Characteristics of Linear Relations Lesson #1: Line Segments on a Cartesian Plane

Lesson 1 and Lesson 2 of this unit are not required for this curriculum, but are included because

1. they are important characteristics of linear relations not covered elsewhere
2. this information is required in higher level math courses such as Calculus

Unit Overview

The graph of a linear relation is represented by a straight line. The line can be infinite or finite depending on the domain and range of the linear relation. In some cases we are only interested in a portion of a line. This portion is called a **line segment**.

We have already studied some of the characteristics of the graph of a linear relation: intercepts, domain, and range. In this unit we study some characteristics of line segments: namely, length, midpoint, distance, and slope. We demonstrate an understanding of slope with respect to rise and run, the slope formula, and rate of change. We then discuss the slopes of parallel and perpendicular lines.

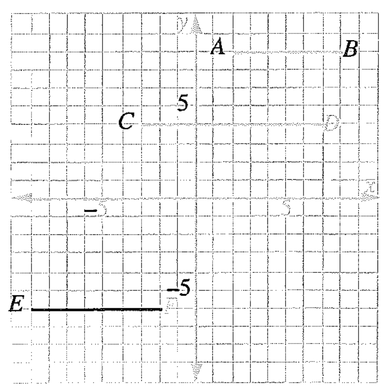
Line Segment

A line segment is the portion of a line between two points on the line.

If the endpoints of a line segment are A and B, we refer to it as line segment AB.

NOTE: Line segment AB may also be written as $\overline{AB}$.

Length of a Horizontal Line Segment



Consider the line segments shown on the grid.

a) Find the length of each line segment by counting.
- length of AB is __6__ units.
- length of CD is __10__ units.
- length of EF is __7__ units.

b) Determine the coordinates of the endpoints of each line segment.
- $AB \rightarrow$ $A(2, 8)$ $B(8, 8)$
- $CD \rightarrow$ $C(-3, 4)$ $D(7, 4)$
- $EF \rightarrow$ $E(-9, -6)$ $F(-2, -6)$

c) Complete the following.
- The difference in the x-coordinates, $x_B - x_A$, is __6__. ← $x_B - x_A = 8 - 2 = 6$
- The difference in the x-coordinates, $x_D - x_C$, is __10__.
- The difference in the x-coordinates, $x_F - x_E$, is __7__.

d) How can the coordinates of the end points of a horizontal line segment be used to find the length of the line segment?

Subtract the x-coordinate of the left endpoint from the x-coordinate of the right endpoint. length = right − left

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Class Ex. #1

a) Line segment AB has endpoints $A(2, 8)$ to $B(-5, 8)$. Determine the length of $\overline{AB}$.

right left

length $= x_A - x_B = 2 - (-5) = 2 + 5 = 7$ 7 units

b) Determine the length of the line segment from $P(a-2, b)$ to $Q(a+4, b)$.

left. right

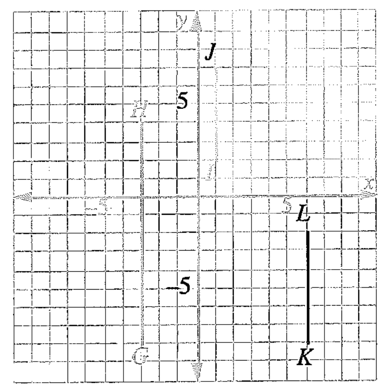
length $= x_Q - x_P = (a+4) - (a-2)$ 6 units

$= a + 4 - a + 2$

$= 6$

Length of a Vertical Line Segment

Consider the line segments shown on the grid.



a) Find the lengths of each line segment by counting.
- length of GH is 12 units.
- length of IJ is 5 units.
- length of KL is 6 units.

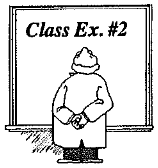
b) Determine the coordinates of the endpoints of each line segment.
- $GH \rightarrow$ $G(-3, -8)$ $H(-3, 4)$
- $IJ \rightarrow$ $I(1, 2)$ $J(1, 7)$
- $KL \rightarrow$ $K(6, -8)$ $L(6, -2)$

c) Complete the following.
- The difference in the y-coordinates, $y_H - y_G$, is 12. ← $4 - (-8) = 4 + 8 = 12$
- The difference in the y-coordinates, $y_J - y_I$, is 5.
- The difference in the y-coordinates, $y_L - y_K$, is 6.

d) How can the coordinates of the end points of a vertical line segment be used to find the length of the line segment?

Subtract the y-coordinate of the lower endpoint from the y-coordinate of the higher endpoint. length = higher - lower

Class Ex. #2



a) Line segment RS has endpoints $R(1, -4)$ to $S(1, -9)$. Determine the length of $\overline{RS}$.

higher lower

length $= y_R - y_S = -4 - (-9) = -4 + 9 = 5$ 5 units

b) Determine the length of the line segment from $P(a, b)$ to $Q(a, b+10)$.

lower higher

length $= y_Q - y_P = (b+10) - b = 10$

10 units

Complete Assignment Questions #1 - #5

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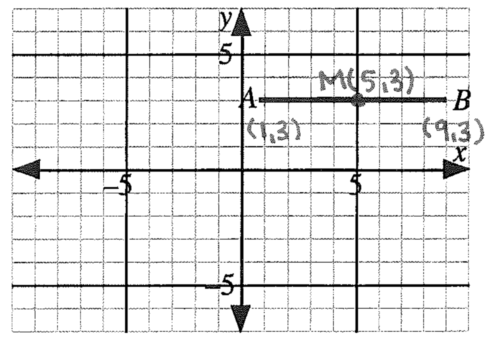
Midpoint

The **midpoint,** ***M*****,** of a line segment on the graph of a linear relation is the point at the centre of the line segment.

Midpoint of a Horizontal Line Segment

Consider the line segment AB shown on the grid.



a) Determine the coordinates of the midpoint by counting. Label the midpoint, M, on the grid and list the coordinates beside it.

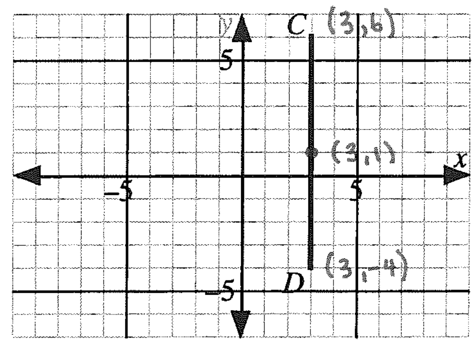
b) List the coordinates of point A and point B on the grid. How can the x-coordinates of points A and B be used to find the coordinates of the midpoint of a horizontal line?

A(1,3) B(9,3) Determine the average (mean) of the x-coordinates.

$M = \frac{1+9}{2} = \frac{10}{2} = 5$

Midpoint of a Vertical Line Segment

Consider the line segment CD shown on the grid.



a) Determine the coordinates of the midpoint by counting. Label the midpoint, M, on the grid and list the coordinates beside it.

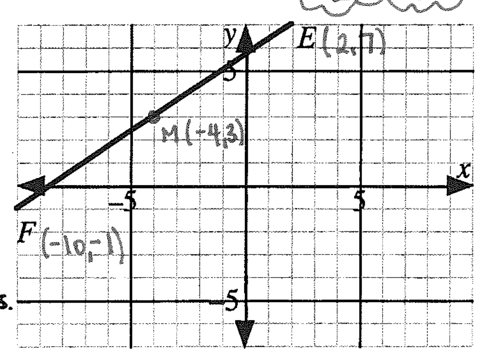
b) List the coordinates of point C and point D on the grid. How can the y-coordinates of points C and D be used to find the coordinates of the midpoint?

C(3,6) D(3,-4) Determine the average (mean) of the y-coordinates.

$M = \frac{6+(-4)}{2} = \frac{2}{2} = 1$

Midpoint of an Oblique (Diagonal) Line Segment

Consider the line segment EF shown on the grid.



a) Use the results from above to determine the midpoint of EF.

M(-4,3)

b) Express in words how to find the midpoint, M, of the line segment joining the points (x_1, y_1) and (x_2, y_2). Determine the average of the x-coordinates and the average of the y-coordinates.

c) Complete the formula to express the relationship in **b)**. $x_M = \frac{x_1+x_2}{2}$ $y_M = \frac{y_1+y_2}{2}$

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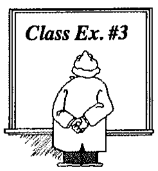
Midpoint of a Line Segment

Consider line segment PQ with endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$.

The midpoint, M, of the line segment has coordinates.

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Line segment PQ can also be written as $\overline{PQ}$.



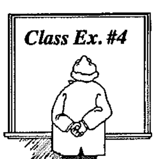
Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

a) $P(4,7)$, $Q(12,3)$ **b)** $E(-5,7)$, $F(-11,-2)$ **c)** $A(w+3, 2w)$, $C(5w-1, 7w+1)$

$M\left(\frac{4+12}{2}, \frac{7+3}{2}\right)$ $M\left(\frac{-5+(-11)}{2}, \frac{7+(-2)}{2}\right)$ $M\left(\frac{w+3+5w-1}{2}, \frac{2w+7w+1}{2}\right)$

$M(8,5)$ $M\left(-8, \frac{5}{2}\right)$ $M\left(\frac{6w+2}{2}, \frac{9w+1}{2}\right)$

$M\left(3w+1, \frac{9w+1}{2}\right)$



Ruby was doing a question in her coordinate geometry homework, and her little brother Max wrote over part of the question as a prank.

$P(5$ $Q(-11,-10)$

Midpoint$(\ ,-6)$

Calculate the missing coordinates.

S1: Missing Midpoint x-coordinate.

$x_M = \frac{5+(-11)}{2} = -3$

S2: Missing Point P y-coordinate.

$\frac{y_P + (-10)}{2} = -6$

$\times 1$

$y_P - 10 = -12$

$+10 \quad +10$

$y_P = -2$

Complete Assignment Questions #6 - #17

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Assignment

Horizontal Length = Right − Left.
Vertical Length = Higher − Lower

1. Determine the length of each line segment.

   a) $A(2,7)$ to $B(5,7)$

   Right − Left $= 5-2 = 3$

   b) $C(-5,3)$ to $D(-5,12)$

   Higher − Lower $= 12-3 = 9$

   c) $I(-3,-8)$ to $J(-3,-3)$

   higher − lower $= 3-(-8) = 3+8 = 11$

   d) $K(7,-10)$ to $L(-35,-10)$

   right − Left $= 7-(-35) = 7+35 = 42$

2. Determine whether each line segment is horizontal or vertical, and write an expression for its length.

   a) $A(p,q)$ to $B(p-4,q)$

   horizontal

   right − Left $= p-(p-4) = p-p+4 \boxed{=4}$

   b) $C(m-3,n+5)$ to $D(m-3,n+12)$

   vertical

   higher − lower $= (n+12)-(n+5) = n+12-n-5$

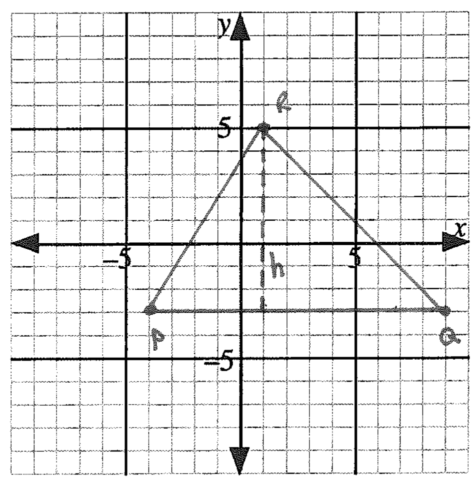
   $\boxed{=7}$

3. A triangle has vertices $P(-4,-3)$, $Q(9,-3)$, and $R(1,5)$.
   - Sketch the triangle on the grid.
   - Calculate the area of the triangle.

   base $= 9-(-4) = 13$ height $= 8$

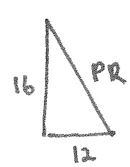
   $A = \frac{1}{2}bh \quad = \frac{1}{2}(13)(8)$

   $= 52$ units^2



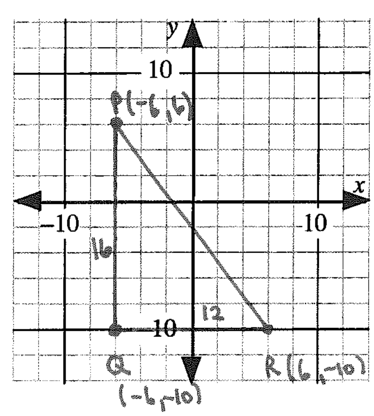
4. On the grid, plot the points $P(-6,6)$, $Q(-6,-10)$, and $R(6,-10)$.

   a) Determine the distance from P to R using the Pythagorean Theorem.



   $PR^2 = PQ^2 + QR^2$
   $= 16^2 + 12^2$
   $= 256 + 144$
   $PR^2 = 400$

   $PR = \sqrt{400}$
   $= 20$ units.



   b) Calculate the area and perimeter of $\triangle PQR$.

   perimeter $= 16+12+20 = 48$ units.

   area $= \frac{1}{2}bh = \frac{1}{2}(12)(16) = 96$ units^2

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5. Rebecca uses quadrant I in a Cartesian plane to describe the location of the bases in a game of high school softball. The four bases form a square. The origin is at home plate. First base is at $(18, 0)$, and the distance between each base is 18 m. The pitcher's mound is located between home plate and second base.

   a) State the coordinates of second base. $(18,18)$

   b) The pitcher stands on the mound 12 m from home plate. If she has to throw a ball to second base, what distance to the nearest tenth of a metre, would she throw the ball?

$OB^2 = AB^2 + OA^2$

$OB^2 = 18^2 + 18^2$

$= 648$

$OB = \sqrt{648}$

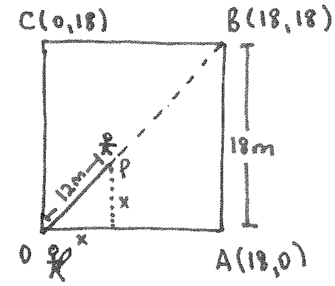
$= 25.45\ldots$

$PB = OB - OP$

$= 25.45\ldots - 12$

$= 13.45\ldots$

She throws the ball 13.5 m.



6. Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

   a) $A(2,6)$, $C(4,16)$ b) $X(-3,-8)$, $Y(-11,0)$ c) $K(15,-17)$, $L(-11,3)$

$M\left(\frac{2+4}{2}, \frac{6+16}{2}\right)$ $\quad$ $M\left(\frac{-3+(-11)}{2}, \frac{-8+0}{2}\right)$ $\quad$ $M\left(\frac{15+(-11)}{2}, \frac{-17+3}{2}\right)$

$M(3,11)$ $\quad$ $M(2,-7)$ $\quad$ $M(2,-7)$

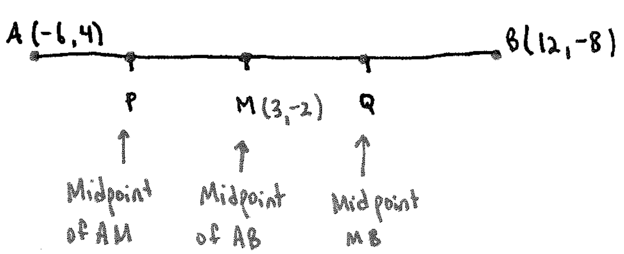
7. Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

   a) $C(3x, 8y)$, $D(7x, -4y)$ b) $S(a+b, a+7b)$, $T(a+b, a-3b)$

$M\left(\frac{3x+7x}{2}, \frac{8y+(-4y)}{2}\right)$ $\quad$ $M\left(\frac{a+b+a+b}{2}, \frac{a+7b+a-3b}{2}\right)$

$M(5x, 2y)$ $\quad$ $M(a+b, a+2b)$

8. Otto was given two points: $A(-6,4)$ and $B(12,-8)$. He was asked to divide $\overline{AB}$ into four equal parts. State the coordinates of the points which will divide $\overline{AB}$ into four equal parts.



$M\left(\frac{-6+12}{2}, \frac{4-8}{2}\right) = M(3,-2)$

$P\left(\frac{-6+3}{2}, \frac{4-2}{2}\right) = P\left(\frac{-3}{2}, 1\right)$

$Q\left(\frac{3+12}{2}, \frac{-2-8}{2}\right) = Q\left(\frac{15}{2}, -5\right)$

The points are $\left(\frac{-3}{2}, 1\right)$, $(3,-2)$, and $\left(\frac{15}{2}, -5\right)$.

