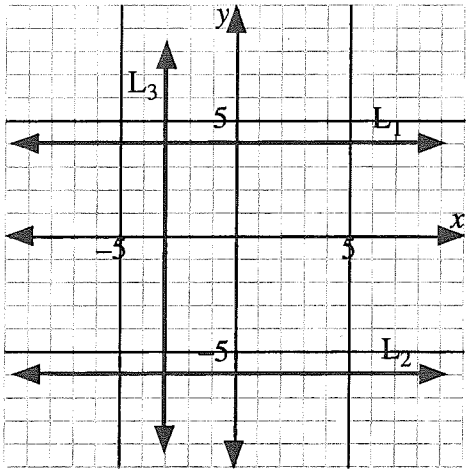
equation: $y = \frac{-7}{10}x + 14$

$b = y_{int} = 14$

Complete Assignment Questions #1 - #5

Horizontal and Vertical Lines

a) State the slope and y-intercept of the horizontal line L_1 shown on the grid.



Slope = 0 $\quad y_{int} = 4$

b) Use the form $y = mx + b$ to determine the equation of the horizontal line L_1.

$y = 0x + 4$

$y = 4$

c) Predict the equation of the horizontal line L_2. Use a graphing calculator to verify.

$y = -6$

d) State the slope and y-intercept of the vertical line L_3 shown on the grid.

Slope is undefined $\quad$ no y-intercept

e) Why can we not use the form $y = mx + b$ to determine the equation of the vertical line L_3?

We do not have a value for m.

f) Predict the equation of the vertical line L_3. Why can we not use a graphing calculator to verify?

$x = -3$ $\quad$ The equation of the line to be graphed needs to be in the form $y = mx + b$.

The equation $y = k$ represents a horizontal line through $(0, k)$.
The equation $x = k$ represents a vertical line through $(k, 0)$.

Determine the equation of the line through the point $(-2, 8)$ and

a) parallel to the y-axis

$x = -2$

b) parallel to the x-axis

$y = 8$

Complete Assignment Questions #6 - #15

Assignment

1. Write the equation of each line

a) with slope 4 and y-intercept -6

$y = 4x - 6$

b) with a y-intercept of 3 and a slope of $-\frac{4}{3}$

$y = -\frac{4}{3}x + 3$

c) passing through the origin with a slope of $-\frac{3}{5}$ $y_{int} = 0$

$y = -\frac{3}{5}x$

d) with y-intercept -5 and parallel to $y = |x$ $m = \text{Slope} = 1$

$y = x - 5$

e) with a y-intercept of -9 and perpendicular to $y = -\frac{2}{3}x + 7$

$m_{\perp} = \text{slope} = \frac{3}{2}$

$y = \frac{3}{2}x - 9$

f) with the same y-intercept as $y = x + 2$ and parallel to $y = \frac{1}{4}x - 6$

$b = y_{int} = 2$

$m = \text{slope} = \frac{1}{4}$

$y = \frac{1}{4}x + 2$

g) through the point $(0, 1)$ and perpendicular to $y = 4x - 2$

$m_{\perp} = \text{perpendicular slope} = -\frac{1}{4}$

$b = y_{int} = 1$

$y = -\frac{1}{4}x + 1$

h) through the point $(0, 4)$ and parallel to $y = \frac{1}{10}x + 24$

$m = \text{parallel slope} = \frac{1}{10}$ $y_{int} = 4$

$y = \frac{1}{10}x + 4$

i) with the same y-intercept as $y = 2x - 3$ and perpendicular to $y = \frac{7}{3}x - 2$

$y_{int} = -3$ $m_{\perp} = -\frac{3}{7}$

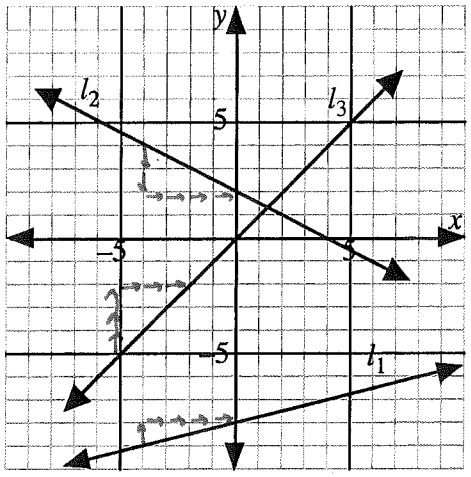
$y = -\frac{3}{7}x - 3$

j) with the same y-intercept as $y = ax + b$ and perpendicular to $y = cx + d$

$y_{int} = b$ $m_{\perp} = -\frac{1}{c}$

$y = -\frac{1}{c}x + b$

2. Every line on the grid passes through points with integer coordinates.
Determine the equation of each line.



line 1 (l_1): slope = $\frac{1}{4}$ $y = \frac{1}{4}x - 8$

$y_{int} = -8$

line 2 (l_2): slope = $\frac{-2}{4} = \frac{-1}{2}$ $y = \frac{-1}{2}x + 2$

$y_{int} = 2$

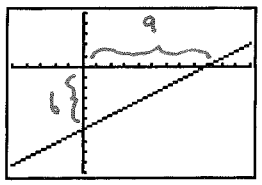
line 3 (l_3): slope = $\frac{3}{3} = 1$ $y = x$

$y_{int} = 0$

3. Each diagram represents the image from the display of a graphing calculator and the window setting used to graph a linear equation. The x- and y-intercepts of each graph are integers.

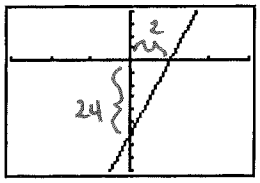
In each case complete the table.

a)
```
WINDOW
Xmin=-5
Xmax=12
Xscl=1
Ymin=-10
Ymax=5
Yscl=1
Xres=1
```



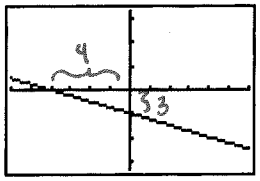
x-intercept	9
y-intercept	-6
slope	$6/9 = 2/3$
equation	$y = \frac{2}{3}x - 6$

b)
```
WINDOW
Xmin=-6
Xmax=6
Xscl=2
Ymin=-36
Ymax=16
Yscl=4
Xres=1
```



x-intercept	2
y-intercept	-24
slope	$\frac{24}{2} = 12$
equation	$y = 12x - 24$

c)
```
WINDOW
Xmin=-6
Xmax=6
Xscl=1
Ymin=-10
Ymax=10
Yscl=3
Xres=1
```



$\frac{\text{rise}}{\text{run}} = \frac{-3}{4}$

x-intercept	-4
y-intercept	-3
slope	$\frac{-3}{4}$ notice how this relates to above values.
equation	$y = -3/4x - 3$

4. Determine the equation of the line which passes through the point $(0, 16)$ and is parallel to the line which passes through $(1, 3)$ and $(4, -6)$.

y int since x=0

$m = \frac{-6-3}{4-1} = \frac{-9}{3} = -3$

parallel slope = -3

$y = -3x + 16$.

$y_{int} = 16$

5. Determine the equation of the line which passes through the point $(0, -1)$ and is perpendicular to the line which passes through $(7, -2)$ and $(12, -3)$.

$y_{int} = -1$

y int since x = 0.

$m = \frac{-3-(-2)}{12-7} = -\frac{1}{5}$

$y = 5x - 1$

perpendicular slope = 5

6. State the equations of the following lines

a) through the point $(-5, 3)$ and parallel to the y-axis $x = -5$

b) through the point $(-5, 3)$ and parallel to the x-axis $y = 3$

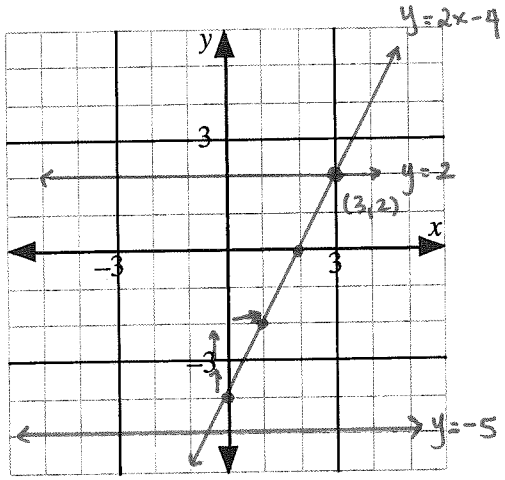
↳ slope = 0

c) through the point $(1, -1)$ and parallel to the x-axis $y = -1$

d) through the point (a, b) and parallel to the y-axis $x = a$

↳ slope is undefined

7. Consider the graph of the function with equation $y = 2$.



a) State the values of m and b.

$m = 0$ $b = 0$

b) Sketch the graph on the grid provided.

c) State the x- and y-intercepts of the graph.

no x-intercepts y-intercept = 2

d) Determine the domain and range of the function.

$D: \{x \mid x \in \mathbb{R}\}$ $R: \{y \mid y \in \mathbb{R}\}$

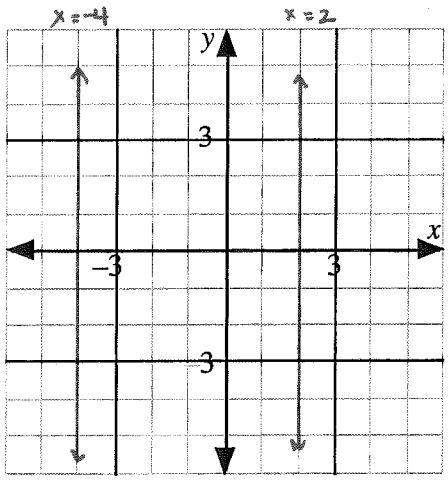
e) On the same grid draw the line with equation $y = 2x - 4$ without using a graphing calculator.

f) State the coordinates of the point of intersection of the two lines.

(3,2)

g) On the grid, draw the line with equation $y = -5$.

8. Consider the graph of the relation with equation $x = -4$.



a) Sketch the graph on the grid provided.

b) State the x- and y-intercepts of the graph.

x-intercept = -4 no y-intercepts

c) Explain why the relation is not a function.

When the input = -4 there are multiple values for the output. The graph of the relation does not pass the vertical line test.

d) Explain why the [Y=] editor key cannot be used to graph $x = -4$.

The equation $x = -4$ cannot be written in the form $y = \ldots$.

e) Determine the domain and range of the relation with equation $x = -4$.

domain $x = -4$ range $y \in \mathbb{R}$.

f) On the grid draw the line with equation $x = 2$.

9. Write the equation of each line

a) parallel to the x-axis through $(3, -9)$ → m = 0, since it is a horizontal line.

$y = -9$

b) parallel to the y-axis through $(3, -9)$

$x = 3$ → m = undefined since it is a vertical line.

c) perpendicular to the x-axis through $(1, 4)$

$x = 1$ m = undefined since it is a vertical line.

d) perpendicular to the y-axis through $(1, 4)$

$y = 4$ → m = 0, since it is a horizontal line

e) the x-axis → m = 0, since it is a horizontal line.

$y = 0$

f) the y-axis

$x = 0$ → m = undefined, since it is a vertical line.

Multiple Choice

10. A line is parallel to the y-axis and passes through the point $(2, -7)$. The equation of the line is → m = undefined

$x = 2$

(A.) $x = 2$
B. $x = -7$
C. $y = 2$
D. $y = -7$

11. A line is parallel to the x-axis and passes through the point $(-6, 10)$. The equation of the line is

↳ horizontal: $m = 0$

A. $x = 10$
B. $x = -6$
C. $y = 10$
D. $y = -6$

$y = 0x + 10$

$y = 10$

12. The line through the origin, perpendicular to the line with equation $y = \frac{2}{3}x$, has equation

A. $y = \frac{2}{3}x$
B. $y = \frac{3}{2}x$
C. $y = -\frac{2}{3}x$
D. $y = -\frac{3}{2}x$

$m_{\perp} = -\frac{3}{2}$

$m = \frac{2}{3}$

$b = 0$

$y = -\frac{3}{2}x + 0$

$y = -\frac{3}{2}x$

13. The point $(2, -1)$ lies on a line with slope 3. The y- intercept of the line is

A. -7
B. -5
C. 5
D. 7

Step 1: $m = 3$, $y = 3x + b$

Step 2: $-1 = 3(2) + b$

$-1 = 6 + b$

$b = -7$

Goal: We need to identify m and b. Problem we must solve for b using point provided.

Numerical Response

14. Consider the line which is perpendicular to the line $y = \frac{1}{3}x + 4$ and has the same y-intercept as $y = 6x - 7$. If the equation of this line is written in the form $y = mx + b$, then the exact value of $m - b$ is _____.

(Record your answer in the numerical response box from left to right)

4			

perpendicular slope $= -3$

y-intercept $= -7$

$y = -3x - 7$ → $m = -3$, $b = -7$

$m - b = -3 - (-7) = 4$

15. Two perpendicular lines intersect on the y-axis. One line has equation $y = 4x + 6$. If the equation of the other line is $y = mx + b$, then the exact value of $m + b$ is _____.

(Record your answer in the numerical response box from left to right)

5	.	7	5

$m = 4$

Perpendicular slope $= -\frac{1}{4}$

to slope $m = 4$

y-intercept $= 6$

equation $y = -\frac{1}{4}x + b$

$m = -\frac{1}{4}$ $b = 6$

$m + b = -\frac{1}{4} + 6$

$= 5.75$

Answer Key

1. **a)** $y = 4x - 6$ **b)** $y = -\frac{4}{3}x + 3$ **c)** $y = -\frac{3}{5}x$ **d)** $y = x - 5$ **e)** $y = \frac{3}{2}x - 9$
f) $y = \frac{1}{4}x + 2$ **g)** $y = -\frac{1}{4}x + 1$ **h)** $y = \frac{1}{10}x + 4$ **i)** $y = -\frac{3}{7}x - 3$ **j)** $y = -\frac{1}{c}x + b$

2. l_1: $y = \frac{1}{4}x - 8$ l_2: $y = -\frac{1}{2}x + 2$ l_3: $y = x$

3. **a)**

x-intercept	9
y-intercept	-6
slope	2/3
equation	$y = \frac{2}{3}x - 6$

b)

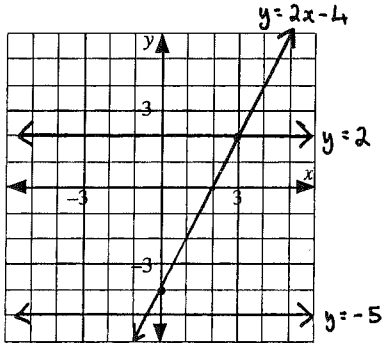
x-intercept	2
y-intercept	-24
slope	12
equation	$y = 12x - 24$

c)

x-intercept	-4
y-intercept	-3
slope	$-3/4$
equation	$y = -\frac{3}{4}x - 3$

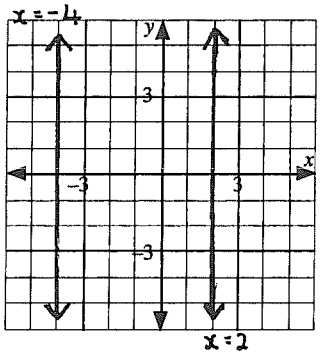
4. $y = -3x + 16$ **5.** $y = 5x - 1$ **6.** **a)** $x = -5$ **b)** $y = 3$ **c)** $y = -1$ **d)** $x = a$

7. **a)** $m = 0, b = 2$
b), e), g) see graph below



c) no *x*-intercept, *y*-intercept = 2
d) domain $x \in R$, range $y = 2$ **f)** (3, 2)

8. **a), f)** see graph below
b) *x*-intercept = –4, no *y*-intercept



c) When the input = –4, there are multiple values for the output. The graph of the relation does not pass the vertical line test.
d) The equation $x = -4$ cannot be written in the form y= . . . .
e) domain $x = -4$, range $y \in R$

9. **a)** $y = -9$ **b)** $x = 3$ **c)** $x = 1$ **d)** $y = 4$ **e)** $y = 0$ **f)** $x = 0$

10. A **11.** C **12.** D **13.** A

14.

4			

15.

5	.	7	5

Equations of Linear Relations Lesson #3:
Writing Equations using $y - y_1 = m(x - x_1)$

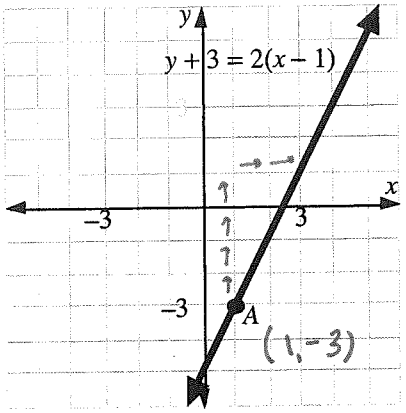
Review

In the last lesson we learned how to write the equation of a straight line using slope y-intercept form, namely $y = mx + b$ where m is the slope and b is the y-intercept.

There is another equally important method exists in writing the equation of straight lines. The investigation below explores this method.

