Functions Lesson #1:
Functions

Review

We have considered six ways in which the relationship between two quantities can be represented.

- in words
- a table of values
- a set of ordered pairs
- a mapping (or arrow) diagram
- an equation
- a graph
- function notation (this unit)

In a **relation** each element of the **domain** (the **input**) is related to an element or elements of the **range** (the **output**).

In this lesson we will study a special type of relation called a **function**.

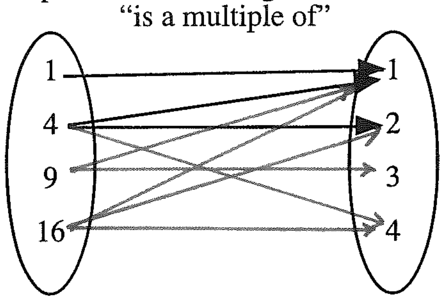
Exploration

To illustrate the concept of function, we will look at two relations described in words with domain $D = \{1, 4, 9, 16\}$ and range $R = \{1, 2, 3, 4\}$.

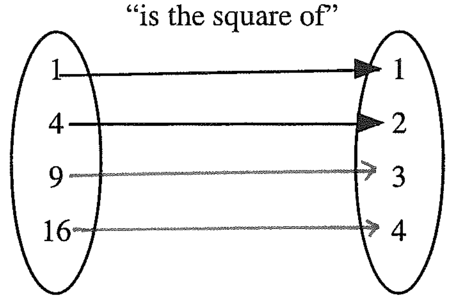
i) "is a multiple of" **ii)** "is the square of"

a) Complete the arrow diagrams.

i) "is a multiple of"



ii) "is the square of"



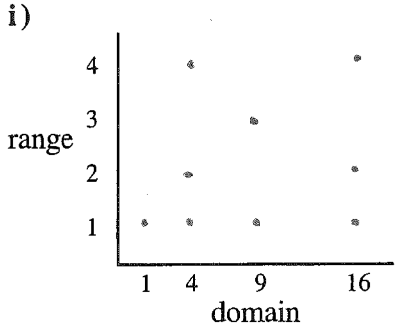
b) Complete the set of ordered pairs.

i) (1, 1), (4, 1), (4,2),(4,4),(9,1),
(9,3),(16,1),(16,2),(16,4)

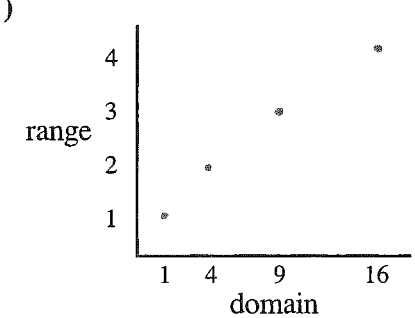
ii) (1,1), (4,2),(9,3),(16,4)

c) Plot the ordered pairs on the grid.

i)



ii)



Function

Summary: one-to-one

A functional relation, or **function**, is a special type of relation in which each element of the domain is related to exactly one element of the range. If any element of the domain is related to more than one element of the range, then the relation is not a function.

Class Ex. #1

In the exploration on the previous page, one of the relations is a function, and the other relation is not a function.

Explain how we can determine which relation is a function by looking at the following:

a) arrow diagrams If only one arrow leaves every element of the domain then the relation is a function

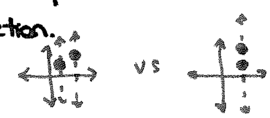
function not a function

note: both examples are still relations.

b) ordered pairs If each x-coordinate has only one corresponding y-coordinate then the relation is a function. (2,3) and (7,9) vs (2,3)(2,6)

Funtion not a function

c) graphs If each point on the horizontal axis has only one point vertically above it then the relation is a function.

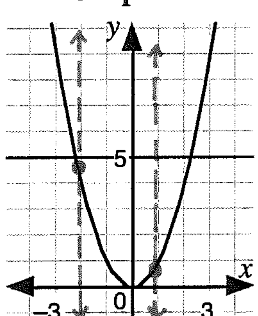


Function vs not a function

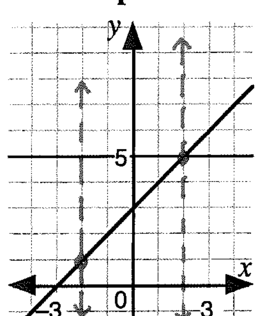
Class Ex. #2

Each of the following is the graph of a relation.

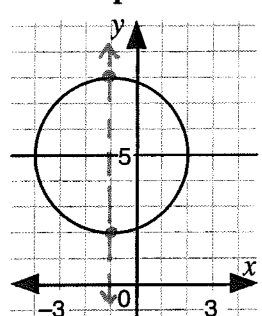
Graph A



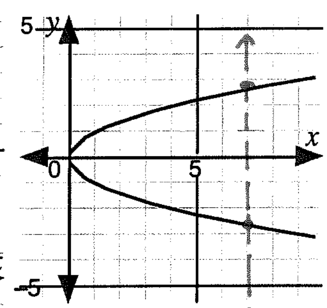
Graph B



Graph C



Graph D



passes the VLT ← cannot find a place that fails → passes the VLT Fails VLT Fails VLT

a) Classify the following statements as true (T) or false (F).

- For each input value there is only one output.
  Use the vertical line test (VLT).
- For each output value there is only one input.
  one-to-one may not satisfied
- The relation is a function.

A	B	C	D
T	T	F	F
F	T	F	T
T	T	F	F

Passes the above two statements. – is a function

Passes the first statement only – is a function

Passes the Second statement only – is not a function.

Passes neith – is not a function

b) From graph C, write two ordered pairs which show that the relation is not a function. Draw a line joining these points.

(-1,3) and (-1,-3)

c) From graph D, write two ordered pairs which show that the relation is not a function. Draw a line joining these points.

(4,2) and (4,-2)

d) On graphs A and B draw a series of vertical lines. Do any of these lines intersect the graph of the relation at more than one point?

no

Vertical Line Test

The vertical line test can be used on the graph of a relation to determine whether the relation is a function or not.

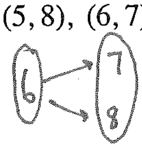
- If every vertical line, drawn on the domain of the relation, intersects the graph exactly once, then the relation is a function.
- If any vertical line intersects the graph more than once, then it is **not** a function.

Class Ex. #3

Determine which of the following are functions. Explain your answers.

Make a quick sketch to visualize

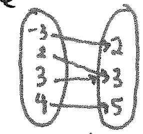
a) (5,8), (6,7), (−5,3), (2,3), (6,8)



No, the input 6 has two outputs.

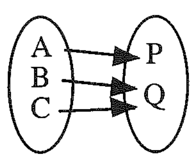
b) (3,3), (2,3), (4,5), (−3,2)

Yes, each input has only one output.



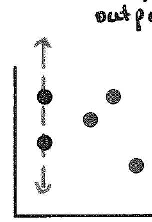
one-to-one

c)



Yes, each input has only one output.
one-to-one

d)

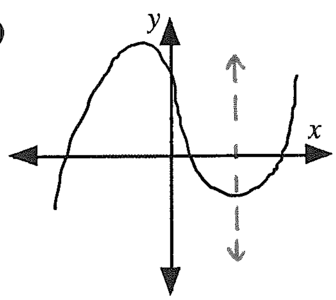


No, one of the inputs has two outputs.
not one-to-one.

e) The relation connecting the provinces and territories of Canada with their capital cities.

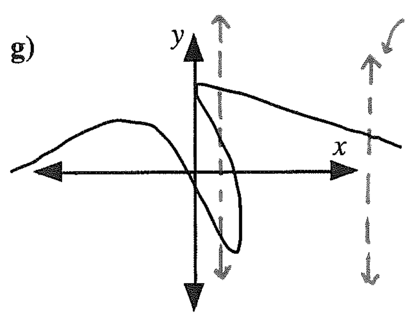
Yes, each input has only one output.

f)



Yes, passes the vertical line test.

g)



Notice that the relation passes the VLT here, but in order to be a function it must pass the test in all cases of possible inputs.

No, fails the vertical line test.

A Function as a Mapping

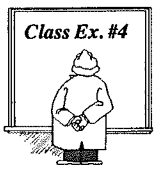
A function from a set D, the domain, to a set R, the range, is a relation in which each element of D is related to exactly one element of R.

If the function f maps an element x in the domain to an element y in the range, we write $f: x \rightarrow y$.

Complete the following for the function "is the square of" on the first page of this lesson.

$1 \rightarrow$ 1 $\quad 4 \rightarrow$ 2 $\quad 9 \rightarrow$ 3 $\quad$ 16 $\rightarrow$ 4

The letter "f" is used in function notation since the word function starts with the letter "f."



Consider the function $f: x \rightarrow 3x + 1$, for domain $\{-1, 0, 1, 2\}$.

Let $x = -1$

a) Complete $\quad -1 \rightarrow -2 \quad 0 \rightarrow 1 \quad 1 \rightarrow 4 \quad 2 \rightarrow 7$

$3(-1) + 1 = -3 + 1 = -2$

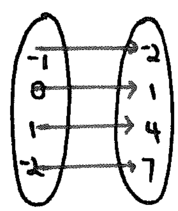
input $\qquad$ output

b) List the elements of the range of the function.

range: $\{-2, 1, 4, 7\}$

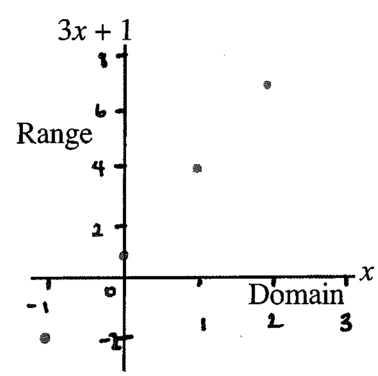
c) Show the function as:

i) an arrow diagram $\qquad$ **ii)** a set of ordered pairs $\qquad$ **iii)** a Cartesian graph.



$(x, 3x+1)$

$(-1, -2)$
$(0, 1)$
$(1, 4)$
$(2, 7)$



At this time we label the top of the vertical axis with $3x + 1$. In the next lesson we will learn function notation which is more commonly used.

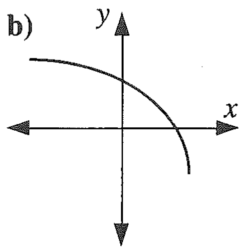
Complete Assignment Questions #1 - #12

Assignment

1. Determine which of the following relations are functions. Give reasons for your answers.

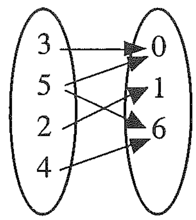
a) $(-1,3)$, $(-2,1)$, $(5,2)$, $(7,3)$ Yes, each input has only one output.

b)



Yes, passes the vertical line test.

c)



No, the input 5 has two outputs.

d)

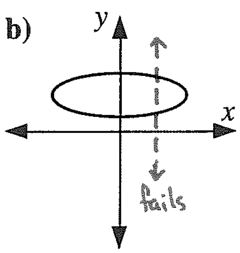
Input (x)	Output (y)
2	3
0	4
–3	5
2	6

No, the input 2 has two outputs.

2. State which of the following relations are functions.

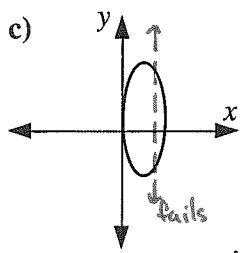
a) $(0,0)$, $(1,2)$, $(2,3)$, $(3,4)$, $(4,3)$ Yes, a function.

b)



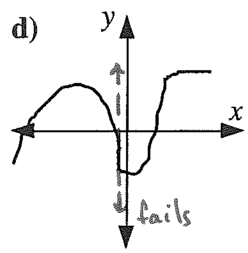
No, only a relation.

c)



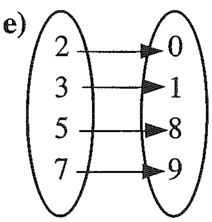
No, only a relation.

d)



No, only a relation.

e)



Yes, a function.

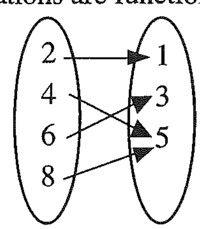
3. State which of the following relations are functions.

a)

Input (x)	Output (y)
0	3
2	4
4	5
6	3

Yes, a function

b)



Yes, a function.

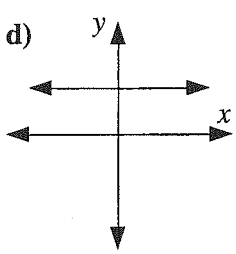
c)

Input (x)	Output (y)
1	5
–1	5
3	5
7	5

Yes, a function.