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9. In each case M is the midpoint of $\overline{AB}$. Determine the value of x.

a) $A(2,6)$, $B(6,x)$, $M(4,-1)$ **b)** $A(3,6)$, $B(x,0)$, $M(0,3)$

$\frac{6+x}{2} = \frac{-1}{1}$

$6+x = -2$
$-6 \quad -6$

$\underline{\underline{x = -8}}$

$\frac{3+x}{2} = \frac{0}{1}$

$3+x = 0$
$-3 \quad -3$

$\underline{\underline{x = -3}}$

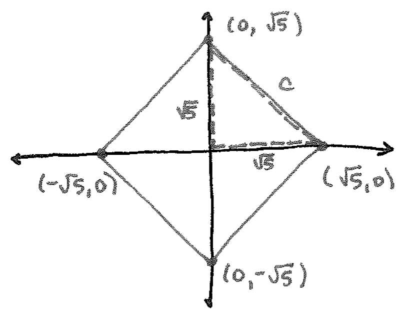
Multiple Choice

10. $ABCD$ is a square with vertices $(\sqrt{5},0)$, $(0,\sqrt{5})$, $(-\sqrt{5},0)$, and $(0,-\sqrt{5})$ respectively. The area of the square, in unit^2, is

A. 5
B. 10
C. 20
D. 100

S1: $c^2 = (\sqrt{5})^2 + (\sqrt{5})^2$
$= 5+5$
$c = \sqrt{10}$

S2: $A = lw$
$= (\sqrt{10})(\sqrt{10})$
$= 10 \text{ units}^2$



11. $P(4,-8)$ and $Q(-2,10)$ are the endpoints of a diameter of a circle. The coordinates of the centre of the circle are

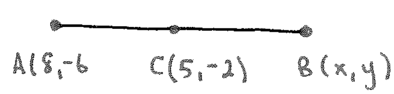
A. $(-3,9)$
B. $(2,2)$
C. $(3,-9)$
D. $(1,1)$

$M\left(\frac{4+(-2)}{2}, \frac{-8+10}{2}\right)$

$M(1,1)$

12. AB is a diameter of a circle; the centre is C. If $A(8,-6)$ and $C(5,-2)$ then B is the point

A. $(2,2)$
B. $(6.5,-4)$
C. $(11,-10)$
D. $(13,-8)$



x_B: $\frac{8+x}{2} = 5$
$8+x = 10$
$x = 2$

y_B: $\frac{-6+y}{2} = -2$
$-6+y = -4$
$y = 2$

13. Which statement is always true?
A. Two line segments of equal length have the same midpoint.
B. Two line segments with the same midpoint are of equal length.
C. A point equidistant from the endpoints of a line segment is the midpoint.
D. None of the above statements is always true.

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Numerical Response

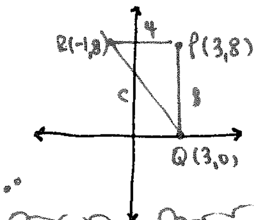
14. To the nearest tenth, the perimeter of $\triangle PQR$ with vertices, $P(3,8)$, $Q(3,0)$, and $R(-1,8)$ is _____.

(Record your answer in the numerical response box from left to right)

2	0	.	9

$QR^2 = PQ^2 + PR^2$
$c^2 = 8^2 + 4^2$
$= 64 + 16$
$= 80$
$c^2 = 80$
$c = \sqrt{80}$
$= 8.9\ldots$

perimeter $= PQ + PR + QR$
$= 8 + 4 + 8.9\ldots$
$= 20.9$



Always make a quick sketch to understand better!

15. The midpoint of line segment ST is $M\left(\frac{1}{2}, -4\right)$. If the coordinates of T are $(-3,3)$, and the coordinates of S are (x, y), the value of x is _____.

(Record your answer in the numerical response box from left to right)

4			

x_s: $2\left(\frac{-3+x}{2}\right) = \left(\frac{1}{2}\right)2$ $\quad -3 + x = 1$ $\quad +3 \quad +3$ $\quad x = 4$

16. The point $M(a,6)$ is the midpoint of $\overline{GH}$ with $G(22,b)$ and $H(6,-8)$. The value of $a+b$ is _____.

(Record your answer in the numerical response box from left to right)

3	4		

$\frac{22+6}{2} = a$
$28 = 2a$
$a = 14$

$\frac{b-8}{2} = 6$
$b - 8 = 12$
$b = 20$

$a + b = 14 + 20 = 34$

17. The midpoint of line segment AB lies on the y-axis. A lies on the x-axis, and B has coordinates $(-4,5)$. The length of AB, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

9	.	4	

$M(0, y)$ $\quad A(x, 0)$ $\quad B(-4, 5)$

$\frac{x-4}{2} = 0$
$x - 4 = 0$
$\boxed{x = 4}$

$\frac{0+5}{2} = y$
$2y = 5$
$\boxed{y = \frac{5}{2}}$

$A(4,0) \quad B(-4,5)$

$d_{AB} = \sqrt{(-4-4)^2 + (5-0)^2} = \sqrt{89}$
$= 9.433\ldots$
$= 9.4$

Answer Key

1. a) 3 b) 9 c) 5 d) 42 **2.** a) horizontal, 4 b) vertical, 7
3. 52 units^2 **4.** a) 20 units b) area = 96 units^2, perimeter = 48 units
5. a) (18,18) b) 13.5 m **6.** a) (3, 11) b) (−7, −4) c) (2, −7)
7. a) $(5x, 2y)$ b) $(a+b, a+2b)$ **8.** $\left(-\frac{3}{2}, 1\right)$, $(3,-2)$, $\left(\frac{15}{2}, -5\right)$
9. a) −8 b) −3 **10.** B **11.** D **12.** A **13.** D

14.

2	0	.	9

15.

4			

16.

3	4		

17.

9	.	4	

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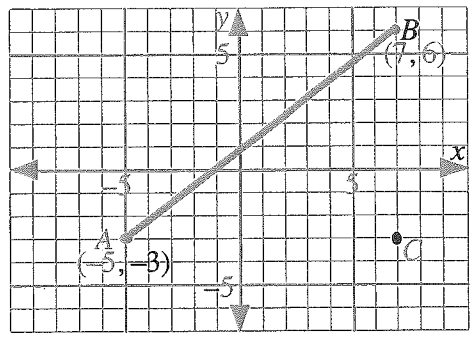
Characteristics of Linear Relations Lesson #2: The Distance Formula

Investigation



Consider line segment AB shown on the grid.

a) Use the Pythagorean theorem to show that the length of AB is 15 units.

AC = 12 $\quad AB^2 = 12^2 + 9^2 = 225$

BC = 9 $\quad AB = \sqrt{225} = 15$

b) Complete the following:

length of $AC = x_B - x_A = 7 - \underline{(-5)} = \underline{12}$

length of $CB = y_B - y_A = \underline{6} - \underline{(-3)} = \underline{9}$

c) Complete the following to verify the length of AB.

(length of AB)^2 = (difference in x-coordinates of B and A)^2 + (difference in y-coordinates of B and A)^2

length of AB $= \sqrt{(\text{difference in } x\text{-coordinates of } B \text{ and } A)^2 + (\text{difference in } y\text{-coordinates of } B \text{ and } A)^2}$

length of AB $= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

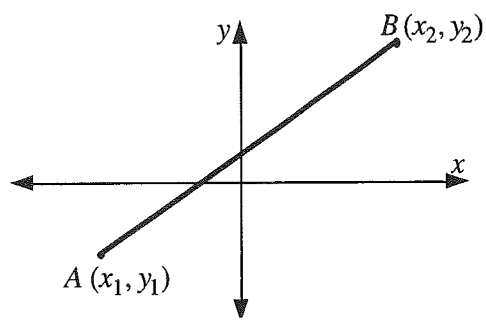
length of AB $= \sqrt{(7 - (-5))^2 + (6 - (-3))^2}$

length of AB $= \sqrt{12^2 + 9^2}$

length of AB $= \sqrt{225}$

length of AB $= 15$

d) Use the same procedure to make a rule for finding the distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$.



(length of AB)^2 = (difference in x-coordinates of B and A)^2 + (difference in y-coordinates of B and A)^2

length of AB $= \sqrt{(\text{difference in } x\text{-coordinates of } B \text{ and } A)^2 + (\text{difference in } y\text{-coordinates of } B \text{ and } A)^2}$

length of AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - x_2)^2}$

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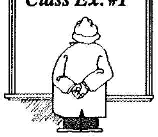
The Distance Formula

To find the length of a line segment on the graph of a linear relation, we can use the distance formula.

To find the distance, d, between points $P(x_1, y_1)$ and $Q(x_2, y_2)$, use

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{or} \quad d_{PQ} = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$$

Class Ex. #1



Find the exact length of the following line segments.

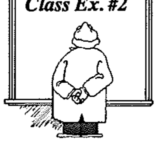
a) $P(2,3)$ to $Q(10,-3)$

$$d_{PQ} = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$$
$$= \sqrt{(10-2)^2 + (-3-3)^2}$$
$$= \sqrt{8^2 + (-6)^2}$$
$$= \sqrt{100}$$
$$= \underline{\underline{10}}$$

b) $G(-25,3)$ to $H(-17,-5)$

$$d_{GH} = \sqrt{(x_H - x_G)^2 + (y_H - y_G)^2}$$
$$= \sqrt{(-17-(-25))^2 + (-5-3)^2}$$
$$= \sqrt{8^2 + (-8)^2}$$
$$= \sqrt{128}$$

Class Ex. #2



Sometimes grids are superimposed on maps to find the distance between two locations. For example, to calculate the distance between two craters, Copernicus and Plato, on the Earth's moon, a coordinate system could be superimposed on a map of the moon. Ordered pairs would then be assigned to Copernicus and Plato to find the distance between the two craters. If the location of Copernicus on the coordinate grid is $(-89, 226)$ and the location of Plato is $(136, 179)$, calculate the distance between the two craters to the nearest unit.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(136-(-89))^2 + (179-226)^2}$$
$$= \sqrt{225^2 + (-47)^2}$$
$$= \sqrt{52834}$$
$$= 229.85\ldots$$

Distance = 230 units.

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Class Ex. #3

a) Explain how we can determine if a triangle is right-angled if we know the length of each side.

If the square of the length of the longest side is equal to the sum of the squares of the lengths of the other two sides, then the triangle is right angled.

b) L is the point $(0,1)$, M is $(-3,-3)$ and N is $(-7,0)$. Prove that $\triangle LMN$ is right-angled.

$d_{LM} = \sqrt{(-3-0)^2+(-3-1)^2}$
$= \sqrt{(-3)^2+(-4)^2}$
$= \sqrt{25}$
$= 5$

$d_{LN} = \sqrt{(-7-0)^2+(0-1)^2}$
$= \sqrt{(-7)^2+(-1)^2}$
$= \underbrace{\sqrt{50}}_{\text{longest side}}$

$d_{MN} = \sqrt{(-7-(-3))^2+(0-(-3))^2}$
$= \sqrt{(-4)^2+(3)^2}$
$= \sqrt{25}$
$= 5$

$(d_{LN})^2 = (\sqrt{50})^2 = 50$
$(d_{LM})^2 + (d_{MN})^2 = 5^2+5^2 = 50$ $\longrightarrow$ since $LN^2 = LM^2 + MN^2$
$\triangle LMN$ is rightangled at M.

Complete Assignment Questions #1 - #10

Assignment

1. Determine the distance between each pair of points.

Remember: It is important to keep the order of the coordinates the same as they are substituted into the distance formula!

a) $A(2,0)$ and $B(7,12)$

$d_{AB} = \sqrt{(7-2)^2+(12-0)^2}$
$= \sqrt{5^2+12^2}$
$= \sqrt{169}$
$= 13$

b) $C(3,7)$ and $D(6,11)$

$d_{CD} = \sqrt{(6-3)^2+(11-7)^2}$
$= \sqrt{3^2+4^2}$
$= \sqrt{25}$
$= 5$

2. Determine the distance, to the nearest hundredth, between each pair of points.

a) $P(4,0)$ and $Q(-2,-7)$

$d_{PQ} = \sqrt{(-2-4)^2+(-7-0)^2}$
$= \sqrt{(-6)^2+(-7)^2}$
$= \sqrt{85}$
$= 9.22$

b) $R(-2.3, 8.9)$ and $S(-3.4,-6.8)$

$d_{RS} = \sqrt{(-3.4-(-2.3))^2+(-6.8-8.9)^2}$
$= \sqrt{(-1.1)^2+(-15.7)^2}$
$= \sqrt{247.7}$
$= 15.74$

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3. Consider the points $P(-2, 2)$, $Q(1, 6)$ and $R(7, 14)$.

a) Calculate the lengths of PQ, QR, and PR. What do you notice?

$d_{PQ} = \sqrt{(1-(-2))^2 + (6-2)^2}$
$= \sqrt{3^2 + 4^2}$
$= \sqrt{25}$
$= 5$

$d_{QR} = \sqrt{(7-1)^2 + (14-6)^2}$
$= \sqrt{6^2 + 8^2}$
$= \sqrt{100}$
$= 10$

$d_{PR} = \sqrt{(7-(-2))^2 + (14-2)^2}$
$= \sqrt{9^2 + 12^2}$
$= \sqrt{225}$
$= 15$

$d_{PQ} + d_{QR} = d_{PR}$

b) What does this mean with regard to the points $P, Q,$ and R?

The points P, Q, R lie on a straight line.

4. A is the point $(6, -2)$, B is $(4, 4)$ and C is $(-3, -5)$.

a) Show that the exact length of AB is $\sqrt{40}$.

$d_{AB} = \sqrt{(4-6)^2 + (4-(-2))^2}$
$= \sqrt{(-2)^2 + 6^2}$
$= \sqrt{40}$

b) Determine the exact lengths of BC and AC and prove that $\angle BAC$ is a right angle.

$d_{BC} = \sqrt{(-3-4)^2 + (-5-4)^2}$
$= \sqrt{(-7)^2 + (-9)^2}$
$= \sqrt{130}$

$d_{AC} = \sqrt{(-3-6)^2 + (-5-(-2))^2}$
$= \sqrt{(-9)^2 + (-3)^2}$
$= \sqrt{90}$

$(d_{BC})^2 = (d_{AB})^2 + (d_{AC})^2$
$(\sqrt{130})^2 = (\sqrt{40})^2 + (\sqrt{90})^2$
$130 = 40 + 90$
$130 = 130$

ΔABC is a right angled triangle at A because $BC^2 = AB^2 + AC^2$.

So $\angle BAC$ is a right angle.

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Remember: the order must be consistent $x_2 - x_1$ and $y_2 - y_1$! second point second point first point

5. A refinery is to be built halfway between the rural towns of Branton and Deer Bridge. A railway is to be built connecting the towns to the refinery. On a Cartesian plane, Branton is located at (1232, 3421) and Deer Bridge is located at (1548, 3753).

M (1390, 3587)

Branton refinery Deer Bridge

a) What are the coordinates of the refinery?

$M\left(\frac{1548+1232}{2}, \frac{3753+3421}{2}\right) = M(1390, 3587)$

b) Determine the length of the railway, to the nearest kilometre, if the grid scale is 1 unit represents 100 metres.

$d = \sqrt{(1548-1232)^2 + (3753+3421)^2}$

$= \sqrt{99856 + 110224}$

$= 458.344\ldots$ units

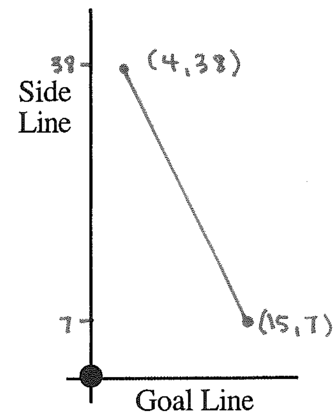
$458.344\ldots \times 100$

$= 45834.484\ldots$ m

46 km

6. In a high school football game, the Chiefs' quarterback scrambles to the Chiefs' 7 yard line, 15 yards from the left sideline. From that position he throws the ball upfield. The pass is caught by a wide receiver who is on the Chiefs' 38 yard line, 4 yards from the left sideline.

The first quadrant of a coordinate grid is superimposed on the football field, with the origin located at the intersection of the Chiefs' goal line and the left side line.



a) With reference to this origin, state the ordered pairs for the location of the quarterback (when he throws the ball) and the wide receiver (when he catches the ball). Mark these points on the grid shown. (15,7) (4,38)

b) Determine the length of the pass (to the nearest yard).

$d = \sqrt{(4-15)^2 + (38-7)^2}$

$= \sqrt{(-11)^2 + 31^2}$

$= \sqrt{1082} = 32.89\ldots$

length of pass = 33 yards

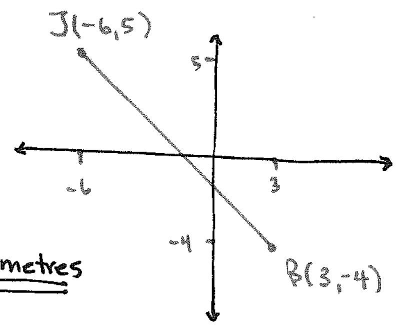
7. At the end of a high school soccer game, Jonas walks 6 blocks west and 5 blocks north from the corner of the soccer field to his house. Beverly walks 3 blocks east and 4 blocks south to reach her house from the same corner of the soccer field.