Investigation *Point-Slope Form*

The graph of $y + 3 = 2(x - 1)$ is shown on the grid.



a) Determine the slope of the graph of $y + 3 = 2(x - 1)$. $m = \frac{4}{2} = 2$

b) List the coordinates of point A on the line. $(1, -3)$

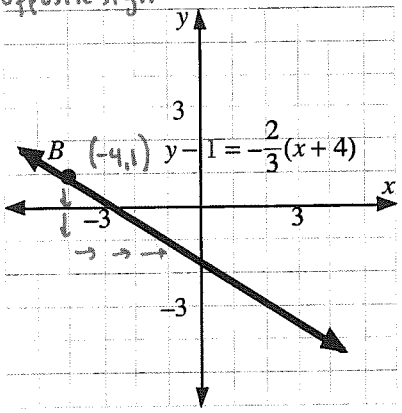
c) Compare your answers in **a)** and **b)** with the numbers in the equation.

$y + 3 = 2(x - 1)$ → slope has same value.

opposite sign of y-coor. same value as x-coordinate, but opposite sign

Thus, the equation is of the form. $y - y_A = m(x - x_A)$.

The graph of $y - 1 = -\frac{2}{3}(x + 4)$ is shown on the grid.



d) Determine the slope of the graph of $y - 1 = -\frac{2}{3}(x + 4)$. $m = \frac{-2}{3}$

e) List the coordinates of point B on the line. $(-4, 1)$

f) Compare your answers in **d)** and **e)** with the numbers in the equation. $y - 1 = \frac{-2}{3}(x + 4)$

slope same.

y same value, but opposite sign

x-coordinate value same, but opposite sign.

g) Consider the graph of the linear equation $y - y_1 = m(x - x_1)$. Based on your observations in **c)** and **f)**, state the slope of the line, and write the coordinates of one point on the line.

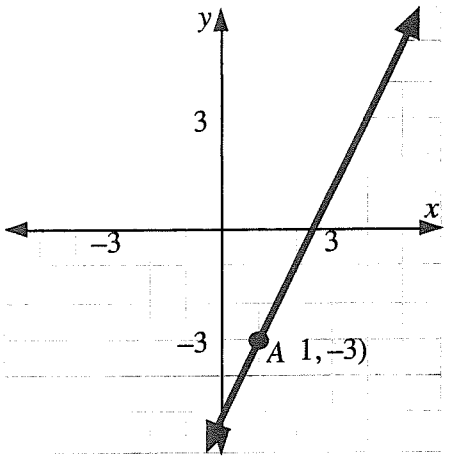
Thus, the equation is of the form $y - y_B = m(x - x_B)$.

slope $= m$

point (x_1, y_2)

Equation of a Line Given the Slope of the Line and a Point on the Line

Consider the line with slope 2 passing through the point $A(1, -3)$. The line is shown on the grid.



Our objective is to determine the equation of the line. In other words, to find a relation between x and y which is satisfied by every point (x, y) on the line.

Let $P(x, y)$ be any point on the line except A.

Using the slope formula we have

$$\frac{y_P - y_A}{x_P - x_A} = m_{AP} \qquad \frac{y-(-3)}{x-1} = 2$$

Cross multiply and solve for y to determine the equation of the line in the form $y = mx + b$.

$$\frac{y+3}{x-1} = \frac{2}{1} \qquad y+3 = 2(x-1) \qquad y+3 = 2x-2 \qquad y = 2x-5$$

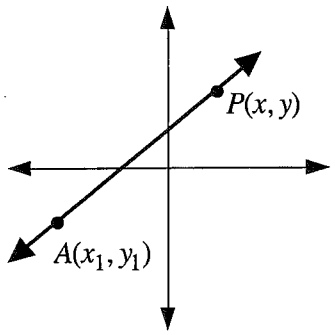
At this point in the exploration, the equation above is valid for all points on the line except A.

Note that the coordinates of A also satisfy the equation, and therefore it is the equation of all points on the line.

In the next section we will use the same procedure to develop a formula for the equation of any line, given the slope of the line and the point on the line.

The Equation of the Line with slope m through the point (x_1, y_1)

Consider the line with slope m passing through the point with coordinates (x_1, y_1).
We will use the same procedure as above to show that the equation of the line can be expressed in the form $y - y_1 = m(x - x_1)$.



Let $P(x, y)$ be any point on the line distinct from A.

Using the slope formula we have

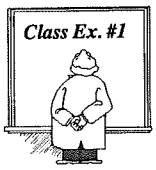
$$m_{AP} = \frac{y_P - y_A}{x_P - x_A} \quad \text{so} \quad m = \frac{y - y_1}{x - x_1}$$

$$m(x - x_1) = y - y_1 \qquad y - y_1 = m(x - x_1)$$

Point-Slope Equation of a Line $\rightarrow$ $y - y_1 = m(x - x_1)$

- The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$ where m is the slope of the line, and (x_1, y_1) represents a point on the line.
- To determine the equation of a line in future math courses, the point-slope equation, $y - y_1 = m(x - x_1)$, is used more frequently than the slope-y-intercept equation, $y = mx + b$.

- The point-slope equation is used when we have **the slope of a line and the coordinates of any point on that line.**
- It is customary to give the final equation in slope y-intercept form or in the general form, $Ax + By + C = 0$ (to be taught in the next lesson).



State the equation, in point-slope form, of the line through the given point and with the given slope.

a) $(6,5)$, 3

$y - 5 = 3(x - 6)$

b) $(1,-1)$, -4

$y - (-1) = -4(x - 1)$

$y + 1 = -4(x - 1)$

c) $(-9,-8)$, $\frac{1}{2}$

$y - (-8) = \frac{1}{2}(x - (-9))$

$y + 8 = \frac{1}{2}(x + 9)$

In each case the slope of a line and a point on the line are given. Determine the equation of the line in slope y-intercept form, $y = mx + b$.

a) $m = 5$, point $(-5, 2)$

$y - 2 = 5(x - (-5))$

$y - 2 = 5(x + 5)$

$y - 2 = 5x + 25$

$y = 5x + 27$

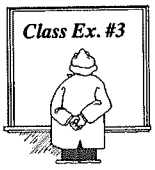
b) $m = -7$, point $(-3, 4)$

$y - 4 = -7(x - (-3))$

$y - 4 = -7(x + 3)$

$y - 4 = -7x - 21$

$y = -7x - 17$



John and Nicki were solving the following quiz question:

" Determine the equation, in slope y-intercept form, of a line with slope -2 passing through the point $(3, -5)$".

$m = -2$

John could only remember the slope y-intercept form $y = mx + b$, but Nicki remembered the point-slope form $y - y_1 = m(x - x_1)$. Complete their work which is started below.

John's work

$y = mx + b$

$y = -2x + b$

$-5 = -2(3) + b$

$-5 = -6 + b$

$1 = b$

$\boxed{b = 1}$ $\longrightarrow$ $y = -2x + 1$

Nicki's work

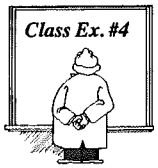
$y - y_1 = m(x - x_1)$

$y - y_1 = -2(x - x_1)$

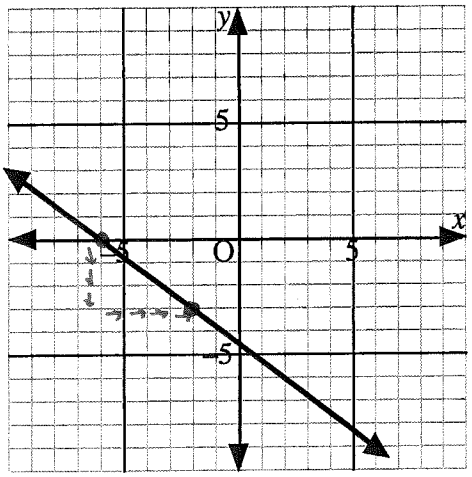
$y - (-5) = -2(x - 3)$

$y + 5 = -2x + 6$

$y = -2x + 1$



The line on the grid passes through at least two points with integer coordinates. Determine the equation of the line in slope y-intercept form.



Point: $(-6,0)$ or $(-2,-3)$

easier point since zero is included!

In order to use point slope form we need:
1 point
1 slope.

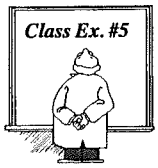
stop! We can solve for slope or get it from graph. $m = -\frac{3}{4}$

$y - y_1 = m(x - x_1)$

$4(y - 0) = \left(-\frac{3}{4}(x + 6)\right)4$ → $4(y-0) = -3(x+6)$

$4y = -3x - 18$

$y = \frac{-3}{4}x - \frac{9}{2}$



In each case, state the slope of the line and write the coordinates of a point on the line.

a) $y + 11 = \frac{1}{7}(x - 4)$

slope $= \frac{1}{7}$ Point $(4,-11)$

b) $y - 9 = -\frac{5}{3}(x - 7)$

slope $= -\frac{5}{3}$ Point $(7,9)$

c) $y = -3(x - 6)$

slope $= -3$ point $(6,0)$

d) $y = -3x - 6$

slope $= -3$ Point $(0,-6)$

Complete Assignment Questions #1 - #9

Assignment

1. State the equation, in point-slope form, of the line through the given point and with the given slope.

a) $(9,3)$, 4

$y - 3 = 4(x - 9)$

b) $(8,-2)$, -3

$y + 2 = -3(x - 8)$

c) $(-5,7)$, 1

$y - 7 = 1(x + 5)$

d) $(0,3)$, $\frac{1}{2}$

$y - 3 = \frac{1}{2}(x - 0)$

$y - 3 = \frac{1}{2}x$

e) $(-7,0)$, $\frac{1}{4}$

$y - 0 = \frac{1}{4}(x + 7)$

$y = \frac{1}{4}(x + 7)$

f) $(-\frac{1}{2},-\frac{5}{4})$, $\frac{6}{5}$

$y + \frac{5}{4} = \frac{6}{5}(x + \frac{1}{2})$

Think about it!
Really all we are doing here is expanding and simplifying here with a final step of isolating for y.

2. Write the following equations in slope y-intercept form $y = mx + b$.

a) $y + 1 = 8(x - 2)$

$y + 1 = 8x - 16$

$-1 \quad -1$

$y = 8x - 17$

b) $y - 3 = -2(x - 7)$

$y - 3 = -2x + 14$

$+3 \quad +3$

$y = -2x + 17$

c) $y - 9 = -11(x + 3)$

$y - 9 = -11x - 33$

$+9 \quad +9$

$y = -11x - 24$

3. Find the equation, in slope y-intercept form, of the line through the given point and with the given slope.

a) $(2,4)$, 6

$y-4=6(x-2)$

$y-4=6x-12$

$+4 \quad +4$

$y=6x-8$

b) $(2,-1)$, 2

$y+1=2(x-2)$

$y+1=2x-4$

$-1 \quad -1$

$y=2x-5$

c) $(0,4)$, -2

$y-4=-2(x-0)$

$y-4=-2x$

$+4 \quad +4$

$y=-2x+4$

d) $(-6,2)$, $\frac{1}{2}$

$y-2=\frac{1}{2}(x+6)$

$y-2=\frac{1}{2}x+3$

$+2 \quad +2$

$y=\frac{1}{2}x+5$

e) $(-7,-7)$, 1

$y+7=1(x+7)$

$y+7=x+7$

$-7 \quad -7$

$y=x$

f) $(0,b)$, m

$y-b=m(x-0)$

$y-b=mx$

$+b \quad +b$

$y=mx+b$.

4. Find the equation, in slope y-intercept form, of the line through the given point and with the given slope.

a) $(2,-5)$, $\frac{1}{4}$

$4(y+5)=\left[\frac{1}{4}(x-2)\right]4$

$4(y+5)=1(x-2)$

$4y+20=x-2$

$-20 \quad -20$

$\frac{4y}{4}=\frac{x-22}{4}$

$y=\frac{1}{4}x-\frac{11}{2}$

b) $(-4,2)$, $-\frac{1}{3}$

$3(y-2)=\left[-\frac{1}{3}(x+4)\right]3$

$3(y-2)=-1(x+4)$

$3y-6=-x-4$

$+6 \quad +6$

$\frac{3y}{3}=\frac{-x+2}{3}$

$y=-\frac{1}{3}x+\frac{2}{3}$

c) $(0,-8)$, $-\frac{3}{4}$

$4(y+8)=\left[-\frac{3}{4}(x-0)\right]4$

$4(y+8)=-3(x)$

$4y+32=-3x$

$-32 \quad -32$

$\frac{4y}{4}=\frac{-3x-32}{4}$

$y=-\frac{3}{4}x-8$

Unlike question 3d) these question must have the denominators simplified through multiplication! Otherwise, it is time-consuming to solve!

5. The point-slope equation of a line is given. State the slope and the coordinates of the point which were used to write the equation.

a) $y-9=-\frac{11}{3}(x+3)$

Slope $=-\frac{11}{3}$ point $(-3,9)$

b) $y+3=\frac{1}{2}x$

Slope $=\frac{1}{2}$ point $(0,-3)$

c) $y-8=-2(x-6)$

slope $=-2$ point $(6,8)$

d) $y=3(x+12)$

$y-0$

Slope $=3$ point $(-12,0)$

e) $y-9=-\frac{5}{3}x$

Slope $=-\frac{5}{3}$ point $(0,9)$

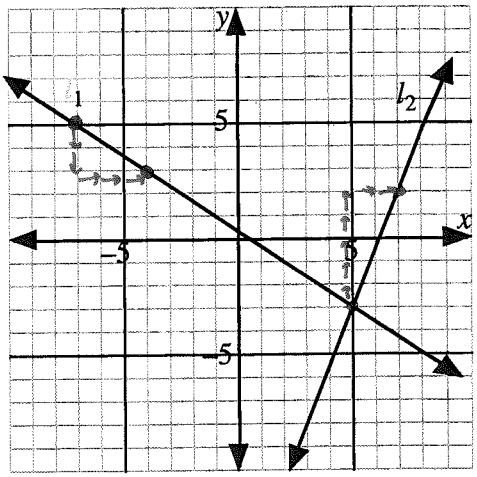
f) $y=\frac{2}{5}(x-0)$

$y-0$

Slope $=\frac{2}{5}$ point $(0,0)$

6. Two lines have been drawn on the grid. Each line passes through at least two points with integer coordinates.

Determine the equation of each line.



line 1 (l_1): Point $(-1, 1)$ $m = -\frac{2}{3}$

$3(y-1) = \left(-\frac{2}{3}(x+1)\right)3$

$3(y-1) = -2(x+1)$

$3y - 3 = -2x - 2$

$+3 \qquad +3$

$\frac{3y}{3} = \frac{-2x+1}{3}$

$y = -\frac{2}{3}x + \frac{1}{3}$

line 2 (l_2): Point $(5, -3)$ $m = \frac{5}{2}$

$2(y+3) = \left(\frac{5}{2}(x-5)\right)2$

$2y + 6 = 5x - 25$

$-6 \qquad -6$

$\frac{2y}{2} = \frac{5x}{2} - \frac{31}{2}$

$y = \frac{5}{2}x - \frac{31}{2}$

Multiple Choice

7. The equation of the line passing through the point $(4, 2)$ with slope -3 is

A. $y = -3x - 14$

B. $y = -3x - 10$

C. $y + 2 = -3(x + 4)$

D. $y - 2 = -3(x - 4)$

$y - 2 = -3(x - 4)$

8. Which of the following linear equations is equivalent to $\left(y - 3 = \left(-\frac{3}{4}(x+7)\right)4\right)4$?

A. $y = -\frac{3}{4}x + \frac{9}{4}$

B. $y = -\frac{3}{4}x - \frac{9}{4}$

C. $y = -\frac{3}{4}x + 10$

D. $y = -\frac{3}{4}x - 10$

$4y - 12 = -3x - 21$

$+12 \qquad +12$

$\frac{4y}{4} = \frac{-3x - 9}{4}$

$y = -\frac{3}{4}x - \frac{9}{4}$

Numerical Response **9.** The equation of the line with an x-intercept of -2 and slope 12 can be written in the form $y - A = C(x - B)$. The value of $A + B + C$ is _____.

(Record your answer in the numerical response box from left to right)

1	0		

Given:

$m = 12$

$P(-2, 0)$ ← x_{int}.