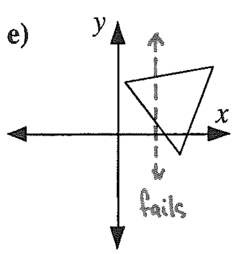
Recall: a relation can passes a VLT, but it must pass the test in a cases of the input.

d)



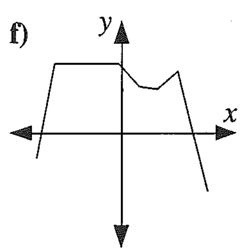
Yes, a function.

e)



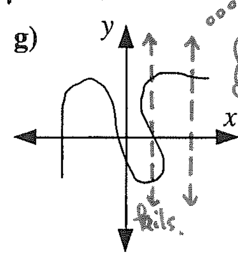
No, only a relation.

f)



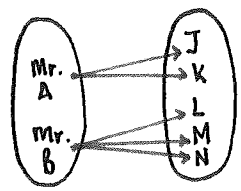
Yes, a function.

g)



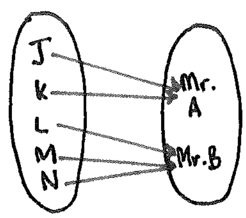
No, only a relation.

4. Mr. A has a son Jim and a daughter Kristen. Mr. B has three daughters, Lauren, Melanie, and Noreen.
   a) Draw an arrow diagram to illustrate the relation "is the father of" from the set of fathers to the set of children. Is the relation "is the father of" a function?



Not a function.

   b) Draw an arrow diagram to illustrate the relation "is the child of" from the set of children to the set of fathers. Is the relation "is the child of" a function?



Yes, the relation is a function.

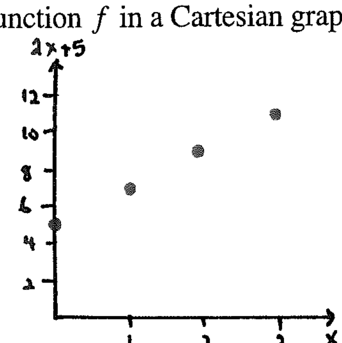
5. The function $f: x \to 2x + 5$ has domain $\{0, 1, 2, 3\}$.

   a) List the elements of the range of the function.

   range: $\{5, 7, 9, 11\}$

   $2(0)+5=5$
   $2(1)+5=7$
   $2(2)+5=9$
   $2(3)+5=11$
   ↑ input ↓ output

   b) Show the function f in a Cartesian graph.



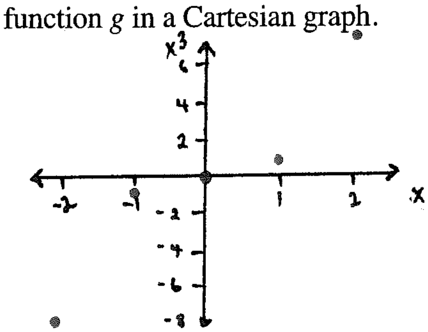
6. The function $g: x \to x^3$ has domain $\{-2, -1, 0, 1, 2\}$.

   a) List the elements of the range of the function.

   range: $\{-8, -1, 0, 1, 8\}$

   $(-2)^3 = -8$
   $(-1)^3 = -1$
   $(0)^3 = 0$
   $(1)^3 = 1$
   $(2)^3 = 8$
   ↑ input ↓ output

   b) Show the function g in a Cartesian graph.



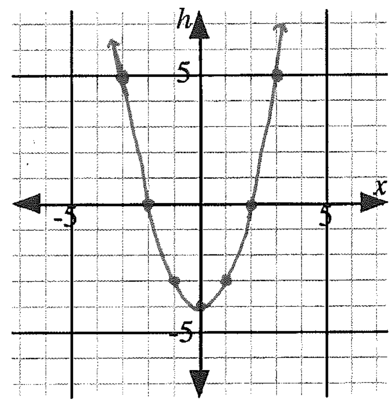
7. Consider the function $f: x \to x^2 - 4$.
   a) Complete the following table of values.

Elements of Domain	3	2	1	–1	–2	–3
Elements of Range	5	0	-3	-3	0	5

Also 0
-4

   b) Plot the ordered pairs on a Cartesian graph.

   c) Draw a smooth curve through the points to illustrate the function $f: x \to x^2 - 4,\ x \in R$.

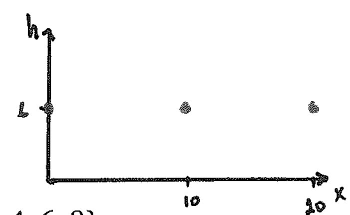


8. The domain of the function $h: x \to 6$ is $\{0, 10, 20\}$.

   a) List the ordered pairs of the graph of the function.

   (0,6), (10,6), (20,6)

   b) Show the function h in a Cartesian graph.



Multiple Choice

9. The function $f: x \to 6 - 2x$ has domain $\{0, 2, 4, 6, 8\}$.
   Which of the following is **not** an element of the range of the function?

   A. –10
   B. 2
   C. 4
   D. –6

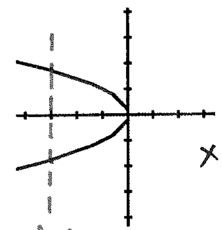
$6-2(0) = 6$
$6-2(2) = 2$
$6-2(4) = -2$
$6-2(6) = -6$
$6-2(8) = -10$

10. Which of the following statements is not always true for a function?

   A. A function is a set of ordered pairs (x, y) in which for every x there is only one y.
   B. A vertical line must not intersect the graph of a function in more than one point.
   C. For every output there is only one input. one-to-one only considers the behaviour
   D. For every element in the domain, there is only one element in the range. of the input having only one output. It does not matter how many inputs an output has.


still a function!

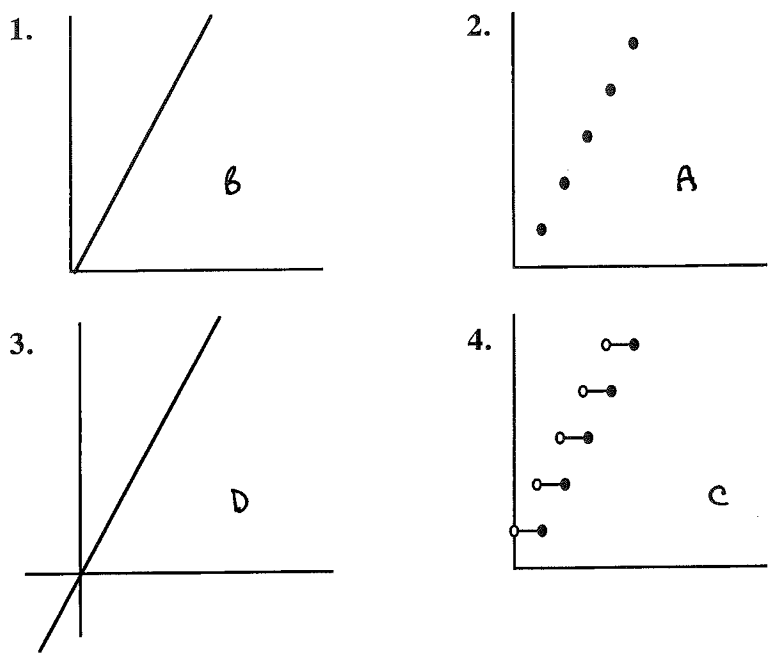
11. Which of the following represents a function?

$\boxed{1}$	"multiply the number by 3 and add 5." ✓	$\boxed{2}$	$y = -x^2$ ✓
$\boxed{3}$	$(9,3), (4,2), (1,1), (0,0). (1,-1), (4,-2), (9,-3)$ ✗	$\boxed{4}$	 ✗ fails

A. 1 only
(B.) 1 and 2 only
C. 1 and 3 only
D. some other combination of 1 - 4

Numerical Response

12. Partial graphs of four functions are shown.



The functions are described as follows:

A: Coffee costs $8 per jar. Graph cost as a function of the number of jars purchased.
discrete – one points and no continuous data.

B: Distance cycled at a constant speed of 8 km/h. Graph distance as a function of time.
Continuous – time is continuous and only positive.

C: Parking costs $8 per hour (or part of an hour). Graph cost as a function of time.
constaint continuous rate and then increases each hour.

D: Set of ordered pairs which satisfy the equation $y = 8x, x \in R$.
Graph y as a function of x.
Continuous – input values are continuous with nolower limit.

Place the graph number for function A in the first box.
Place the graph number for function B in the second box.
Place the graph number for function C in the third box.
Place the graph number for function D in the last box.

(Record your answer in the numerical response box from left to right) $\boxed{2\,|\,1\,|\,4\,|\,3}$

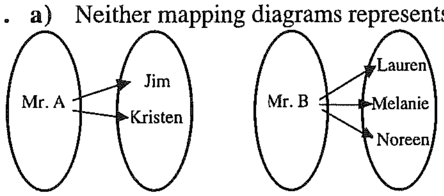
Answer Key

1. a) function: each first coordinate has only one second coordinate
b) function: vertical lines intersects the graph exactly once
c) not a function: the input 5 has two outputs
d) not a function: the input 2 has two outputs

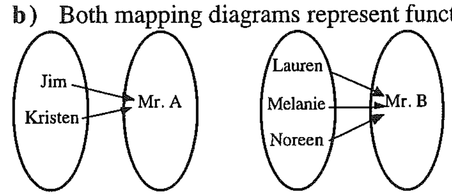
2. a) function **b)** not a function **c)** not a function **d)** not a function **e)** function

3. a) function **b)** function **c)** function **d)** function **e)** not a function
f) function **g)** not a function

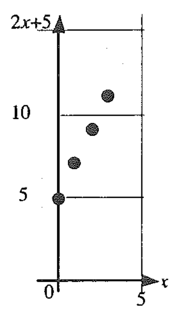
4. a) Neither mapping diagrams represents a function



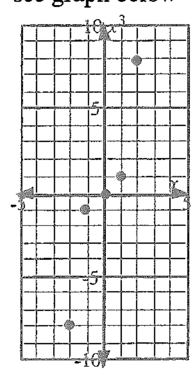
b) Both mapping diagrams represent functions



5. a) $\{5, 7, 9, 11\}$
b) see graph below

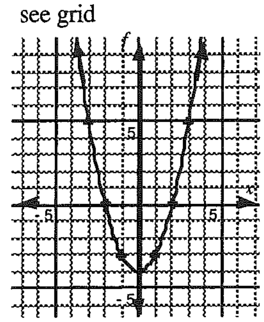


6. a) $\{-8, -1, 0, 1, 8\}$
b) see graph below

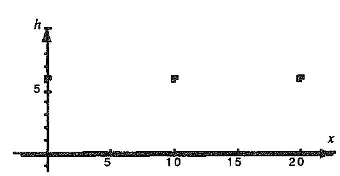


7. a) see table **b)** see grid **c)** see grid

Elements of Domain	3	2	1	−1	−2	−3
Elements of Range	5	0	−3	−3	0	5



8. a) $\{(0, 6), (10, 6), (20, 6)\}$ **b)**



9. C **10.** C **11.** B **12.**

2	1	4	3

Functions Lesson #2:
Function Notation - Part One

Mapping Notation

In the previous lesson we discovered some ways in which functions can be represented:

- in words
- a table of values
- a set of ordered pairs
- a mapping (or arrow) diagram
- an equation
- a graph
- function notation (this unit)

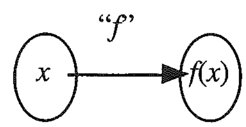
A **function** was defined in mapping notation as follows:

"A function from a set D, the domain, to a set R, the range, is a relation in which each element of D is related to exactly one element of R.
If the function f maps an element x in the domain to an element y in the range, we write $f: x \to y$."

Consider the function $f: x \to 2x + 3$ defined on the set of real numbers.
Under this function we know that $5 \to 2(5) + 3$ ie $5 \to 13$.
We say that under the function f, the **image** of 5 is 13.
We also say that the **value of the function** is 13 when $x = 5$.

Function Notation

In most math courses, function notation is used to replace the mapping notation $f: x \to 2x+3$. Under a function f, the image of an element x in the domain is denoted by $f(x)$, which is read "f of x".



In the example above, the function f can be defined by the formula $f(x) = 2x + 3$.
The notation $f(x) = 2x + 3$ is called **function notation**.

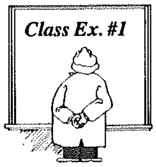
We showed above, that, under the function f, the image of 5 is 13. We write $f(5) = 13$.

<u>mapping notation</u>	<u>function notation</u>	<u>equation of graph of function</u>
$f: x \to 2x+3$	$f(x) = 2x + 3$	$y = 2x + 3$
$f: 5 \to 2(5)+3$	$f(5) = 2(5) + 3$	$y = 2(5) + 3$
$f: 5 \to 13$	$f(5) = 13$	$y = 13$

The symbol $f(x)$ is read as "f at x" or "f of x".
$f(x)$ provides a formula for the function f, and also represents the <u>value</u> of the function for a given value of x.

In function notation:

- $f(x)$ does <u>not</u> mean f times x.
- The "name" of the function is f.
- Values of the independent variable represent the **inputs** of a function and are shown on the **horizontal axis**.
- Values of the dependent variable represent the **outputs** of a function and are shown on the **vertical axis**.