N W E S

a) Taking the corner of the soccer field as the origin, list the coordinates of each home.

J(-6,5) B(3,-4)

b) If a block represents 135 metres, determine the direct distance, to the nearest metre, between their homes.



$d_{JB} = \sqrt{(3-(-6))^2 + (-4-5)^2}$

$= \sqrt{9^2 + (-9)^2}$

$= \sqrt{162}$

$= 12.727\ldots$

$12.727\ldots \times 135$

$= 1718.26\ldots$

1718 metres

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Multiple Choice

8. The distance between the points $(2,-1)$ and $(6,2)$ is

(A.) 5
B. $\sqrt{7}$
C. $\sqrt{17}$
D. 25

$d = \sqrt{(6-2)^2 + (2-(-1))^2}$

$= \sqrt{4^2 + 3^2}$

$= \sqrt{25}$

$= 5$

Explore: It does not matter which point is considered 1st and which is second. It only matters that we are consistent!

$d = \sqrt{(2-6)^2 + (-1-2)^2}$

$= \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5$

Same answer!

9. A circle with its centre at the origin passes through the point $(-6,-8)$. The radius of the circle is

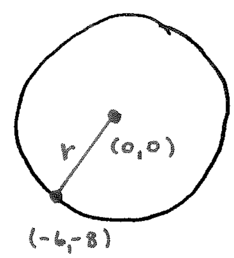
A. 6
B. 8
(C.) 10
D. 100

$d_r = \sqrt{(0-(-6))^2 + (0-(-8))^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{100}$

$= 10$



the distance from (-6,-8) to (0,0) is 10 units and represents the radius.

Numerical Response

10. The diameter, to the nearest tenth, of the circle with centre $(-5,9)$ which passes through $(4,-3)$ is _____.

(Record your answer in the numerical response box from left to right)

3	0	.	0

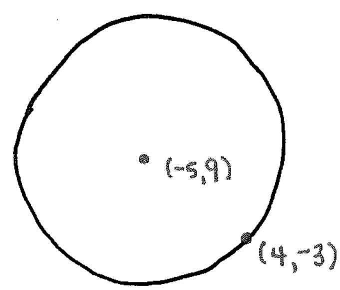
Goal: To solve for radius and then x2 as the diameter is twice the radius.

Step 1: radius $= d_r = \sqrt{(4-(-5))^2 + (-3-9)^2}$

$= \sqrt{(9)^2 + (-12)^2}$

$= \sqrt{225}$

$= 15$



S2: $15 \times 2 = 30$ The diameter is 30 units.

Answer Key

1. **a)** 13 **b)** 5 **2.** **a)** 9.22 **b)** 15.74

3. **a)** $PQ = 5$, $QR = 10$, $PR = 15$. $PQ + QR = PR$ **b)** The points P, Q and R lie on a straight line.

4. **b)** $BC = \sqrt{130}$, $AC = \sqrt{90}$.
Since $BC^2 = AB^2 + AC^2$, triangle ABC must be right angled at A so angle BAC is a right angle.

5. **a)** (1390, 3587) **b)** 46 km

6. **a)** $Q(15,7)$ $W(4,38)$ **b)** 33 yards

7. **a)** $J(-6,5)$ $B(3,-4)$ **b)** $\sqrt{162}$ blocks ~ 1718 m.

8. A **9.** C **10.**

3	0	.	0

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Characteristics of Linear Relations Lesson #3: Slope of a Line Segment

A trucker driving up a hill with a heavy load may be concerned with the steepness of the hill. When building a roof, a builder may be concerned with the steepness (or pitch) of the roof. A skier going down a hill may be concerned with the steepness of the ski hill.

In mathematics, the term **slope** is used to describe the steepness of a line segment.

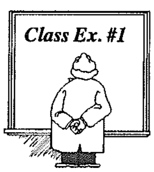
Slope of a Line Segment

The **slope** of a line segment is a measure of the steepness of the line segment.

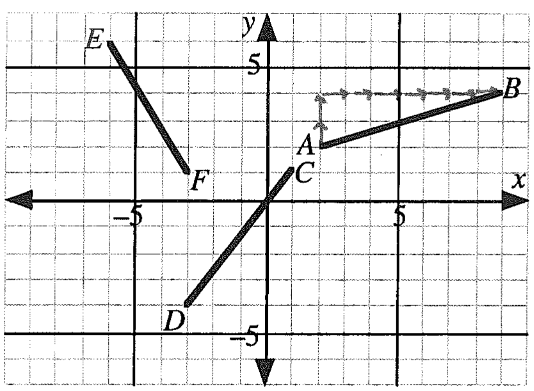
It is the ratio of **rise** (the change in vertical height between the endpoints) over **run** (the change in horizontal length between the endpoints).

$$\text{Slope} = \frac{\textbf{rise}}{\textbf{run}}$$

- the **rise** is POSITIVE if we count UP, and NEGATIVE if we count DOWN.
- the **run** is POSITIVE if we count RIGHT, and NEGATIVE if we count LEFT.



Each line segment on the grid has endpoints with integer coordinates. Complete the table below.



Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB	2	7	$\frac{2}{7}$
CD	-5	-4	$\frac{-5}{-4} = \frac{5}{4}$
EF	-5	3	$\frac{-5}{3} = -\frac{5}{3}$

positive ↑
positive →

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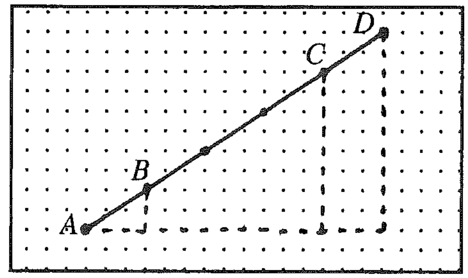
Investigation #1 *Investigating the Slope of Line Segments*

a) Complete the chart. Write the slopes in simplest form.

Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB	2	3	$\frac{2}{3}$
AC	8	12	$\frac{8}{12} = \frac{2}{3}$
AD	10	15	$\frac{10}{15} = \frac{2}{3}$
BC	6	9	$\frac{6}{9} = \frac{2}{3}$



b) How are the slopes of the line segments related?

They are the same.

Slope of a Line

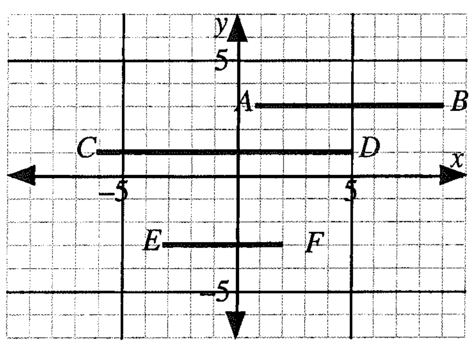
The slopes of all line segments on a line are equal.
The slope of a line representing the graph of a linear relation can be found using

$$\text{slope} = \frac{\text{rise}}{\text{run}} \text{ for any two points on the line.}$$

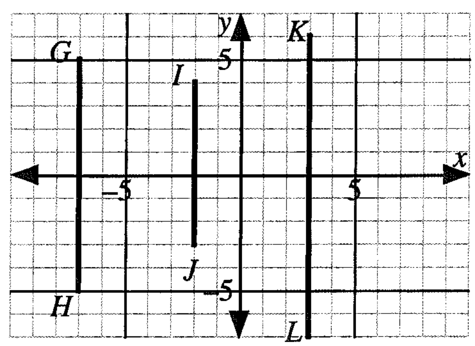
Investigation #2 *Slopes of Horizontal and Vertical Line Segments*

Consider the line segments in Grid 1 and Grid 2 below.

Grid 1



Grid 2



a) Determine the slopes of all the line segments in Grid 1. zero ← ie: $\frac{\text{rise}}{\text{run}}$ $\text{slope}_{AB} = \frac{0}{8} = 0$

b) Determine the slopes of all the line segments in Grid 2.
undefined ← ie $\frac{\text{rise}}{\text{run}}$ $\text{slope}_{GH} = \frac{10}{0} = \text{undefined}$

c) Complete the following statements.

- Horizontal line segments have a slope of zero.
- Vertical line segments have an undefined slope.

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Investigation #3 *Positive and Negative Slopes*

a) Each line on the grids passes through at least two points with integer coordinates. Calculate the slope of each of the lines.

positive ↑ and negative ↓
positive → negative ←

Remember on a Cartesian Plane
- the **rise** is POSITIVE if we count UP, and NEGATIVE if we count DOWN
- the **run** is POSITIVE if we count RIGHT, and NEGATIVE if we count LEFT

Grid 1 **Grid 2**

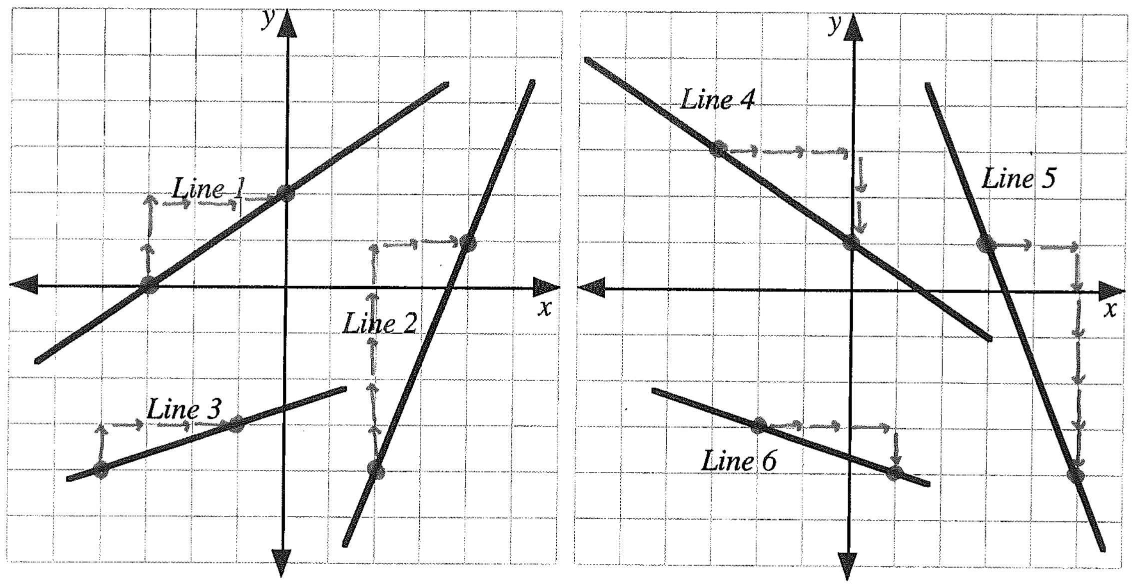

Table For Grid 1

Line	Slope
1	$\frac{2}{3}$
2	$\frac{5}{2}$
3	$\frac{1}{3}$

Table For Grid 2

Line	Slope
4	$-\frac{2}{3}$
5	$-\frac{5}{2}$
6	$-\frac{1}{3}$

b) Compare the slopes of:

- Line 1 and Line 4
- Line 2 and Line 5
- Line 3 and Line 6

The slopes are opposite in sign.

c) Complete the following statements.

- A line which rises from left to right has a positive slope.
- A line which falls from left to right has a negative. slope.

Complete Assignment Questions #1 and #2

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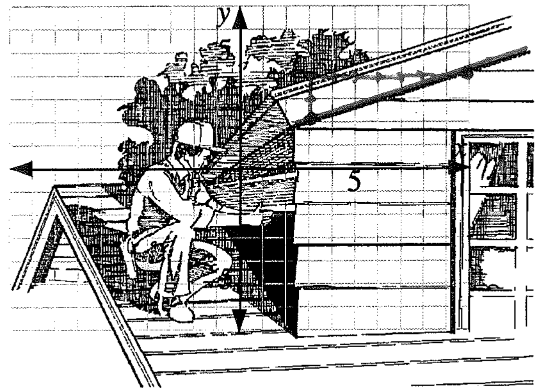
Class Ex. #2

A grid has been superimposed on the sketch.



a) Estimate the pitch (slope) of the roof to the right of the worker's head.

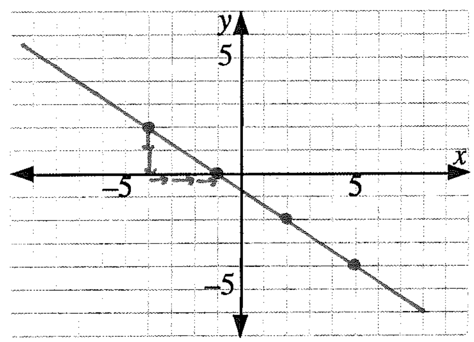
$\frac{2}{7}$ ... $\frac{\text{positive}\uparrow}{\text{positive}\rightarrow}$

b) Could the grid be used to estimate the pitch of the roof the worker is standing on? Explain.

No, the roof is not on the same plane as the grid.

Class Ex. #3

Draw a line segment on the grid which passes through the point $(-4, 2)$ and has a slope of $-\frac{2}{3}$. The line segment must be long enough to cross both the x-axis and the y-axis.



Write the coordinates of three other points on the line segment which have integer coordinates.

$(-1,0), (2,-2), (5,-4)$

Think about it!

Slope $= -\frac{2}{3}$ → $\frac{\text{negative}\downarrow}{\text{positive}\rightarrow}$, $\frac{\text{positive}\uparrow}{\text{negative}\leftarrow}$

Note: either use of the slope will draw the same line segment!

Step 1: Plot point $(-4,2)$

S2: Use the plotted point and slope $-\frac{2}{3}$ to plot an additional point

S3: Connect and extend the straight line to draw a line segment.

Class Ex. #4

A line segment has a slope of $-\frac{5}{7}$ and a rise of 12. Calculate the run as an exact value.

$\frac{\text{rise}}{\text{run}} = \frac{-5}{7}$

Let rise = 12

$\frac{12}{\text{run}} = \frac{-5}{7}$

$\frac{84}{-5} = \frac{-5\,\text{run}}{-5}$

$\text{run} = \frac{-84}{5}$

Complete Assignment Questions #3 - #13

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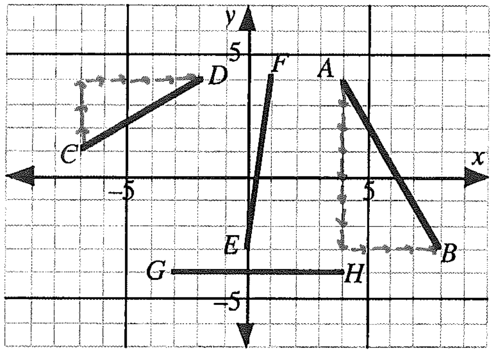
Assignment

1. Each line segment on the grid has endpoints with integer coordinates. Complete the table.



Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB	-7	4	-7/4
CD	3	5	3/5
EF	7	1	7/1 = 7
GH	0	7	0/7 = 0

2. Every line on the grid passes through at least two points with integer coordinates. Calculate the slope of each of the lines.

slope of Line 1 : $\frac{1}{2}$

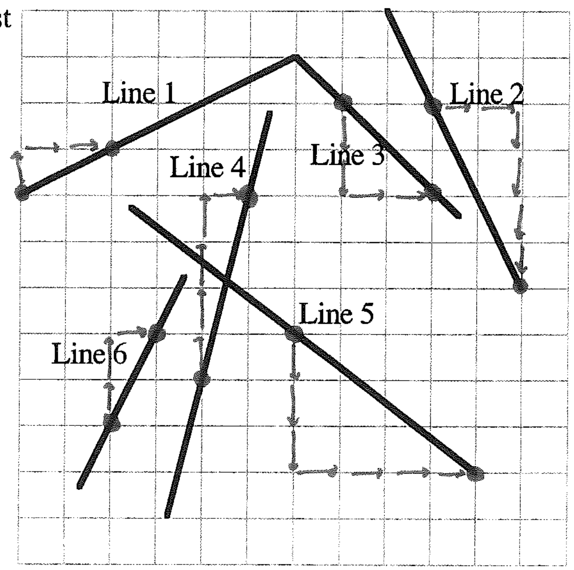
slope of Line 2: $\frac{-4}{2} = -2$

slope of Line 3: $\frac{-2}{2} = -1$

slope of Line 4: $\frac{4}{1} = 4$

slope of Line 5: $\frac{-3}{4}$

slope of Line 6: $\frac{2}{1} = 2$



Recall: Slope Signs $\frac{\text{positive}\uparrow}{\text{positive}\rightarrow}$ and $\frac{\text{negative}\downarrow}{\text{negative}\leftarrow}$

3. Draw a line segment on the grid which passes through the point (–5, –2) and has a slope of $\frac{2}{3}$. The line segment must be long enough to cross both the *x*-axis and the *y*-axis.