$y - 0 = 12(x + 2)$

$A = 0 \quad B = 12 \quad C = -2$

Let $A = 0$, $B = 12$, $C = -2$

$A + B + C = 0 + 12 - 2 = 10$

Answer Key

1. **a)** $y - 3 = 4(x - 9)$ **b)** $y + 2 = -3(x - 8)$ **c)** $y - 7 = 1(x + 5)$
d) $y - 3 = \frac{1}{2}x$ **e)** $y = \frac{1}{4}(x + 7)$ **f)** $y + \frac{5}{4} = \frac{6}{5}\left(x + \frac{1}{2}\right)$

2. **a)** $y = 8x - 17$ **b)** $y = -2x + 17$ **c)** $y = -11x - 24$

3. **a)** $y = 6x - 8$ **b)** $y = 2x - 5$ **c)** $y = -2x + 4$ **d)** $y = \frac{1}{2}x + 5$ **e)** $y = x$ **f)** $y = mx + b$

4. **a)** $y = \frac{1}{4}x - \frac{11}{2}$ **b)** $y = -\frac{1}{3}x + \frac{2}{3}$ **c)** $y = -\frac{3}{4}x - 8$

5. **a)** $m = -\frac{11}{3}$, $P(-3, 9)$ **b)** $m = \frac{1}{2}$, $P(0, -3)$ **c)** $m = -2$, $P(6, 8)$ **d)** $m = 3$, $P(-12, 0)$
e) $m = -\frac{5}{3}$, $P(0, 9)$ **f)** $m = \frac{2}{5}$, $P(0, 0)$

6. $l_1 \Rightarrow 2x + 3y - 1 = 0$ or $y = -\frac{2}{3}x + \frac{1}{3}$ $l_2 \Rightarrow 5x - 2y - 31 = 0$ or $y = \frac{5}{2}x - \frac{31}{2}$

7. D **8.** B **9.**

1	0		

Equations of Linear Relations Lesson #4:
The General Form Equation $Ax + By + C = 0$

Review

In Lesson #2 and Lesson #3 of this unit we studied two forms of the equation of a straight line. The form of these equations are

- the slope y-intercept form which can also be writen as _____
- the point-slope form which can also be written as _____

General Form of the Equation of a Line $\rightarrow Ax + By + C = 0$

The **general form** of the equation of a line is an equation where all the terms are collected to the left side of the equation. The right side of the equation is zero.
It has the following characteristics:

- It is written as $Ax + By + C = 0$, where A, B, and C are expressed as **integers** if possible, and A is usually positive.
- It allows us to write equations for oblique lines, horizontal lines, and vertical lines.
- In some texts, the form $Ax + By + C = 0$ is referred to as **standard form**.

The following equations are written in either point slope form or slope y-intercept form. Convert the equations to general form, $Ax + By + C = 0$, where A, B, and C are integers.

a) $y + 2 = \frac{1}{2}(x - 3)$

$2(y+2) = \left[\frac{1}{2}(x-3)\right]2$

$2(y+2) = 1(x-3)$

$2y + 4 = x - 3$

$x - 2y - 7 = 0$

b) $y = \frac{2}{3}x + 7$

$3(y) = \left(\frac{2}{3}x + 7\right)3$

$3y = 2x + 7(3)$

$3y = 2x + 21$

$2x - 3y + 21 = 0$

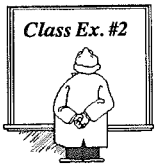
c) $y = -\frac{1}{4}x + \frac{3}{5}$

$20(y) = \left(-\frac{1}{4}x + \frac{3}{5}\right)20$

$20y = (5)(-1x) + (4)(3)$

$20y = -5x + 12$

$5x + 20y - 12 = 0$



Determine the equation, in general form, of the line with slope $-\frac{3}{7}$ passing through $(2, -1)$.

$y + 1 = -\frac{3}{7}(x - 2)$

$7(y+1) = \left(-\frac{3}{7}(x-2)\right)7$

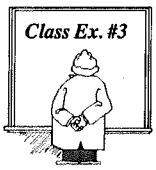
$7(y+1) = -3(x-2)$

$7y + 7 = -3x + 6$

$3x + 7y + 1 = 0$

Determining the Slope and y-intercept from Ax + By + C = 0

Given the equation of a line in general form, $Ax + By + C = 0$, the slope and y-intercept can be found by converting the equation into slope y-intercept form, $y = mx + b$.



Determine the slope and y-intercept of the graph of the following lines.

Goal: Convert into slope y-intercept form, since these are the values in which we are solving for.

a) $2x - 5y + 25 = 0$

$-2x \quad -2x$

$-5y = -2x - 25$

$\frac{-5y}{-5} = \frac{-2x - 25}{-5}$

$y = \frac{2}{5}x + 5$

Slope $= \frac{2}{5}$ $y_{int} = 5$

b) $6x + 2y - 15 = 0$

$-6x \quad -6x$

$2y - 15 = -6x$

$+15 \quad +15$

$\frac{2y}{2} = \frac{-6x + 15}{2}$

$y = -3x + \frac{15}{2}$

Slope $= -3$ $y_{int} = \frac{15}{2}$

Complete Assignment Questions #1 - #4

The lines $3x - 4y + 8 = 0$ and $5x - ky - 6 = 0$ have the same y-intercept. Determine the value of k.

Goal: Convert into slope y-intercept form to isolate both y-intercepts. This will allow us to let $y_{int_1} = y_{int_2}$.

$3x - 4y + 8 = 0$

$+4y \quad +4y$

$\frac{3x + 8}{4} = \frac{4y}{4}$

$y = \frac{3}{4}x + 2$

y_{int_1}

$5x - ky - 6 = 0$

$+ky \quad +ky$

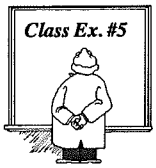
$\frac{5x - 6}{k} = \frac{ky}{k}$

$y = \frac{5}{k}x - \frac{6}{k}$ ← y_{int_2}

$y_{int_1} = y_{int_2}$

$\frac{2}{1} = \frac{-6}{k}$

$\frac{2k}{2} = \frac{-6}{2}$ $k = -3$



Which of the following lines is/are perpendicular to the line $4x - 2y + 9 = 0$?

i) $6x + 3y - 1 = 0$ **ii)** $x + 2y - 12 = 0$ **iii)** $5x + 10y = 0$

$3y = -6x + 1$

$y = -2x + \frac{1}{3}$

Slope $= -2$

$2y = -x + 12$

$y = -\frac{1}{2}x + 6$

Slope $= -\frac{1}{2}$

$10y = -5x$

$y = -\frac{1}{2}x$

Slope $= -\frac{1}{2}$

Recall: In order to be perpendicular to $4x - 2y + 9 = 0$ the slopes must be negative reciprocals to one another!

Given Line: $4x - 2y + 9 = 0$

$4x + 9 = 2y$

$y = 2x + \frac{9}{2}$

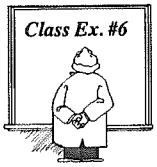
Slope $= 2$

Thus, ii and iii are perpendicular because $-\frac{1}{2}$ is a negative reciprocal of 2.

Use the following information to answer Class Ex. #6.

A student made the following statements about the line with equation $2y = 5x + 12$.

Statement 1: The line has a slope of 5.
Statement 2: The line is parallel to $10x - 4y + 13 = 0$.
Statement 3: The line passes through $(-2, 1)$.



Which of the above statement(s) is/are true?

A. 1 and 2 only
B. 1 and 3 only
C. 2 and 3 only
D. some other combination of statements 1, 2, and 3

$2y = 5x + 12$

$y = \frac{5}{2}x + 6$

Slope $= \frac{5}{2}$ | $y_{int} = 6$

Statement 1 is false.

$10x - 4y + 13 = 0$

$10x + 13 = 4y$

$y = \frac{5}{2}x + \frac{13}{4}$

Slope $= \frac{5}{2}$

Statement 2 is True.

$2y$	$5x + 12$
$2(1)$	$5(-2) + 12$
2	$-10 + 12$
	2

LS = RS

Statement 3 is true.

Complete Assignment Questions #5 - #16

Assignment

1. Convert the following equations to general form $(Ax + By + C = 0)$ where A, B, and C are integers.

a) $y - 4 = 7(x - 1)$

$y - 4 = 7x - 7$

$-y \quad +4 \qquad -y \quad +4$

$7x - y - 3 = 0$

b) $y = -2x + 9$

$-9 \quad +2x \qquad +2x \quad -9$

$2x + y - 9 = 0$

c) $y = mx + b$

$-y \qquad -y$

$mx - y + b = 0$

d) $4(y) = \left(-\frac{3}{4}x + 5\right)4$

$4y = -3x + 20$

$3x + 4y - 20 = 0$

e) $2(y + 8) = \left[-\frac{3}{2}(x - 5)\right]2$

$2(y + 8) = -3(x - 5)$

$2y + 16 = -3x + 15$

$+3x \quad -15 \qquad +3x \quad -15$

$3x + 2y + 1 = 0$

f) $12(y) = \left(\frac{5}{3}x - \frac{1}{4}\right)12$

12 is the LCM of 3 and 4

$12y = 4(5x) - 3(1)$

$12y = 20x - 3$

$20x - 12y - 3 = 0$

2. Find the equation, in general form, of the line through the given point and with the given slope.

a) $(6,1)$, 3

$y - y_1 = m(x - x_1)$

$y - (1) = 3(x - (6))$

$y - 1 = 3(x - 6)$

$y - 1 = 3x - 18$

$-y + 1 \quad -y + 1$

$0 = 3x - y - 17$

$\boxed{3x - y - 17 = 0}$

b) $(-9,-2)$, $\frac{2}{5}$

$y - (-2) = \frac{2}{5}(x - (-9))$

$5(y+2) = 5\left(\frac{2}{5}\right)(x+9)$

$5(y+2) = 2(x+9)$

$5y + 10 = 2x + 18$

$-5y - 10 \quad -5y - 10$

$0 = 2x - 5y + 8$

$\boxed{2x - 5y + 8 = 0}$

c) $(0,0)$, $\frac{4}{3}$

$y - (0) = \frac{4}{3}(x - (0))$

$y - 0 = \frac{4}{3}(x - 0)$

$y = \frac{4}{3}x$

$3(y) = 3\left(\frac{4}{3}x\right)$

$3y = 4x$

$-3y \quad -3y$

$0 = 4x - 3y$

$\boxed{4x - 3y = 0}$

Think about it! This question could not be more clear! It is asking you to solve for the slope and the y-intercept. We have a form for that!!

3. Determine the slope and y-intercept of the graph of the following lines.

a) $x + y - 11 = 0$

$y = -1x + 11$

$m = \text{slope} = -1$

$b = y_{int} = 11$

b) $3x - 2y + 30 = 0$

$3x + 30 = 2y$

$y = \frac{3}{2}x + 15$

$m = \frac{3}{2}$

$b = 15$

c) $3x + 6y - 7 = 0$

$6y = -3x + 7$

$y = -\frac{1}{2}x + \frac{7}{6}$

$m = -\frac{1}{2}$

$b = 7/6$

4. Determine the slope, y-intercept, and x-intercept of the graph of the following lines.

a) $2x + y - 6 = 0$

b) $5x - 2y + 20 = 0$

c) $4x - 5y - 3 = 0$

Step 1:

a) $y = -2x + 6$

y_{int}: $b = 6$

Slope: $m = -2$

b) $5x + 20 = 2y$

$y = \frac{5}{2}x + 10$

$b = 10$

$m = \frac{5}{2}$

c) $4x - 3 = 5y$

$y = \frac{4}{5}x - \frac{3}{5}$

$b = -\frac{3}{5}$

$m = \frac{4}{5}$

Step 2

x_{int}:

a) Let $y = 0$

$2x + (0) - 6 = 0$

$2x = 6$

$x = 3$ $\quad x_{int} = 3$

b) Let $y = 0$

$5x - 2(0) + 20 = 0$

$5x = -20$

$x = -4$ $\quad x_{int} = -4$

c) Let $y = 0$

$4x - 5(0) - 3 = 0$

$4x = 3$

$x = \frac{3}{4}$ $\quad x_{int} = \frac{3}{4}$

5. Consider the lines $x - 2y + 1 = 0$ and $4x + ky - 8 = 0$.

a) Determine the value of k if the lines have the same slope.

b) Determine the value of k if the lines have the same y-intercept.

Goal: Convert into slope y-intercept form, allowing us to solve for slope and then let them equal each other.

$y_{int} = \frac{1}{2}$ $\quad$ $y_{int} = \frac{8}{k}$

$x - 2y + 1 = 0$

$x + 1 = 2y$

$y = \frac{1}{2}x + \frac{1}{2}$

Slope $= \frac{1}{2}$

$4x + ky - 8 = 0$

$ky = -4x + 8$

$y = \frac{-4}{k}x + \frac{8}{k}$

Slope $= -\frac{4}{k}$

$\frac{1}{2} = -\frac{4}{k}$

$k = -8$

$\frac{1}{2} = \frac{8}{k}$

$k = 16$

6. Consider the lines $3x - 5y - 15 = 0$ and $ax + 2y - 6 = 0$.

a) Determine the value of a if the lines have the same slope.

b) Determine the value of a if the lines have the same x-intercept.

Goal: Convert into $y = mx + b$ to identify the slopes of each line, letting them equal each other to solve for a.

$3x - 5y - 15 = 0$

$3x - 15 = 5y$

$y = \frac{3}{5}x - 3$

Slope $= \frac{3}{5}$

$ax + 2y - 6 = 0$

$2y = -ax + 6$

$y = -\frac{a}{2}x + 3$

Slope $= -\frac{a}{2}$

$\frac{3}{5} = -\frac{a}{2}$

$6 = -5a$

$a = -\frac{6}{5}$

$3x - 5y - 15 = 0$ Let $y = 0$

$3x - 5(0) - 15 = 0$

$3x = 15$

$x = 5$

$x_{int} = 5$

$ax + 2y - 6 = 0$ Let $y = 0$

$ax + 2(0) - 6 = 0$

$ax = 6$

$x = \frac{6}{a}$

$x_{int} = \frac{6}{a}$

$5 = \frac{6}{a}$ $\quad a = \frac{6}{5}$

Multiple Choice **7.** The equation of the line passing through the orgin with slope $-\frac{1}{2}$ is

(0,0) point $\quad$ slope.

A. $x + 2y = 0$

B. $x - 2y = 0$

C. $2x + y = 0$

D. $2x - y = 0$

$y - 0 = -\frac{1}{2}(x - 0)$

$2(y) = \left(-\frac{1}{2}x\right)2$

$2y = -x$

$x + 2y = 0$

Recall: The x term should be positive and there are no denominators!

8. Match each equation on the left with the correct characteristic of the graph of the equation on the right. Each characteristic may be used once, more than once, or not at all.

Note: Just by converting i-v to $y=mx+b$ form we can address parts A, B, D, E, and F.

Still we must check for C: Point $(-10,-4)$ and G: x-intercept $\frac{5}{2}$.

Equation		Characteristic	
i)	$6x-2y+5=0$ E	**A.**	Slope $=-\frac{1}{3}$
ii)	$2x-5y=0$ C	**B.**	y-intercept $=-\frac{5}{2}$
iii)	$x+3y+6=0$ A	**C.**	Passes through $(-10,-4)$
iv)	$x-4y+10=0$ E	**D.**	Slope $=0$
v)	$2x-y-5=0$ G	**E.**	y-intercept $=\frac{5}{2}$
		F.	Perpendicular to $y=\frac{5}{2}x-3$ ↳ perpendicular $m=-\frac{2}{5}$
		G.	x-intercept $=\frac{5}{2}$

i) $6x+5=2y$ → $y=3x+\frac{5}{2}$ → $m=3$, $y\text{-int}=b=\frac{5}{2}$

Check
C: $y=3(-10)+\frac{5}{2}=-\frac{55}{2}$ ✓✓
G: $6x-2(0)+5=0$, $6x=5$, $x=\frac{5}{6}$ ✓✓

ii) $2x-5y=0$ → $y=\frac{2}{5}x$ → $m=\frac{2}{5}$, $y\text{-int}=0$

Check
C: $y=\frac{2}{5}(-10)=-4$ ✓✓
G: $2x-5(0)=0$, $x=0$ ✓✓

iii) $x+3y+6=0$ → $3y=-x-6$ → $y=-\frac{1}{3}x-2$ → $m=-\frac{1}{3}$, $b=-2$

Check
C: $y=-\frac{1}{3}(-10)-2=\frac{4}{3}$ ✓✓
G: $x+3(0)+6=0$, $x=-6$. ✓✓

iv) $x-4y+10=0$ → $y=\frac{1}{4}x+\frac{5}{2}$ → $m=\frac{1}{4}$, $b=\frac{5}{2}$

Check
C. $(-10,0)$ ✓✓
G: $x_{int}=-10$ ✓✓

v. $2x-y-5=0$ → $y=2x-5$ → $m=2$, $b=-5$

Check.
C. $(-10,-25)$ ✓✓
G. $0=2x-5$, $5=2x$, $x_{int}=\frac{5}{2}$ ✓✓

9. The slope of the line with equation $6x+5y-1=0$ is

A. $-\frac{6}{5}$ ← Slope.

B. $-\frac{5}{6}$

C. $\frac{6}{5}$

D. $\frac{1}{5}$

$5y=-6x+1$

$y=-\frac{6}{5}x+\frac{1}{5}$ } Convert to slope y-int. form!

10. Which line has a y-intercept of 1?

↳ convert all to slope y-int. form!!