Consider the function $f(x) = x^2 + 5$ and $g(x) = 4 - x$. Evaluate:

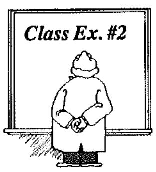
a) $f(3) = (3)^2 + 5$
$= 9 + 5$
$= 14$

b) $g(1) = 4 - (1)$
$= 3$

c) $f(-2) = (-2)^2 + 5$
$= 4 + 5$
$= 9$

d) $g(-2) = 4 - (-2)$
$= 4 + 2$
$= 6$

e) $f(0) - g(0) = ((0)^2 + 5) - (4 - (0))$
$= 5 - 4$
$= 1$



Consider the function f defined by $f(x) = 5x^3 - 2x, x \in R$. Determine:

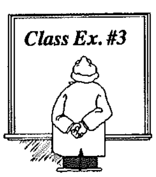
a) $f(-3)$
$= 5(-3)^3 - 2(-3)$
$= -129$

b) the value of f when $x = 2$
$f(2) = 5(2)^3 - 2(2)$
$= 36$

c) the image of 7 under f
$f(7) = 5(7)^3 - 2(7)$
$= 1701$

e) an expression for $f(a)$
$f(a) = 5(a)^3 - 2(a)$
$= 5a^3 - 2a$

f) an expression for $f(2x)$
$f(\ ) = 5(2x)^3 - 2(2x)$
$= 5(8x^3) - 2(2x)$
$= 40x^3 - 4x$



If $P(x) = 4x^2 - 6x + 1$, determine a simplified expression for $P(x - 3)$.

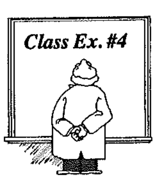
$P(x - 3) = 4(x - 3)^2 - 6(x - 3) + 1$
$= 4(x^2 - 6x + 9) - 6x + 18 + 1$
$= 4x^2 - 24x + 36 - 6x + 19 = \underline{4x^2 - 30x + 55}$

Side Work

	x	-3
x	x^2	$-3x$
-3	$-3x$	$+9$

$x^2 - 6x + 9$

Complete Assignment Questions #1 - #7



Consider the function $f(x) = 10x - 3$, $x \in R$.

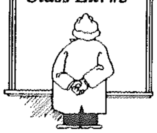
a) Determine the value of x if $f(x) = 47$.

$(47) = 10x - 3$
$+3 \qquad +3$
$\frac{50}{10} = \frac{10x}{10}$
$\underline{x = 5}$

b) Solve the equation $f(x) = -23$.

$(-23) = 10x - 3$
$-20 = 10x$
$\underline{x = -2}$

Class Ex. #5



Consider the function $f(x) = x^2 - 5, x \in R$.

a) Evaluate $f(4)$.

$f(4) = (4)^2 - 5$
$= 16 - 5$
$= 11$

b) Solve the equation $f(x) = 4$.

$(4) = x^2 - 5$
$+5 \qquad +5$
$9 = x^2$
$x = \sqrt{9} = \pm 3$

c) Solve the equation $f(t) = 75$, where $t > 0$. Answer as an exact value and as a decimal to the nearest hundredth.

$f(t) = t^2 - 5$
$(75) = t^2 - 5$
$+5 \qquad +5$
$t^2 = 80$
$t = \pm\sqrt{80} = \pm 8.944\ldots$

Exact Value is $+\sqrt{80}$.
Decimal is $+8.94$.

Notice the $-\sqrt{80}$ and -8.94 is not include in answer since $t > 0$.

Complete Assignment Questions #8 - #13

Assignment

1. Each statement refers to the function f whose graph has equation $y = f(x)$. Circle the correct choice.

a) f is the *name* / *value* of the function.
b) The values of x represent the *inputs* / *outputs* of the function.
c) The values of $f(x)$ represent the *inputs* / *outputs* of the function.
d) The values of y represent the *inputs* / *outputs* of the function.
e) x represents the *independent* / *dependent* variable of the function.
f) $f(x)$ represents the *independent* / *dependent* variable of the function.
g) y represents the *independent* / *dependent* variable of the function.

2. If $f(x) = 5x - 7$, determine:

a) $f(2)$
$f(2) = 5(2) - 7$
$f(2) = 3$

b) $f(-3) = 5(-3) - 7$
$= -15 - 7$
$= -22$

c) $f(0) = 5(0) - 7$
$= 0 - 7$
$= -7$

3. Function g is defined by $g(x) = 6 - x^2$. Evaluate

a) $g(4) = 6 - (4)^2$
$= 6 - 16$
$= -10$

b) $g(-6) = 6 - (-6)^2$
$= 6 - 36$
$= -30$

c) $g(\sqrt{3}) = 6 - (\sqrt{3})^2$
$= 6 - 3$
$= 3$

4. A function f is defined by the formula $f(x) = x^3 + 1$. Find

a) the image of 2 under f
$f(2) = (2)^3 + 1$
$= 8 + 1$
$= 9$

b) the value of f at -7.
$f(-7) = (-7)^3 + 1$
$= -343 + 1$
$= -342$

c) an expression for $f(a)$
$f(a) = (a)^3 + 1$
$= a^3 + 1$

5. If $f(x) = x^3 - 2x^2 - x - 5$, evaluate

a) $f(5) = (5)^3 - 2(5)^2 - (5) - 5$
$= 125 - 2(25) - 5 - 5$
$= 65$

b) $f(-3) = (-3)^3 - 2(-3)^2 - (-3) - 5$
$= -27 - 2(9) + 3 - 5$
$= -47$

6. Consider the function f defined by $f(x) = 8 - 2x,\ x \in R$. Determine

a) $f(4) = 8 - 2(4)$
$= 8 - 8$
$= 0$

b) the value of f when $x = -4$
$f(-4) = 8 - 2(-4)$
$= 8 + 8$
$= 16$

c) the image of 0.5 under f
$f(0.5) = 8 - 2(0.5)$
$= 8 - 1$
$= 7$

d) an expression for $f(2t)$
$f(2t) = 8 - 2(2t)$
$= 8 - 4t$

e) an expression for $f(a + 3)$
$f(a+3) = 8 - 2(a+3)$
$= 8 - 2a - 6$
$= 2 - 2a$

	x	-5
x	x^2	$-5x$
-5	$-5x$	25

7. If $F(x) = 3x^2 - 2x - 9$, determine a simplified expression for

a) $F(-x) = 3(-x)^2 - 2(-x) - 9$
$= 3x^2 + 2x - 9$

b) $F(x - 5) = 3(x - 5)^2 - 2(x - 5) - 9$
$= 3(x^2 - 10x + 25) - 2x + 10 - 9$
$= 3x^2 - 30x + 75 - 2x + 10 - 9$
$= 3x^2 - 32x + 76$

8. a) If $f(x) = 5x - 7$, then determine the value of x if $f(x) = 43$.

$(43) = 5x - 7$
$+7 \qquad +7$
$\frac{50}{5} = \frac{5x}{5}$

$x = 10$

b) If $g(x) = 6x + 3$, then determine the value of x if $g(x) = -24$.

$(-24) = 6x + 3$
$+3 \qquad -3$
$\frac{-27}{6} = \frac{6x}{6}$

$x = \frac{-27}{6} = \frac{-9}{2}$ $\div 3$

c) If $g(t) = 56 - 3t$, then determine the value of t if $g(t) = 11$.

$(11) = 56 - 3t$
$-56 \quad -56$
$3t = 45$

$t = 15$

d) If $h(x) = -3x + 1$, then determine the value of x if $h(x) = 22$.

$(22) = -3x + 1$
$-1 \qquad -1$
$21 = -3x$

$x = \frac{21}{-3} = -7$

e) If $P(x) = 50 - 3x^2$, then determine the values of x if $P(x) = -25$.

$(-25) = 50 - 3x^2$
$-50 \quad -50$
$3x^2 = 75$
$x^2 = 25$

$x = \sqrt{25} = \pm 5$

9. Consider the function f defined by $f(x) = 6x - 15$. Find

a) $f(0)$

$= 6(0) - 15$
$= 0 - 15$
$= -15$

b) an expression for $f(2x + 1)$

$= 6(2x+1) - 15$
$= 12x + 6 - 15$
$= 12x - 9$

c) the solution to the equation $f(x) = 27$

$(27) = 6x - 15$
$+15 \qquad +15$
$42 = 6x$ ← divide both sides by 6.
$x = 7$

10. A function C is defined by $C(x) = \sqrt{x}$ where $x \geq 0$. → must only be values of zero or positive numbers!

a) Evaluate

i) $C(16) = \sqrt{16}$
$= 4$

ii) $C\left(\frac{1}{36}\right) = \sqrt{\frac{1}{36}}$
$= \frac{1}{6}$

iii) $\frac{C(100)}{C(4)}$
$= \frac{\sqrt{100}}{\sqrt{4}}$
$= \frac{10}{2}$
$= 5$

b) If $C(x) = 9$, find x.

$\sqrt{(x)} = 9$
$x = 9^2$ ← square both sides to simplify the $\sqrt{\ }$ and isolate the x-value!
$= 81$

11. A function g is defined by the formula $g(t) = t + 12$.

a) Calculate the value of $g(4) + g(-2)$.

$= [(4) + 12] + [(-2) + 12]$

$= 16 + 10$

$= 26$

b) If $g(a^2) = 48$, determine all possible values of a.

$(48) = (a^2) + 12$

$48 = a^2 + 12$

$-12 \quad -12$

$36 = a^2$

$a = \sqrt{36}$

$= \pm 6$

Multiple Choice **12.** If $f(x) = 3x - 1$ and $f(t) = 8$, then $t =$

A. $\frac{7}{3}$

B. 3

C. $\frac{11}{3}$

D. 23

$(8) = 3x - 1$

$+1 \quad +1$

$\frac{9}{3} = \frac{\cancel{3}x}{\cancel{3}}$

$x = 3$

Numerical Response **13.** A function f is defined by the formula $f(x) = 8\sqrt{x}$, $x \in R$. The value of $f(144)$ is ______ .

(Record your answer in the numerical response box from left to right)

9	6		

$f(144) = 8\sqrt{(144)} = 8(12) = 96$

Further assignment questions on Function Notation - Part One will appear in the assignment of the next lesson, Function Notation - Part Two.

Answer Key

1. a) name **b)** inputs **c)** outputs **d)** outputs
e) independent **f)** dependent **g)** dependent

2. a) 3 **b)** -22 **c)** -7 **3. a)** -10 **b)** -30 **c)** 3

4. a) 9 **b)** -342 **c)** $a^3 + 1$ **5. a)** 65 **b)** -47

6. a) 0 **b)** 16 **c)** 7 **d)** $8 - 4t$ **e)** $2 - 2a$

7. a) $3x^2 + 2x - 9$ **b)** $3x^2 - 32x + 76$

8. a) 10 **b)** $-\frac{9}{2}$ **c)** 15 **d)** -7 **e)** ± 5

9. a) -15 **b)** $12x - 9$ **c)** $x = 7$ **10. a) i)** 4 **ii)** $\frac{1}{6}$ **iii)** 5 **b)** 81

11. a) 26 **b)** ± 6 **12.** B **13.**

9	6		

Functions Lesson #3:
Function Notation - Part Two

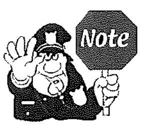
Graphing a Function

Consider the function $f(x) = 3x + 1$. The values of x represent the inputs and make up the domain of the function. The values of $f(x)$ represent the outputs and make up the range of the function.

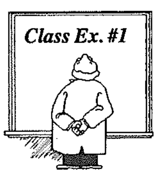
In previous lessons, we have used y to represent the outputs and the range of a relation. We can therefore write the function $f(x) = 3x + 1$ in x-y notation as $y = 3x + 1$.

The function $f(x) = 3x + 1$ can be written in x-y notation as shown.

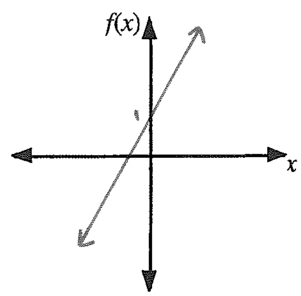
Function notation	x-y notation
$f(x) = 3x + 1$	$y = 3x + 1$

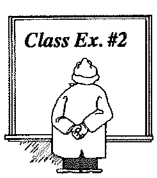


- Values of the independent variable represent the **inputs** of a function and are shown on the **horizontal axis**.
- Values of the dependent variable represent the **outputs** of a function and are shown on the **vertical axis**.



Use a graphing calculator to sketch the graph of the function $f(x) = 3x + 1$.





a) In each case, express the relation given in function notation as an equation in two variables.

i) $f(x) = 7x - 23$ **ii)** $g(t) = t^2 - 2t + 35$

$y = 7x - 23$ $y = t^2 - 2t + 35$

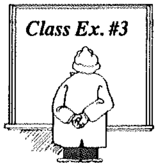
b) Express the relation $y = 11x - 15$ in function notation.

$f(x) = 11x - 15$

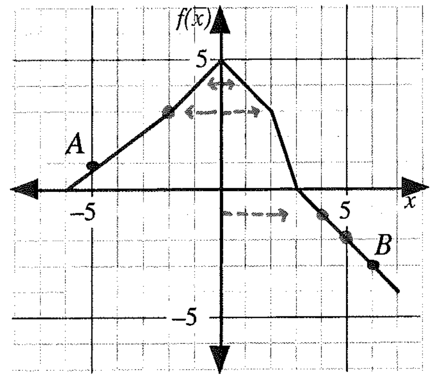
c) The graph of the function defined by $y = f(x)$ has equation $y = 4 - 3x$. Express the equation in function notation.