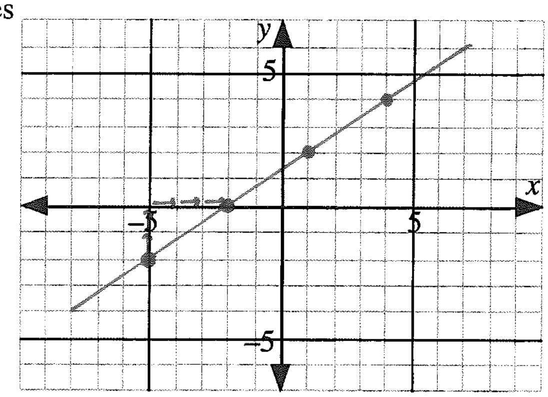
Write the coordinates of three other points on the line segment which have integer coordinates.

(-2,0)

(1,2)

(4,4)

slope = $\frac{\text{rise}}{\text{run}} = \frac{2}{3}$ $\frac{\text{positive}\uparrow}{\text{positive}\rightarrow}$



Notice the graph does not have arrows on each end. This make it a line segment.

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4. Repeat question #3 for line segments with the given slope passing through the given point.

$\frac{\text{positive} \uparrow}{\text{positive} \rightarrow}$ ← rising

a) slope $= \frac{2}{5}$, $(2, 1)$

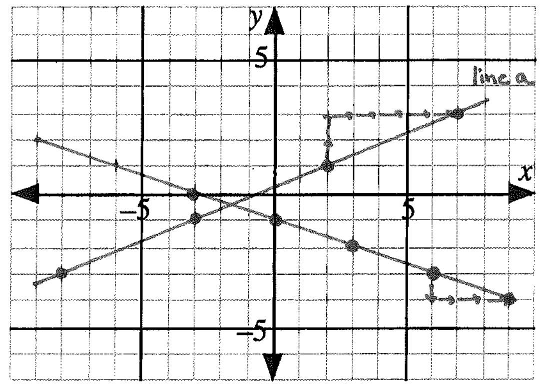
(−8,−3),(−3,−1),(7,3)

b) slope $= -\frac{1}{3}$, $(6, -3)$

$\frac{\text{negative} \downarrow}{\text{positive} \rightarrow}$

↑ falling

(−3,0),(0,−1),(3,−2)



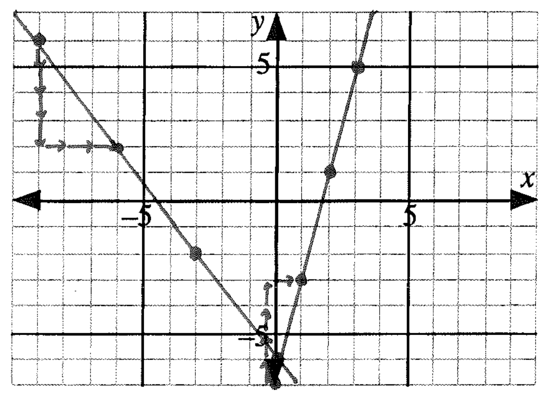
c) slope $= -\frac{4}{3}$, $(-9, 6)$

(−6,2),(−3,−2),(0,−6)

d) slope $= 4$, $(0, -7)$

slope $= \frac{\text{rise}}{\text{run}} = 4 = \frac{4}{1}$

(1,−3),(2,1),(3,5)



Recall: Slope = 0 is a horizontal line.

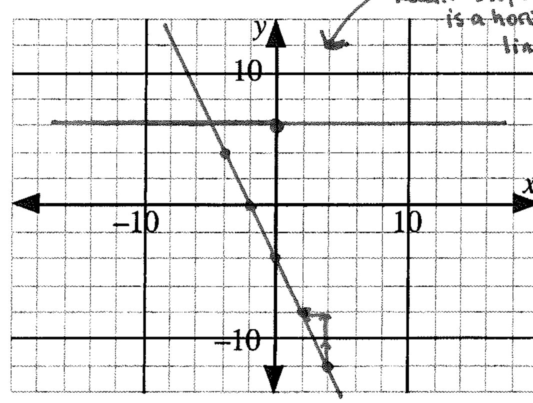
e) slope $= -2$, $(4, -12)$

slope $= \frac{-2}{1}$ or $\frac{2}{-1}$ → $\frac{4}{-2}$ since the scale goes up by 2 each time!

(2,−8),(0,−4),(−2,0)

f) slope $= 0$, $(0, 6)$

(1,6),(2,6),(3,6)



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5. P has coordinates $(-1, 2)$. Find two positions for point Q so that the slope of PQ is

a) 2 $\frac{\text{rise}}{\text{run}} = \frac{2}{1}$

$(0,4)$, $(1,6)$

Think about it!

$(-1+1, 2+2)$

$(-1,2)$

b) -3 $\frac{\text{rise}}{\text{run}} = \frac{-3}{1}$

$(0,-1)$, $(1,-4)$

$(-1+1, 2-3)$

c) $\frac{1}{3}$ $\frac{\text{rise}}{\text{run}} = \frac{1}{3}$

d) $-\frac{2}{5}$ $\frac{\text{rise}}{\text{run}} = \frac{-2}{5}$

$(4,0)$, $(9,-2)$

e) 0 $\frac{\text{rise}}{\text{run}}$ = horizontal line

$(0,2)$, $(1,2)$

f) undefined $\frac{\text{rise}}{\text{run}}$

undefined

$\div$ by 0

$(-1,1)$, $(-1,0)$

6. Two of three measures are given for rise, run, and slope.
Calculate the value of the third measure in each of the following.

a) slope $= \frac{5}{7}$ and run $= 49$

$\frac{\text{rise}}{\text{run}} = \frac{5}{7}$ $\qquad$ $\frac{\text{rise}}{49} = \frac{5}{7}$

Let run $= 49$ $\qquad$ $7\text{rise} = 245$

$\boxed{\text{rise} = 35}$

b) slope $= -\frac{3}{8}$ and rise $= 15$

$\frac{\text{rise}}{\text{run}} = \frac{-3}{8}$ $\qquad$ $\frac{15}{\text{run}} = \frac{-3}{8}$

Let rise $= 15$ $\qquad$ $-3\text{run} = 120$

$\boxed{\text{run} = -40}$

c) slope $= -\frac{6}{11}$ and run $= 33$

$\frac{\text{rise}}{\text{run}} = \frac{-6}{11}$ $\qquad$ $\frac{\text{rise}}{33} = \frac{-6}{11}$

Let run $= 33$ $\qquad$ $\frac{11\text{rise}}{11} = \frac{-198}{11}$

$\boxed{\text{rise} = -18}$

d) slope $= \frac{3}{4}$ and rise $= 15$

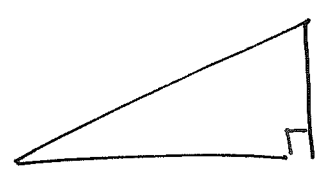
$\frac{\text{rise}}{\text{run}} = \frac{3}{4}$ $\qquad$ $\frac{15}{\text{run}} = \frac{3}{4}$

Let rise $= 15$ $\qquad$ $\frac{3\text{run}}{3} = \frac{60}{3}$

$\boxed{\text{run} = 20}$

7. A ramp which has been set up by skateboarders has a slope of $\frac{2}{3}$. Calculate the height of the ramp if the it has a base length of 1.5 metres.

$\frac{2}{3} = \frac{h}{1.5}$ $\qquad$ $3 = 3h$ $\qquad$ $h = 1$ $\qquad$ heigh = 1 metre



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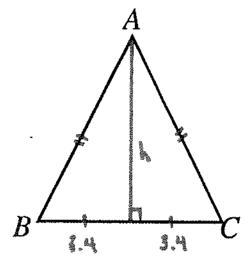
8. Triangle ABC is isosceles with $AB = AC$ and $BC = 6.8$ cm. Calculate the area of the triangle if the slope of $AC = -\frac{5}{4}$.



$$\frac{h}{-3.4} = \frac{-5}{4}$$

$$4h = 17$$

$$h = \frac{17}{4}$$

$$\text{Area} = \frac{1}{2}bh$$

$$= \frac{1}{2}(6.8)\left(\frac{17}{4}\right)$$

$$= 14.45$$

$\underline{\underline{14.45 \text{ cm}^2}}$

Multiple Choice **9.** The slope of $\overline{PQ}$ is

negative ↓
positive →

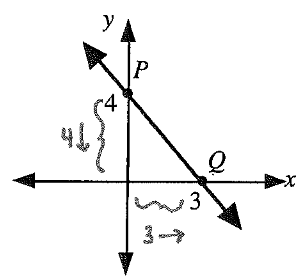
A. $\frac{3}{4}$

B. $-\frac{3}{4}$

C. $\frac{4}{3}$

D. $-\frac{4}{3}$

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = -\frac{4}{3}$$



10. The point $(-4, 0)$ is on a line which has a slope of $-\frac{2}{5}$. The next point with integer coordinates on the line to the right of $(-4, 0)$ is

A. $(-9, -2)$

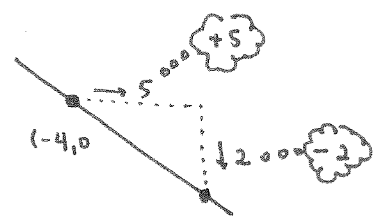
B. $(-9, 2)$

C. $(1, -2)$

D. $(-2, -5)$

point: $(-4+5, 0-2)$

$= (1, -2)$



11. P is a point in quadrant I, Q is a point in quadrant II, R is a point in quadrant III, and S is a point in quadrant IV.

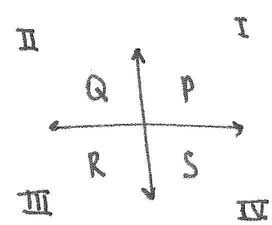
Which one of the following statements must be true?

A. Line segment PQ has a positive slope. maybe

B. Line segment QR has a positive slope. maybe

C. Line segment PR has a positive slope. ✓

D. Line segment QS has a positive slope. negative



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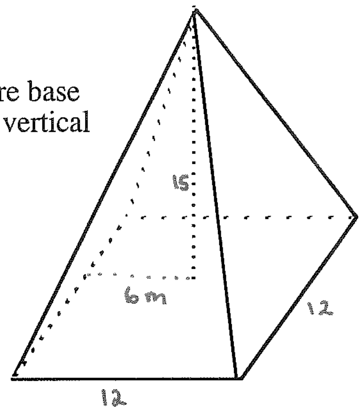
Use the following information to answer questions #12 and #13.

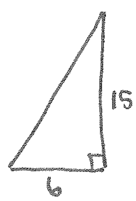
A pyramid has a square base of length 12 m and a vertical height of 15 m.



Numerical Response 12. A beetle starts to climb the pyramid starting from the midpoint of one of the faces. To the nearest tenth, the slope of the beetle's climb is _____.

(Record your answer in the numerical response box from left to right)

2	.	5	



Slope $= \frac{15}{6} = 2.5$

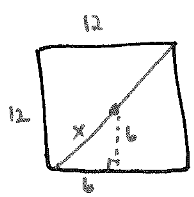
Slope is positive meaning the slope in the diagram should rise. And it does!

13. A fly starts to climb the pyramid along one of the edges. To the nearest tenth, the slope of the fly's climb is _____.

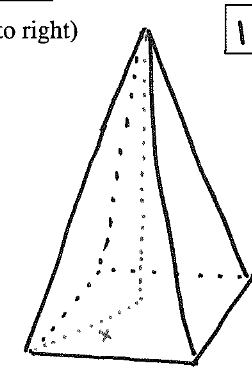
(Record your answer in the numerical response box from left to right)

1	.	8	

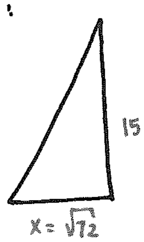
base:



$x^2 = 6^2 + 6^2$
$= 72$
$x = \sqrt{72}$



slope :



Slope $= \frac{15}{\sqrt{72}} = 1.767\ldots$

Slope $= \underline{1.8}$

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Answer Key

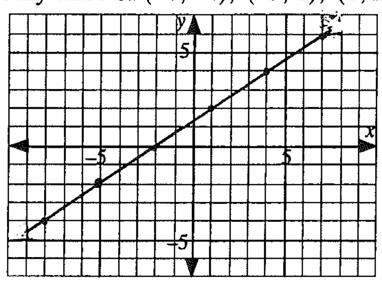
1.

Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB	−7	4	−7/4
CD	3	5	3/5
EF	7	1	7/1 = 7
GH	0	7	0/7 = 0

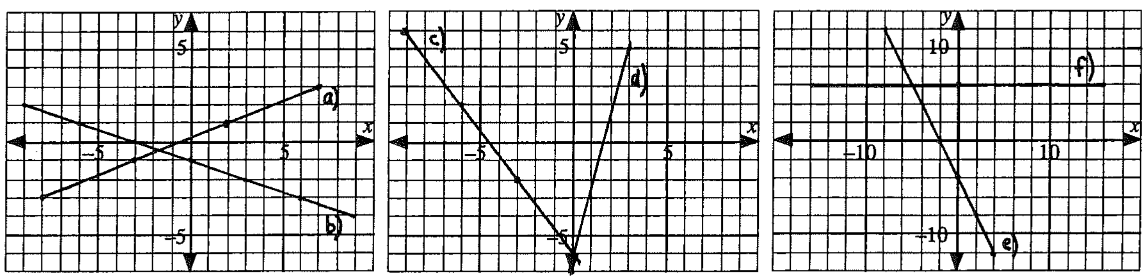
2. slope of line 1 = $\frac{1}{2}$, slope of line 2 = −2, slope of line 3 = −1

slope of line 4 = 4, slope of line 5 = $-\frac{3}{4}$, slope of line 6 = 2

3. Any three of (−8, −4), (−2, 0), (1, 2), (4, 4)



4.



a) (−8, −3), (−3, −1), (7, 3)

b) Any 3 of (−9, 2), (−6, 1), (−3, 0) (0, −1), (3, −2), (9, −4)

c) (−6, 2), (−3, −2), (0, −6)

d) (1, −3), (2, 1), (3, 5)

e) Many possible answers including (2, −8), (0, −4), (−2, 0)

f) Many possible answers including (1, 6), (2, 6), (3, 6)

5. Many possible answers, including any two from:

a) (−3, −2), (−2, 0), (0, 4), (1, 6) **b)** (−3, 8), (−2, 5), (0, −1), (1, −4),
c) (2, 3), (5, 4), (−4, 1), (−7, 0) **d)** (−11, 6), (−6, 4), (4, 0), (9, −2)
e) (−3, 2), (−2, 2), (0, 2), (1, 2) **f)** (−1, 1), (−1, 0), (−1, −1), (−1, 3)

6. a) rise = 35 **b)** run = −40 **c)** rise = −18 **d)** run = 20

7. 1 metre **8.** 14.45 cm^2 **9.** D **10.** C **11.** C

12.

2	.	5	

13.

1	.	8	

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Characteristics of Linear Relations Lesson #4: The Slope Formula

Review

Complete the following statements.

a) Slope is the measure of the _______________ of a line.

b) Slope is the ratio of the vertical change (called the __________) over the horizontal change (called the __________).

c) A line segment which rises from left to right has a ______________ slope.

d) A line segment which falls from left to right has a ______________ slope.

e) A horizontal line segment has a slope of _____.

f) A vertical line segment has an ______________ slope.

g) The slopes of all line segments on a line are ________ .

Developing the Slope Formula

a) Calculate the slope of line segment AB using slope = $\frac{\text{rise}}{\text{run}}$.

$\frac{3}{6} = \frac{1}{2}$

b) List the coordinates of the endpoints of line segment AB.

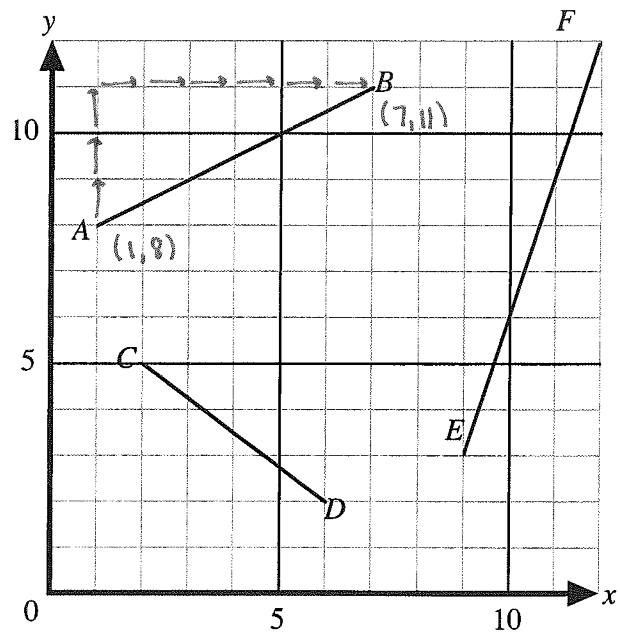
$A(1, 8)$ $B(7, 11)$

c) How can the rise of line segment AB be determined using y_B and y_A?