A. $x+5y+1=0$ → $y=-\frac{1}{5}x-\frac{1}{5}$, $b=-\frac{1}{5}$ ✗

B. $x+3y+3=0$ → $y=-\frac{1}{3}x-1$, $b=-1$ ✗

C. $x-2y+2=0$ → $y=\frac{1}{2}x+1$, $b=1$ ✓

D. $2y=3x+1$ → $y=\frac{3}{2}x+\frac{1}{2}$, $b=\frac{1}{2}$ ✗

11. The slope of a line perpendicular to the line $x + 3y + 8 = 0$ is

A. -8

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. 3

$3y = -x - 8$

$y = -\frac{1}{3}x - \frac{8}{3}$

Slope $= -\frac{1}{3}$ ⊥ Slope $= 3$

negative reciprocal

12. The line $2y + 3x + 6 = 0$ intersects the y-axis at P.
The slope of the line joining P to $Q(6, -2)$ is

A. $-\frac{5}{6}$

B. $\frac{1}{6}$

C. $-\frac{1}{6}$

D. $-\frac{2}{3}$

$2y = -3x - 6$

$y = -\frac{3}{2}x - 3$

$y_{int} = -3$

$P(0, -3)$

y_{int} occurs when $x = 0$

$y = -\frac{3}{2}(0) - 3$

$y = -3$

$m_{PQ} = \frac{-2 - (-3)}{6 - 0}$

$= \frac{1}{6}$

13. The lines with equations $ay = 4x + 9$ and $y = 5x - 7$ are perpendicular.
The value of a is

A. $\frac{4}{5}$

B. $-\frac{4}{5}$

C. $-\frac{5}{4}$

D. -20

Goal: Convert both equations to $y = mx + b$ form, allowing us to isolate for m. Given that the slopes are perpendicular we should be able to use $(m)(-\frac{1}{m}) = -1$ to solve for a.

$y = \frac{4}{a}x + \frac{9}{9}$

Slope $= \frac{4}{a}$

Slope $= 5$

$(\frac{4}{a})(5) = -1$

$\frac{20}{a} = -1$

$20 = -a$

$a = -20$

Use the following information to answer the next question.

Consider the following statements about all the lines in the form $kx + 4y - 8 = 0$, where $k \in R$.

Statement 1: The lines have the same slope. ✗
Statement 2: The lines have the same y-intercept. ✓
Statement 3: The lines have the same x-intercept. ✗

14. Which of the above statement(s) is/are true?

A. 1, 2, and 3

B. 1 only

C. 2 only

D. 3 only

$4y = -kx + 8$

$y = -\frac{k}{4}x + 2$

$m = \text{slope} = -\frac{k}{4}$

$b = y_{int} = 2$

x_{int}: Let $y = 0$

$kx + 4(0) - 8 = 0$

$kx - 8 = 0$

$x = \frac{8}{k}$

Think about it! $m = -\frac{k}{4}$, which has a variable in it allowing the slope to change.

Think about it! The x_{int} also has a variable!!

$b = 2$ with no variable therefore no change

15. Line L has equation $5x-3y+21=0$. A is the point $(-6,-3)$, B is $(3,-2)$, and C is $(-3,2)$. Which of these points lie on line L?

A. A only
B. A and B only
C. A and C only
D. B and C only

$5x-3y+21$	0
$5(-6)-3(-3)+21$	
$-30+9+21$	
0	0

LS = RS ✓✓

$5x-3y+21$	0
$5(3)-3(-2)+21$	
$15+6+21$	
42	0

LS ≠ RS ✗✗

$5x-3y+21$	0
$5(-3)-3(2)+21$	
$-15-6+21$	
0	0

LS = LS ✓✓

Numerical Response **16.** Given that the line joining the points $(2,3)$ and $(8,-q)$, where $q \in W$, is perpendicular to the line $3x-2y-5=0$, then the value of q is _____ .

(Record your answer in the numerical response box from left to right)

1			

Goal: We want to apply the perpendicular slope theory $(m)\left(-\frac{1}{m}\right)=1$.
So let us solve for the slopes in both cases as our first step!

Step 1:

$$m=\frac{-q-3}{8-2}=\boxed{\frac{-q-3}{6}}$$

Step 2: $3x-2y-5=0$

$3x-5=2y$

$y=\frac{3}{2}x-\frac{5}{2}$

Slope $=\boxed{\frac{3}{2}}$

Step 3:

$$\left(\frac{-q-3}{6}\right)\left(\frac{3}{2}\right)=-1$$

$\frac{3(-q-3)}{12}=-1$

$3(-q-3)=-12$

$-3q-9=-12$

$3=3q$

$q=1$

Answer Key

1. **a)** $7x-y-3=0$ **b)** $2x+y-9=0$ **c)** $mx-y+b=0$
d) $3x+4y-20=0$ **e)** $3x+2y+1=0$ **f)** $20x-12y-3=0$

2. **a)** $3x-y-17=0$ **b)** $2x-5y+8=0$ **c)** $4x-3y=0$

3. **a)** slope $=-1$, y-int $=11$ **b)** slope $=\frac{3}{2}$, y-int $=15$ **c)** slope $=-\frac{1}{2}$, y-int $=\frac{7}{6}$

4. **a)** slope $=-2$, y-int $=6$, x-int $=3$
b) slope $=\frac{5}{2}$, y-int $=10$, x-int $=-4$
c) slope $=\frac{4}{5}$, y-int $=-\frac{3}{5}$, x-int $=\frac{3}{4}$

5. **a)** -8 **b)** 16 **6.** **a)** $-\frac{6}{5}$ **b)** $\frac{6}{5}$

7. A
8. i) E ii) C iii) A iv) E v) G
9. A **10.** C **11.** D **12.** B
13. D **14.** C **15.** C **16.** 1

Equations of Linear Relations Lesson #5:
Further Practice with Linear Equations

Writing Linear Equations

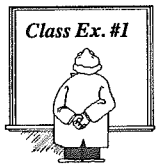
Linear equations can be written in different forms:

$Ax + By + C = 0$ → General form of a linear equation.

$y = mx + b$ → Slope y-intercept form of a linear equation.

$y - y_1 = m(x - x_1)$ → Point-slope form of a linear equation.

The slope y-intercept form is used when we are given the slope of a line and the y-intercept.
The point-slope form is used when we are given the slope of a line and any point on the line.
In many cases, either the point or the slope of the line must be determined from the information given before the equation can be used.



Given $P(3,-1)$ and $Q(-2,-6)$, determine the equation, in general form, of a line passing through the two points.

$m_{PQ} = \frac{-6+1}{-2-3}$

$= \frac{-5}{-5}$

$= 1$

$y - y_1 = m(x - x_1)$

$y + 1 = m(x - 3)$ ← Let $x = 3$, $y = -1$ (point)

$y + 1 = 1(x - 3)$ ← Let $m = 1$ (slope)

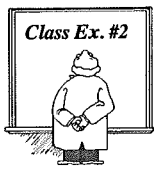
These should be done in one step, but separated for understanding!

$y + 1 = x - 3$

$0 = x - y - 4$

$x - y - 4 = 0$

Note: P(3,-1) seems easier, the values are smaller and there is only 1 negative!



Determine the equation, in general form, of a line through the point $(5,0)$ and perpendicular to the line with equation $3x - 5y + 17 = 0$.

$3x + 17 = 5y$

$y = \frac{3}{5}x + \frac{17}{5}$

Slope $= \frac{3}{5}$

↓

perpendicular slope $= \frac{-5}{3}$

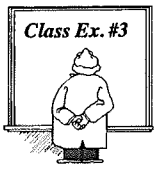
$y - y_1 = m(x - x_1)$

$3(y - 0) = \left[\frac{-5}{3}(x - 5)\right]3$

$3y = -5(x - 5)$

$3y = -5x + 25$

$5x + 3y - 25 = 0$



Find the equation, in general form, of the line perpendicular to the line $5x - 7y - 10 = 0$ and with the same x-intercept as $x - 2y - 12 = 0$.

use point/slope/y-int. form

$5x - 10 = 7y$

$y = \frac{5}{7}x - \frac{10}{7}$

Slope $= \frac{5}{7}$

perpendicular Slope $= \frac{-7}{5}$

x-intercept: let $y = 0$

$x - 2(0) - 12 = 0$

$x = 12$

$x_{int} = 12$ or $P(12, 0)$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-7}{5}(x - 12)$

$5(y - 0) = -7(x - 12)$

$5y = -7x + 84$

$7x + 5y - 84 = 0$

Complete Assignment Questions #1 - #14

Assignment

Note: Which point you choose to sub into an equation is entirely up to you because both will provide you the same result! However, points with smaller numerical values or with less negatives often result in fewer errors! Choose wisely!!!

1. Find the equation, in general form, of the line through each pair of points. use point/slope/y-int. form

a) (7, 5) and (6, 1) ← easier math using this point.

$m = \frac{1-5}{6-7} = \frac{-4}{-1} = 4$

$y - y_1 = m(x - x_1)$

$y - 1 = 4(x - 6)$

$y - 1 = 4x - 24$

$0 = 4x - y - 23$

$4x - y - 23 = 0$

b) (3, –7) and (–5, 9)

$m = \frac{9+7}{-5-3} = \frac{16}{-8} = -2$

$y - 9 = -2(x + 5)$

$y - 9 = -2x - 10$

$2x + y + 1 = 0$

c) (–3, 4) and (11, 25) easier math

$m = \frac{25-4}{11+3} = \frac{21}{14} = \frac{3}{2}$

$y - 4 = \frac{3}{2}(x + 3)$

$2(y - 4) = 3(x + 3)$

$2y - 8 = 3x + 9$

$3x - 2y + 17 = 0$

d) (10, –15) and (–2, –12) easier math

$m = \frac{-}{-} = \frac{3}{-12} = -\frac{1}{4}$

$y + 12 = -\frac{1}{4}(x + 2)$

$4(y + 12) = -1(x + 2)$

$4y + 48 = -x - 2$

$x + 4y + 50 = 0$

e) (4, –7) and (3, –7)

$m = \frac{-7+7}{3-4} = \frac{0}{-1} = 0$

$y + 7 = 0(x - 3)$

$y + 7 = 0$

f) (–5, –8) and (–4, –10)

$m = \frac{-10+8}{-4+5} = \frac{-2}{1} = -2$

$y + 8 = -2(x + 5)$

$y + 8 = -2x - 10$

$2x + y + 18 = 0$

2. Identify the lines in **1.** which are
   i) parallel — b and f (same slope)
   ii) perpendicular — a and d (negative reciprocals)

3. Write the equation of each line in general form

a) with slope $\frac{2}{7}$ and an x-intercept of -6 (point/slope/y-int.)

$m = \frac{2}{7}$ $P(-6, 0)$

$y - y_1 = m(x - x_1)$

$7(y - 0) = \frac{2}{7}(x + 6)\,7$

$7(y - 0) = 2(x + 6)$

$7y = 2x + 12$

$2x - 7y + 12 = 0$

b) with a y-intercept of $-\frac{8}{3}$ and a slope of 7 (point/slope/y-int.)

$m = 7$ $P(0, -\frac{8}{3})$

$y = mx + b$

$3(y) = \left(7x - \frac{8}{3}\right)3$

$3y = 21x - 8$

$21x - 3y - 8 = 0$

c) through the point $(2, 0)$ and perpendicular to $3x - 5y + 19 = 0$ (p/s/y-int)

Side Work.

Perpendicular Slope.

$3x - 5y + 19 = 0$

$3x + 19 = 5y$

$y = \frac{3}{5}x + \frac{19}{5}$

$m_{\perp} = -\frac{5}{3}$

$m_{\perp} = -\frac{5}{3}$ $P(2, 0)$

$y - 0 = -\frac{5}{3}(x - 2)$

$3y = -5(x - 2)$

$3y = -5x + 10$

$5x + 3y - 10 = 0$

d) through the point $(3, -6)$ and parallel to $5x + 3y + 9 = 0$ (point/slope/y-int)

Side Work.

$3y = -5x - 9$

$y = -\frac{5}{3}x - 3$

Slope $= -\frac{5}{3}$

parallel slope $= -\frac{5}{3}$

$m = -\frac{5}{3}$ $P(3, -6)$

$y + 6 = -\frac{5}{3}(x - 3)$

$3(y + 6) = -5(x - 3)$

$3y + 18 = -5x + 15$

$5x + 3y + 3 = 0$

4. Write the equation of each line in general form

a) perpendicular to $y = x$ and with the same x-intercept as $y = 2x + 10$ (point/slope/y-int.)

Side Work.

m = Slope $= 1$

$m_{\perp} = -1$

$P(-5, 0)$

$0 = 2x + 10$

$2x = -10$

$x = -5$

$y - y_1 = m(x - x_1)$

$y - 0 = -1(x + 5)$

$y = -x - 5$

$x + y + 5 = 0$

b) parallel to $2x - 3y + 7 = 0$ and with the same y-intercept as $5x - 3y - 12 = 0$ (point/slope/y-int)

Side Work

$2x + 7 = 3y$

$y = \frac{2}{3}x + \frac{7}{3}$

Slope $= \frac{2}{3}$

$5x - 12 = 3y$

$y = \frac{5}{3}x - 4$

y-int $= -4$

$y = mx + b$

$3(y) = \left(\frac{2}{3}x - 4\right)3$

$3y = 2x - 12$

$2x - 3y - 12 = 0$

parallel slope $= \frac{2}{3}$

5. Write the equation of each line in general form point / slope / yint

a) perpendicular to $6x - 2y + 5 = 0$ and with the same y-intercept as $x - y + 8 = 0$

Side Work

$6x + 5 = 2y$

$y = 3x + \frac{5}{3}$

m = Slope = 3

$x + 8 = y$

$y = x + 8$

$y_{int} = 8$

$m_{\perp} = -\frac{1}{3}$

$y_{int} = 8$

$y = mx + b$

$3(y) = \left[-\frac{1}{3}x + 8\right]3$

$3y = -x + 24$

$x + 3y - 24 = 0$

b) with the same x-intercept as $9x - 2y + 18 = 0$ and through the point $(4, -5)$

Side Work Point / Slope / yint.

$9x + 18 = 0$

$9x = -18$

$x_{int} = -2$

$P(-2, 0)$

$m = \frac{-5 - 0}{4 + 2}$

$= -\frac{5}{6}$.

$m = -\frac{5}{6}$

Point $(-2, 0)$

$y - 0 = -\frac{5}{6}(x + 2)$

$6y = -5(x + 2)$

$6y = -5x - 10$

$5x + 6y + 10 = 0$

6. Line l contains the point $A(7, 9)$ and is parallel to a line which contains the points $B(-4, 5)$ and $C(8, -1)$. Determine the equation of line l in the form $y = mx + b$.

Side Work Point / slope / yint.

$m_{BC} = \frac{-}{-} = -\frac{6}{12} = -\frac{1}{2}$

parallel slope $= -\frac{1}{2}$

$m = -\frac{1}{2}$

$A(7, 9)$

This time we are going to just distribute the $\frac{1}{2}$ first!

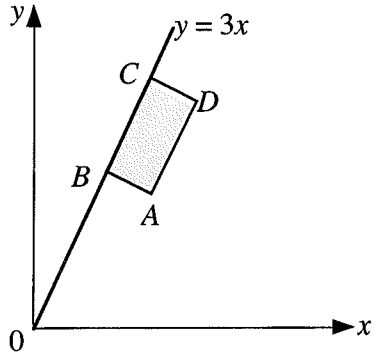
$y - y_1 = m(x - x_1)$

$y - 9 = -\frac{1}{2}(x - 7)$

$y - 9 = -\frac{1}{2}x + \frac{7}{2}$

$y = -\frac{1}{2}x + \frac{25}{2}$

7. A Cartesian plane is placed on the plan of a farm. The farmhouse is at the origin, and $ABCD$ represents a rectangular field of wheat. A farm road, with equation $y = 3x$, runs from the farmhouse along one side of the field.



a) If the point A has coordinates $(2, 4)$, determine the equation of AD.

Side Work. point / slope / yint.

AD is parallel to BC

$m_{AD} = 3$

Given (2, 4)

$y - 4 = 3(x - 2)$

$y - 4 = 3x - 6$

$y = 3x - 2$ or $3x - y - 2 = 0$

b) Determine the equation of AB. point / slope / yint.

Side Work

$AB \perp AD$

So $m_{AB} = -\frac{1}{3}$

Given (2, 4)

$y - 4 = -\frac{1}{3}(x - 2)$

$3(y - 4) = -1(x - 2)$

$3y - 12 = -x + 2$

$x + 3y - 14 = 0$

8. A child with a fixed amount of money can buy 2 bags of chips and 5 cans of pop, or 3 bags of chips and 2 cans of pop. A linear relationship exists between the number of bags of chips, x, and the number of cans of pop, y, which can be bought.

a) Write the coordinates of two points which lie on the graph of this linear relationship.

(2,5) and (3,2)

b) Determine the equation of the linear relationship. (point/slope) / y-int.

Side Work.

$m = \frac{2-5}{3-2} = \frac{-3}{1} = -3$

$y-5 = -3(x-2)$

$y-5 = -3x+6$

$y = -3x+11$ or $3x+y-11=0$

Multiple Choice

9. The equation of the line through the point $(7,-4)$ and perpendicular to the line with equation $5x-4y+13=0$, can be written in the form

(Point / Slope / y-int.)