$f(x) = 4 - 3x$

Summary
$y = f(x)$



The graph of a function f is shown.



a) Complete

i) $f(5) =$ -2 ii) $f(-2) =$ 3 iii) $f(4) =$ -1

b) Write the ordered pairs associated in **a**).

i) (5, -2) ii) (-2, 3) iii) (4, -1)

c) State the value(s) of x if

i) $f(x) = -1$ ii) $f(x) = 3$ iii) $f(x) = 4$

x = 4 x = ± 2 x = ± 1

d) Use the notation in **a**) to make a statement about the points A and B on the graph.

f(-5) = 1 f(6) = -3

e) Write the x- and y- intercepts of the graph using function notation.

x_{int}: f(-6) = 0, f(3) = 0 y_{int}: f(0) = 5

Think about it!
f(6) = -3
input output
(6, -3)
input output.

f) Complete the following statements.

- The domain of f is $\{x \mid$ -6 $\le x \le$ 7 $, x \in R\}$
- The range of f is $\{f(x) \mid$ -4 $\le f(x) \le$ 5 $, f(x) \in R\}$

Complete Assignment Questions #1 - #12

Assignment

y = f(x)

1. In each case, express the relation given in function notation as an equation in two variables.

in x and y

a) $f(x) = 10 - 3x$ b) $g(x) = 12x^2 - 5$ c) $P(t) = 2t + 9$

y = 10 - 3x y = 12x² - 5 y = 2t + 9

2. Express the following relations in function notation.

a) $y = 17x - 9$ b) $y = 4v + 25$ c) $x + 2y + 6 = 0$

f(x) = 17x - 9 f(v) = 4v + 25 2y = -x - 6, y = $-\frac{1}{2}$ - 3 f(x) = $-\frac{1}{2}$ - 3

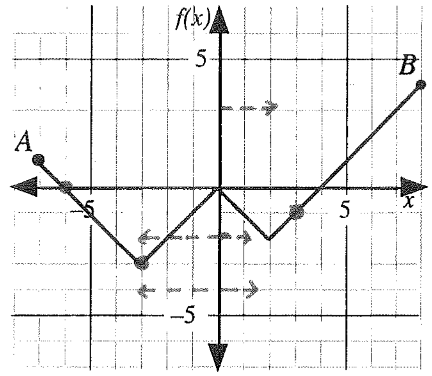
3. a) The graph of the function defined by $y = f(x)$ has equation $y = 0.5x - 0.25$
Express the equation in function notation.

f(x) = 0.5x - 0.25

b) The graph of the velocity function defined by $v = f(t)$ has equation $v = 4.9t^2$.
Express the equation in function notation.

f(t) = 4.9t²

4. The graph of a function f is shown.



a) Complete

i) $f(3) =$ −1 **ii)** $f(-3) =$ −3 **iii)** $f(-6) =$ 0

b) Write the ordered pairs associated with **a)**.

i) (3,−1 **ii)** (−3,−3) **iii)** (−6,0)

c) State the value(s) of x if

i) $f(x) = 3$ **ii)** $f(x) = -2$ **iii)** $f(x) = -4$

x = 7 x = −4, −2, 2 x = no solution.

d) Use the notation in a) to make a statement about the points A and B on the graph.

f(−7) = 1 f(8) = 4

Think about it!

f(−7) = 1

input output

(−7, 1)

input output

e) Write the x- and y- intercepts of the graph using function notation.

Xint: f(−6) = 0 Yint: f(4) = 0 Shared xint: yint: f(0) = 0

f) Complete the following statements.

- The domain of f is $\{x \mid$ −7 $\leq x \leq$ 8 $, x \in R\}$
- The range of f is $\{f(x) \mid$ −3 $\leq f(x) \leq$ 4 $, f(x) \in R\}$

5. The function $g(x) = 3x^2 - 4$ has a domain $\{-2, -1, 0, 1, 2\}$.

a) State the range of g.

$g(-2) = 3(-2)^2 - 4 = 8$

$g(-1) = 3(-1)^2 - 4 = -1$

$g(0) = 3(0)^2 - 4 = -4$

$g(1) = 3(1)^2 - 4 = -1$

$g(2) = 3(2)^2 - 4 = 8$

range: $\{-4, -1, 8\}$

b) Solve the equation $g(x) = -1$.

Let $g(x) = -1$

$(-1) = 3x^2 - 4$

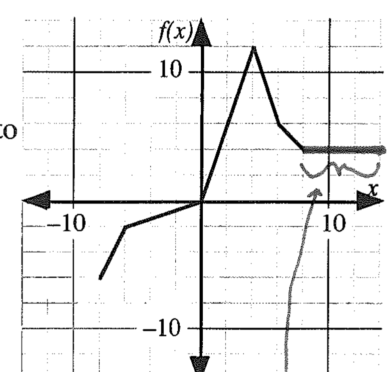
$+4 \qquad +4$

$\frac{3x^2}{3} = \frac{3}{3}$

$x^2 = 1 \longrightarrow x = \sqrt{1} = \underline{\underline{\pm 1}}$

6. Consider the graph of the function f shown below.

a) Complete the table.



x	$f(x)$	Ordered Pair
2	6	(2, 6)
0	0	(0, 0)
−6	−2	(−6, −2)
8	4	(8, 4)
−8	−6	(−8, −6)
10	4	(10, 4)

b) Explain why the solution to the equation $f(x) = 4$ has an infinite number of solutions.

The horizontal line where f(x) = 4 (ie y = 4) has an infinite number of input values between x = 8 and x = 14.

ie 9 and 9.1 and 9.01 and 9.001 and so on.

7. Given that $f(x) = 9 - 2x$

a) evaluate $f(-3)$

$f(-3) = 9 - 2(-3)$
$= 9 + 6$
$= 15$

b) find the value of $f(t) + f(-t)$

$f(t) + f(-t) = [9-2(t)] + [9-2(-t)]$
$= 9 - 2t + 9 + 2t$
$= 18$

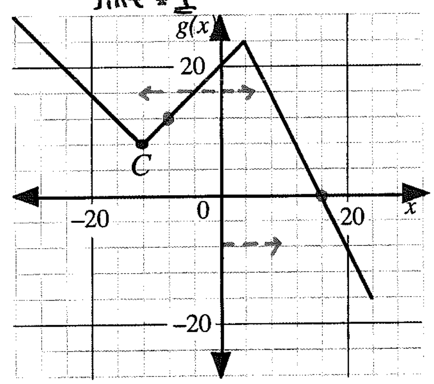
c) calculate the x-intercept and the y-intercept on the graph of f.

$y = 9 - 2x$

x_{int} Let $y = 0$
$(0) = 9 - 2x$
$2x = 9$
$x_{int} = \frac{9}{2}$

y_{int} Let $x = 0$
$y = 9 - 2(0)$
$= 9$
$y_{int} = 9$

8. The graph of a function is shown.



a) A student is asked to make a statement about point C on the graph. The student states that $f(-3) = 2$.

i) Explain **two** errors in the student's statement.

The name of the graph is g $\boxed{\text{not}}$ f.

The scale is 4 units per box not 1 unit per box.

ii) Write a correct statement using function notation about point C.

$g(-12) = 8$

Recall:
$g(-12) = 8$
input output.
$(-12, 8)$
input output

b) Give the solution to the following equations.

i) $g(x) = -8$ $x = 20$

ii) $g(x) = 16$. $x = -20, -4, 8$

c) State the value of **i)** $g(-8)$ 12 **ii)** $g(16)$ 0

d) State the domain and range of the function.

domain: $\{x \mid -32 \leq x \leq 24, x \in \mathbb{R}\}$
range: $\{g(x) \mid -16 \leq g(x) \leq 28, g(x) \in \mathbb{R}\}$

e) The equation $g(a) = b$ has **exactly two** solutions. Explain clearly how to use the graph to determine values of a and b, and provide two sets of answers to the problem.

A horizontal line must intersect the graph at exactly two points.

This occurs when $g(x) = 24$ and when $g(x) = 8$.

Solution 1: $b = 24$ when $a = -28$ or 4.

Solution 2: $b = 8$ when $a = -12$ or 12.

9. Consider the function $f(x) = 1 - x^2$, where x is an integer.

a) Evaluate $f(2) - f(-1)$

$= [1-(2)^2] - [1-(-1)^2]$

$= (1-4) - (1-1)$

$= -3 - 0$

$= -3$

b) Given that $f(a) = -8$, calculate all possible values of a.

$f(a) = 1 - (a)^2$

$(-8) = 1 - a^2$

$-1 \quad -1$

$a^2 = 9$

$a = \sqrt{9} = \pm 3$

Multiple Choice

10. The graph of the function $f(x) = 4^x$, $x \in R$, intersects the y-axis at

A. $(0, 0)$
B. $(0, 1)$
C. $(0, 4)$
D. no point

Let $x = 0$

$f(0) = 4^{(0)}$
$= 1$

Thus, $(0, 1)$

Use the following information to answer the next question.

Function P is such that $P(5) = -1$.
Two students each make a statement about the function P.

- Rose states "When the domain value is 5, the related range value is -1." ✓
- Susan states "The point $(-1, 5)$ is on the graph of $y = P(x)$." ✗

11. Which of the following is true?

A. Both statements are correct.
B. Both statements are incorrect.
C. Rose is correct and Susan is incorrect.
D. Susan is correct and Rose is incorrect.

Recall:
$P(5) = -1$
↑ ↓
input output
$(5, -1)$
↑ ↓
input output

Numerical Response 12. Consider the graph of the function $f(x) = 5x - 11$. The x-intercept of the graph of f is located at $(a, 0)$. The value of a is ____ .

(Record your answer in the numerical response box from left to right)

2	.	2	

Remember:) $y = f(x)$ So we can treat this statement Let $f(x)=0$ the same as Let $y=0$

Let $f(x) = 0$

$(0) = 5x - 11$

$+11 \quad +11$

$\frac{11}{5} = \frac{5x}{5}$

$x = 2.2$

Answer Key

1. **a)** $y = 10 - 3x$ **b)** $y = 12x^2 - 5$ **c)** $y = 2t + 9$

2. **a)** $f(x) = 17x - 9$ **b)** $f(v) = 4v + 25$ **c)** $f(x) = -\frac{1}{2}x - 3$

3. **a)** $f(x) = 0.5x - 0.25$ **b)** $f(t) = 4.9t^2$

4. **a)** **i)** -1 **ii)** -3 **iii)** 0

b) **i)** $(3, -1)$ **ii)** $(-3, -3)$ **iii)** $(-6, 0)$

c) **i)** 7 **ii)** $-4, -2, 2$ **iii)** no solution **d)** A is $f(-7) = 1$, B is $f(8) = 4$

e) x-intercepts can be represented in function notation by; $f(-6) = 0, f(0) = 0, f(4) = 0$

y-intercept can be represented in function notation by $f(0) = 0$

f) $-7 \le x \le 8$, $-3 \le f(x) \le 4$

5. **a)** Range = $\{-4, -1, 8\}$ **b)** $x = \pm 1$

6. See table below.

x	$f(x)$	Ordered Pair
2	6	(2, 6)
0	0	(0, 0)
−6	−2	(−6, −2)
8	4	(8, 4)
−8	−6	(−8, −6)
10	4	(10, 4)

b) The horizontal line where $f(x) = 4$ has an infinite number of input values between 8 and 14.

7. **a)** 15 **b)** 18 **c)** x-int = $\frac{9}{2}$, y-int = 9

8. **a)** **i)** The name of the function is g not f. The scale is 4 units per box, not 1 unit per box.

ii) $g(-12) = 8$

b) **i)** $x = 20$ **ii)** $x = -20, -4, 8$

c) **i)** 12 **ii)** 0

d) Domain = $\{x \mid -32 \le x \le 24, x \in R\}$, $\{g(x) \mid -16 \le g(x) \le 28\}, g(x) \in R$

e) A horizontal line must intersect the graph at exactly two points.

This occurs when $g(x) = 24$ and when $g(x) = 8$.

Solution 1: $b = 24$ when $a = -28$ or 4.

Solution 2: $b = 8$ when $a = -12$ or 12

9. **a)** -3 **b)** ± 3 **10.** B **11.** C **12.**

2	.	2	

Functions Lesson #4:
Function Notation and Problem Solving

Using Function Notation

In the previous unit we solved problems about relations defined by an equation.
In this lesson we will solve problems where function notation is used to define the relation.

On page 256, assignment question #6, we had the following scenario.

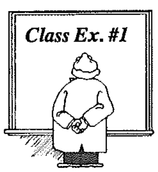
" A candle manufacturer found that their "Long-Last" candles melted according to the formula $h = -2t + 12$, where h is the height of the candle, in cm, after t hours."

The relation between height and time is described by an **equation.**

The relation is a function because for each input there is only one output, and so it can be described using the **function notation** below.

