TRIGONOMETRIC EQUATIONS & IDENTITIES UNIT REVIEW

Name: _____

Block:_____

Date:_____

Total _____ = _____%

95

All answers rounded to 2 decimal places unless otherwise stated.
Show all work where possible.

1. Within the domain $0 \leq x < 2\pi$, how many solutions are there for the following equations.

   a) $\cos 5x = 0.78$

   a) _____
   1 mark

   b) $\sin 3x = \frac{1}{\sqrt{2}}$

   b) _____
   1 mark

   c) $\tan 4x = 12$

   c) _____
   1 mark

   d) $\tan bx = 5$

   d) _____
   1 mark

2. For the equation $\cos 2x = -0.32$

   a) solve for x, where $0 \leq x < 2\pi$

   a) _____
   2 marks

   b) determine the general solution.

   b) _____
   2 marks

3. For the equation $3\sin x = x$, solve for x and determine all solutions.

3) _____
2 marks

4. For the equation $\cos^2 x + 4\cos x - 2 = 0$

a) solve for x, where $0^\circ \leq x < 360^\circ$

a) _____
2 marks

b) determine the general solution.

b) _____
2 marks

5. Solve the following equations **<u>algebraically</u>**, where $0° \leq x < 360°$.
Give answers as exact solutions.

a) $\sqrt{3}\tan x = -1$

a) _____
2 marks

b) $\sec x = -2$

b) _____

2 marks

c) $\cos x = \sqrt{3} - \cos x$

c) _____

2 marks

d) $3\tan^2 x = 1$

d) _____

3 marks

6. Solve the following equations **algebraically**, where $0 \leq x < 2\pi$.
Give answers as exact solutions.

a) $2\sin^2 x + \sin x = 0$

a) _____

3 marks

b) $2\cos^2 x + 3\cos x + 1 = 0$

b) _____

3 marks

c) $\sqrt{2}\cos^2 x - \cos x = 0$

c) _____

3 marks

d) $\sin 2x = -\frac{1}{2}$

d) _____

3 marks

e) $\cos\frac{1}{2}x = -\frac{\sqrt{3}}{2}$

e) _____

3 marks

7. Simplify using the sum and difference identities.

a) $\cos\left(\frac{3\pi}{2} - x\right)$

a) _____

2 marks

b) If $\tan x$ is -1, simplify $\tan\left(\frac{\pi}{3} + x\right)$. Answer exact value and rationalize denominator.

b) _____

3 marks

c) $\sin\left(\frac{\pi}{3} - x\right) - \cos\left(\frac{\pi}{6} + x\right)$

c) _____

3 marks

d) $\cos 3x \cos 5x - \sin 3x \sin 5x$

d) _____
2 marks

e) $\sin 6.7 \cos 2.3 + \cos 6.7 \sin 2.3$

e) _____
2 marks

f) Find the exact value of $\sin\left(\frac{17\pi}{12}\right)$ Hint: use sum identity

f) _____
3 marks

g) Find the exact value of $\cos(A - B)$ given:

$\tan A = \frac{2}{5}$ if angle A is in quadrant 1 and $\cos B = -\frac{2}{3}$ if angle B is in quadrant 3.

g) _____

3 marks

8. Write the following as a single trigonometric function.

a) $\cos 24 \sin 24$

a) _____

2 marks

b) $1 - 2\sin^2 \frac{\pi}{6}$

b) _____

2 marks

c) $\frac{1}{2}\sin 12\theta \cos 12\theta$

c) _____
2 marks

d) $\frac{2\tan 4\theta}{5-5\tan^2 4\theta}$

d) _____
2 marks

9. If $\cos\theta = -\frac{1}{3}$ and θ is in Quadrant III. Evaluate $\sin 2\theta$.

9. _____
3 marks

10. Determine all restrictions for $\frac{\csc\theta}{1 - \cos\theta}$.

10. _____

2 marks

11. Prove the following identities.

a) $\cot\theta \sec\theta \sin\theta = 1$

3 marks

b) $\cot\theta + \tan\theta = \csc\theta \sec\theta$

3 marks

c) $\dfrac{\sin\theta + \tan\theta}{\cos\theta + 1} = \tan\theta$

3 marks

d) $\cos^2\theta = \dfrac{\cot^2\theta}{1 + \cot^2\theta}$

3 marks

e) $\dfrac{1 + \tan\theta}{1 + \cot\theta} = \dfrac{1 - \tan\theta}{\cot\theta - 1}$

3 marks

f) $\dfrac{\tan\theta}{\cos\theta - \sec\theta} = -\csc\theta$

3 marks

g) $\dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{1 - \cos\theta}{\sin\theta}$

3 marks

h) $\dfrac{\sin 2\theta}{1 - \cos 2\theta} = \cot\theta$

3 marks

A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Reciprocal and Quotient Identities:

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Addition Identities:

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double Angle Identities:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$