A. $y+4=\frac{5}{4}(x-7)$

B. $y=-\frac{4}{5}(x+7)$

C. $y+4=-\frac{4}{5}(x-7)$

D. $y+4=\frac{4}{5}(x-7)$

Side Work.

$5x+13=4y$

$y=\frac{5}{4}x+\frac{13}{4}$

slope $= \frac{5}{4}$

perpendicular slope $= -\frac{4}{5}$

$y+4=-\frac{4}{5}(x-7)$

10. A line passing through the point $(0,3)$ is perpendicular to the line $x-2y-5=0$. The equation of the line is

A. $2x+y-3=0$

B. $2x+y+3=0$

C. $x-2y+6=0$

D. $2x-y+3=0$

Side Work.

$x-5=2y$

$y=\frac{1}{2}x-\frac{5}{2}$

m = slope $= \frac{1}{2} \rightarrow m_{\perp} = -2$

Point / (slope / y-int) $y=mx+b$

$y=-2x+3$

$2x+y-3=0$

Note: If you ever forget that slope y-intercept is easiest here to use, you can still use point slope form!

11. Which of the following linear relations is not equivalent to the other three?

A. $y-4=-\frac{1}{3}(x+6)$ $\quad 3(y-4)=-(x+6)$ $\quad 3y-12=-x-6$ $\quad x+3y-6=0$

B. $x+3y+2=0$

C. the line passing through $(0,2)$ and $(6,0)$ $\quad m=\frac{0-2}{6-0}=-\frac{1}{3}$ $\quad y=-\frac{1}{3}x+2$

D. $y=-\frac{1}{3}x+2$ $\quad 3y=-x+2(3)$ $\quad 3y=-x+6$ $\quad x+3y-6=0$

12. A line passing through the point $(0, 3)$ is parallel to the line $x - 2y - 5 = 0$. The equation of the line is

point | slope | y int.

A. $2x + y - 3 = 0$
B. $2x + y + 3 = 0$
C. $x - 2y + 6 = 0$
D. $2x - y + 3 = 0$

Side Work.

$x - 5 = 2y$

$y = \frac{1}{2}x - \frac{5}{2}$

Slope $= \frac{1}{2}$

Parallel slope $= \frac{1}{2}$

$y = \frac{1}{2}x + 3$

$2y = x + 6$

$0 = x - 2y + 6$

$x - 2y + 6 = 0$

13. The image of $y = 2x + 7$ after a counterclockwise rotation of $90°$ about the origin is

A. $y = -\frac{1}{2}x + \frac{7}{2}$
B. $y = \frac{1}{2}x - \frac{7}{2}$
C. $y = -\frac{1}{2}x - \frac{7}{2}$
D. $y = -2x - 7$

$y_{int} = 7$

$x_{int}: 0 = 2x + 7$

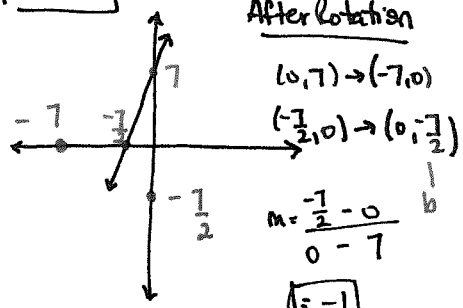
$-7 = 2x$

$x = -\frac{7}{2}$

$\left(-\frac{7}{2}, 0\right)$

P (0,0) = P(0,7)

$y = 7$ because of y_{int}



After Rotation

$(0,7) \to (-7,0)$

$\left(-\frac{7}{2},0\right) \to \left(0,-\frac{7}{2}\right)$

$m = \frac{-\frac{7}{2} - 0}{0 - 7} = -\frac{1}{2}$

b

m

Thus, After Rotation $y = -\frac{1}{2}x - \frac{7}{2}$

Numerical Response **14.** The line through the points $(-3, 4)$ and $(-1, -2)$ has equation $y + ax + b = 0$, where a and b are integers. The value of $a + b$ is _____.

(Record your answer in the numerical response box from left to right)

8			

point | slope | y int.

$m = \frac{-2 - 4}{-1 + 3}$

$= -\frac{6}{2}$

$= -3$

$m = -3$

Point $(-1, -2)$

$y + 2 = -3(x + 1)$

$y + 2 = -3x - 3$

$y = -3x - 5$

$y + 3x + 5 = 0$

a b

$a + b$

$= 3 + 5$

$= 8$

Answer Key

1. a) $4x - y - 23 = 0$ **b)** $2x + y + 1 = 0$ **c)** $3x - 2y + 17 = 0$
d) $x + 4y + 50 = 0$ **e)** $y + 7 = 0$ **f)** $2x + y + 18 = 0$

2. i) b and f **ii)** a and d

3. a) $2x - 7y + 12 = 0$ **b)** $21x - 3y - 8 = 0$ **c)** $5x + 3y - 10 = 0$ **d)** $5x + 3y + 3 = 0$

4. a) $x + y + 5 = 0$ **b)** $2x - 3y - 12 = 0$

5. a) $x + 3y - 24 = 0$ **b)** $5x + 6y + 10 = 0$

6. $y = -\frac{1}{2}x + \frac{25}{2}$

7. a) $y = 3x - 2$ or $3x - y - 2 = 0$ **b)** $y = -\frac{1}{3}x + \frac{14}{3}$ or $x + 3y - 14 = 0$

8. a) $(2, 5)$ and $(3, 2)$ **b)** $3x + y - 11 = 0$

9. C **10.** A **11.** B **12.** C **13.** C **14.**

8			

Equations of Linear Relations Lesson #6:
Graphing Linear Equations

Graphing Linear Equations Without Technology

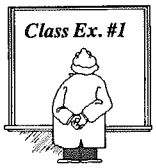
Linear equations can be written in different forms:

$Ax + By + C = 0$ → General form of a linear equation.

$y = mx + b$ → Slope y-intercept form of a linear equation.

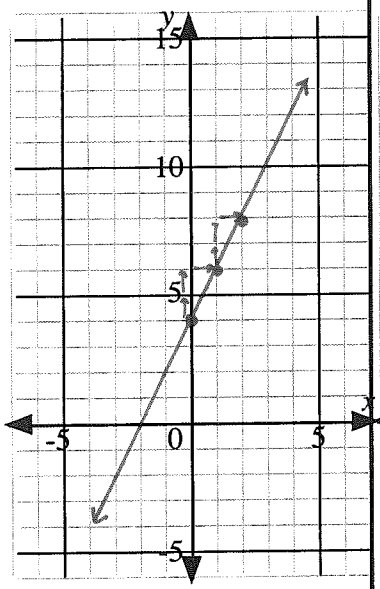
$y - y_1 = m(x - x_1)$ → Point-slope form of a linear equation.

The method used to graph a linear relation without technology depends on the form in which the linear equation is written.



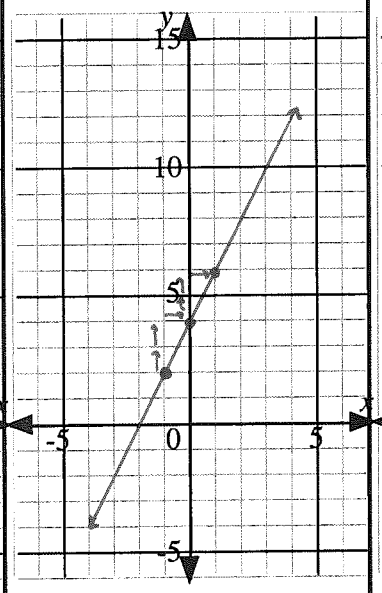
Without altering the form in which the linear equation is written, explain the different strategies used to graph (without technology) each of the following linear relations. Draw the graph of each linear relation on the grid provided.

a) $y = 2x + 4$



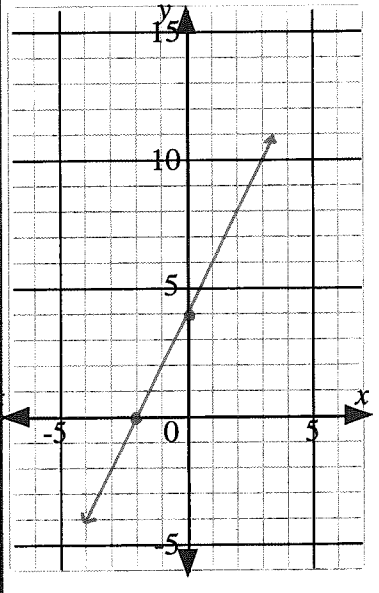
Start at the y-intercept (4) and use the slope as $\frac{\text{rise}}{\text{run}} = \frac{2}{1}$ to mark more points. Join the points and extend the line.

b) $y - 2 = 2(x + 1)$



Start at the point (-1, 2) and use the slope as $\frac{\text{rise}}{\text{run}} = \frac{2}{1}$ to mark more points. Join the points and extend the line.

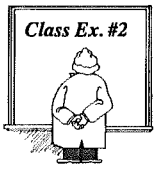
c) $2x - y + 4 = 0$



Determine the x-intercept by making $y = 0$ and the y-intercept by making $x = 0$. Join the points and extend the line.

x_{int}: $2x + 4 = 0 \Rightarrow x = -2$

y_{int}: $-y + 4 = 0 \Rightarrow y = 4$



Match each linear relation to its graph.

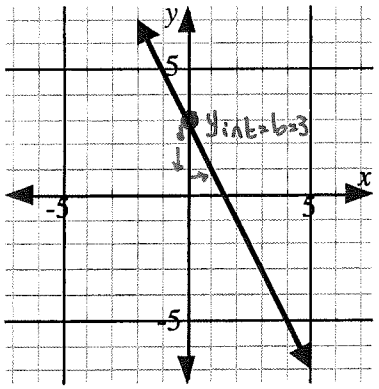
Equation 1:
$2x - y + 3 = 0$ D

Equation 2:
$y = -2x + 3$ A

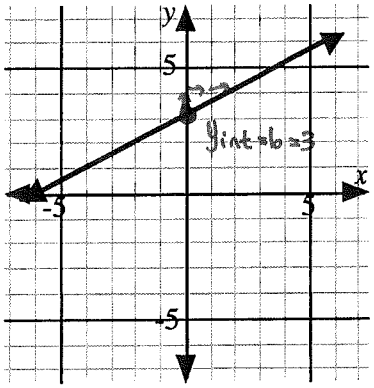
Equation 3:
$y + 1 = -\frac{1}{2}(x + 4)$ C

Equation 4:
$2y - x - 6 = 0$ B

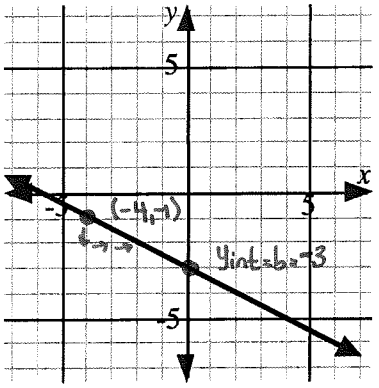
Graph A



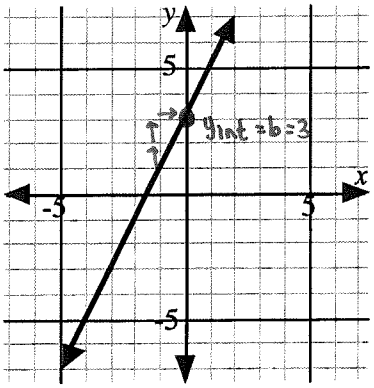
Graph B



Graph C



Graph D

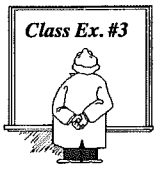


Equation 1: $2x - y + 3 = 0$ $m = 2$
$y = 2x + 3$ $b = 3$

Equation 2: $y = -2x + 3$ $m = -2$ $b = 3$

Equation 3: $y + 1 = -\frac{1}{2}(x + 4)$ $m = -\frac{1}{2}$
Point $(-4, -1)$

Equation 4: $2y - x - 6 = 0$
$2y = x + 6 \rightarrow y = \frac{1}{2}x + 3$ $m = \frac{1}{2}$ $b = 3$

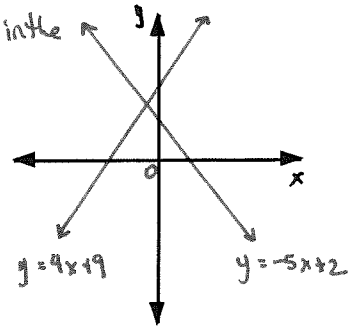


Graphing Linear Equations With Technology

- Explain the strategy used to graph (with technology) the linear relations.
  $y + 8 = -5(x - 2)$ and $4x - y + 9 = 0$

To enter into graphing calculator or otherwise rewrite each equation in the form of $y = \ldots$

$y = -5(x - 2) - 8$ or $y = -5x + 2$
$y = 4x + 9$



- State an appropriate window to show x- and y-intercepts, and draw the graph of both linear relations on the same grid.

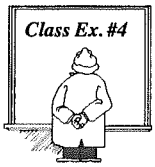
$x: [-5, 5, 1]$ $y: [-5, 15, 2]$

- Determine the x and y-intercepts of $4x - y + 9 = 0$.

$x_{int} = -\frac{9}{4}$ $y_{int} = 9$

Complete Assignment Questions #1 - #6

Determining Linear Relationships from Tables of Data

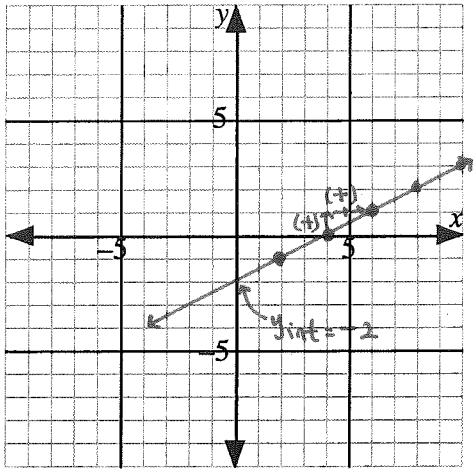


Consider the following data points expressed in a table of values

x	2	4	6	8	10
y	–1	0	1	2	3

a) Plot the data points on the grid to verify that there is a linear relationship.

b) Join the points together and determine the slope of the line.



c) Determine the equation of the linear relationship in the form $y = mx + b$.

$m = \frac{1}{2}$

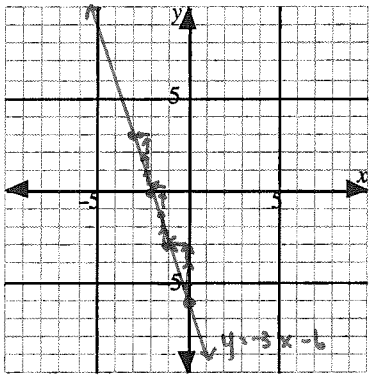
$y = -\frac{1}{2}x - 2$

$y_{int} = -2$

Complete Assignment Questions #7 - #10

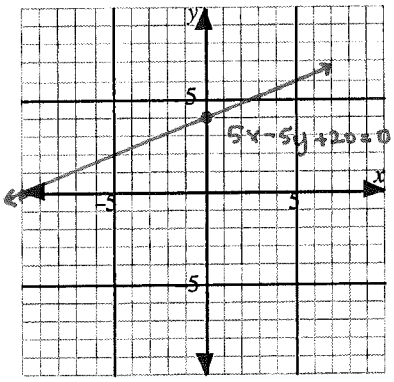
Assignment

1. Without using technology or without altering the form in which the linear equation is written, explain how to graph $y = -3x - 6$ on a grid.
Draw the graph on the grid provided.



Plot the y-intercept (0, -6). Since the slope, -3, equals rise over run move up 3 and left 1, plotting another point. Repeat process for two more points and draw a line through the points.

2. Without using technology or without altering the form in which the linear equation is written, explain how to graph $2x - 5y + 20 = 0$ on a grid.
Draw the graph on the grid provided.

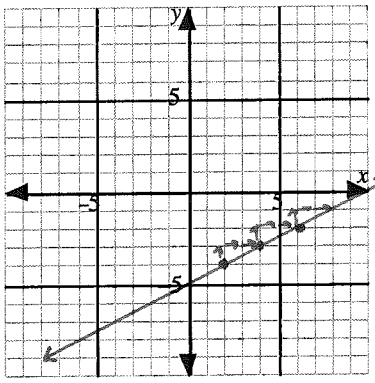


Plot the x-intercept and the y-intercept and join the points.

x_{int}: $2x + 20 = 0$ $2x = -20$ $x_{int} = -10$

y_{int}: $-5y + 20 = 0$ $20 = 5y$ $y_{int} = 4$

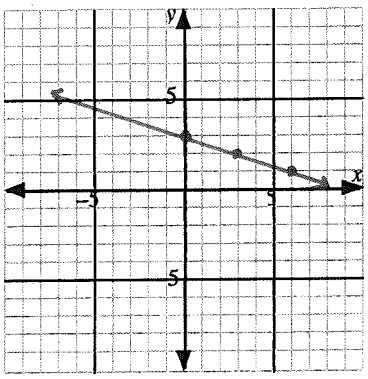
3. Without using technology or without altering the form in which the linear equation is written, explain how to graph $y + 4 = \frac{1}{2}(x - 2)$ on a grid.
Draw the graph on the grid provided.