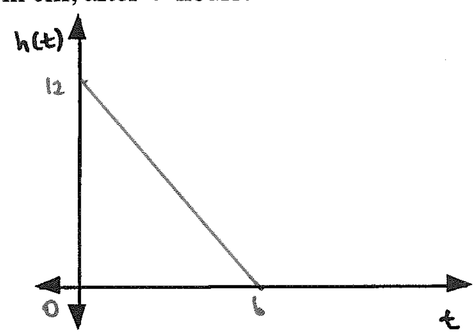
" A candle manufacturer found that their "Long-Last" candles melted according to the formula $h(t) = -2t + 12$, where h is the height of the candle, in cm, after t hours."

In this example, the notation $h(4)$ is a simplified way of representing the height of the candle after four hours.



A candle manufacturer found that their "Long-Last" candles melted according to the formula $h(t) = -2t + 12$, where h is the height of the candle, in cm, after t hours.

a) Use a graphing calculator to sketch the graph of the function and show the graph on the grid



b) Determine the value of $h(5)$.

$h(5) = -2(5) + 12 = 2$

c) Write in words the meaning of $h(5)$.

The height of the candle after 5 hours is 2cm.

d) Evaluate the following, and explain the significance of each.

i) $h(0) = -2(0) + 12$
$= 12$

ii) $h(6) = -2(6) + 12$
$= 0$

iii) $h(8) = -2(8) + 12$
$= -4$

The starting height, $t = 0$, is 12cm.

The height after 6 hours, $t = 6$, is 0cm (burned down to nothing).

Has no meaning in the context of the question.

e) How long will it take for the candle to burn down to a height of 7 cm?

$(7) = -2t + 12$ $2t = 5$ $t = 2.5$ 2.5 hours.

f) Suggest an appropriate domain and range for the function.

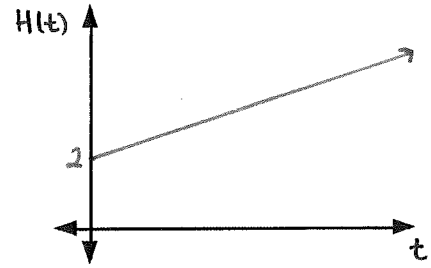
domain: $\{t \mid 0 \le t \le 6, t \in R\}$ range: $\{h(t) \mid 0 \le h(t) \le 12, h(t) \in R\}$

max = 12
min = 0
max = 6
min = 0

Complete Assignment Questions #1 - #4

Assignment

1. Ivory the botanist treated a 2 cm plant with a special growth fertilizer. With this fertilizer, the plant grew at a rate modelled by the function $H(t) = \frac{5}{3}t + 2$, where $H(t)$ represents the height of the plant in cm after t days.



a) Use a graphing calculator to sketch the graph of the function and show the graph on the grid.

b) Determine the value of $H(3)$.

$H(3) = \frac{5}{3}(3) + 2 = 7$

c) Write in words the meaning of $H(3)$.

After 3 days the height is 7cm.

d) Evaluate the following.

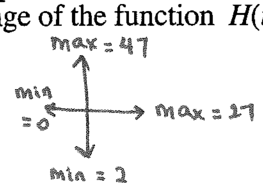
i) $H(0) = \frac{5}{3}(0) + 2 = 2$ **ii)** $H(6) = \frac{5}{3}(6) + 2 = 12$ **iii)** $H(21) = \frac{5}{3}(21) + 2 = 37$

e) How long will it take for the plant to reach a height of 21 cm?

$(21) = \frac{5}{3}t + 2$ $19 = \frac{5}{3}t$ $57 = 5t$ $t = 11.4$ 11.4 days.

f) It takes 27 days for the plant to mature (to reach maximum height). State the domain and range of the function $H(t)$.

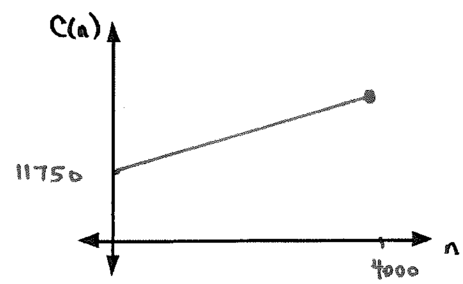
$H(27) = \frac{5}{3}(27) + 2 = 47$



domain : $\{t \mid 0 \le t \le 27, t \in \mathbb{R}\}$

range : $\{H(t) \mid 2 \le H(t) \le 47, H(t) \in \mathbb{R}\}$

2. The cost to Inner Technology of producing IT graphing calculators can be modelled by the function $C(n) = 11750 + 32n$, where $C(n)$ represents the cost in dollars of producing n calculators.



a) Sketch the graph of the function for a maximum of 4000 calculators.

b) Determine the value of $C(30)$.

$C(30) = 11750 + 32(30) = 12710$

c) Write in words the meaning of $C(30)$.

It will cost \$12710 to produce 30 calculators.

d) Evaluate $C(0)$ and explain its significance.

$C(0) = 11750 + 32(0) = 11750$. There are fixed costs of \$11750 before any calculators are produced.

e) How many calculators can be produced for $31 270?

$(31\,270) = 11750 + 32n$ $-11750 \quad -11750$ $\frac{32n}{32} = \frac{19520}{32}$ $n = 610$ 610 calculators.

f) Last month IT produced 2 600 calculators and spent $14 000 on advertising. If there are other fixed monthly costs of $24 500, and each calculator sells for $165, how much profit would be made if all the calculators are sold? Profit = Revenue − Costs.

Step 1: Revenue.

Revenue $= 2600(165)$
$= \boxed{\$429\,000}$

Step 2: Cost

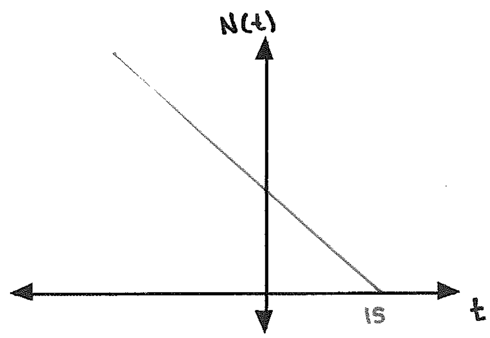
Total Cost $= C(2600) + 14000 + 24500$
$= [11750 + 32(2600)] + 38500$
$= 94950 + 38500$
$= \boxed{\$133\,450}$

Step 3: Solve for Profit.

Profit $= \$429000 - \$133450$
$= \$295\,550$.

3. Over the last 10 years, data was recorded for the number of cups of hot chocolate sold at BGB Senior High School. It was found from the data that the warmer the weather, the less cups of hot chocolate were sold. The data can be modelled by the formula $N(t) = 150 - 10t$, where $N(t)$ is the daily number of cups of hot chocolates sold when the average daily temperature is t °C.

a) Sketch the graph of the function on the grid provided.



b) Determine the value of $N(-5)$.

$N(-5) = 150 - 10(-5) = 200$

c) Write in words the meaning of $N(-5)$.

200 cups of hot chocolate were sold when the average temp is −5°C.

d) What was the average temperature if 190 cups of hot chocolate were sold?

$(190) = 150 - 10t$ $-150 \quad -150$ $\frac{10t}{10} = \frac{-40}{10}$ $t = -4$ $-4°C$

e) Explain how to estimate the lower limit of the domain of the relation.

Estimate the minimum average daily temperature.

f) Suggest an appropriate domain and range for the function $N(t)$ if BGB High School is located in the Okanagan, British Columbia.

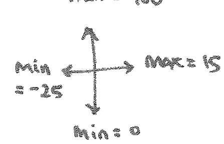
domain: $\{t \mid -25 \le t \le 15, t \in R\}$

range: $\{N(t) \mid 0 \le N(t) \le 400, N(t) \in W\}$

since it is a count number.
no partial cups.

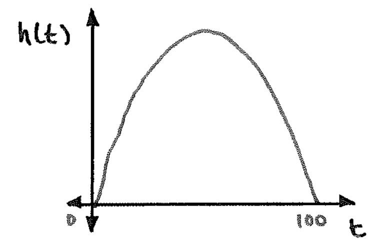
Side Work.

$N(-25) = 150 - 10(-25)$
$= 150 + 250$
$= 400.$



4. A special type of weather balloon follows a path which can be represented by the formula $h(t) = -9t^2 + 900t$, where $h(t)$ is the height in cm after t minutes.

a) Sketch the graph of the function on the grid.



b) Determine the value of $h(30)$ and $h(70)$.

h(30) = 18 900 h(70) = 18900

c) Does $h(30) = h(70)$? Do they mean the same thing? Explain.

They are equal, but do not mean the same thing. h(30) is the height after 30 mins and h(70) is the height after 70 mins.

d) Evaluate the following, and explain their significance in the context of the question.

i) $h(0)$ = 0 **ii)** $h(100)$ = 0 **iii)** $h(110)$ = −9900

Initial height = 0 m After 100 min the balloon has landed on the ground. No meaning since the balloon has already landed.

e) What is the highest point the balloon will reach?

h(50) = 22500cm or 225m.

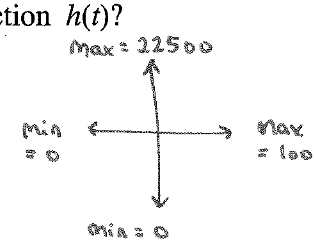
f) When will the balloon land?

100 minutes after taking off.

g) Suggest an appropriate domain and range for the function $h(t)$?

domain: $\{t \mid 0 \le t \le 100, t \in R\}$

range: $\{h(t) \mid 0 \le h(t) \le 22500, h(t) \in R\}$



Answer Key

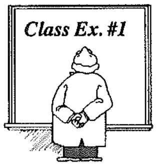
1. **b)** 7 **c)** After 3 days the height is 7 cm. **d)** i) 2 ii) 12 iii) 37 **e)** 11.4 days
f) domain $\{t \mid 0 \le t \le 27, t \in R\}$ range $\{H(t) \mid 2 \le H(t) \le 47, H(t) \in R\}$

2. **b)** 12710 **c)** It costs \$12 710 to produce 30 calculators.
d) $C(0) = 11\,750$. There are fixed costs of \$11750 before any calculators are produced.
e) 610 **f)** \$295 550

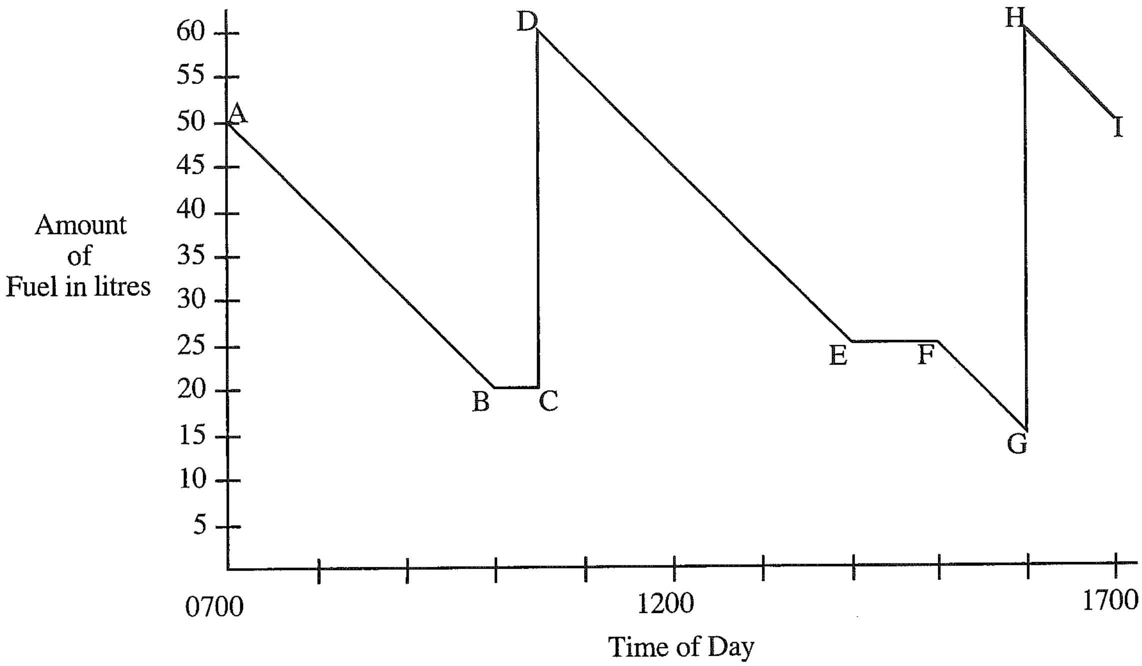
3. **b)** 200 **c)** 200 cups are sold when the average temperature is −5°C. **d)** −4°C
e) Estimate the minimum average daily temperature.
f) Answers may vary. domain $\{t \mid -20 \le t \le 15, t \in R\}$ range $\{N(t) \mid 0 \le N(t) \le 350, N(t) \in W\}$

4. **b)** both = 18 900
c) They are equal but do not represent the same thing. $h(30)$ is the height after 30 minutes. and $h(70)$ is the height after 70 minutes
d) i) 0 Initial height = 0 m ii) 0 After 100 min the balloon has landed on the ground.
iii) −9900 this has no meaning since the balloon has already landed
e) 22 500 cm = 225 m **f)** after 100 min
g) domain $\{t \mid 0 \le t \le 100, t \in R\}$ range $\{h(t) \mid 0 \le h(t) \le 22\,500, h(t) \in R\}$

Functions Lesson #5:
Interpreting Graphs of Functions



The Carter Family are driving to the Yukon for a family vacation. The graph represents the amount of fuel (in litres) in the gas tank of their car on the first day of their journey.