$y_B - y_A = 11 - 8 = 3$

d) How can the run of line segment AB be determined using x_A and x_B?

$x_B - x_A = 7 - 1 = 6$



e) Use your results from **c)** and **d)** to write a formula which describes how the slope of line segment AB can be calculated using its endpoints.

$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_B - y_A}{x_B - x_A}$

f) Calculate the slope of line segment AB using the formula in **e)**.

$\text{Slope} = \frac{11-8}{7-1} = \frac{3}{6} = \frac{1}{2}$

g) Calculate the slope of the line segments CD and EF using the method in **a)** and verify using the formula from **e)**.

$\text{slope of } CD = \frac{y_D - y_C}{x_D - x_C} = \frac{2-5}{6-2} = \frac{-3}{4}$

$\text{slope of } EF = \frac{y_F - y_E}{x_F - x_E} = \frac{12-3}{12-9} = \frac{9}{3} = 3$

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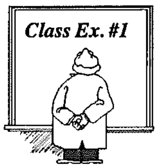
The Slope Formula

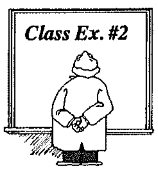
In mathematics the letter "m" is used to represent slope. If the graph of a linear relation passes through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the slope of this line can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{or} \qquad m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P}$$



Find the slope of a line which passes through the points $G(-3, 8)$ and $H(7, -2)$.

$$m_{GH} = \frac{y_H - y_G}{x_H - x_G} = \frac{-2 - 8}{7 - (-3)} = \frac{-10}{10} = 1$$



Eleanor, Bonnie, and Carl are calculating the slope of a line segment with endpoints $E(15, 8)$ and $F(-10, 6)$. Their work is shown below.

	Eleanor	Bonnie	Carl
Step 1:	$m_{\overline{EF}} = \frac{-10-15}{6-8}$	$m_{\overline{EF}} = \frac{6-8}{15-(-10)}$	$m_{\overline{EF}} = \frac{8-6}{15-10}$
Step 2:	$= \frac{-25}{-2}$	$= \frac{-2}{25}$	$= \frac{2}{5}$
Step 3:	$m_{\overline{EF}} = \frac{25}{2}$	$m_{\overline{EF}} = -\frac{2}{25}$	$m_{\overline{EF}} = \frac{2}{5}$

Since their answers are all different, at least two of the students have made errors in their calculations. Describe all the errors which have been made and determine the correct slope.

Eleanor used $\frac{x_F - x_E}{y_F - y_E}$ instead of $\frac{y_F - y_E}{x_F - x_E}$ ← Eleanor used $\frac{\text{run}}{\text{rise}}$ instead of $\frac{\text{rise}}{\text{run}}$!

Bonnie used $\frac{y_F - y_E}{x_E - x_F}$ instead of $\frac{y_F - y_E}{x_F - x_E}$ ← Bonnie was not consistant! she flipped her coordinates!

Carl attempted to use $\frac{y_E - y_F}{x_E - x_F} = \frac{8 - 6}{15 - (-10)}$ ← Carl replaced x_F by 10 not -10.

Complete Assignment Questions #1 - #5

correct slope $= \frac{6-8}{-10-15} = \frac{-2}{-25} = \boxed{\frac{2}{25}}$

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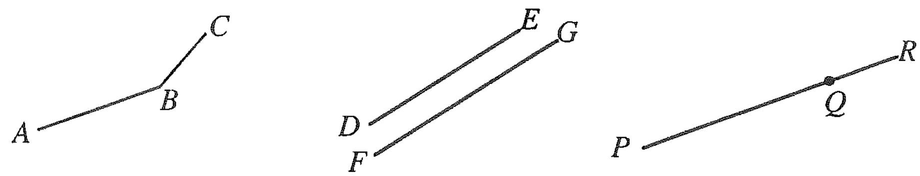
Collinear Points

Two lines in a plane can either be



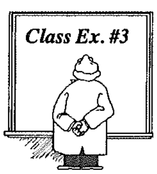
at an angle

parallel and distinct

parallel and form a straight line.

Points that lie on the same straight line are said to be **collinear**, i.e. P, Q, and R are collinear.

If three points P, Q, and R are collinear then $m_{PQ} = m_{QR} = m_{PR}$.
Proving that any two of these three slopes are equal is sufficient for the third to be equal and for the points to be collinear.



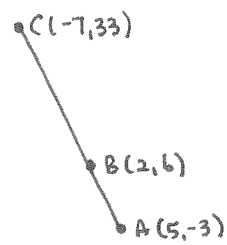
Consider points $A(5,-3)$, $B(2,6)$, and $C(-7,33)$.

a) Prove that the points A, B, and C are collinear.

$m_{AB} = \dfrac{y_B - y_A}{x_B - x_A} = \dfrac{6-(-3)}{2-5} = \dfrac{9}{-3} = -3$

$m_{BC} = \dfrac{y_C - y_B}{x_C - x_B} = \dfrac{33-6}{-7-2} = \dfrac{27}{-9} = -3$

Since $m_{AB} = m_{BC}$ the points A, B, and C are collinear.



b) Find the value of y if the point $D(-4, y)$ lies on line segment AC.

$m_{AD} = -3 \longleftarrow$ This is true since $m_{AC} = -3$ and point D lies on it.

$m_{AD} = \dfrac{y_D - y_A}{x_D - x_A} = \dfrac{y-(-3)}{-4-5} = \dfrac{y+3}{-4-5}$

$\dfrac{y+3}{-9} = \dfrac{-3}{1}$

$y + 3 = 27$
$ -3 \quad -3$

$\underline{\underline{y = 24}}$

Complete Assignment Questions #6 - #12

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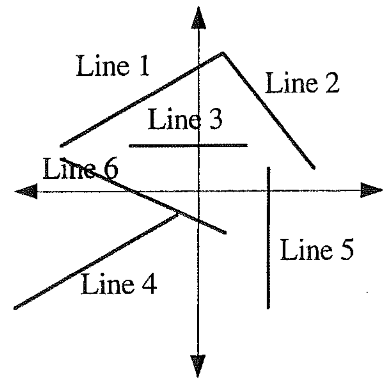
Assignment



1. State whether the slope of each line is positive, negative, zero, or undefined.

Line 1: positive since it is rising from left to right.
Line 2: negative since it is falling from left to right.
Line 3: zero since it's horizontal.
Line 4: positive ... rising....
Line 5: undefined since it is vertical
Line 6: negative ... falling ....

Recall: We must be consistent. The point we decide to be first must be first for both x- and y-values. Otherwise, we will be unable to solve for slope.

2. Use the slope formula to calculate the slope of the line segment with the given endpoints.

a) $A(12,-2)$ (second) and $B(0,3)$ (first)

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{3-(-2)}{0-12} = -\frac{5}{12}$$

b) $C(-2,3)$ (second) and $D(2,-2)$ (first)

$$m_{CD} = \frac{y_D - y_C}{x_D - x_C} = \frac{-2-3}{2-(-2)} = -\frac{5}{4}$$

c) $P(-15,-2)$ (second) and $O(0,0)$ (first)

$$m_{PO} = \frac{y_O - y_P}{x_O - x_P} = \frac{0-(-2)}{0-(-15)} = \frac{2}{15}$$

d) $S(36,-41)$ (second) and $T(-20,-27)$ (first)

$$m_{ST} = \frac{y_T - y_S}{x_T - x_S} = \frac{-27-(-41)}{-20-36} = \frac{14}{-56} = -\frac{1}{4}$$

e) $U(-172,-56)$ (second) and $V(-172,32)$ (first)

$$m_{UV} = \frac{y_V - y_U}{x_V - x_U} = \frac{32-(-56)}{-172-(-172)} = \frac{88}{0} = \text{undefined}$$

f) $K(8,-41)$ (second) and $L(397,-41)$ (first)

$$m_{KL} = \frac{y_L - y_K}{x_L - x_K} = \frac{-41-(-41)}{397-8} = \frac{0}{389} = 0$$

3. Use the slope formula to calculate the slope of the line passing through the given points.

Again consistent points are key here!

a) $(3,-6)$ (second) and $(8,4)$ (first)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-6)}{8-3} = \frac{10}{5} = 2$$

b) $(-12,7)$ (second) and $(0,-2)$ (first)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-7}{0-(-12)} = \frac{-9}{12} = -\frac{3}{4}$$

c) $(-3,-8)$ (second) and $(1,5)$ (first)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-8)}{1-(-3)} = \frac{13}{4}$$

d) $(21,1)$ (second) and $(-4,-9)$ (first)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-9-1}{-4-21} = \underbrace{\frac{-10}{-25}} = \frac{2}{5}$$

Slope must always be simplified to lowest terms!

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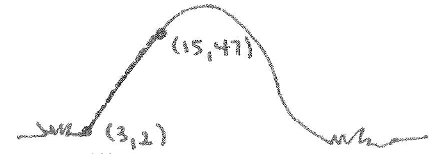
4. A coordinate grid is superimposed on a cross-section of a hill. The coordinates of the bottom and the top of a straight path up the hill are, respectively, $(3,2)$ and $(15,47)$, where the units are in metres.



a) Calculate the slope of the hill.

$m = \frac{47-2}{15-3} = \frac{45}{12} = \frac{15}{4}$

b) Calculate the coordinates of the midpoint of the path up the hill.

Recall: The averages of both the x- and y-coordinates

$M\left(\frac{15+3}{2}, \frac{47+2}{2}\right) = M\left(9, \frac{49}{2}\right)$

Notice the slope of the path is rising. This will make our final solution positive! A great way to estimate if the solution is correct!

c) Calculate the length of the path to the nearest tenth of a metre.

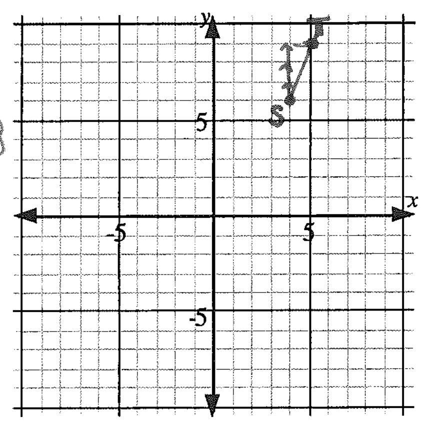
$d = \sqrt{(15-3)^2 + (47-2)^2}$

$= \sqrt{12^2 + 45^2}$

$= \sqrt{2169} = 46.57\ldots$

length = 46.6 m.

5. The line segment joining each pair of points has the given slope. Determine each value of k and draw the line segment on the grid.



a) $S(4,6)$ and $T(5,k)$ slope = 3

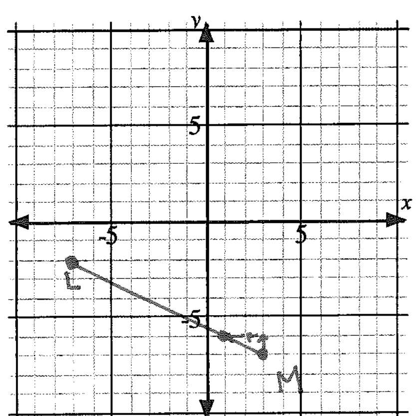
$\frac{3}{1}$ → positive ↑ positive →

$m = \frac{k-6}{5-4} = 3$

$k - 6 = 3$
$+6 \quad +6$

$\boxed{k=9}$ ← Can use to check plotted point w.

b) $L(k,-2)$ and $M(3,-7)$ slope $= -\frac{1}{2}$



Check Work!

$m = -\frac{1}{2}$ $\frac{1}{-2}$ → positive ↑ negative ←

$\frac{-7-(-2)}{3-k} = -\frac{1}{2}$

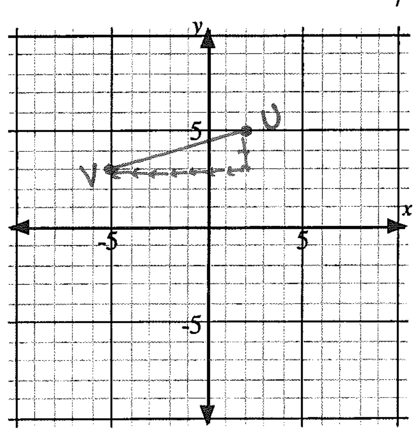
$-5(2) = -1(3-k)$

$-10 = -3 + k$

$\boxed{k=-7}$

c) $U(2,5)$ and $V(k,3)$ slope $= \frac{2}{7}$

remember! $\frac{-}{-} = +$ negative ↓ negative ← is also a positive slope.



Check Work!

$\frac{3-5}{k-2} = \frac{2}{7}$

$-2(7) = 2(k-2)$

$-14 = 2k - 4$

$-10 = 2k$

$\boxed{k=-5}$

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6. Consider points $P(4,-9)$, $Q(-1,-7)$, and $R(-11,-3)$.

a) Use the slope formula to prove that the points P, Q, and R are collinear.

$m_{PQ} = \dfrac{-7-(-9)}{-1-4} = -\dfrac{2}{5}$ $\qquad$ $m_{QR} = \dfrac{-3-(-7)}{-11-(-1)} = \dfrac{4}{-10} = -\dfrac{2}{5}$

Since $m_{PQ} = m_{QR}$ the points of P, Q, and R are collinear.

b) Use the distance formula to prove that the points P, Q, and R are collinear.

$d_{PQ} = \sqrt{(-1-4)^2 + (-7-(-9))^2} = \sqrt{(-5)^2 + 2^2} = \sqrt{29} = 5.385\ldots$

sum = 16.155...

$d_{QR} = \sqrt{(-11-(-1))^2 + (-3-(-7))^2} = \sqrt{(-10)^2 + 4^2} = \sqrt{116} = 10.770\ldots$

$d_{PR} = \sqrt{(-11-4)^2 + (-3-(-9))^2} = \sqrt{(-15)^2 + 6^2} = \sqrt{261} = 16.155\ldots$

Since $d_{PR} = d_{PQ} + d_{QR}$, the points P, Q, and R are collinear.

7. Consider points $A(8,-7)$, $B(-8,-3)$, and $C(-24,1)$.

Think About It! If you are asking yourself... : So what if two line segments have the same slope ... what makes them any different then parallel lines? * They have the same slope and they share a point! *

a) Prove that the points A, B, and C are collinear.

$m_{AB} = \dfrac{-3-(-7)}{-8-8} = \dfrac{4}{-16} = -\dfrac{1}{4}$ $\qquad$ $m_{BC} = \dfrac{1-(-3)}{-24-(-8)} = \dfrac{4}{-16} = -\dfrac{1}{4}$

Since $m_{AB} = -\frac{1}{4}$ and $m_{BC} = -\frac{1}{4}$ we can say $m_{AB} = m_{BC}$ and the points A, B, and C are collinear.

b) Does the point $D(-2,-4)$ lie on line segment AC? Explain.

$m_{AD} = \dfrac{-4-(-7)}{-2-8} = -\dfrac{3}{10}$

Since $m_{AD} \neq m_{AB}$ the point D does not lie on line segment AC.

c) Find the value of k if the point $E(k,k)$ lies on line segment AC.

$m_{AE} = m_{AB} = -\frac{1}{4}$

$\dfrac{k-(-7)}{k-8} = -\dfrac{1}{4}$

$4(k+7) = -1(k-8)$

$4k+28 = -k+8$

$5k = -20$

$\underline{k = -4}$

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8. A private jet has crashed in the desert at the point $P(-10, 17)$. A search party sets out in an all terrain vehicle from A_1, passing in a straight line through A_2. A helicopter sets out from B_1 and flies in a straight line through B_2.
If the search parties continue in these directions, will either of them discover the crashed plane?

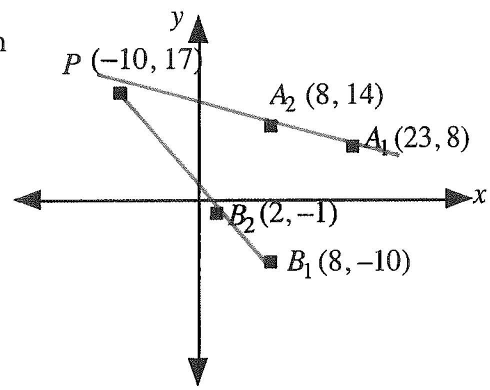

Diagram not to scale

$m_{A_1A_2} = \frac{14-8}{8-23} = \frac{6}{-15} = -\frac{2}{5}$

$m_{A_2P} = \frac{17-14}{-10-8} = \frac{3}{-18} = -\frac{1}{6}$

Slopes $m_{B_1B_2}$ and m_{B_2P}

$m_{B_1B_2} = \frac{-1-(-10)}{2-8} = \frac{9}{-6} = -\frac{3}{2}$

$m_{B_2P} = \frac{17-(-1)}{-10-2} = \frac{18}{-12} = -\frac{3}{2}$

Since $m_{A_1A_2} \neq m_{A_2P}$ the points $A_1, A_2,$ and P do not lie on a straight line. The search party in the all terrain vehicle will not discover the plane.

