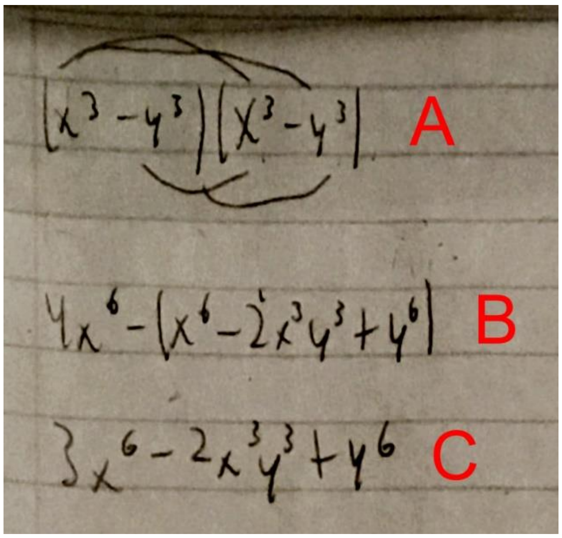
“Factor the expressions below if possible.”

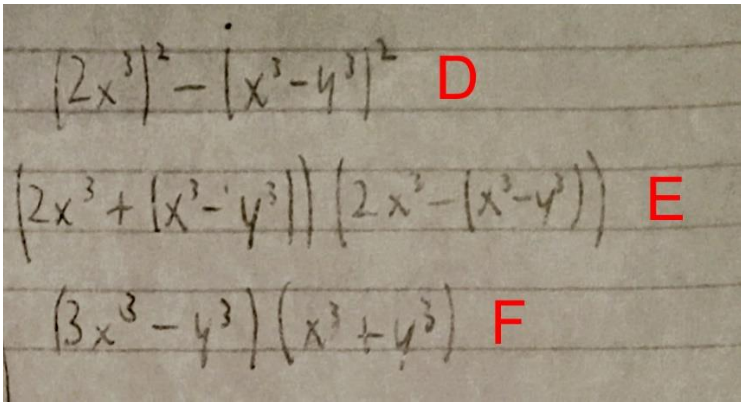
i) $4x^6-(x^3-y^3)^2$

Below is my initial, incorrect method for solving the problem. Here, I went against the goal of factoring and simplified the polynomial instead. The proper method involves difference of squares factoring, which I will go over after covering my original solution.



For part A, I expanded the second binomial term, $(x^3-y^3)^2$, using FOIL, which is a multiplication method for binomial polynomials which stands for Front, Outer, Inner, Last. Some may call it Front, Outside, Inside, Last, instead. This refers to the order to multiply the individual numbers and variables; Front multiplies the first number of each term, Outer multiplies the leftmost and rightmost numbers, Inner multiplies the two center numbers, and Last multiplies the last number of each term.

Part B is the end result of FOILing $(x^3-y^3)^2$, after combining like terms and inserting it back into the greater expression, and part C is the result of fully simplifying the expression, incorrectly, as no part of the second binomial term beyond x^6 had its value properly flipped by the subtraction sign. This is primarily a result of my lack of attention.



Above is the correct solution. Part D involves rewriting $4x^6$ to enable the use of the difference of squares method of factoring, which can be demonstrated by x^2-y^2. Because of the subtraction sign and the fact that both terms are perfect squares, it can be factored into (x-y)(x+y), which, if FOILed, yields x^2-y^2. With the expression here, the difference of squares is much more evident through substitution, by replacing $2x^3$ with A and x^3+y^3 with B, resulting in A^2-B^2.

Part E is simply the result of factoring by difference of squares, which, if we kept substituting, would look like (A+B)(A-B), whereas part F is the result of combining like terms without multiplying. The second binomial term can technically be factored even further using a technique called the sum of cubes, and while this was not taught to me, it deserves a mention.

x^3+y^3 would be factored into $(x^3-y^3)(x^2-xy+y^2)$, making the final result $(3x^3-y^3)(x^3-y^3)(x^2-xy+y^2)$.