The graph of the journey is divided into eight line segments.

a) With reference to the journey, explain what is happening between:

i) A and B Driving at a constant rate for 3hours. ← based on constant rate of fuel consuption.

ii) B and C 30 min break ← based on no fuel consumption.

iii) C and D Refulling ← based on rapid spike in fuel levels.

b) What is the rate of fuel consumption (in litres per hour) between D and E?

fuel used = 60 - 25 = 35 L   rate = $\frac{35}{3.5}$ = 10 L/hour.
time = 3.5 hours.

c) Which line segment represents the car being refueled for the second time?

GH ← again it is a rapid spike. (vertical line.)

d) Calculate the total time when the car was driven.

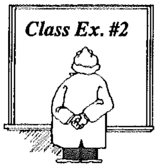
0700 → 1700 hours with a 30min and 1 hour break.

8.5 hours.

e) If fuel costs 85¢ per litre, calculate the cost of the fuel used for the first day of the journey.

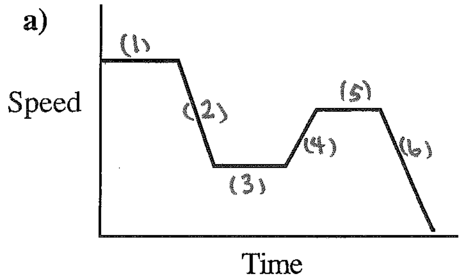
$$\text{fuel used: } \underbrace{(50-20)}_{AB} + \underbrace{(60-25)}_{DE} + \underbrace{(25-15)}_{FG} + \underbrace{(60-50)}_{HI} = 85\text{ L}$$

cost = 85 × 0.85 = \$72.25

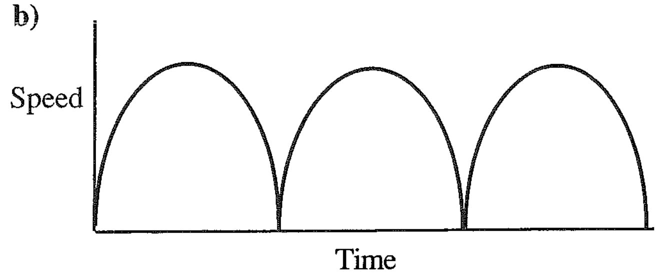


Suggest a possible scenario for each of the following graphs:

a)



b)



(1) Car travelling at a constant speed

(2) slows down for a construction zone

(3) steady speed through the construction zone

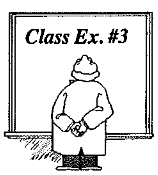
(4) speeds up leaving the construction zone

(5) steady speed (slower than (1))

(6) slows to a stop (lights)

A pendulum swinging back and forth.

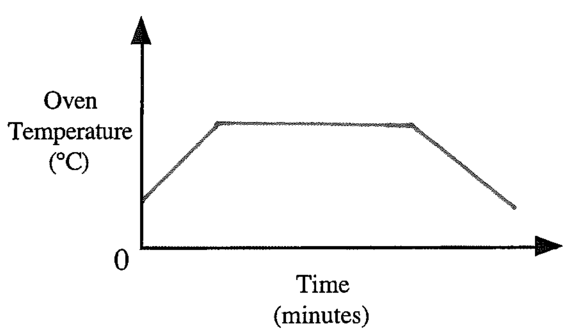
Sketching a Graph

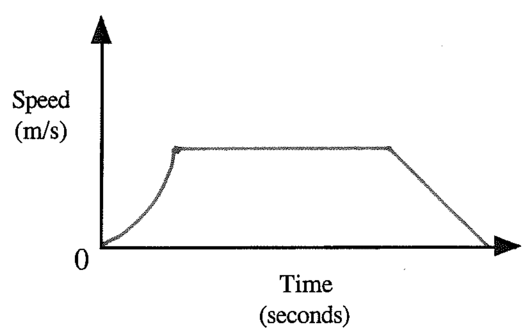


Sketch a graph with no scale for each of the following

a) the oven temperature when baking a pie

b) Ben taking part in a 100 m sprint

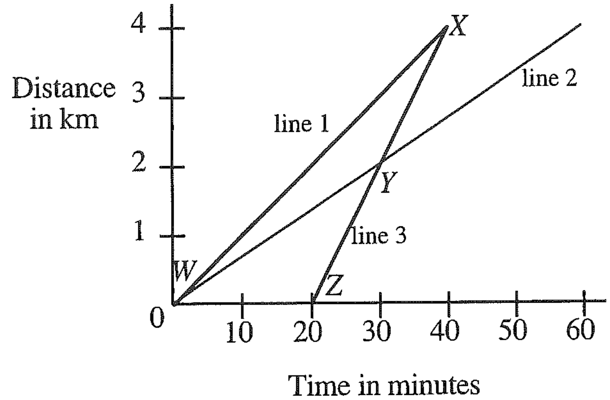




Complete Assignment Questions #1 - #12

Assignment

1. Amanda, Brittany, and Chelsea, each follow the same route to school. One morning Amanda cycles to school, Brittany walks to school, and Chelsea runs to school.
Lines 1, 2, and 3, on the graph represent the three routes.



a) Complete the table below.

	Line 1	Line 2	Line 3
Distance (km)	4	4	4
Time (hrs)	2/3	1	1/3
Rate (km/hr)	6	4	12
Student	C	B	A

b) Explain what is happening at the following points.

i) *W* Chelsea and Brittany leave home at the same time.

ii) *X* Chelsea and Amanda arrive at school

iii) *Y* Amanda overtakes Brittany

iv) *Z* Amanda leaves home 20 minutes after Brittany and chelsea.

c) How can you tell from the steepness of the lines which line represents the route of each student?

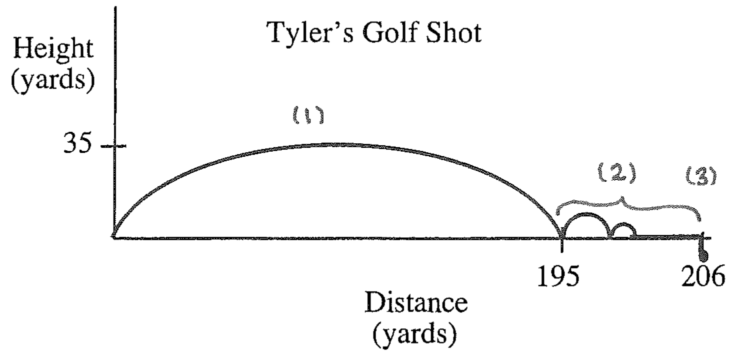
The steeper the line the less time is taken to travel to school.

The steepest slope represents the cyclist Amanda. (line 3)

The next steepest slope represents the runner Chelsea (line 2)

The remaining line represent the walker Brittany. (line 1)

2. Tyler, a member of St. Andrews High School golf team, hits a golf ball. The graph shows the path of the ball. Describe Tyler's golf shot.



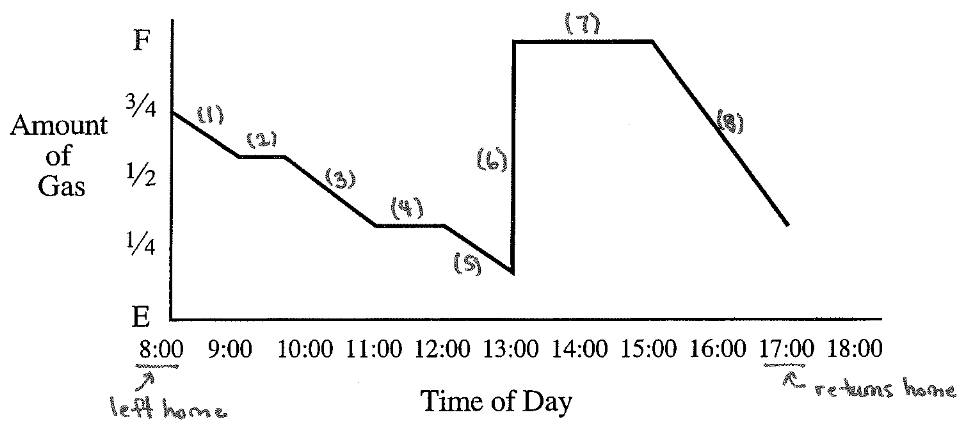
(1) Tyler hits the ball into the air for a distance of 195 yards.

(2) The ball bounces twice and roles to the hole.

(3) The ball drops into the hole.

The golf shot travels a total of 206 yards and has a maximum height of 35 yards.

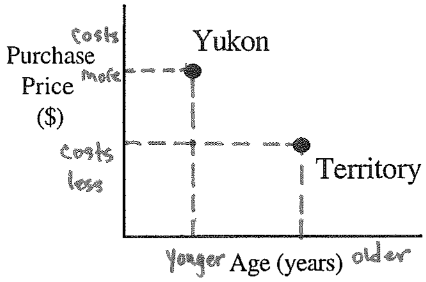
3. Dar sells medical supplies. The graph shows the amount of gasoline in his car during a particular day.

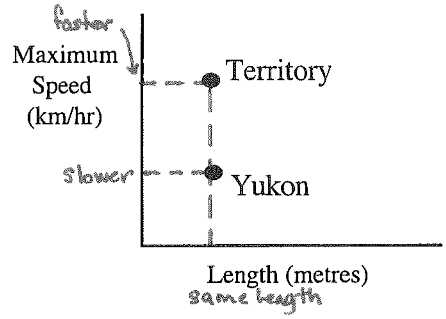


Describe how Dar may have spent the day.

(1) Dar left home at 8am. He started with $\frac{3}{4}$ tank of gas in his car. He drove for about 1 hour.
(2) Dar had a meeting for about $\frac{1}{2}$ hour.
(3) He drove for about $1\frac{1}{2}$ hours.
(4) Dar had a meeting for an hour (2nd meeting).
(5) He drove for a further hour.
(6) He refuelled at 1pm.
(7) Dar had a lunch meeting for about 2 hours.
(8) He drove home and arrived about 5pm with about $\frac{1}{4}$ tank of gas left in his car.

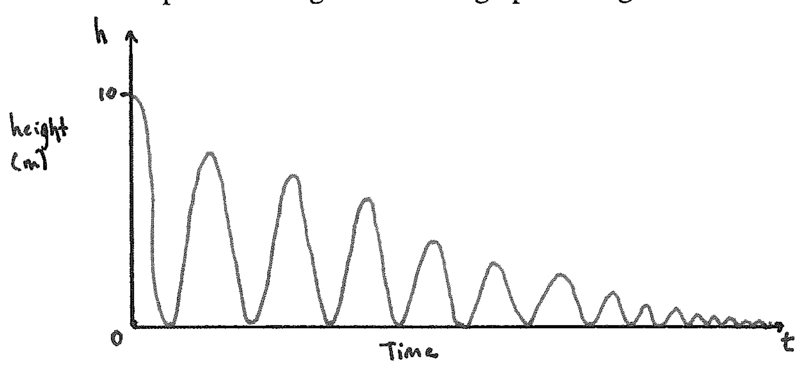
4. The two graphs shown compare two yachts: the Yukon and the Territory. The first graph compares the yachts by age and cost. The second graph compares the boats by speed and length. Describe the comparison between the two yachts.





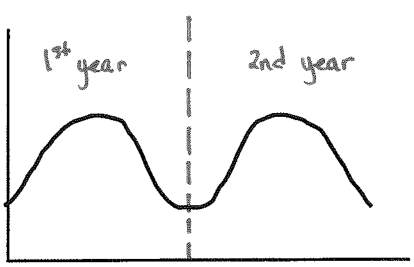
The Territory is older than the Yukon and its purchase price was less. Both the Territory and the Yukon are the same length, but the Territory can achieve a greater maximum speed.

5. A super ball is dropped from a 10 m building. On each bounce, it bounces back to 80% of its previous height. Create a graph of height as a function of time.



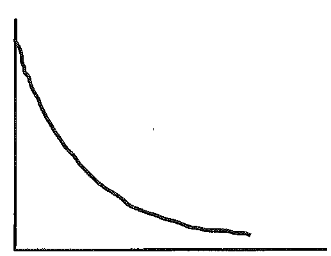
6. Suggest a possible scenario for each of the following graphs.

a)



The number of hours of day light per day over a period of two years for a location in the northern hemisphere.