Since $m_{B_1B_2} = m_{B_2P}$ the points $B_1, B_2,$ and P are collinear. The search party in the helicopter will discover the plane.

Multiple Choice

9. The slope of the line segment joining $E(5,-1)$ and $F(3,7)$, is

A. -3
B. -4
C. $-\frac{1}{3}$
D. $-\frac{1}{4}$

$m_{EF} = \frac{7-(-1)}{3-5} = \frac{8}{-2} = -4$

10. If the line segment joining $(2,3)$ and $(8,k)$ has slope $-\frac{2}{3}$, then $k=$

A. -1
B. -3
C. -6
D. 7

$\frac{k-3}{8-2} = -\frac{2}{3}$

$3(k-3) = -2(6)$
$3k-9 = -12$
$3k = -3$
$k = -1$

11. One endpoint of a line segment is $(1,6)$. The other endpoint is on the x-axis. If the slope of the line segment is -3, then the midpoint of the line segment is

A. $(4,6)$
B. $(2,3)$
C. $(-10,3)$
D. $\left(\frac{1}{2}, \frac{15}{2}\right)$

Step 1: Solve for the x-coordinate

$(1,6)$ and $(x,0)$

$m = \frac{0-6}{x-1} = -3$

$-6 = -3(x-1)$
$-6 = -3x+3$
$3x = 9$
$\boxed{x=3}$

Step 2: Use x-coordinate with remaining information to solve for midpoint.

$(1,6)$ and $(3,0)$

$M\left(\frac{1+3}{2}, \frac{6+0}{2}\right)$

$M(2,3)$

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Numerical Response **12.** $P(3,6)$, $Q(8,-2)$, and $R(-6,0)$, are the vertices of a triangle. T is the midpoint of QR. The slope of the line PT, to the nearest tenth, is _____.

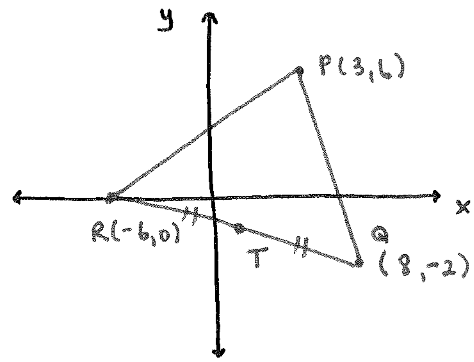
(Record your answer in the numerical response box from left to right)

3	.	5	

T is the midpoint of RQ.

$$T\left(\frac{-6+8}{2}, \frac{0+(-2)}{2}\right) = T(1,-1)$$

$$m_{PT} = \frac{y_T - y_P}{x_T - x_P} = \frac{-1-6}{1-3} = \frac{-7}{-2} = 3.5$$



Answer Key

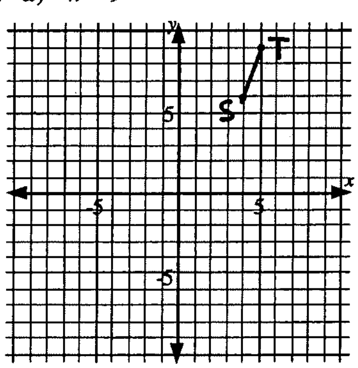
1. Line 1 - positive, Line 2 - negative, Line 3 - zero, Line 4 - positive, Line 5 - undefined, Line 6 - negative

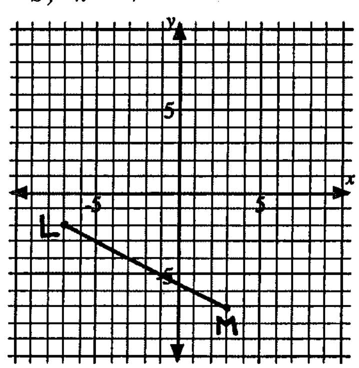
2. **a)** $-\frac{5}{12}$ **b)** $-\frac{5}{4}$ **c)** $\frac{2}{15}$ **d)** $-\frac{1}{4}$ **e)** undefined **f)** 0

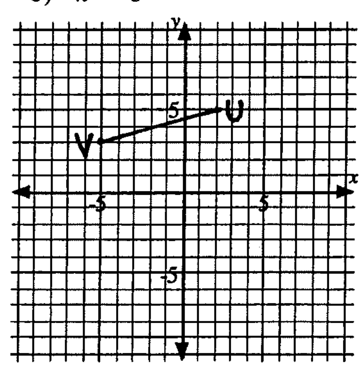
3. **a)** 2 **b)** $-\frac{3}{4}$ **c)** $\frac{13}{4}$ **d)** $\frac{2}{5}$

4. **a)** $\frac{15}{4}$ **b)** $\left(9, \frac{49}{2}\right)$ **c)** 46.6 m.

5. **a)** $k = 9$ **b)** $k = -7$ **c)** $k = -5$







6. **a)** $m_{PQ} = -\frac{2}{5}$, $m_{QR} = -\frac{2}{5}$. Since $m_{PQ} = m_{QR}$, the points P, Q and R are collinear.
b) $PQ = \sqrt{29}$, $QR = 2\sqrt{29}$, $PR = 3\sqrt{29}$. Since $PQ + QR = PR$, the points P, Q and R are collinear.

7. **a)** $m_{AB} = -\frac{1}{4}$, $m_{BC} = -\frac{1}{4}$. Since $m_{AB} = m_{BC}$, the points A, B and C are collinear.
b) $m_{AD} = -\frac{3}{10}$ Since $m_{AD} \neq m_{AB}$, the point D does not lie on line segment AC. **c)** $k = -4$

8. $m_{A_1A_2} = -\frac{2}{5}$, $m_{A_2P} = -\frac{1}{6}$. Since $m_{A_1A_2} \neq m_{A_2P}$, the search party in the all terrain vehicle will not discover the plane.
$m_{B_1B_2} = -\frac{3}{2}$, $m_{B_2P} = -\frac{3}{2}$. Since $m_{B_1B_2} = m_{B_2P}$, the search party in the helicopter will discover the plane.

9. B **10.** A **11.** B **12.**

3	.	5	

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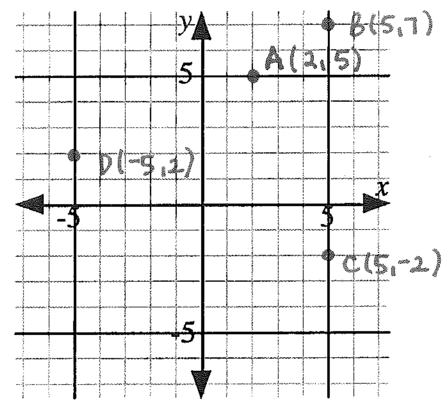
Characteristics of Linear Relations Lesson #5: Parallel and Perpendicular Lines

Review of Transformations

In earlier mathematics courses we studied transformations: translations, reflections, and rotations. In order to investigate parallel and perpendicular line segments, we will review translations and rotations.

On the grid, show the image of the point $A(2,5)$ after the following transformations. In each case write the coordinates of the image.



a) A translation 3 units right and 2 units up.

$A(2,5) \rightarrow \quad B(5,7)$

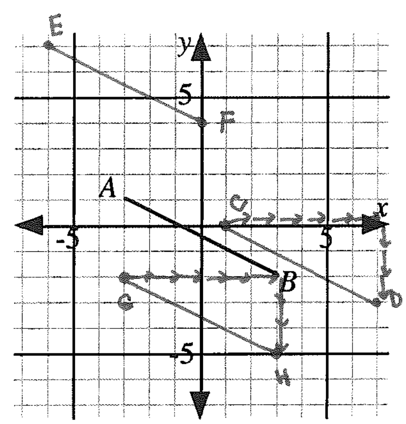
b) A 90° clockwise rotation about the origin.

$A(2,5) \rightarrow \quad C(5,-2)$

c) A 90° counterclockwise rotation about the origin.

$A(2,5) \rightarrow \quad D(-5,2)$

Investigating Parallel Line Segments



a) On the grid, show the image of line segment AB after the following transformations.

i) A translation 4 units right and 1 unit down to form line segment CD.

ii) A translation 3 units left and 6 units up to form line segment EF.

iii) A translation 3 units down to form line segment GH.

b) Calculate the slope of each of the line segments.

$m_{AB} = \frac{-3}{6} = -\frac{1}{2}$ $\qquad m_{CD} = \frac{-3}{6} = -\frac{1}{2}$

$m_{EF} = \frac{-3}{6} = -\frac{1}{2}$ $\qquad m_{GH} = \frac{-3}{6} = -\frac{1}{2}$

c) The four line segments are parallel. Make a conjecture about the slopes of parallel line segments.

Parallel line segments have the same slope.

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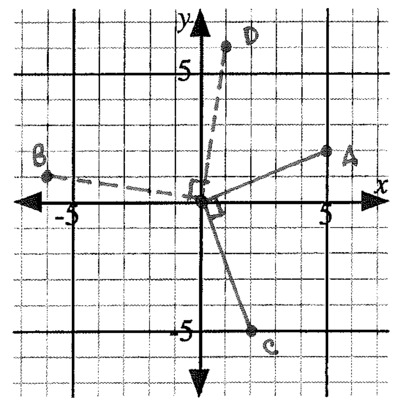
Investigating Perpendicular Line Segments

a) i) On the grid, plot the point $A(5, 2)$ and draw the line joining the point to the origin, O.

ii) Rotate the line through an angle of 90° clockwise about O and show the image on the grid.

iii) Find the slopes of the two perpendicular lines and multiply them together.



$m_{OA} = \frac{2-0}{5-0} = \frac{2}{5}$

$m_{OC} = \frac{-5-0}{2-0} = -\frac{5}{2}$

$m_{OA} \times m_{OC} = \left(\frac{2}{5}\right)\left(\frac{-5}{2}\right) = \frac{-10}{10} = \underline{\underline{-1}}$

b) Repeat part **a)** for the point $B(-6, 1)$.

$m_{OB} = \frac{1-0}{-6-0} = -\frac{1}{6}$

$m_{OD} = \frac{6-0}{1-0} = 6$

$m_{OB} \times m_{OD} = \left(-\frac{1}{6}\right)\left(\frac{6}{1}\right) = -\frac{6}{6} = \underline{\underline{-1}}$

c) Make a conjecture about the slopes of perpendicular line segments.

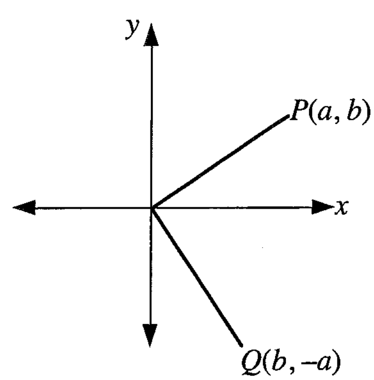
The slopes of perpendicular line segments have a product equal to $\underline{\underline{-1}}$.

d) Complete the following to prove the conjecture in **c)**.
Under a rotation of 90° clockwise about O, $P(a, b) \rightarrow Q(b, -a)$.

$m_{OP} = \frac{y_P - y_O}{x_P - x_O} = \frac{b-0}{a-0} = \frac{b}{a}$

$m_{OQ} = \frac{y_Q - y_O}{x_Q - x_O} = \frac{-a-0}{b-0} = -\frac{a}{b}$

$m_{OP} \times m_{OQ} = \left(\frac{b}{a}\right)\left(\frac{-a}{b}\right) = \frac{-ab}{ab} = -1$



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Parallel Lines and Perpendicular Lines

Recall that the slope of any line segment within a line represents the slope of the line.

Consider then two lines with slopes m_1 and m_2.

- The lines are **parallel** if they have the same slope, i.e. $m_1 = m_2$.
- The lines are **perpendicular** if the product of the slopes is -1,
  i.e. $m_1 \times m_2 = -1$ or $m_1 m_2 = -1$ or $m_1 = -\dfrac{1}{m_2}$
- For perpendicular lines, each slope is the negative reciprocal of the other provided neither slope is equal to zero.

Class Ex. #1

Consider line segment AC with a slope of $\dfrac{3}{4}$.

a) Write the slope of line segment GH which is parallel to AC.

$m_{GH} = \frac{3}{4}$

b) Write the slope of line segment BF which is perpendicular to AC.

$m_{BF} = -\frac{4}{3}$

Check Work.

$\frac{3}{4} \times -\frac{4}{3} = \frac{-12}{12} = -1$ ✓✓

Class Ex. #2

The slopes of two lines are given.
Determine if the lines are parallel, perpendicular, or neither.

a) $m_1 = \dfrac{1}{4}$, $m_2 = \dfrac{3}{12} = \dfrac{1}{4}$

parallel since slopes are equal

b) $m_1 = \dfrac{5}{7}$, $m_2 = \dfrac{14}{10} = \dfrac{7}{5}$

neither → not parallel because not equal

→ not perpendicular because not $-\frac{7}{5}$

Complete Assignment Questions #1 - #6

Class Ex. #3

If P is the point $(4, 7)$ and Q is the point $(6, -2)$, find the slope of a line segment

a) parallel to line segment PQ **b)** perpendicular to line segment PQ

$m_{PQ} = \frac{-2-7}{6-4} = \frac{-9}{2}$

a) $-\frac{9}{2}$

b) $\frac{2}{9}$

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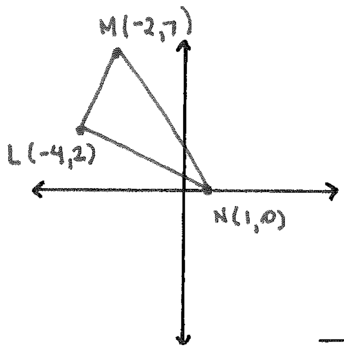
Class Ex. #4

$\triangle LMN$ has coordinates $L(-4,2)$, $M(-2,7)$, and $N(1,0)$. Use slopes to show that the triangle is right-angled at L.



$m_{LM} = \dfrac{7-2}{-2-(-4)} = \dfrac{5}{2}$ $\qquad m_{LN} = \dfrac{0-2}{1-(-4)} = \dfrac{-2}{5}$

$m_{LM} \times m_{LN} = \left(\dfrac{5}{2}\right)\left(\dfrac{-2}{5}\right) = \dfrac{-10}{10} = -1$

Since the product of the slopes is -1, the lines LM and LN are perpendicular.
The triangle at point L is right-angled.

Class Ex. #5

Two lines have slopes of $-\dfrac{3}{4}$ and $\dfrac{k}{5}$ respectively. Find the value of k if the lines are

a) parallel

$-\dfrac{3}{4} = \dfrac{k}{5}$

$\dfrac{-15}{4} = \dfrac{4k}{4}$

$k = \dfrac{-15}{4}$

b) perpendicular

$\left(\dfrac{-3}{4}\right)\left(\dfrac{k}{5}\right) = -1$

$\dfrac{-3k}{20} = \dfrac{-1}{1}$

$\dfrac{-3k}{-3} = \dfrac{-20}{-3}$

$k = \dfrac{20}{3}$

Complete Assignment Questions #7 - #16

Assignment

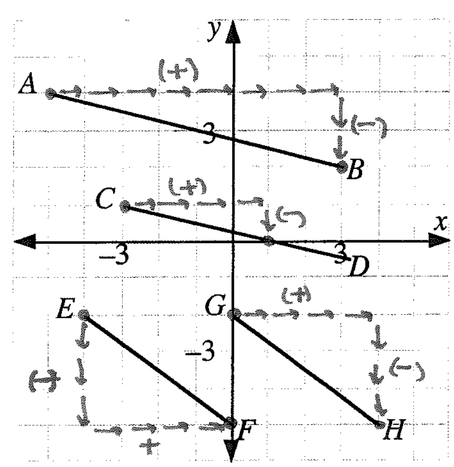
1. AB is parallel to CD. EF is parallel to GH.

a) Determine the slopes of the following pairs of parallel line segments using $m = \dfrac{\text{rise}}{\text{run}}$.

Line Segment	Slope
AB	$\frac{-2}{8} = \frac{-1}{4}$
CD	$\frac{-1}{4}$

Line Segment	Slope
EF	$\frac{-3}{4}$
GH	$\frac{-3}{4}$

Recall:
positive ↑
positive →
and
negative ↓
negative ←



Notice all of the line segments are falling from left to right. We can check the signs of our work! They all must have negative slopes and they do!!

b) Observe the slopes of the pairs of parallel lines in **a)**.
Write a rule in reference to the slopes of parallel lines.

Lines which are parallel have the same slope.