Plot the point $(2,-4)$. Since the slope equals $\frac{1}{2}$ move 1 up and 2 right and plot another point. Repeat for two more points.

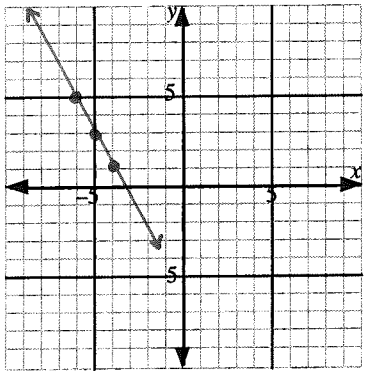
4. Without altering the form in which the linear equation is written, draw the graph (without technology) of each of the following linear relations on the grid provided.

a) $y = -\frac{1}{3}x + 3$



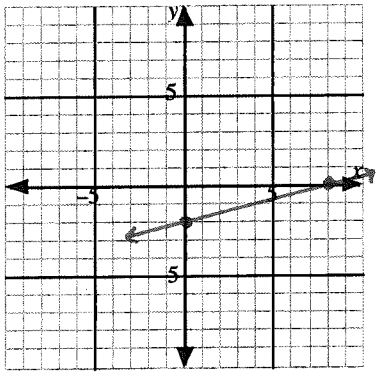
Points $(0,3)$ $(3,2)$ $(6,1)$

b) $y - 5 = -2(x + 6)$



Points $(-6,5)$ $(-5,3)$ $(-4,1)$

c) $x - 4y - 8 = 0$



Points $(8,0)$ $(0,-2)$

x_{int}: $x - 8 = 0$ $x_{int} = 8$

y_{int}: $-4y - 8 = 0$ $-8 = 4y$

$y_{int} = -2$

5. Match each linear relation to its graph.

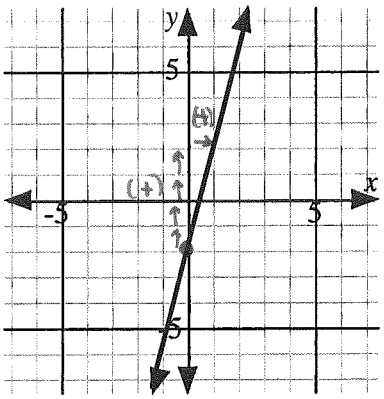
Equation 1:
$y - 6 = -4(x + 1)$ B

Equation 2:
$x - 4y - 8 = 0$ C

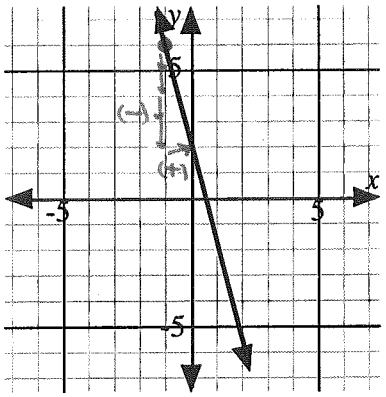
Equation 3:
$4x - y - 2 = 0$ A

Equation 4:
$y = -\frac{1}{4}x + 2$ D

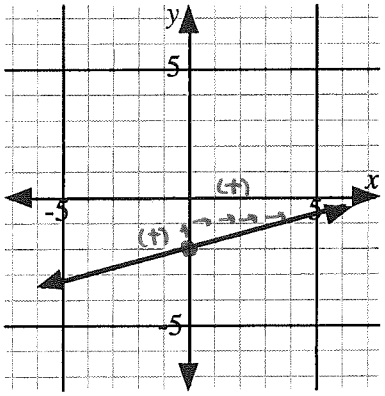
Graph A



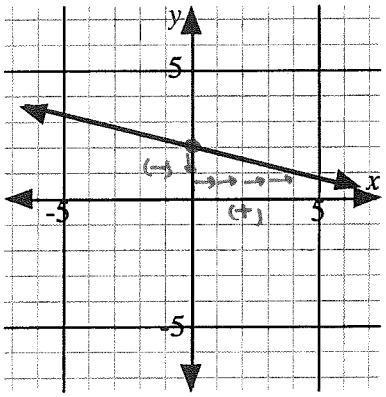
Graph B



Graph C



Graph D



Equation 1: Point $(-1, 6)$ slope $= -4$

Equation 2: $x - 8 = 4y \rightarrow y = \frac{1}{4}x - 2$

$y_{int} = -2$ slope $\frac{1}{4}$

Equation 3: $4x - 2 = y \rightarrow y = 4x - 2$

$y_{int} = -2$ Slope $= 4$

Equation 4: Slope $= -\frac{1}{4}$ $y_{int} = 2$

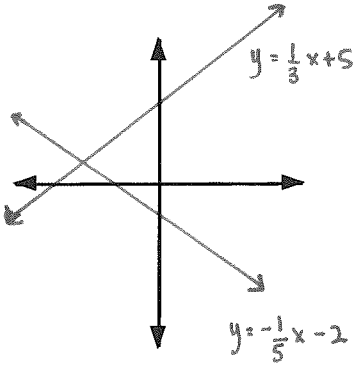
6. a) Explain the strategy used to graph (with technology) the linear relations

$x + 5y + 10 = 0$ and $y - 3 = \frac{1}{3}(x + 6)$.

Solve each equation for y, then input Y_1 and Y_2 into the equation editor of the graphing calculator.

$5y = -x - 10$ $\quad y = \frac{1}{3}(x + 6) + 3$

$y = -\frac{1}{5}x - 2$ $\quad$ or $y = \frac{1}{3}x + 5$



b) State an appropriate window to show x- and y-intercepts, and draw the graph of both linear relations on the grid.

$X: [-20, 10, 5]$ $\quad Y: [-6, 10, 2]$

c) Determine the x- and y-intercepts of each graph.

$x + 5y + 10 = 0$ $\quad y - 3 = \frac{1}{3}(x + 6)$

$x_{int} = -10$ $\quad y_{int} = -2$ $\quad x_{int} = -15$ $\quad y_{int} = 5$

2nd TRACE 2 ... 2nd TRACE 1 ...

7. Consider the following data points expressed in a table of values

a)

x	–1	0	1	2	3
y	–3	–1	1	3	5

b)

x	–6	–3	0	3	6
y	6	5	4	3	2

c)

x	0	1	2	3	4
y	1	4	9	16	25

In each case, determine the equation of the linear relationship if it exists.

Linear

$m = \frac{-3+1}{-1-0} = \frac{-2}{-1} = 2$

$b = -1$

$y = 2x - 1$

Linear

$m = -\frac{1}{3}$ $b = 4$

$y = -\frac{1}{3}x + 4$

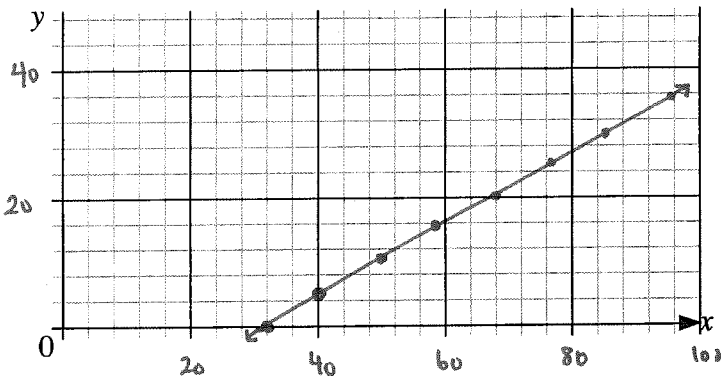
non-linear.

8. The following data is taken from a continuous linear relationship involving two quantities x and y.

x	32	41	50	59	68	77	86	95
y	0	5	10	15	20	25	30	35

point | slope | y int.

a) Plot the data on the grid and obtain, in general form, the equation of the linear relation which is represented by the data.



$m = \frac{5-0}{41-32} = \frac{5}{9}$

$y - 0 = \frac{5}{9}(x - 32)$

$9y = 5(x - 32)$

$9y = 5x - 160$

$9y + 160 = 5x$ $x = \frac{9}{5}y + 32$

b) Rewrite the equation of the linear relation

i) in terms of x (i.e. $y = ...$)

$y = \frac{5}{9}(x - 32)$

or

$y = \frac{5}{9}x - \frac{160}{9}$

ii) in terms of y (i.e. $x = ...$)

$x = \frac{9}{5}y + 32$

c) The formulas in **b)** are well known in the scientific field. Can you suggest what scientific variables are represented by x and y?

x is temperature in F°

y is temperature in °C.

Use the following information to answer the questions #9 and #10.

The following data is taken from a continous linear relationship involving two quantities, x and y.

x	-10	–5	5	10	20
y	-4	–3	–1	0	2

Multiple Choice **9.** When the data is graphed on a grid, the slope of the line is

A. 5
B. –5
C. $\frac{1}{5}$
D. $-\frac{1}{5}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{-4+3}{-10+5} = \frac{-1}{-5} = \frac{1}{5}$

Numerical Response **10.** The equation of the linear relationship can be written in the form $Ax - 5y + C = 0$. The value of $A - C$ is _____ .

point/slope y-int.

Step1: Given using $P(10, 0)$ $\quad m = \frac{1}{5}$

Step2: $Ax - 5y + C = 0$

$y - 0 = \frac{1}{5}(x - 10)$

$y = \frac{1}{5}(x - 10)$

$5(y) = \left(\frac{1}{5}x - 2\right)5$

$5y = x - 10$

$x - 5y - 10 = 0$

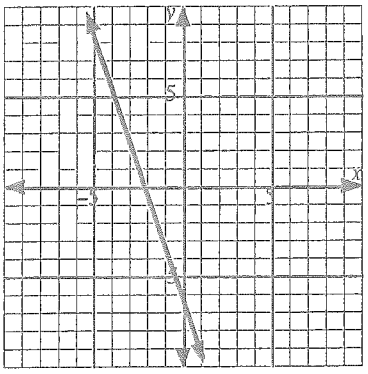
A C

Step3: $A - C$

$1 - -10 = 11$

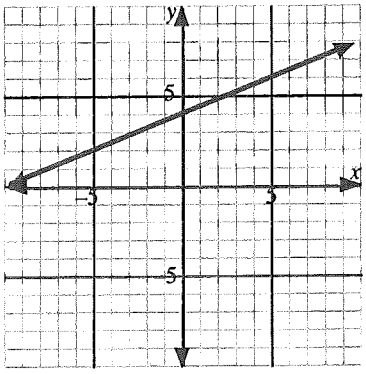
Answer Key

1.



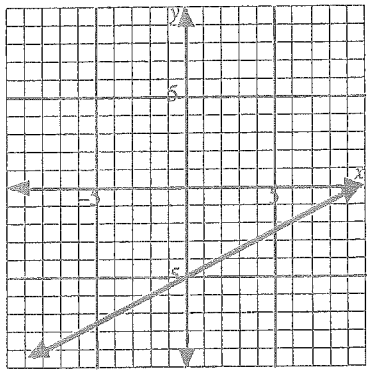
Plot the y-intercept $(0, -6)$. Since the slope, -3, equals rise over run, move 3 up and 1 left and plot another point. Repeat for two more points and draw a line through the points.

2.



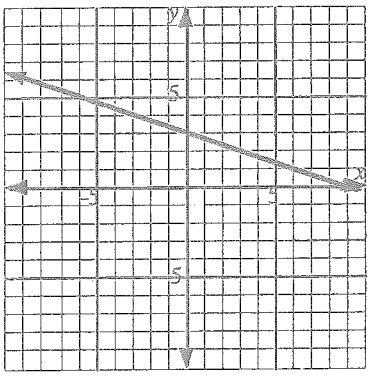
Plot the x-intercept $(-10, 0)$ and the y-intercept $(0, 4)$. Draw a line through these two points.

3.

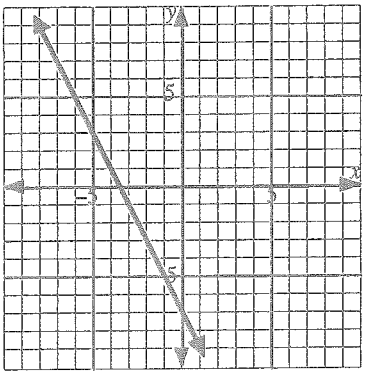


Plot the point $(2, -4)$. Since the slope equals 1/2, move 1 up and 2 right and plot another point. Repeat for two more points and draw a line through the points.

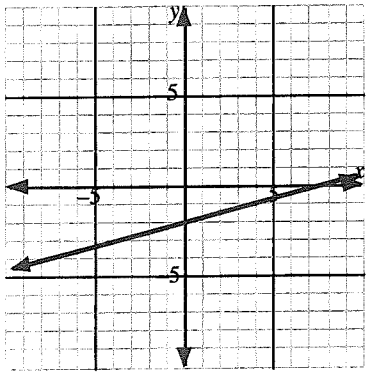
4. a)



b)



c)



5. 1B, 2C, 3A, 4D.

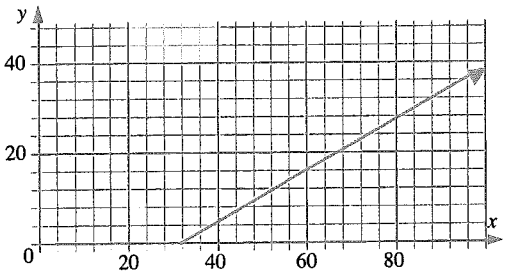
6. a) Solve each equation for y. Then input Y_1 and Y_2 into the equation editor of the graphing calculator. Press Graph.

b) x:[–20, 10, 5] y:[–6, 10, 2], answers may vary.

c) For $x + 5y + 10 = 0$, x-int $= -10$, y-int $= -2$, and for $y - 3 = \frac{1}{3}(x + 6)$, x-int $= -15$, and y-int $= 5$.

7. a) $y = 2x - 1$ **b)** $y = -\frac{1}{3}x + 4$ **c)** not linear

8. a) $5x - 9y - 160 = 0$



b) i) $y = \frac{5}{9}x - \frac{160}{9}$, or $y = \frac{5}{9}(x - 32)$

ii) $x = \frac{9}{5}y + 32$

c) x is temperature in °F, and y is temperature in °C.

9. C **10.**

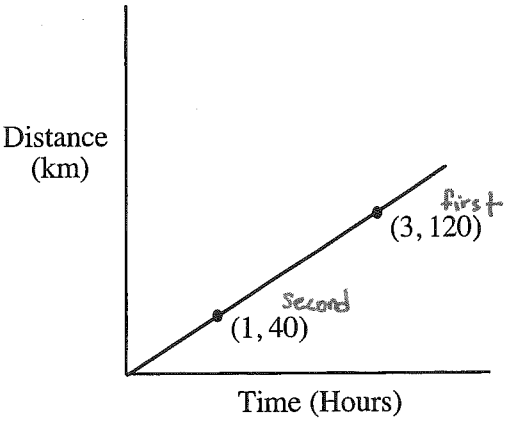
1	1		

Equations of Linear Relations Lesson #7:
Slope as a Rate of Change

Rate of Change

Part One

The graph shown represents the distance travelled by a car as a linear function of time.



a) Complete the following to calculate the slope of the line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{120 - 40}{3 - 1} = \frac{80}{2} = 40$$

b) The slope represents a **rate of change**: a change in distance divided by a change in time.

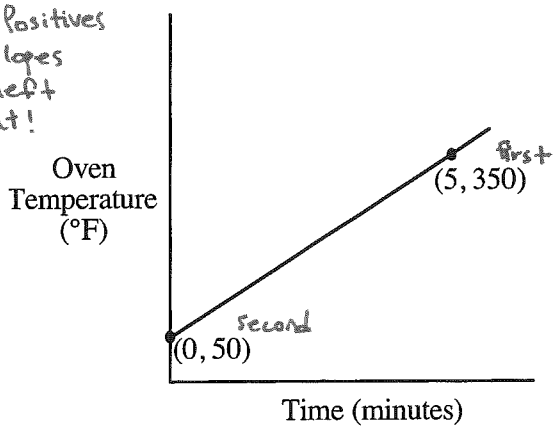
Complete the following statement by choosing the correct alternative and filling in the blank.

The distance is (increasing/decreasing) at the rate of 40 km per hour.

↳ Think about it! How does this relate to the sign of the slope? Positives slopes rise from left to right!

Part Two

The graph shown represents the temperature of an oven as a linear function of time.



a) Calculate the slope of the line.

$$m = \frac{350 - 50}{5 - 0} = \frac{300}{5} = 60$$

b) The slope represents a **rate of change**: a change in temperature divided by a change in time.