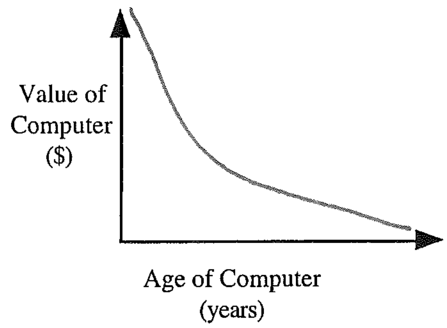
b)



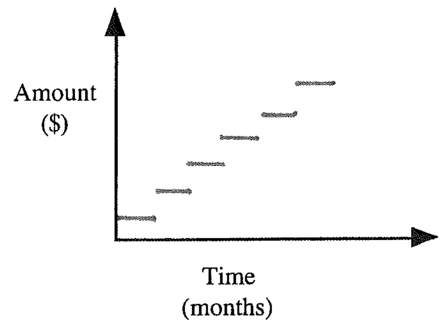
The value of a car depreciating over time.

7. Sketch a graph with no scale to represent each of the following.

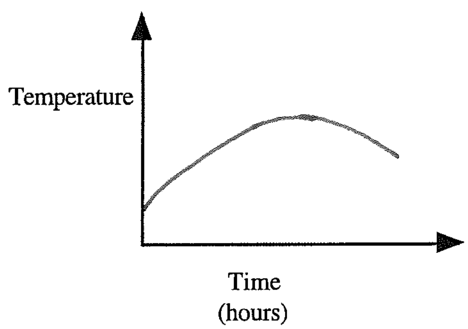
a) A computer's value compared to its age in years.



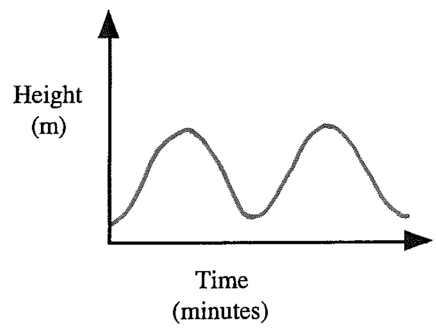
b) The amount of your savings if you save $10 every month for a period of six months.



c) The air temperature during a spring day from 6:00 a.m. to 6:00 p.m.



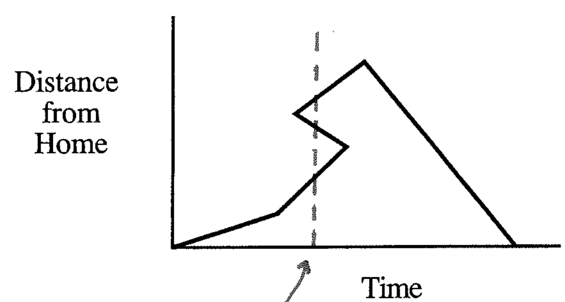
d) You are sitting in the bottom chair of a ferris wheel. Graph your height above the ground during two rotations of the wheel.



8. A student drew the following graph to represent a journey. Explain why the graph must be incorrect.

The graph is not a function.

The person cannot be at three different places at the same time.



Problem: If this maybe 3pm, how can the person be 3 different distances from home at the same time?

Matching Match each description on the left with the best graph on the right. Each graph may be used once, more than once, or not at all.

<u>Description</u>

9. Sketch a graph of a person's height as a function of their age.

E

10. The number of hours of daylight in a given town in northern BC depends on the day of the year. Sketch a graph of the number of hours of daylight as a function of day of the year.

D

11. Sketch a graph of the area of a circle as a function of its radius.

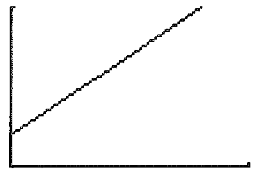
F

12. You start driving at a constant speed with a full tank of gas. Sketch a graph of litres of gas in the tank as a function of distance travelled.

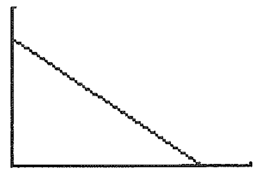
B

<u>Graph</u>

A.

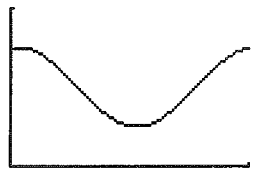


B.

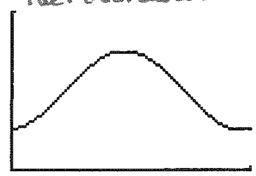


As distance is travelled fuel decreases.

C.

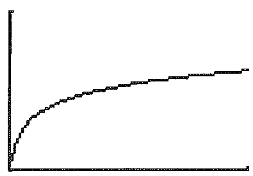


D.



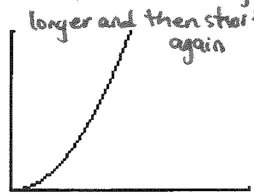
Moves from shorter days to longer and then shorter again

E.



A person grows rapidly as a small child and then slows as they are reaching adulthood.

F.



The area of a circle directly increases or decreases based on the length of a radias.

Answer Key (Answers may vary)

1. a) see table below

	Line 1	Line 2	Line 3
Distance (km)	4	4	4
Time (hrs)	$\frac{2}{3}$	1	$\frac{1}{3}$
Rate (km/hr)	6	4	12
Student	Chelsea	Brittany	Amanda

b) i) Chelsea and Brittany leave home at the same time.

ii) Chelsea and Amanda arrive at school.

iii) Amanda overtakes Brittany.

iv) Amanda leaves home 20 minutes after Brittany and Chelsea.

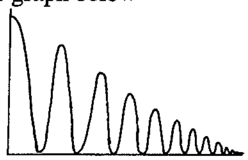
c) The steeper the graph, the less time is taken to travel to school. The steepest slope represents the cyclist Amanda, the next steepest slope represents the runner Chelsea, and the remaining line represents the walker Brittany.

2. Tyler hits the ball through the air for a distance of 195 yards. The ball bounces twice and rolls into the hole. The golf shot travelled a total of 206 yards, and had a maximum height of 35 yards.

3. Dar left home at 8:00 AM with $\frac{3}{4}$ tank of gas in his car. He drove for about one hour, had a meeting for about $\frac{1}{2}$ hour, drove for about $1\frac{1}{2}$ hours, had a second meeting for one hour, and drove for about one hour. He refueled at 1 pm and had a lunch meeting for about 2 hours. He then drove home and arrived about 5 p.m. with a quarter tank of gas left.

4. The Territory is older than the Yukon, and its purchase price was less. Both the Territory and the Yukon are the same length, but the Territory can achieve a greater maximum speed.

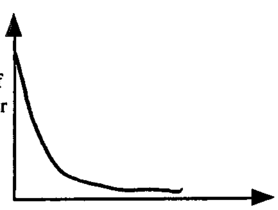
5. see graph below



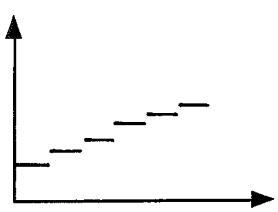
6. a) The number of hours of daylight per day over a period of two years for a location in the northern hemisphere.

b) The value of a car depreciating over time.

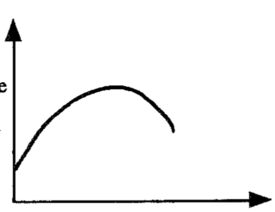
7. a)



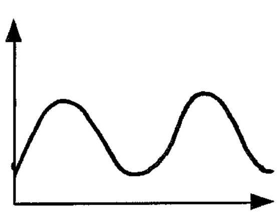
b)



c)



d)



8. The graph is not a function. The person cannot be at three different places at the same time.

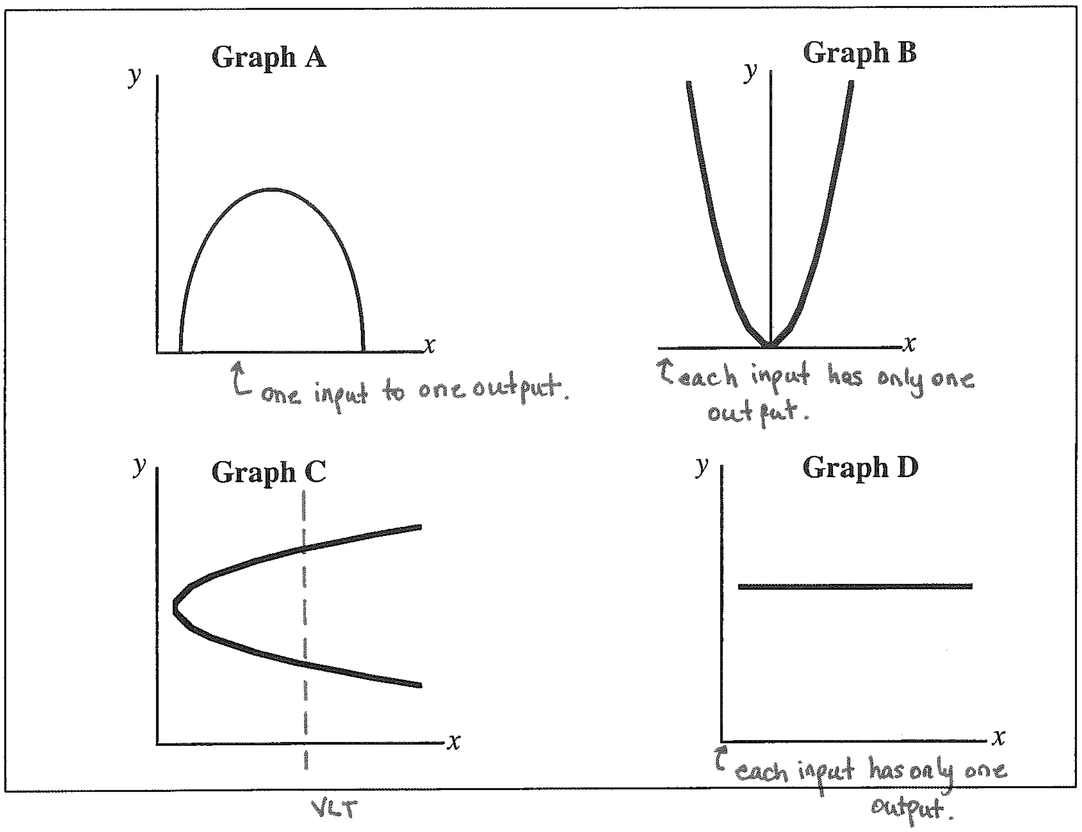
9. E **10.** D **11.** F **12.** B

Functions Lesson #6:
Practice Test

1. Which of the following cannot be used to represent a function?

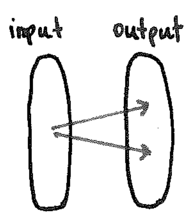
   A. Graph
   B. Table of Values
   C. Ordered Pairs
   D. Coordinate

Use the following information to answer the next question.



2. Which of the graphs show a relation which is not a function?

   A. Graph A ✓
   B. Graph B ✓
   C. Graph C ← Problem: Does not pass the vertical line test (VLT). It has two outputs for one input.
   D. Graph D ✓



Numerical Response **1.** The function $f(x) = 2 + x^2$ has domain $\{-3, -2, -1, 0, 1, 2\}$.

The difference between the largest element of the range and the smallest element of the range is ______ .

(Record your answer in the numerical response box from left to right) | 9 | | | |

STEP 1: Solve for all outputs in order to analyze range.

STEP 2: range: $\{2, 3, 6, 11\}$

$f(-3) = 2 + (-3)^2 = 2 + 9 = 11$

$f(-2) = 2 + (-2)^2 = 2 + 4 = 6$

$f(-1) = 2 + (-1)^2 = 2 + 1 = 3$

$f(0) = 2 + (0)^2 = 2$

$f(1) = 2 + (1)^2 = 3$

$f(2) = 2 + (2)^2 = 6$

STEP 3: largest element − smallest element.
$= 11 - 2$
$= \underline{\underline{9}}$

Use the following information to answer the next question.

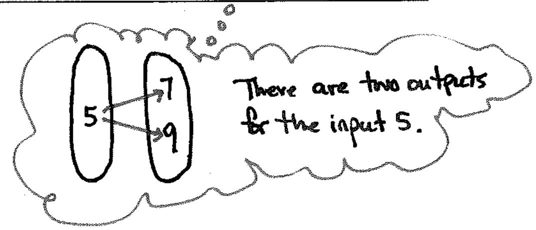
Four relations are represented by the sets of ordered pairs shown.

I. $\{(1,1), (2,2), (3,3)\}$ ✓ **II.** $\{(-1,0), (0,-1)\}$ ✓

III. $\{(2,6), (3,-8), (-2,6), (0,0)\}$ ✓ **IV.** $\{(3,4), (5,7), (5,9), (7,10)\}$ ✗

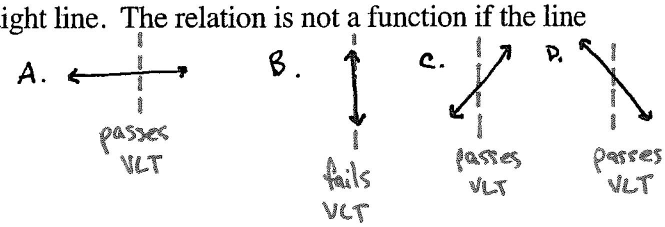
3. The relations which are functions are

A. I only
B. I and II only
C. I, II, III only
D. some other combination of I, II, III, IV



4. The graph of a relation results in a straight line. The relation is not a function if the line

A. is horizontal ✓
B. is vertical ✗
C. increases from left to right ✓
D. decreases from left to right ✓



5. The function $f(x) = 3 - 2x^2$ has domain $\{-6, -4, 0, 2, 5\}$.
Which of the following is an element of the range of the function?