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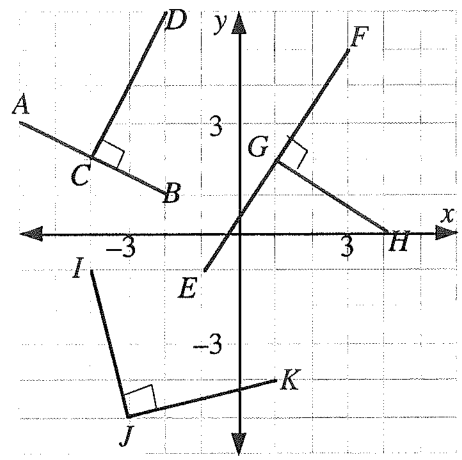
2.a) Determine the slopes of the following pairs of perpendicular line segments using $m = \frac{\text{rise}}{\text{run}}$.

Line Segment	Slope	Line Segment	Slope	Line Segment	Slope
AB	$\frac{-2}{4} = \frac{-1}{2}$	*EF*	$\frac{6}{4} = \frac{3}{2}$	*IJ*	$\frac{-4}{1} = -4$
CD	$\frac{4}{2} = 2$	*GH*	$\frac{-2}{3}$	*JK*	$\frac{1}{4}$



b) Multiply the slopes of the pairs of perpendicular line segments.

$m_{AB} \times m_{CD}$	$m_{EF} \times m_{GH}$	$m_{IJ} \times m_{JK}$
$-\frac{1}{2} \times \frac{2}{1} = -\frac{2}{2}$	$(\frac{3}{2})(\frac{-2}{3}) = -\frac{6}{6}$	$(\frac{-4}{1})(\frac{1}{4}) = \frac{-4}{4}$
$= -1$	$= -1$	

c) Write a rule in reference to the slope of two lines which are perpendicular to each other.

The product of the slopes of perpendicular lines is -1.

3. The slopes of two line segments are given. Determine if the lines are parallel, perpendicular, or neither.

Important: In order to compare slopes it is best to simplify to lowest terms first!

a) $m_{AB} = \frac{8}{20}$, $m_{PQ} = \frac{2}{5}$ $\quad = \frac{2}{5}$ parallel

b) $m_{AB} = \frac{3}{2}$, $m_{PQ} = -\frac{2}{3}$ perpendicular

c) $m_{AB} = \frac{1}{6}$, $m_{PQ} = \frac{2}{12}$ $\quad = \frac{1}{6}$ parallel

d) $m_{AB} = \frac{7}{8}$, $m_{PQ} = \frac{8}{7}$ neither, sign must also be opposite.

e) $m_{AB} = \frac{9}{3}$, $m_{PQ} = -\frac{1}{3}$ $\quad = \frac{3}{1}$ perpendicular

f) $m_{AB} = -5$, $m_{PQ} = \frac{1}{5}$ perpendicular

g) $m_{AB} = \frac{4}{8}$, $m_{PQ} = 2$ $\quad = \frac{1}{2}$ neither, sign must also be opposite

h) $m_{AB} = -\frac{12}{2}$, $m_{PQ} = -6$ $\quad = -6$ parallel

i) $m_{AB} = -\frac{5}{2}$, $m_{PQ} = -\frac{2}{5}$ neither, sign must also be opposite.

4. The slopes of some line segments are given.

$m_{AB} = 6 \qquad m_{CD} = \frac{1}{6} \qquad m_{EF} = -6 \qquad m_{GH} = 6 \qquad m_{IJ} = -6 \qquad m_{KL} = \frac{1}{6}$

Which pairs of lines are parallel to each other?

AB and GH EF and IJ CD and KL

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5. The slopes of some line segments are given. $\frac{1}{2}$

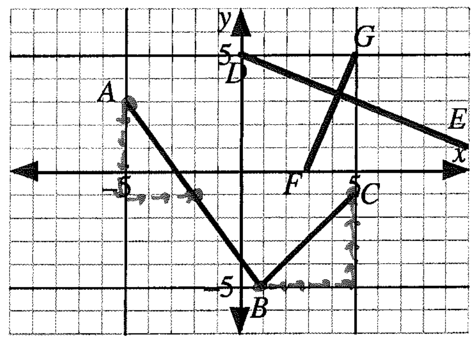
$m_{RS} = -2$ $\quad m_{UV} = \frac{1}{4}$ $\quad m_{EF} = 0.5$ $\quad m_{ZT} = 2$

$m_{PQ} = -4$ $\quad m_{KL} = -\frac{1}{2}$ $\quad m_{MN} = 4$ $\quad m_{XY} = -\frac{1}{4}$

Which pairs of lines are perpendicular to each other?

RS and EF UV and PQ ZT and KL MN and XY

6. The four line segments have endpoints with integer coordinates. In each case determine whether the two intersecting line segments are perpendicular.



$m_{AB} = \frac{-8}{6} = \frac{-4}{3}$ and $m_{BC} = \frac{4}{4} = 1$

Thus, AB and BC are not perpendicular.

$m_{DE} = \frac{-4}{10} = \frac{-2}{5}$ and $m_{FG} = \frac{5}{2}$ ← Yes DE and FG are perpendicular.

7. A, B, and C are the points $(0, 4)$, $(-3, 1)$, and $(5, -2)$ respectively. Determine the slope of a line

a) parallel to line segment AB

$m_{AB} = \frac{1-4}{-3-0} = 1$ $\quad m = 1$

b) perpendicular to line segment AB

$m = -1$

c) parallel to line segment BC

$m_{BC} = \frac{-2-1}{5-(-3)} = \frac{-3}{8}$ $\quad m = -\frac{3}{8}$

d) perpendicular to line segment AC

$m_{AC} = \frac{-2-4}{5-0} = \frac{-6}{5}$ $\quad m = \frac{5}{6}$

8. ΔABC has vertices $A(3, 5)$, $B(-2, -5)$, $C(-5, 1)$.

a) Explain how we can determine if $\triangle ABC$ is a right triangle.

Determine the slope of each side of the triangle. If two of the slopes are negative reciprocals, the triangle is a right triangle.

b) Determine if $\triangle ABC$ is a right triangle.

$m_{AB} = \frac{-5-5}{-2-3} = \frac{-10}{-5} = 2$

$m_{BC} = \frac{1-(-5)}{-5-(-2)} = \frac{6}{-3} = -2$

$m_{AC} = \frac{1-5}{-5-3} = \frac{-4}{-8} = \frac{1}{2}$

$m_{BC} \times m_{AC} = \left(\frac{-2}{1}\right)\left(\frac{1}{2}\right) = \frac{-2}{2} = -1$

$BC \perp AC$ → $\angle ACB$ is a right angle

Thus, ΔABC is a right triangle at C.

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9. The vertices of two triangles are given.
Determine if either of the triangles is right-angled.

a) $\Delta PQR \rightarrow P(-3,3),\ Q(-1,1),\ R(-5,-1)$ b) $\Delta ABC \rightarrow A(-7,9),\ B(3,13),\ C(7,3)$

$m_{PQ} = \frac{1-3}{-1-(-3)} = \frac{-2}{2} = -1$

$m_{QR} = \frac{-1-1}{-5-(-1)} = \frac{-2}{-4} = \frac{1}{2}$

$m_{PR} = \frac{-1-3}{-5-(-3)} = \frac{-4}{2} = 2$

No, the slopes are not negative reciprocals.
Δ PQR is not right angled.

$m_{AB} = \frac{13-9}{3-(-7)} = \frac{4}{10} = \frac{2}{5}$

$m_{BC} = \frac{3-13}{7-3} = \frac{-10}{4} = \frac{-5}{2}$

$m_{AC} = \frac{3-9}{7-(-7)} = \frac{-6}{14} = \frac{-3}{7}$

$m_{AB} \times m_{BC} = \left(\frac{2}{5}\right)\left(\frac{-5}{2}\right) = -1$

Δ ABC is right angled at B.

10. The slopes of parallel lines are given.
Determine the value of the variable.

Recall: Parallel slope must be equal $m_1 = m_2$

a) $4, \frac{k}{3}$ b) $-2, \frac{2}{n}$ c) $\frac{5}{6}, 3m$ d) $\frac{3}{4}, -\frac{w}{6}$

a) $\frac{4}{1} = \frac{k}{3}$ $\quad \underline{k = 12}$

b) $\frac{-2}{1} = \frac{2}{n}$ $\quad -2n = 2 \quad \underline{n = -1}$

c) $\frac{5}{6} = \frac{3m}{1}$ $\quad 18m = 5 \quad \underline{m = \frac{5}{18}}$

d) $\frac{3}{4} = -\frac{w}{6}$ $\quad -4w = 18 \quad w = \frac{18}{-4} = \underline{\frac{-9}{2}}$

11. The slopes of perpendicular lines are given.
Determine the value of the variable.

Recall: Perpendicular slopes must be negative reciprocals of each other $m \times -\frac{1}{m} = -1$ and multiply to the value of -1.

a) $\frac{1}{3}, 3h$ b) $4, \frac{8}{p}$ c) $-5, \frac{s}{2}$ d) $-\frac{3}{4}, -\frac{q}{6}$

a) $\left(\frac{1}{3}\right)\left(\frac{3h}{1}\right) = -1$ $\quad \frac{3h}{3} = -1 \quad \underline{h = -1}$

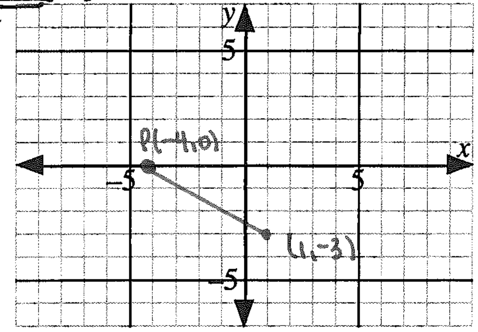
b) $\left(\frac{4}{1}\right)\left(\frac{8}{p}\right) = -1$ $\quad \frac{32}{p} = \frac{-1}{1} \quad \underline{p = -32}$

c) $\left(\frac{-5}{1}\right)\left(\frac{s}{2}\right) = -1$ $\quad \frac{-5s}{2} = \frac{-1}{1} \quad -5s = -2 \quad \underline{s = 2/5}$

d) $\left(\frac{-3}{4}\right)\left(-\frac{q}{6}\right) = -1$ $\quad \frac{+3q}{24} = \frac{-1}{1} \quad +3q = -24 \quad \underline{q = -8}$

12. $P(-4,0)$ and $R(1,-3)$ are opposite vertices of a rhombus $PQRS$. Find the slope of diagonal QS.

IMPORTANT: The diagonals of a rhombus are perpendicular!



$m_{RP} = \frac{-3-0}{1-(-4)} = \frac{-3}{5}$

↓

$m_{QS} = \frac{5}{3}$

Must be a negative reciprocal
Check work! $\left(\frac{-3}{5}\right)\left(\frac{5}{3}\right) = -1$

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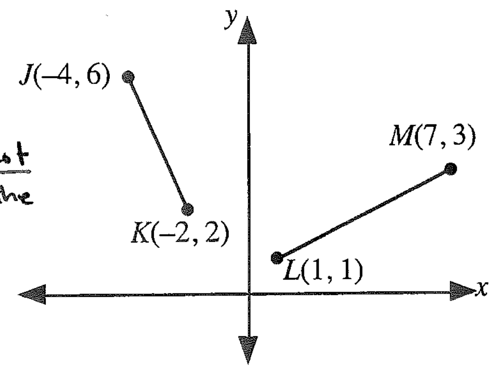
13. a) Show that when line segments JK and ML are extended until they intersect, they will not meet at right angles.



$m_{JK} = \frac{2-6}{-2-(-4)} = \frac{-4}{2} = -2$

$m_{ML} = \frac{1-3}{1-7} = \frac{-2}{-6} = \frac{1}{3}$

Since the slopes are not negative reciprocals the lines will not meet at a right angle!

b) If the y-coordinate of M is changed, the line segments, when extended, will meet at right angles. To what value should the y-coordinate of M be changed?

Goal: To meet at right angle m_{ML} must equal $\frac{1}{2}$, which is the negative reciprocal of -2.

$(\frac{1}{2})(\frac{-2}{1}) = -1$

$m_{ML} = \frac{y-1}{7-1} = \frac{1}{2}$

$2(y-1) = 6$

$2y - 2 = 6$

$2y = 8$

$y = 4$

Interesting: Consider which method you prefer. Which method seems to have less chance of errors and is faster?

14. Given that $A, B,$ and C are the points $(-3, 3)$, $(0, 6)$, and $(5, 1)$ respectively, prove that triangle ABC is right angled by using

a) the slope formula

Goal: check $m_{AB} \times m_{BC} = -1$

$m_{AB} = \frac{6-3}{0-(-3)} = \frac{3}{3} = 1$

$m_{BC} = \frac{1-6}{5-0} = \frac{-5}{5} = -1$

$m_{AC} \times m_{BC} = (1)(-1) = -1$ ✓✓

So, $AB \perp BC$

Thus, $\triangle ABC$ is right angled at B.

b) the distance formula

Goal: Check $AC^2 = AB^2 + BC^2$

$d_{AB} = \sqrt{(0-(-3))^2 + (6-3)^2} = \sqrt{18}$

$d_{BC} = \sqrt{(5-0)^2 + (1-6)^2} = \sqrt{50}$

$d_{AC} = \sqrt{(5-(-3))^2 + (1-3)^2} = \sqrt{68}$

$AC^2 = AB^2 + BC^2$

$(\sqrt{68})^2 = (\sqrt{18})^2 + (\sqrt{50})^2$

$68 = 18 + 50$

$68 = 68$

Since $AC^2 = AB^2 + BC^2$, $\triangle ABC$ is right angled at B.

Multiple Choice

15. A and B are the points $(1, 2)$ and $(-2, 3)$ respectively. A line perpendicular to AB will have slope

A. -3

B. $-\frac{1}{3}$

C. 3

D. $\frac{1}{3}$

$m_{AB} = \frac{3-2}{-2-1} = -\frac{1}{3}$

$m_{\perp} = 3$

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Numerical Response 16. The line segment joining $U(-3, p)$ and $V(-6, 5)$ is perpendicular to the line segment joining $X(4, 2)$ and $Y(9, 0)$. The value of p, to the nearest tenth, is ______.

(Record your answer in the numerical response box from left to right)

1	2	.	5

Goal: Since we are told that UV is perpendicular to XY, we can use the understanding of $m_{uv} \times m_{xy} = -1$ to solve for p.

Step 1: Solve for m_{uv} and m_{xy} and simplify as far as possible

$m_{uv} = \frac{5-p}{-6-(-3)} = \boxed{\frac{5-p}{-3}}$ $m_{xy} = \frac{0-2}{9-4} = \boxed{\frac{-2}{5}}$

Step 2: $m_{uv} \times m_{xy} = -1$: $\left(\frac{5-p}{-3}\right)\left(\frac{-2}{5}\right) = -1$

Step 3: Solve for p : $\frac{-2(5-p)}{-15} = \frac{-1}{1}$

$-2(5-p) = +15$

$-10 + 2p = +15$
$+10 \quad +10$

$\frac{2p}{2} = \frac{25}{2}$

$p = 12.5$

Answer Key

1. a) slope $AB = -\frac{1}{4}$ slope $CD = -\frac{1}{4}$ slope $EF = -\frac{3}{4}$ slope $GH = -\frac{3}{4}$
b) Lines which are parallel have the same slope

2. a) slope $AB = -\frac{1}{2}$ slope $EF = \frac{3}{2}$ slope $IJ = -4$
slope $CD = 2$ slope $GH = -\frac{2}{3}$ slope $JK = \frac{1}{4}$
b) All the products are -1. **c)** The product of the slopes is -1.

3. a) parallel **b)** perpendicular **c)** parallel
d) neither **e)** perpendicular **f)** perpendicular
g) neither **h)** parallel **i)** neither

4. *AB* and *GH*, *CD* and *KL*, *EF* and *IJ*.

5. *RS* and *EF*, *UV* and *PQ*, *ZT* and *KL*, *MN* and *XY*.

6. *AB* and *BC* are not perpendicular. *DE* and *FG* are perpendicular.

7. a) 1 **b)** -1 **c)** $-\frac{3}{8}$ **d)** $\frac{5}{6}$

8. a) Determine the slope of each side of the triangle. If two of the slopes are negative reciprocals of each, then the triangle is a right triangle.
b) $m_{BC} = -2$, $m_{AC} = \frac{1}{2}$. Since the slopes are negative reciprocals, the triangle is a right triangle.