What units are used to represent this rate of change? °F/minute

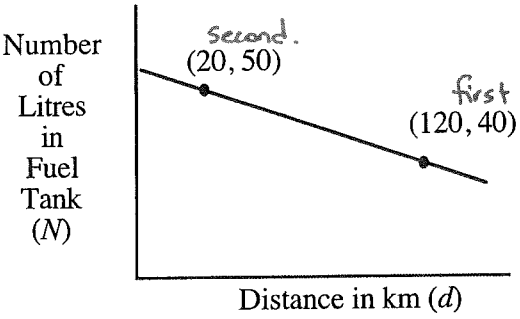
c) Complete the following statement.

The temperature is (increasing/decreasing) at the rate of 60 °F per minute.

↳ positive slopes rise from left to right!

Part Three

The graph shown represents the amount of fuel in the gas tank of a car as a function of the distance travelled.



a) Calculate the slope of the line.

$m = \frac{40 - 50}{120 - 20} = \frac{-10}{100} = -\frac{1}{10}$

b) The slope represents a **rate of change**: a change in the amount of fuel in the fuel tank divided by a change in distance.

What units are used to represent this rate of change? L/km

c) Complete the following statements. → Think about it!

The amount of fuel in the tank is (increasing/decreasing). Circle one.

The rate of change of fuel in the fuel tank is $-1/10$ L per km.

The amount of fuel is decreasing at the rate of $1/10$ L per km.

A slope that is negative makes for a falling line from left to right!

d) Determine the equation of the line in the form $N = mD + b$. (point/slope)/y int.

$N - N_1 = m(D - D_1)$

$N - 50 = -\frac{1}{10}(D - 20)$

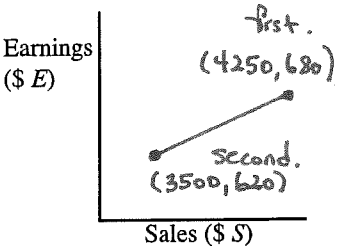
$N - 50 = -\frac{1}{10}D + 2$

$N = -\frac{1}{10}D + 52$

Tyrone is paid a base salary per week plus commission for selling electrical appliances. Last week his sales totalled \$3 500, and he earned \$620. This week he earned \$680 for sales of \$4 250.

a) On a grid, plot ordered pairs to represent this information.

b) Calculate the slope of the line segment joining the ordered pairs.



$m = \frac{680 - 620}{4250 - 3500} = \frac{60}{750} = 0.08$

c) Explain what the slope of the graph represents.

He earns 8 cents for every dollar in sales.

d) State as a percent, the rate of commission which Tyrone is paid. 8%

e) Calculate his weekly base salary.

\$620 = 8% of \$3500 + B

620 = 280 + B

B = 340

base salary = \$340

f) How does the answer to **e)** relate to the graph?

It is the E-intercept of the graph.

g) Write the equation in the form $E = mS + b$.

$E = 0.08S + 340$

h) Two weeks ago Tyrone earned \$486. Calculate his sales for that week.

$486 = 0.8S + 340$

$146 = 0.8S$

$S = \frac{146}{0.08} = 1825$

Sales = \$1825

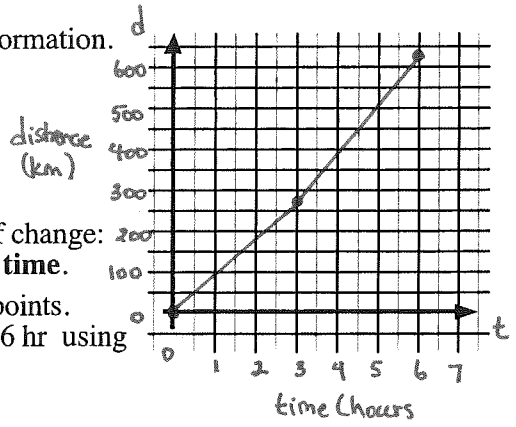
Average Speed

Biyen is taking part in a long distance car race. After 3 hours he had travelled 270 km, and after 6 hours he had travelled 630 km.

a) On a grid, plot ordered pairs to represent this information.

b) Calculate the slope of the line segment joining the ordered pairs.

$m = \frac{630 - 270}{6 - 3} = \frac{360}{3} = 120$



c) The slope of the line segment represents a rate of change: **a change in distance divided by a change in time.**

This rate is the **average speed** between the two points.

State the average speed of the car from 3 hr to 6 hr using appropriate units. 120 km/h

d) On the grid, plot the point $(0, 0)$ and determine the average speed of the car during the first 3 hr of the race.

$m = \frac{270 - 0}{3 - 0} = 90$ 90 km/h.

e) By looking at the grid and without doing any calculations, how can we tell that the average speed during the first 3 hr was less than the average speed during the next 3 hr?

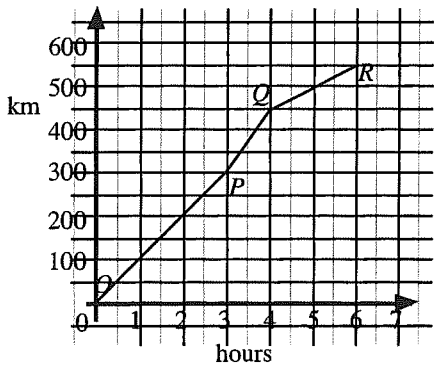
The slope is less steep during the first 3h.

On a graph of distance as a function of time, the slope of a line segment joining two points represents the average speed between the two points.

Complete Assignment Questions #1 - #10

Assignment

1. The distances and times are recorded at certain points on a journey. Calculate the average speed



a) between O and P $\frac{300}{3} = 100$ km/h

b) between P and Q $\frac{450 - 300}{4 - 3} = 150$ km/h

c) between Q and R $\frac{550 - 450}{6 - 4} = 50$ km/h

d) for the whole journey $\frac{550}{6} = 91\frac{2}{3}$ km/h.

2. Absolute Value Computer Company was formed in 2014. By Jan 2016, the company had sold 520 000 computers and by July 2016, the company had sold 610 000 computers. Calculate the average rate of change stating appropriate units.

$\frac{610\,000 - 520\,000}{7 - 1} = 15\,000$ computers per month.

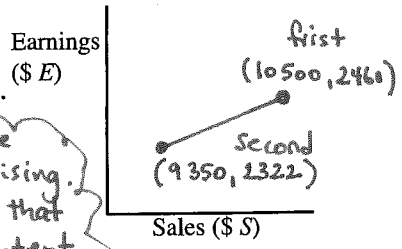
3. In 2017, the transit authority in a large city reported 17 678 465 passenger journeys. In 2012, the number of passenger journeys was 21 520 075. Calculate the average rate of change stating appropriate units.

$\frac{17\,678\,465 - 21\,520\,075}{2017 - 2012} = -768\,322$ passenger journeys per year.

Question: what does the negative mean? Journeys are decreasing!

4. Shanna is paid a base salary per month plus commission for working in a clothing store. In January her sales totalled \$10 500 and she earned \$2 460. In February her sales totalled \$9 350, and she earned \$2 322.

a) On a grid, plot ordered pairs to represent this information.



b) Calculate the slope of the line segment joining the ordered pairs.

$m = \frac{2460 - 2322}{10500 - 9350} = \frac{138}{1150} = 0.12$

Notice the slope is positive and rising. Also, notice that the context of the question is also increasing!

c) Explain what the slope of the graph represents.

She earns 12 cents for every dollar of sales.

d) State, as a percent, the rate of commission which Shanna is paid.

12%

e) Calculate her monthly base salary.

$2460 = 12\%$ of $10500 + B$

$2460 = 1260 + B$

$B = 1200$ base salary = \$1200

f) How does the answer to **e)** relate to the graph?

It is the E-intercept on the vertical axis.

g) Write the equation in the form $E = mS + b$.

$E = 0.12S + 1200$

h) In March, her sales were \$11 200. Calculate her earnings for March.

$E = 0.12(11\,200) + 1200$

$= 1344 + 1200 = 2544$ earnings = \$2544.

i) In April, her rate of commission was increased by 1%. If her earnings were \$70 less than in March, calculate the value of her sales in April.

earnings = $2544 - 70 = \$2474$

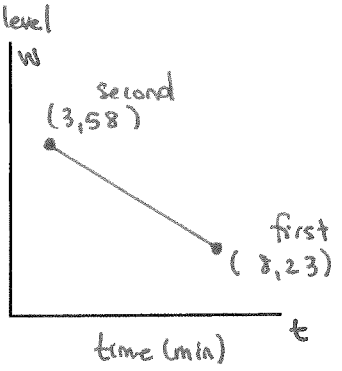
$2474 = 0.135 + 1200$

$0.13S = 1274$

$S = \frac{1274}{0.13} = 9800$ Sales = \$9800

5. Water is leaking out of the bottom of a barrel at a constant rate. After 3 min the water level is 58 cm and after 8 min the water level is 23 cm.

a) On a grid, plot ordered pairs (time, water level) to represent this information.



b) Calculate the slope of the line segment joining the ordered pairs.

$m = \frac{23 - 58}{8 - 3} = \frac{-35}{5} = -7$

c) Explain what the slope of the graph represents.

The rate at which the water level is changing. in cm/min.

d) Complete the following.

The water level is changing at the rate of -7 cm per min.

e) Determine an equation to represent this information in the form $W = mt + b$, where W represents the water level in cm and t represents the time in minutes.

$m = -7$ Point $(8, 23)$ point/slope/y-int.

$W - W_1 = m(t - t_1)$

$W - 23 = -7(t - 8)$ → $W - 23 = -7t + 56$

$W = -7t + 79$

f) After how many minutes will the barrel be empty?

$0 = -7t + 79$

$7t = 79$ → $t = 11\frac{2}{7}$ $11\frac{2}{7}$ mins

g) What was the water level in the barrel when it started leaking?

$t = 0$ $W = 79$ 79 cm

$W = -7(0) + 79$

$W = 79$ ← simply the b-value of $y = mx + b$.

h) The barrel is cylindrical in shape with a radius of 20 cm.

i) Calculate the volume (in terms of π) of water in the barrel after 3 min and after 8 min.

3 min $(3, 23200\pi)$ $V = \pi r^2 h$

$= \pi(20)^2(58)$

$= 23200\pi \text{ cm}^3$

8 min $(8, 9200\pi)$ $V = \pi r^2 h$

$= \pi(20)^2(23)$

$= 9200\pi \text{ cm}^3$

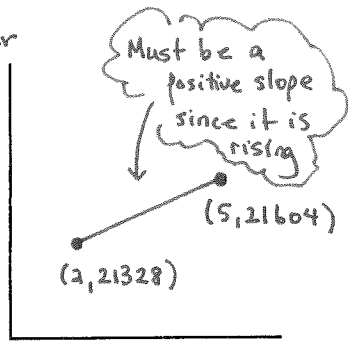
ii) Calculate the rate (in terms of π) at which water is leaking out of the barrel.

↳ ⟷ slope since the rate is decreasing!

$\frac{23200\pi - 9200\pi}{3 - 8} = -2800\pi \text{ cm}^3/\text{min}$.

Thus, decreasing at a rate of $2800\pi \text{ cm}^3/\text{min}$

6. Jack rented a car from Absolute Value Rent-a-Car Company. After driving for two hours, the odometer reading was 21 328 and after five hours the odometer reading was 21 604. He completed his journey after six hours.



a) On a grid, plot ordered pairs to represent this information.

b) Calculate the slope of the line segment joining the ordered pairs.

$m = \frac{21604 - 21328}{5 - 2} = \frac{276}{3} = 92$

Recall: Since the value of the rate of change (slope) is positive, the relationship must be increasing.

c) Explain what the slope of the graph represents.

... The average speed between 2h and 5h is 92 km/h.

d) Assuming a constant rate of driving for the whole journey, determine the odometer reading at the start of the journey.

$21328 - 2(92) = 21144$

e) Determine an equation to represent this information in the form $f(t) = mt + b$, where $f(t)$ represents the odometer reading after t hours.

$f(t) = 92t + 21144$

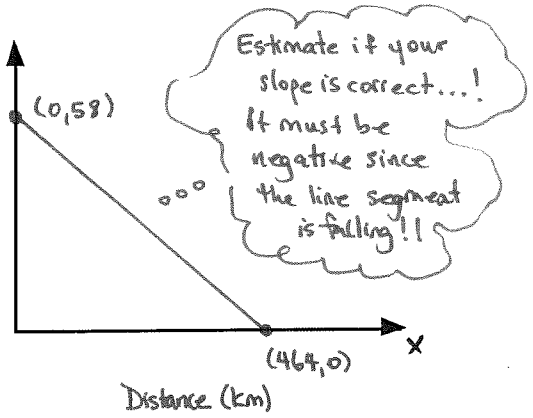
f) State an appropriate domain and range for f.

$f(6) = 92(6) + 21144 = 21696$

domain: $\{t \mid 0 \le t \le 6, t \in R\}$

range: $\{f(t) \mid 21144 \le f(t) \le 21696\}$

7. To test the gas consumption of a new SUV, Jana filled up the gas tank of 58 L and drove the SUV until it was empty. She drove the SUV for 464 km.



a) Sketch the graph on the grid provided with distance travelled on the horizontal axis.

b) Write an equation in the form $y = mx + b$ which represents the volume of fuel in the tank as a function of distance.

$m = \frac{0 - 58}{464 - 0} = -\frac{1}{8}$ $\qquad y = -\frac{1}{8}x + 58$

c) State the slope of the graph, and explain what it represents.

m = Slope $= -\frac{1}{8}$. The volume of fuel in the gas tank is decreasing at a rate of $\frac{1}{8}$ L/km.

d) Determine the distance travelled after using 12 litres of gas.

$\frac{1\text{ km}}{\frac{1}{8}\text{ L}} \times 12\text{ L} = 96\text{ km}$

e) How many litres of gas are used by the SUV when it has travelled 200 km?

$200\text{ km} \times \frac{\frac{1}{8}\text{ L}}{1\text{ km}} = 25\text{ L}$ $\qquad$ 25 litres

8. After 7 days of heavy rainstorms, the water level in a river peaked at 2.85 m above the regular level. Four days later, the water dropped to 2.25 m above the regular level.

a) Assuming the water level falls at a constant rate, determine a function $h(t)$, which describes the height of the river above regular level as a function of time. Take $t = 0$ at peak water level.

Given: $(0, 2.85)$ ← initial time = 0 Solve: $m = \frac{2.25 - 2.85}{4 - 0}$ $h(t) = -0.15t + 2.85$

$(4, 2.25)$

↑ input ↓ output.

$= \frac{-0.6}{4} = -0.15$

b) State the slope of the graph of the function and explain what it represents. Think about it!

m = slope = -0.15. It is the rate at which the water level is changing.

negative value of rate of change makes sense here! It must be negative! The level is falling.

c) Determine an appropriate domain and range for h.

Let $h = 0$, when the height of the river returns to regular level.

$0 = -0.15t + 2.85$

$0.15t = 2.85$

$t = \frac{2.85}{0.15}$

$= 19$

domain: $\{t \mid 0 \le t \le 19, t \in R\}$

range: $\{h \mid 0 \le h \le 2.85, h \in R\}$

Multiple Choice

9. A repair company charges a fixed call-out fee for any service call, plus a fixed rate per hour for the length of the repair. A three hour repair costs \$155, and a four and a half hour repair costs \$215. The fixed call-out fee and the cost of a seven hour repair are respectively

A. \$35 and \$285
B. \$40 and \$285
C. \$35 and \$315
D. \$40 and \$315

S1: Given: second $(3, 155)$ first $(4.5, 215)$ Solve: $m = \frac{215 - 155}{4.5 - 3}$ $= 40$

S2: (Point/slope)/y-int $y - 155 = 40(x - 3)$ $y - 155 = 40x - 120$ $y = 40x + 35$

S3: when $x = 7$ $y = 40(7) + 35 = 315$

Numerical Response

10. The temperature at the top of a mine shaft is 18° C. 250 metres below the surface, the temperature is 18.8° C. To the nearest tenth, the rate of temperature increase in °C per km is _____ .

(Record your answer in the numerical response box from left to right)

3	.	2	

Given: second: $(0, 18)$ first: $(250, 18.8)$ Solve: $m = \frac{18.8 - 18}{250 - 0}$ $= 0.0032$

0.0032°C per m

3.2°C per km

Answer Key

1. **a)** 100 km/h **b)** 150 km/h **c)** 50 km/h **d)** $91\frac{2}{3}$ km/h

2. 15 000 computers per month

3. –768 322 passenger journeys per year

4. **b)** 0.12 **c)** the rate of commission (earnings per sales)
d) 12% **e)** $1200
f) it is the intercept on the vertical axis **g)** $E = 0.12S + 1200$ **h)** $2544 **i)** $9800

5. **b)** –7
c) the rate at which the water level is changing in cm/min
d) –7 cm/min **e)** $W = -7t + 79$ **f)** $11\frac{2}{7}$ min
g) 79 cm **h)** **i)** 23200π cm^3, 9200π cm^3 **ii)** 2800π cm^3/min

6. **b)** 92
c) the average speed between 2h and 5h is 92 km/h
d) 21144 **e)** $f(t) = 92t + 21144$
f) domain $0 \le t \le 6, t \in R$, range $21144 \le f(t) \le 21696, f(t) \in R$

7. **b)** $y = -\frac{1}{8}x + 58$
c) slope $= -\frac{1}{8}$, the volume of fuel in the gas tank is decreasing at the rate of $\frac{1}{8}$ L/km
d) 96 km **e)** 25 litres

8. **a)** $h(t) = -0.15t + 2.85$
b) slope = –0.15, it represents the rate at which the water level is changing in metres per day
c) domain $0 \le t \le 19, t \in R$, range $0 \le h(t) \le 2.85, h(t) \in R$

9. C **10.**

3	.	2	

Equations of Linear Relations Lesson #8:
Practice Test

1. The slope of the line with equation $3y = 2x - 12$ is

point / slope / yint

$y = \frac{2}{3}x - 4$ ← divided both sides by 3.