A. 35
B. 27
C. 1
D. −29

$f(-6) = 3 - 2(-6)^2 = -69$

$f(-4) = 3 - 2(-4)^2 = -29$

$f(0) = 3 - 2(0)^2 = 3$

$f(2) = 3 - 2(2)^2 = -5$

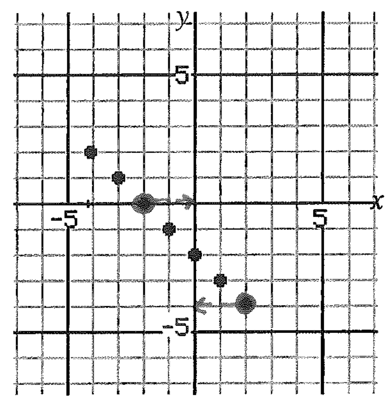
$f(5) = 3 - 2(5)^2 = -47$

↑ input ↓ output

Recall:
- each input relates to domain
- each output relates to range.

Use the following information to answer the next question.

The diagram shows the graph of the function $y = f(x)$.



6. The value of $f(-2) + f(2)$ is

input/ x-coor. input/ x-coor.

$f(-2) + f(2) = 0 + (-4) = -4$

A. -4
B. -2
C. 0
D. $f(0)$

Use the following information to answer questions #7 and #8.

Consider the function $P(x) = 5x + 2$.

7. The value of $P(6)$ is

input

$P(6) = 5(6) + 2$
$= 30 + 2$
$= 32$

A. $\frac{8}{5}$
B. $\frac{4}{5}$
C. 32
D. 58

8. If $P(b) = 6$, then $b =$

input output input.

$(6) = 5b + 2$
$-2 \qquad -2$

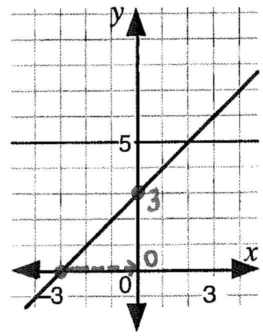
$\frac{5b}{5} = \frac{4}{5}$

$b = \frac{4}{5}$

A. $\frac{8}{5}$
B. $\frac{4}{5}$
C. $\frac{5}{4}$
D. 32

Use the following information to answer the next two questions.

The graph of a function $y = f(x)$ is shown.



Numerical Response **2.** The value of $f(-3) + f(0)$ is ______ .

0 + 3 = 3

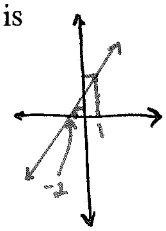
(Record your answer in the numerical response box from left to right)

3			

9. If $f(a) = 1$ and $f(b) = 4$, then the value of $b - 2a$ is

A. -3
B. -1
C. 2
D. 5

$b - 2a$
$= (1) - 2(-2)$
$= 1 + 4$
$= 5$



Side Work.

$f(a) = 1$, $a = -2$

$f(b) = 4$, $b = 1$

10. Consider the graph of the function $f(x) = 4x - 10$. The x-intercept of the graph of f is

A. -10
B. -2.5
C. 2.5
D. 10

x_{int}: Let $f(x) = 0$

Remember: $f(x)$ is y so treat in same manner

$(0) = 4x - 10$
$+10 \quad +10$

$\frac{4x}{4} = \frac{10}{4}$

$x = 2.5$

Numerical Response **3.** Consider the graph of the function $P(x) = 2x^2 - 16$.
The x-intercepts of the graph of P are located at $(m, 0)$ and $(-m, 0)$.
The value of m, to the nearest tenth, is ______ .

(Record your answer in the numerical response box from left to right)

2	.	8	

x_{int} = Let $P(x) = 0$

$P(x) = 2x^2 - 16$
$(0) = 2x^2 - 16$
$+16 \quad +16$

$\frac{16}{2} = \frac{2x^2}{2}$

$x^2 = 8$

$x = \sqrt{8} = 2.828...$

Use the following information to answer the next question.

Consider the following functions.

1. $p(x) = x^2 - 4x - 2$	**2.** $p(x) = \frac{1}{3}x + 14$
3. $p(x) = 3x^2 + x$	**4.** $p(x) = 7 - 5x$

11. For each function evaluate $p(-3)$, and put the expressions in order from greatest to least. The order is

- **A.** 4312
- **B.** 3412
- **C.** 3124
- **D.** none of the above

1. $p(\ ) = (\ )^2 - 4(\ ) - 2 = 9 + 12 - 2 = 19$

2. $p(\ ) = \frac{1}{3}(\ ) + 14 = -1 + 14 = 13$

3. $p(\ ) = 3(\ )^2 + (\ ) = 27 - 3 = 24$

4. $p(\ ) = 7 - 5(\ ) = 7 + 15 = 22$

greatest ⟶ least.

24	22	19	13
↓	↓	↓	↓
3	4	1	2

12. Given a function g defined by $g(x) = px + q$ with $g(0) = 2$ and $g(1) = 3$ then

- **A.** $p = 1,\ q = 0$
- **B.** $p = 3,\ q = 2$
- **C.** $p = 1,\ q = 2$
- **D.** $p = 3,\ q = 0$

Step 1: Solve for q using $g(0) = 2$

$g(0) = p(0) + q = 2$

$= 0 + q = 2$

$\boxed{q = 2}$

Step 2: Solve for p using $g(1) = 3$ and $q = 2$.

$g(1) = p(1) + q = 3$

$p + q = 3$

Let $q = 2$

$p + (2) = 3$
$\quad -2 \quad -2$

$\boxed{p = 1}$

13. If $f(x) = 3^x$ and $f(-a) = \frac{1}{81}$, then $a =$

- **A.** 4
- **B.** −4
- **C.** −27
- **D.** 27

Recall: Negative exponent law.

$3^{(-a)} = \frac{1}{81}$

$\frac{1}{3^a} = \frac{1}{81}$

$3^a = 81$

$3^a = 3^4$

$\boxed{a = 4}$

Cross multiply to reduce denominators to 1.

14. If $g(x) = \frac{2}{3}x + 6$, an expression for $g(2x-1)$ is

A. $\frac{4}{3}x + 5$

B. $\frac{4}{3}x + \frac{16}{3}$

C. $\frac{4}{3}x + 11$

D $\frac{8}{3}x + 5$

$g(2x-1) = \frac{2}{3}(2x-1) + 6$

$= \frac{4}{3}x - \frac{2}{3} + 6$ ... $\frac{-2}{3} + 6 = \frac{-2}{3} + \frac{18}{3} = \frac{16}{3}$

$= \frac{4}{3}x + \frac{16}{3}$

Numerical Response **4.** If $f(x) = 1 - 2x - 5x^2$, and if $f(x+2)$ is written in the form $ax^2 + bx + c$, the value of $a - b - c$ is ______ .

(Record your answer in the numerical response box from left to right)

4	0		

Side Work.

	x	2
x	x^2	2x
2	2x	4

$f(x+2) = 1 - 2(x+2) - 5(x+2)^2$

$= 1 - 2x + 4 - 5(x^2 + 4x + 4)$

$= 1 - 2x - 4 - 5x^2 - 20x - 20$

$= -5x^2 - 22x - 23$

Numerical Response **5.** $f(a) = \frac{a}{a+4}$. The exact value of $f(5) - f(5^{-1})$ written as a rational number in simplest form is $\frac{p}{q}$. The value of p is ______ .

(Record your answer in the numerical response box from left to right)

3	2		

$f(5) = \frac{(5)}{(5)+4}$

$= \boxed{\frac{5}{9}}$

$f(5^{-1}) = \frac{(5^{-1})}{(5^{-1})+4}$

$= \frac{\frac{1}{5}}{\frac{1}{5} + \frac{20}{5}}$

$= \boxed{\frac{1}{21}}$ ... $\frac{\frac{1}{5}}{\frac{21}{5}} = \frac{1}{\cancel{5}} \times \frac{\cancel{5}}{21} = \frac{1}{21}$

$f(5) - f(5^{-1})$

$= \frac{5}{9} - \frac{1}{21}$

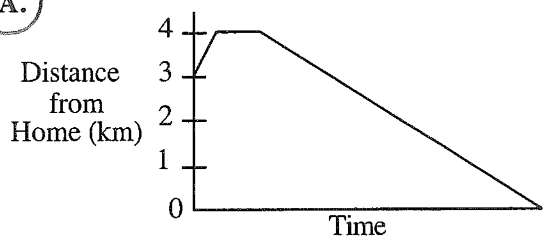
$= \frac{32}{63}$ ← p

Use the following information to answer the next question.

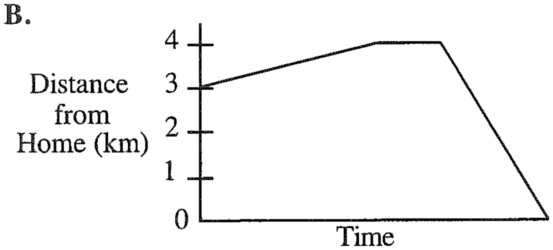
Melanie leaves school and runs to the shop. She spends some time in the shop and walks home.

15. Which graph best describes Melanie's distance from home starting from when she left school?

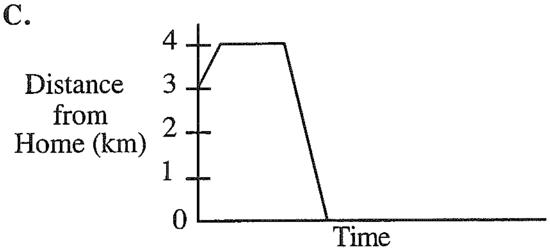
A.



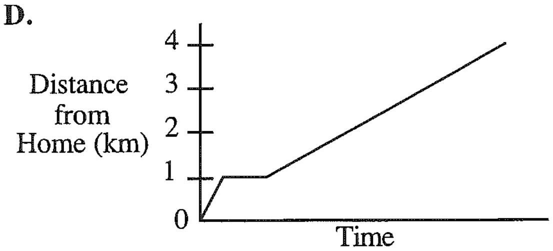
B.



C.



D.

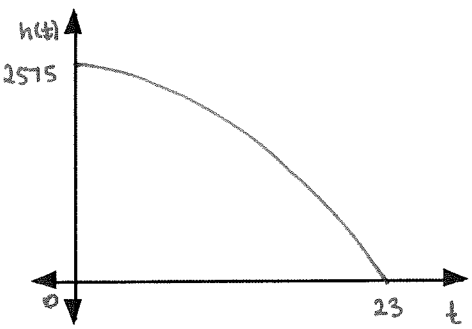


Written Response - 5 marks

1. As part of an experiment, a 50 kg steel ball is dropped from a Canadian Air Force jet.

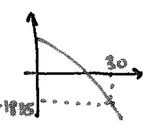
The height of the steel ball above the ground can be described by part of the graph of the function $h(t) = 2575 - 4.9t^2$, where $h(t)$ is the height, in metres, of the steel ball after t seconds.

- Sketch the graph of the function on the grid.



- Determine the value of $h(30)$ and explain why $h(30)$ does not represent the height of the ball after 30 seconds?

$h(30) = 2575 - 4.9(30)^2$
$= 2575 - 4410$
$= -1835$



The ball has already hit the ground, so the function no longer represents the height of the ball.

- At what height is the ball dropped from the jet?

$h(t)_{int}$: $t = 0 \leftarrow$ initial time.

$h(0) = 2575 - 4.9(0)^2 = 2575 - 0 = 2575.$ $\underline{\underline{2575m}}$

- How long (to the nearest second) will it take the ball to make contact with the ground?

t_{int}: $h(t) = 0 \leftarrow$ ground level.

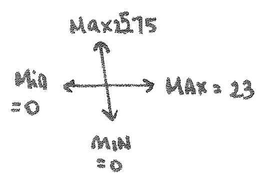
$(0) = 2575 - 4.9t^2$
$\frac{4.9t^2}{4.9} = \frac{2575}{4.9}$ $\rightarrow$ $t^2 = 525.510...$
$t = \sqrt{525.510...}$
$t = 22.924...$

$\underline{\underline{23 \text{ seconds}}}$

- Suggest an appropriate domain and range for the function $h(t)$.

domain: $\{t \mid 0 \le t \le 23, t \in R\}$

range: $\{h(t) \mid 0 \le h(t) \le 2575, h(t) \in R\}$



Answer Key

1. D	2. C	3. C	4. B	5. D	6. A	7. C	8. B
9. D	10. D	11. B	12. C	13. A	14. B	15. A	

Numerical Response

1. 9
2. 3
3. 2.8
4. 40
5. 32

Written Response

1. • –1835. The ball has already hit the ground, so the function no longer represents the height of the ball.
   • 2575 m
   • 23 seconds
   • domain $\{t \mid 0 \le t \le 23, t \in R\}$ range $\{h(t) \mid 0 \le h(t) \le 2575, h(t) \in R\}$