9. a) ΔPQR is <u>not</u> a right triangle **b)** ΔABC is a right triangle

10. a) $k = 12$ **b)** $n = -1$ **c)** $m = \frac{5}{18}$ **d)** $w = -\frac{9}{2}$

11. a) $h = -1$ **b)** $p = -32$ **c)** $s = \frac{2}{5}$ **d)** $q = -8$

12. $m_{QS} = \frac{5}{3}$

13. a) $M_{JK} = -2$, $M_L = \frac{1}{3}$. The product of the slopes does not equal -1. **b)** $y_M = 4$

14. a) $m_{AB} = 1$, $m_{BC} = -1$ Since the product of the slopes $= -1$, AB and BC are perpendicular. Triangle *ABC* is right angled at *B*.
b) $AB = \sqrt{18}$, $BC = \sqrt{50}$, $AC = \sqrt{68}$. $AC^2 = 68$. $AB^2 + BC^2 = 68$.
$AC^2 = AB^2 + BC^2$ so the Pythagorean theorem is satisfied and the triangle is right angled at *B*.

15. C **16.**

1	2	.	5

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Characteristics of Linear Relations Lesson #6: Practice Test

Recall: length = largest − smallest.

1. Which of the following horizontal or vertical line segments has the greatest length?
   - A. PQ with $P(2,9)$ and $Q(7,9)$. horizontal $7-2=5$
   - B. RS with $R(4,11)$ and $S(4,4)$. vertical $11-4=7$
   - C. TV with $T(-6,1)$ and $V(2,1)$. horizontal $2-(-6)=8$
   - D. WZ with $W(-5,-5)$ and $Z(-5,-11)$. vertical $-5-(-11)=6$

2. The exact distance between the points $\underbrace{(8,3)}_{}$ (second point) and $(5,-2)$ (first point) is
   - A. 5.83
   - B. $\sqrt{34}$
   - C. 8
   - D. 34

$$d=\sqrt{(5-8)^2+(-2-3)^2}$$
$$=\sqrt{(-3)^2+(-5)^2}$$
$$=\sqrt{9+25}=\sqrt{34}$$

Numerical Response 1. To the nearest hundredth, the distance between $A(-2,3)$ (second) and $B(-5,-6)$ (first) is ______.
(Record your answer in the numerical response box from left to right)

9	.	4	9

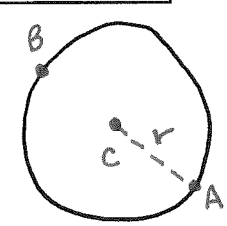
$$d=\sqrt{\underbrace{(-5-(-2))^2}_{x\text{-coodinates}}+\underbrace{(-6-3)^2}_{y\text{-coodinates.}}}=\sqrt{(-3)^2+(-9)^2}=\sqrt{90}=9.49$$

Use the following information to answer the next two questions.

> The points $A(4,2)$ and $B(-2,6)$ are on the circumference of a circle.
>
> The line segment AB passes through the centre of the circle.

3. The centre of the circle is the point
   - A. $(2,8)$
   - B. $(3,-2)$
   - C. $(1,4)$
   - D. $(6,-4)$

$$M\left(\frac{4+(-2)}{2},\frac{2+6}{2}\right)$$
$$M(1,4)$$



Numerical Response 2. The area of the circle, to the nearest whole number, is ______.
(Record your answer in the numerical response box from left to right)

4	1		

STEP1: $r=d_{AC}=\sqrt{(1-4)^2+(4-2)^2}=\sqrt{(-3)^2+2^2}=\sqrt{13}$

STEP2: $A=\pi r^2=\pi(\sqrt{13})^2=13\pi=40.84\ldots$

Numerical Response 3. PQ is the diameter of a circle, the centre is R. If $Q(6.7,\cancel{4.5})$ and $R(8.5,\cancel{7.9})$ then the x–coordinate of P is ______.
(Record your answer in the numerical response box from left to right)

1	0	.	3

$$x_R=\frac{x_P+x_Q}{2}\qquad 8.5=\frac{x+6.7}{2}\qquad 17=x+6.7\qquad x=10.3$$

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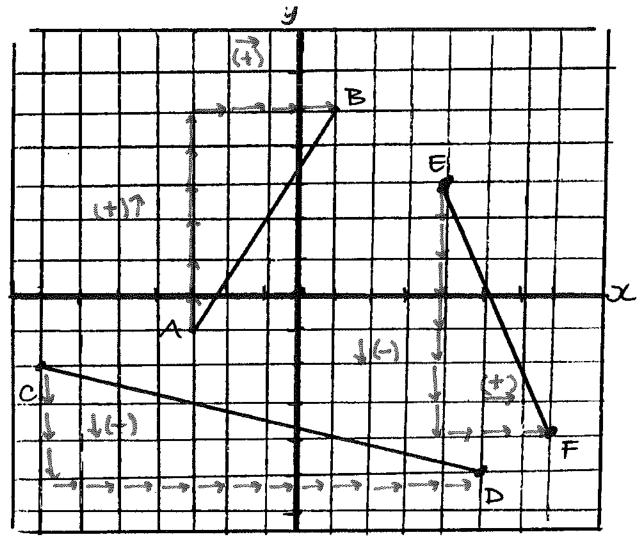
Note: In this question we still use the Midpoint formula, but only for the x-coordinate.

Use the following information to answer questions #4 - #6.

Nadine practiced for a math quiz by drawing line segments on a grid and then determining their slopes. Her line segments and grid are shown.



① Recall: direction and signs

$m = \frac{\text{positive}\uparrow}{\text{positive}\rightarrow}$

$m = \frac{\text{negative}\downarrow}{\text{negative}\leftarrow}$

② Recall:

- Slopes that rise left to right must be positive
- Slopes that fall left to right must be negative

Matching Match each line segment on the left with the slope on the right. Each slope may be used once, more than once, or not at all.

Line Segment		**Slope**			
4.	*AB* $m_{AB} = \frac{6}{4} = \boxed{\frac{3}{2}}$ E	A.	-4	D.	$\frac{3}{7}$
5.	*CD* $m_{CD} = \frac{-3}{12} = \boxed{-\frac{1}{4}}$ F	B.	$-\frac{3}{2}$	E.	$\frac{3}{2}$
6.	*EF* $m_{EF} = \boxed{-\frac{7}{3}}$ C	C.	$-\frac{7}{3}$	F.	$-\frac{1}{4}$

7. Consider $\overline{AB}$ joining $A(6,-4)$ and $B(-4,-4)$, and $\overline{CD}$ joining $C(1,-9)$ and $D(1,1)$. Which one of the following statements about these line segments is true?

   A. $\overline{AB}$ and $\overline{CD}$ have the same slope and are equal in length.
   B. $\overline{AB}$ and $\overline{CD}$ have the same slope and are unequal in length.
   C. $\overline{CD}$ has a length of 10 units and a slope of zero.
   D. $\overline{AB}$ and $\overline{CD}$ have the same midpoint and are equal in length.

Slopes: $m_{AB} = \frac{-4-(-4)}{-4-6} = \frac{0}{-10} = 0$

$m_{CD} = \frac{1-(-9)}{1-1} = \frac{10}{0}$ = undefined

Lengths: horizontal Line Length = Largest − Smallest. $6-(-4)=10$

Vertical Line Length = $1-(-9)=10$

Midpoint of AB $= M\left(\frac{6+(-4)}{2}, \frac{-4+(-4)}{2}\right) = M(1,-4)$ Midpoint of CD $= M\left(\frac{1+1}{2}, \frac{-9+1}{2}\right) = M(1,-4)$

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Use the following information to answer questions #8 - #10.

Line Segment AB	Line Segment PQ
$A(-2,4)$ $B(2,-6)$	$P(7,1)$ $Q(-3,-3)$
second first	second first

8. Which of the following statements is correct about the line segments?

- **A.** The length of line segment AB is greater than the length of line segment PQ.
- **B.** The length of line segment AB is less than the length of line segment PQ.
- **C.** The length of line segment AB is equal to the length of line segment PQ.
- **D.** Not enough information is given to calculate the lengths of the line segments.

$$d_{AB} = \sqrt{(2-(-2))^2 + (-6-4)^2} = \sqrt{4^2 + (-10)^2} = \sqrt{116}$$

$$d_{PQ} = \sqrt{(-3-7)^2 + (-3-1)^2} = \sqrt{(-10)^2 + (-4)^2} = \sqrt{116}$$

9. Which of the following statements is correct about the line segments?

- **A.** The slope of line segment AB is ~~positive~~ and the slope of line segment PQ is negative.
- **B.** The slope of line segment AB is ~~positive~~ and the slope of line segment PQ is positive.
- **C.** Line segment AB is ~~parallel~~ to line segment PQ.
- **D.** Line segment AB is perpendicular to line segment PQ.

$$m_{AB} = \frac{-6-4}{2-(-2)} = \frac{-10}{4} = \boxed{\frac{-5}{2}}$$ negative, falling slope

$$m_{PQ} = \frac{-3-1}{-3-7} = \frac{-4}{-10} = \frac{2}{5}$$ positive, rising slope

Check:

parallel: No, $-\frac{5}{2} \neq \frac{2}{5}$

perpendicular: Yes, $\left(-\frac{5}{2}\right)\left(\frac{2}{5}\right) = -\frac{10}{10} = -1$

10. Which of the following statements is correct about the line segments?

- **A.** The midpoint of AB has an x-coordinate greater than the midpoint of PQ. ✗
- **B.** The midpoint of AB has an y-coordinate greater than the midpoint of PQ. ✗
- **C.** The midpoints of AB and PQ are the same point. ✗
- **D.** The line segment joining the midpoints is horizontal.

Midpoint of AB $= M\left(\frac{-2+2}{2}, \frac{4+(-6)}{2}\right) = M(0,-1)$

Midpoint of PQ $= M\left(\frac{7+(-3)}{2}, \frac{1+(-3)}{2}\right) = M(2,-1)$

Slope of Midpoints $= \frac{-1-(-1)}{2-0} = 0$

Recall: All horizontal lines must have slopes of zero.

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11. K and L are the points (4, 7) and (–1, – 3) respectively.
second first
The slope of a line perpendicular to KL is

A. $\frac{1}{2}$
B. $-\frac{1}{2}$
C. 2
D. –2

$m_{KL} = \frac{-3-7}{-1-4} = \frac{-10}{-5} = 2$

Thus, the perpendicular slope of 2 is $-\frac{1}{2}$. must be the negative reciprocal.

Check Work! $(2)(-\frac{1}{2}) = -1$ ✓

Use the following information to answer questions #12–#15.

Quadrilateral $PQRS$ has vertices $P(-4,-6)$, $Q(-6,-2)$, $R(0, 1)$, and $S(2,-3)$.

12. The slope and length of line segment PQ are respectively

A. –2 and $\sqrt{20}$
B. $-\frac{1}{2}$ and $\sqrt{20}$
C. –2 and $\sqrt{116}$
D. $-\frac{1}{2}$ and $\sqrt{116}$

Slope:
$m_{PQ} = \frac{-2-(-6)}{-6-(-4)} = \frac{4}{-2} = \boxed{-2}$

Length:
$d_{PQ} = \sqrt{(-6+4)^2 + (-2+6)^2} = \sqrt{(-2)^2+4^2} = \boxed{\sqrt{20}}$

13. The slope and length of line segment QR are respectively

A. 2 and $\sqrt{37}$
B. $\frac{1}{2}$ and $\sqrt{37}$
C. 2 and $\sqrt{45}$
D. $\frac{1}{2}$ and $\sqrt{45}$

Slope:
$m_{QR} = \frac{1-(-2)}{0-(-6)} = \frac{3}{6} = \boxed{\frac{1}{2}}$

Length:
$d_{QR} = \sqrt{(0-(-6))^2 + (1-(-2))^2} = \sqrt{6^2+3^2} = \boxed{\sqrt{45}}$

14. Consider the following statements:

$m_{PQ} = -2$ $m_{QR} = \frac{1}{2}$
I. PQ is parallel to SR. ✓ **II.** QR is perpendicular to SR. ✓
III. The lengths of PQ and SR are the same. ✓
Length $= \sqrt{20}$

Which of the following is correct?

A. Statement **I** is false.
B. Statement **II** is false.
C. Statement **III** is false.
D. None of the above statements is false.

Slope of SR:
$m_{SR} = \frac{1-(-3)}{0-2} = \frac{4}{-2} = \boxed{-2}$

length of SR:
$d_{SR} = \sqrt{(0-2)^2 + (1-(-3))^2} = \sqrt{2^2+4^2} = \boxed{\sqrt{20}}$

15. Which of the following most completely describes Quadrilateral $PQRS$?

A. rectangle **B.** square **C.** parallelogram **D.** rhombus

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Think about it! Two sets of information parallel sides that are also perpendicular. and side length that are equal! X2. $\sqrt{45}$ $\sqrt{20}$.

Numerical Response 4. Two lines have slopes of $-\frac{2}{3}$ and $\frac{15}{t}$, respectively.
If the lines are perpendicular, then the value of t must be _____ .
(Record your answer in the numerical response box from left to right)

1	0		

$\underbrace{\left(\frac{-2}{3}\right)\left(\frac{15}{t}\right)}=-1$ $\qquad \frac{-30}{3t}=\frac{-1}{1}$ $\qquad \frac{-30}{-3}=\frac{-3t}{-3}$ $\qquad t=10$

In order to be perpendicular slope must multiply to -1!

Numerical Response 5. The line segment joining $K(-1, y)$ and $L(-2, 8)$ is parallel to the line segment joining $M(4, 4)$ and $N(0, 5)$. The value of y, to the nearest hundredth, is _____.

(Record your answer in the numerical response box from left to right)

7	.	7	5

Goal: Since we know that $m_{KL} = m_{MN}$ given that they are parallel, we must first slove for each and then let them equal one another in order to isolate for y!

Step 1: $m_{KL}=\frac{8-y}{-2-(-1)}=\boxed{\frac{8-y}{-1}}$ $\qquad m_{MN}=\frac{5-4}{0-4}=\boxed{-\frac{1}{4}}$

Step 2: $\frac{8-y}{-1}=-\frac{1}{4}$

Step 3: $4(8-y)=1$
$32-4y=1$
$-32 \qquad -32$
$4y=31$
$y=7.75$

Written Response - 5 marks

1. Consider the points $A(-2, 6)$, $B(2, 0)$, $C(0, 2)$, and $D(6, -6)$.

- Three of these points lie on the same straight line and one point does not. **Explain** how you could algebraically determine which of these points is not collinear with the other three.

Use the slope formula to determine the slopes of AB, AC, and CD. If the slopes are all different, then A is the point which is not collinear with the other three.

If, on the other hand, only one of the slopes is different, then the point that is connected to A in the line segment with the different slope is the point that is not collinear with the other three.

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- **Determine** algebraically which of the points $A(-2,6)$, $B(2,0)$, $C(0,2)$, and $D(6,-6)$ is not collinear with the other three.

2 choices: 1) slope same with shared point. ← less work!
2) distance same with shared point.

Notice that two of the slopes are the same!

$$m_{AB} = \frac{0-6}{2-(-2)} = \frac{-6}{4} = \boxed{\frac{-3}{2}}$$

$$m_{AC} = \frac{2-6}{0-(-2)} = \frac{-4}{2} = \boxed{-2}$$

$$m_{AD} = \frac{-6-6}{6-(-2)} = \frac{-12}{8} = \boxed{\frac{-3}{2}}$$

Thus, A, B, and D are collinear.

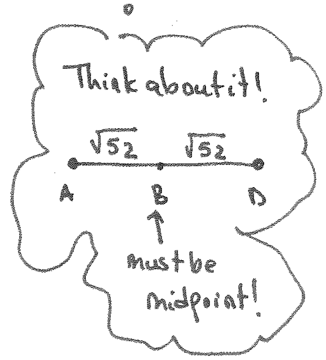
C is not collinear with the other 3 points given the slope −2.

- Without using the midpoint formula determine algebraically that B is the midpoint of line segment AD.

Since we know that A, B, and D lie on the same straight line, B will be the midpoint of AD if we can show using the distance formula that AB = BD in length.

$$d_{AB} = \sqrt{(2-(-2))^2 + (0-6)^2} = \sqrt{4^2 + (-6)^2} = \sqrt{52}$$

$$d_{BD} = \sqrt{(6-2)^2 + (-6-0)^2} = \sqrt{4^2 + (-6)^2} = \sqrt{52}$$



Thus, AB = BD so B is the midpoint of AD.

Answer Key

1. C	2. B	3. C	4. E	5. F	6. C	7. D	8. C
9. D	10. D	11. B	12. A	13. D	14. D	15. A	

Numerical Response

1. | 9 | . | 4 | 9 |
2. | 4 | 1 | | |
3. | 1 | 0 | . | 3 |
4. | 1 | 0 | | |
5. | 7 | . | 7 | 5 |

Written Response

1. • Use the slope formula to determine the slopes of AB, AC, AD. If the slopes are all different, then A is the point which is not collinear with the other three. If, on the other hand, only one of the slopes is different, then the point that is connected to A in the line segment with the different slope is the point that is not collinear with the other three.
- *C*
- Since we know that A, B, and D lie on the same straight line, B will be the midpoint of AD if we can show using the distance formula that $AB = BD$ in length.

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