A. 2
B. $\frac{2}{3}$
C. -4
D. -12

2. The y-intercept of the graph of the line with equation $y = 5x - 10$ is

Recall: $y = mx + b$ (slope, yint)

$y_{int} = b = -10$

A. 2
B. 5
C. 10
D. -10

3. Which equation represents a line with a slope of 3 and a y-intercept of -4?

A. $y = -4x + 3$
B. $y = -\frac{1}{3}x - 4$
C. $y = 3x - 4$
D. $y = 3x + 4$

Let $m = 3$, $b = -4$

point / slope / yint.

$y = mx + b$.

$y = (3)x + (-4)$

$y = 3x - 4$

4. Which of the following is the equation of a line perpendicular to $5y + x + 6 = 0$?

Goal: Isolate the slope of given equation and then take the negative reciprocal.

A. $y = 5x$
B. $y = x$
C. $y = \frac{1}{5}x$
D. $y = -\frac{1}{5}x$

Step1: $5y = -x - 6$ $\quad$ $y = \frac{-1}{5}x - \frac{6}{5}$ $\quad$ Slope $= -\frac{1}{5}$

S2: Negative reciprocal $\quad$ Slope$_{\perp} = 5$

5. Which of these ordered pairs can be found on the graph of the line $3x - 5y - 4 = 0$?

i) $(8, 4)$ $\quad$ ii) $(-3, 1)$ $\quad$ iii) $(0, -0.8)$ $\quad$ iv) $(-2, 2)$

A. i) and ii) only
B. i) and iii) only
C. i), ii), and iii) only
D. some other combination of i), ii), iii), and iv)

i) $3(8) - 5(4) - 4 = 0$ ✓✓ $\quad$ ii) $3(-3) - 5(1) - 4 = 0$ ××

iii) $3(0) - 5(-0.8) - 4 = 0$ ✓✓ $\quad$ iv) $3(-2) - 5(2) - 4 = -20$ ××

∴ For both i) and iii) the LS = RS

6. The point of intersection of the line $9x - 3y + 9 = 0$ and the y-axis is

A. $(0, 9)$
B. $(0, 3)$
C. $(0, -1)$
D. $(0, -3)$

y_{int}: Let $x = 0$ → $-3y + 9 = 0$

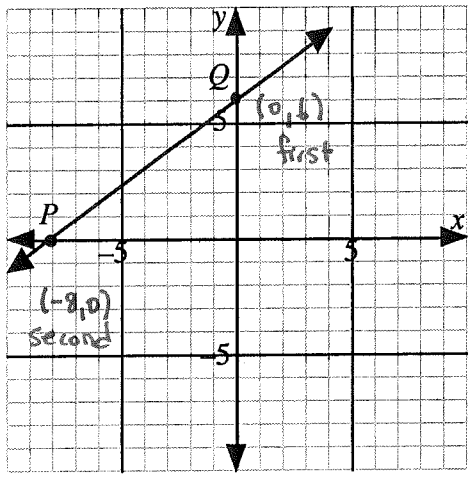
$9(0) - 3y + 9 = 0$ → $9 = 3y$

$y = 3$

$(0, 3)$

*Use the following information to answer the next **two** questions.*

The x- and y-intercepts of the graph shown are integers.



7. The equation of the line PQ is

A. $3x + 4y + 24 = 0$
B. $3x + 4y + 32 = 0$
C. $3x - 4y + 24 = 0$
D. $3x - 4y + 32 = 0$

$m_{PQ} = \frac{6 - 0}{0 + 8} = \frac{6}{8} = \frac{3}{4}$

A slope and/or rate of change must always be simplified to lowest terms.

Point / slope / y_{int}

$P(-8, 0)$

$Q(0, 6)$

$y = mx + b$

$y = \frac{3}{4}x + 6$

$4y = 3x + 24$

$0 = 3x - 4y + 24$

$3x - 4y + 24 = 0$

Numerical Response

1. Given that the line above passes through $(7.2, k)$, the value of k, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right)

1	1	.	4

Let $x = 7.2$

$y = k$

$3(7.2) - 4(k) + 24 = 0$

$21.6 - 4k + 24 = 0$ ← subtract 21.6 and 24 from both sides.

$\frac{4k}{4} = \frac{45.6}{4}$ ← multiply both sides by −1.

$k = 11.4$

8. If the lines $ax+by+c=0$ and $dx+ey+f=0$ are parallel, then

A. $ae-bd=0$
B. $ae+bd=0$
C. $ad-be=0$
D. $ad+be=0$

Goal: If the slopes are parallel, then isolate the slopes, let them be equal to each other, and then simplify until they are equal to zero.

Step 1: $by=-ax-c$

$y=\left[\frac{-a}{b}\right]x-\frac{c}{b}$

Step 2: $ey=-dx-f$

$y=\left[\frac{-d}{e}\right]x-\frac{f}{e}$

S3: $-\frac{a}{b}=-\frac{d}{e}$

$-ae=-bd$

$ae-bd=0$

Numerical Response

2. Given that the line joining the points $(2,3)$ and $(8,-q)$ and where $q \in W$, is perpendicular to the line $3x-2y-5=0$, then the value of q is _____.

(Record your answer in the numerical response box from left to right)

1			

Point slope / y int.

Step 1: $3x-5=2y$

$y=\frac{3}{2}x-\frac{5}{2}$

$m=\left[\frac{3}{2}\right]$ and $m_{\perp}=-\frac{2}{3}$

Step 2: $m=\frac{-q-3}{8-2}=-\frac{2}{3}$

$\frac{-q-3}{6}=\frac{-2}{3}$

$3(-q-3)=-2(6)$

$-3q-9=-12$

$+9 \quad +9$

$\frac{3}{3}=\frac{3q}{3}$

$q=1$

9. The equations of four straight lines are

1) $7x-y=0$ 2) $7x+y-6=0$ 3) $x-7y+4=0$ 4) $x+7y-2=0$

Which pairs of lines are perpendicular?

A. 1) and 2) only
B. 1) and 4) only
C. both 1) and 4) and 2) and 3)
D. both 1) and 2) and 2) and 3)

1.) $7x=y$, $y=7x$ $\quad m=7$

2.) $y=-7x+6$ $\quad m=-7$

3.) $x+4=7y$, $y=\frac{1}{7}x+\frac{4}{7}$ $\quad m=\frac{1}{7}$

4.) $7y=-x-2$ $\quad y=-\frac{1}{7}x-\frac{2}{7}$ $\quad m=-\frac{1}{7}$

Recall: $(-7)\left(\frac{1}{7}\right)=-1$ and $(7)\left(-\frac{1}{7}\right)=-1$

10. The line passing through the points $(-5,-2)$ and $(-2,-1)$ has equation

A. $x+y+3=0$
B. $x+3y+5=0$
C. $x-3y+1=0$
D. $x-3y-1=0$

Point / slope / y int.

$m=\frac{-1+2}{-2+5}=\left[\frac{1}{3}\right]$

$y+1=\frac{1}{3}(x+2)$

$3(y+1)=x+2$

$3y+3=x+2$

$0=x-3y-1$

$x-3y-1=0$

Numerical Response

3. The lines $3x-y+2=0$ and $5x-By+26=0$ and where $B \in W$, intersect on the y-axis. The value of B is _____.

(Record your answer in the numerical response box from left to right)

1	3		

$3x+2=y$

$y=3x+2$

y int $=b=2$

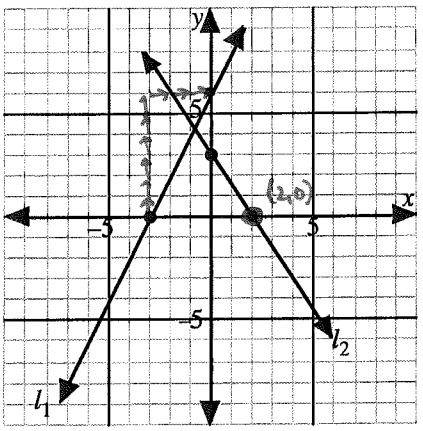
$(0,2)$ lies on $5x-By+26=0$

$5(0)-B(2)+26=0$

$26=2B$

$B=13$

11. Which equation represents a line which is perpendicular to line l_1 and has the same x-intercept as line l_2 ?



A. $x+2y-2=0$
B. $x+2y+2=0$
C. $2x+y-4=0$
D. $2x+y+4=0$

Step 1:
Line 1 (l_1):

$m_{l_1} = \frac{\text{rise}}{\text{run}} = \frac{6}{3} = 2$ $\quad m_{\perp} = -\frac{1}{2}$

Step 2:
Line 2 (l_2): (point / slope / y_{int}.)

$x_{int} = 2$

↓

Point (2, 0)

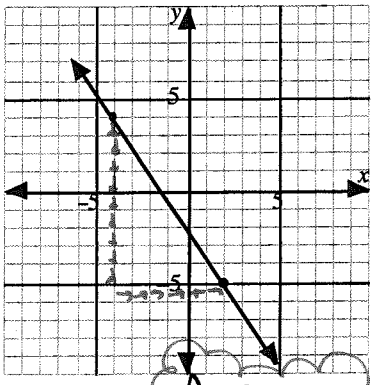
Step 3:

$y-0 = -\frac{1}{2}(x-2)$ → $2y = -x+2$

$2y = -1(x-2)$ → $x+2y-2=0$

Numerical Response

4. The equation of the line shown in the diagram is $Ax+2y+C=0$. The value of $\frac{A}{C}$, to the nearest hundredth, is _____.



$m = \frac{\text{rise}}{\text{run}} = \frac{9}{-6} = -\frac{3}{2}$

(point / slope / y_{int}.)

Point (2, -5)

$y+5 = -\frac{3}{2}(x-2)$

$2(y+5) = -3(x-2)$

$2y+10 = -3x+6$

$3x+2y+4=0$

$A=3$ $C=4$ $\Rightarrow$ $\frac{A}{C} = \frac{3}{4} = 0.75$

(Record your answer in the numerical response box from left to right)

0	.	7	5

Notice the trend! Usually when a slope value must be solved for given an equation we must convert it into $y=mx+b$ form. first!

12. The equation of AB is $x-2y+4=0$.
AB cuts the y-axis at C.
CD is perpendicular to AB.

$x+4 = 2y$

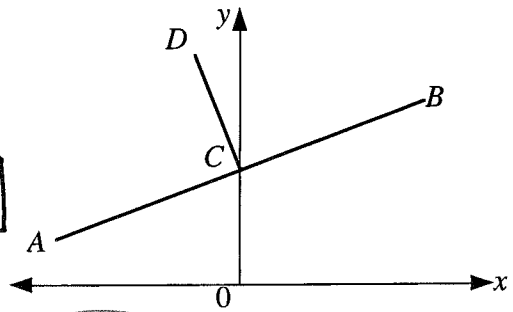
The equation of CD is

$y = \frac{1}{2}x+2$

A. $x+2y-2=0$
B. $2x+y-2=0$
C. $2x-y+2=0$
D. $2x+y-4=0$

$m_{AB} = \frac{1}{2}$ $\quad m_{CD} = -2$

C (0, 2)



$y-2 = -2(x-0)$ (point / slope / y_{int}.)

$y-2 = -2x$

$2x+y-2=0$

13. Consider $\triangle PQR$ in which side PQ has slope $\frac{1}{3}$ and R has coordinates $(-4, 7)$. The equation of the altitude from R to PQ (the line drawn from R to PQ, perpendicular to PQ), is

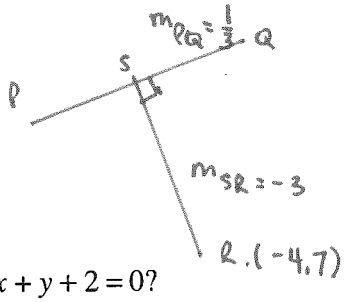
A. $x + 3y = 25$
B. $3x + y = 19$
C. $3x + y = -5$
D. $3x + y = -19$

$m_{SR} = -3$

$y - 7 = -3(x + 4)$

$y - 7 = -3x - 12$

$3x + y + 5 = 0 \longrightarrow 3x + y = -5$



14. Which of the following lines is/are perpendicular to the line $9x + y + 2 = 0$?

i) $9y + x = 2$ ii) $9y - x = 2$ iii) $y = 9x + 2$ iv) $9y = x - 2$

$y = -9x - 2$, $m = -9$

A. i) and iii) only
B. ii) only
C. iv) only
D. some other combination of i), ii), iii), and iv)

i) $9y = -x + 2$, $y = -\frac{1}{9}x + \frac{2}{9}$, $m = -\frac{1}{9}$

ii) $9y = x + 2$, $y = \frac{1}{9}x + \frac{2}{9}$, $m = \frac{1}{9}$

iii) $m = 9$

iv) $y = \frac{1}{9}x - \frac{2}{9}$, $m = \frac{1}{9}$

ii and iv are perpendicular!

15. The line l_1 passes through the points $(-3, 5)$ and $(-2, -1)$.

Which of the following statements is true?

i) l_1 passes through $(4, -37)$.
ii) l_1 has an x–intercept of $-\frac{13}{6}$.
iii) l_1 is perpendicular to $y = \frac{1}{6}x + 2$.

A. i) and ii) only
B. i) and iii) only
C. ii) and iii) only
D. i), ii), and iii)

i)

y	$-6x - 13$
-37	$-6(4) - 13$
	-37

LS = RS ✓

ii) x_{int}: Let $y = 0$ $\quad 0 = -6x - 13$, $6x = -13$, $x = -13/6$

iii) $m_{l_1} = -6$

Side work.

$m = \frac{-1-5}{-2+3} = -6$

$y + 1 = -6(x + 2)$

$y + 1 = -6x - 12$

$y = -6x - 13$

Numerical Response

5. The temperature at sea level is 12.1 °C. At the top of a mountain, 6 400 m above sea level, the temperature is –29.5 °C. To the nearest tenth, the rate of temperature decrease, in °C per km, is _____.

(Record your answer in the numerical response box from left to right)

6	.	5	

$m = \frac{-29.5 - 12.1}{6400 - 0} = 0.0065\ ^\circ C \text{ per m} = 6.5\ ^\circ C \text{ per km}$

Written Response - 5 marks

1. Consider the points $P(-7,-2)$, $Q(2,1)$, $R(-2,-7)$, and $S(8,3)$.

- Show that the equation of the line, L_1, through S and perpendicular to PQ is $y=-3x+27$.

(point/slope) y-int.

$m_{PQ} = \frac{1+2}{2+7} = \frac{3}{9} = \frac{1}{3}$

$y-3 = -3(x-8)$

$y-3 = -3x+24$

$y = -3x+27$

$m_{\perp} = -3$

$S(8,3)$

Check work

$(\frac{1}{3})(-3) = \frac{-3}{3} = -1$

- Determine the equation of the line, L_2, through R and parallel to PQ. Give the answer in in slope y-intercept form.

(Point/slope) y-int.

$m_{PQ} = \frac{1+2}{2+7} = \frac{3}{9} = \frac{1}{3}$

$y+7 = \frac{1}{3}(x+2)$

$m_{parallel} = \frac{1}{3}$

$y = \frac{1}{3}x + \frac{2}{3} - 7$

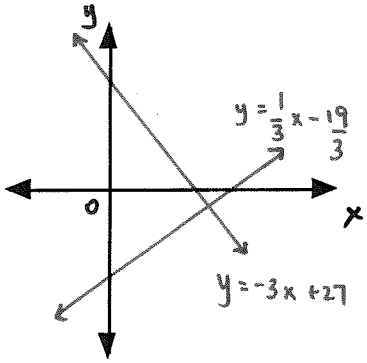
$R(-2,-7)$

$y = \frac{1}{3}x - \frac{19}{3}$

- Draw both lines on the grid, and state a suitable window which shows x- and y-intercepts for each graph.

$x:[-5,25,5]$

$y:[-10,35,5]$



Answer Key

1. B	2. D	3. C	4. A	5. B
6. B	7. C	8. A	9. C	10. D
11. A	12. B	13. C	14. D	15. D

Numerical Response

1.	1	1	.	4	2.	1				3.	1	3		
4.	0	.	7	5	5.	6	.	5						

Written Response

1. • $y=-3x+27$ • $y=\frac{1}{3}x-\frac{19}{3}$ • x:[−5, 25, 5] and y:[−10, 35, 5]