Lesson 3: Arithmetic Growth and Decay

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Arithmetic Sequences Lesson #3:
Arithmetic Growth and Decay

pg. 527

Generating Number Patterns Exhibiting Arithmetic Growth

d is positive (increasing in value)

Many real-life scenarios can be represented by a pattern of numbers which exhibit arithmetic growth.

Class Ex. #1

BP Birthplace Forest program in Calgary enables parents to honour their children by planting a tree when their child is born. At the same time, this shows concern for the urban environment by encouraging the growth of city forests. Some of the trees which have been planted are evergreen trees which grow an average of 12 to 18 inches per year. The program was launched in the year 2000.

$t_n = a + (n-1)d$

a) An evergreen tree, 6 inches tall was planted in June 2007 and has a growth rate of 15 inches per year. Two students were asked to determine the height of the tree in June 2018. Joel formed an arithmetic sequence beginning 6, 21, 36, ... and Jenna formed an arithmetic sequence beginning 21, 36, 51 ... . Use the formulas in the previous lesson to determine each student's answer to the problem.

6, 21, 36...
2007 2008 2009

2018 → 12th term

Joel: $a = 6$, $d = 15$

$t_{12} = 6 + (12-1)(15)$
$= 171$ inches

21, 36, 51...
2008 2009 2010

2018 → 11th term

Jenna: $a = 21$, $d = 15$

$t_{11} = 21 + (11-1)(15)$
$= 171$ inches

b) Determine the formula for the general term of Joel's arithmetic sequence.

$t_n = 6 + (n-1)(15)$
$t_n = 6 + 15n - 15$
$t_n = 15n - 9$

assumes 2007 is term 1

c) Determine the formula for the general term of Jenna's arithmetic sequence.

$t_n = 21 + (n-1)(15)$
$t_n = 21 + 15n - 15$
$t_n = 15n + 6$

assumes 2008 is term 1

d) Explain why the formulas in **b)** and **c)** are different.

Joel started his sequence in 2007, Jenna started hers in 2008

e) If the tree continues to grow at the same rate, in which year will it reach a height of 28 ft?

Convert to inches
$28 \times 12 = 336$ in

$t_n = 15n + 6$
$336 = 15n + 6$
$330 = 15n$
$n = 22$ years after 2007
∴ in 2029, it will reach a height of 28 ft.

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Generating Number Patterns Exhibiting Arithmetic Decay

d is negative (values decrease)

Many real-life scenarios can be represented by a pattern of numbers which exhibit arithmetic decay or arithmetic depreciation.

A printing press was bought in the year 1999. It depreciates (goes down) in value by the same amount each year. Five years after its purchase, the printing press had a value of \$311 000. It had a scrap value of \$2900 in the year 2017.

term 1 = in 2000

a) Use an arithmetic sequence to determine the annual depreciation. d

$t_5 = 311\,000$ $\qquad$ $t_{18} = 2900$

$t_5 = a + (5-1)d$ $\qquad$ $t_{18} = a + (18-1)d$

$311000 = a + 4d$ $\qquad$ $2900 = a + 17d$

$$\begin{array}{rl} & 311000 = a + 4d \\ - & (2900 = a + 17d) \\ \hline & 308100 = -13d \\ & d = -23700 \end{array}$$

The printing press decreases in value by \$23700 each year

b) Determine the purchase price of the printing press.

↳ find a first

$t_5 = 311000$

$311000 = a + 4d$

$311000 = a + 4(-23700)$

$311000 = a - 94800$

$a = 405800$

↖ 1 year after 1999

in 1999 $= 405800 + 23700$

$= \$429\,500$

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Relating Arithmetic Sequences to Linear Functions

We can relate arithmetic sequences to linear functions over the natural numbers. 1, 2, 3, 4... (no fractions)
Consider the following example:

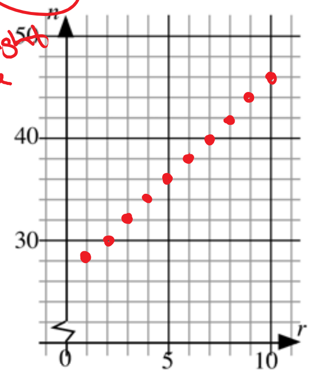
A pile of bricks is arranged in rows. There are 28 bricks in the first row, and the number of bricks in each successive row is two more than in the previous row.

a) Complete the table of values showing the number of bricks in each of the first 10 rows.

Row Number, r	Number of Bricks, n
1	28
2	30
3	32
4	34
5	36
6	38
7	40
8	42
9	44
10	46

b) Plot the ordered pairs on the grid. and classify the relationship as linear or non-linear.

straight line



do not connect the dots – data is discrete (no in-between values)

c) Determine if the range is an arithmetic sequence.

range : 28, 30, 32, ... , 44, 46

arithmetic – each value increases by a common difference of 2

d) State the domain of the relationship.

domain : 1, 2, 3, ... , 9, 10

e) Explain why the graph does not have an intercept on the vertical axis?

no such thing as row 0

f) Write the equation for the number of bricks in a row, n, as a function of the row number, r.

$t_n = a + (n-1)d$

$t_n = 28 + (n-1)(2)$

$t_n = 28 + 2n - 2$

$t_n = 2n + 26$

\# of bricks → $t_n = 2n + 26$ ← row #

$\boxed{n = 2r + 26}$

Complete Assignment Questions #1 - #8

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