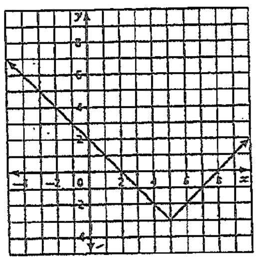
Chapter 1 Review

1.1 Horizontal and Vertical Translations, pages 1–8

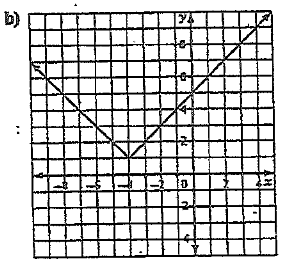
1. Write an equation to represent each translation of the function $y = |x|$.

a)



Equation: ______________

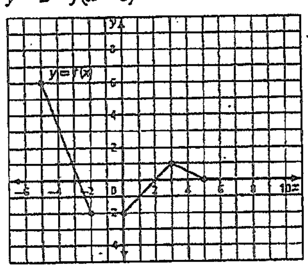
b)



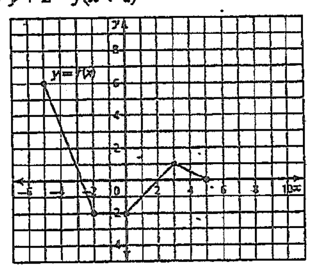
Equation: ______________

2. For $y = f(x)$ as shown, graph the following.

a) $y - 2 = f(x - 3)$



b) $y + 2 = f(x + 1)$



1.2 Reflections and Stretches, pages 9–17

3. The key point $(12, -5)$ is on the graph of $y = f(x)$. Determine the coordinates of its image point under each transformation.

a) $y = -f(x)$

$(x, y) \rightarrow$

$(12, -5) \rightarrow$

b) $y = f(-4x)$

$(x, y) \rightarrow$

$(12, -5) \rightarrow$

c) $y = 2f\left(\frac{1}{3}x\right)$

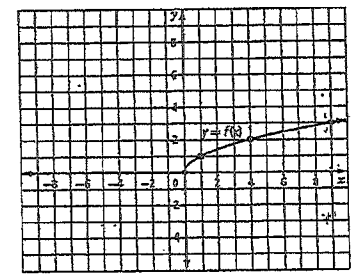
$(x, y) \rightarrow$

$(12, -5) \rightarrow$

4. Describe the following transformations of $y = f(x)$ and sketch a graph of each transformation.

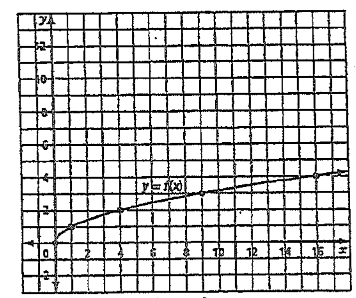
a) $y = -f(-x)$

Description:



b) $y = 3f(2x)$

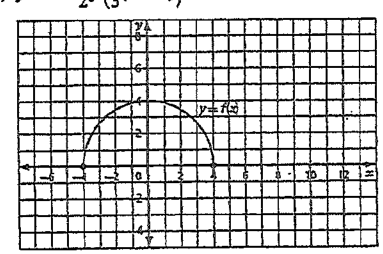
Description:



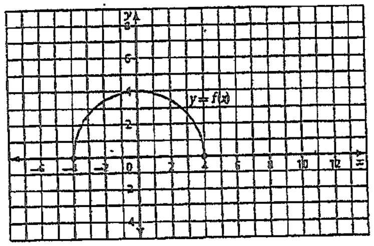
1.3 Combining Transformations, pages 18–25

5. The graph of the function $y = f(x)$ is given. Graph each of the following transformations of the function. Show each stage of the transformation in a different colour.

a) $y - 5 = \frac{1}{2}f\left(\frac{2}{3}(x - 6)\right)$



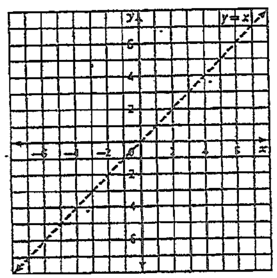
b) $y = -f(4x + 12)$



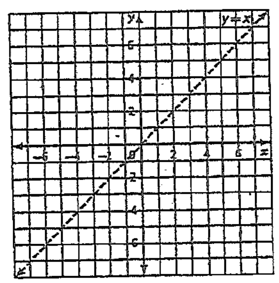
1.4 Inverse of a Relation, pages 26–34

6. Determine algebraically the inverse of each function. If necessary, restrict the domain so that the inverse of $f(x)$ is also a function. Verify by sketching the graph of the function and its inverse.

a) $f(x) = -\frac{1}{2}x + 5$



b) $f(x) = 2(x - 1)^2$



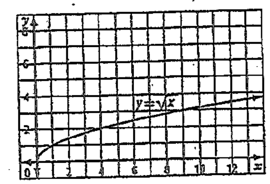
Chapter 2 Radical Functions

2.1 Radical Functions and Transformations

KEY IDEAS

Base Radical Function

- The base radical function $y = \sqrt{x}$ has the following graph and properties:
  - x-intercept of 0
  - y-intercept of 0
  - domain: $\{x \mid x \geq 0, x \in R\}$
  - range: $\{y \mid y \geq 0, y \in R\}$
  - The intercepts and domain and range suggest an endpoint at (0, 0), and no right endpoint.
- The graph is shaped like half of a parabola. The domain and range indicate that the half parabola is in the first quadrant.



Transforming Radical Functions

The base radical function $y = \sqrt{x}$ is transformed by changing the values of the parameters a, b, h, and k in the equation $y = a\sqrt{b(x - h)} + k$. The parameters have the following effects on the base function:

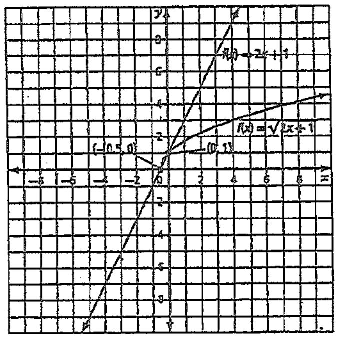
a	• vertical stretch by a factor of $\lvert a \rvert$ • if a is $a < 0$, the graph of $y = \sqrt{x}$ is reflected in the x-axis
b	• horizontal stretch by a factor of $\frac{1}{\lvert b \rvert}$ • if b is $b < 0$, the graph of $y = \sqrt{x}$ is reflected in the y-axis
h	• horizontal translation • $(x - h)$ means the graph of $y = \sqrt{x}$ moves h units right. For example, $y = \sqrt{x - 1}$ means that the graph of $y = \sqrt{x}$ moves 1 unit right. • $(x + h)$ means the graph of $y = \sqrt{x}$ moves h units left. For example, $y = \sqrt{x + 5}$ means that the graph of $y = \sqrt{x}$ moves 5 units left. This translation has the opposite effect than many people think. It is a common error to think that the + sign moves the graph to the right and the − sign moves the graph to the left. This is not the case.
k	• vertical translation • $+k$ means the graph of $y = \sqrt{x}$ moves k units up • $-k$ means the graph of $y = \sqrt{x}$ moves k units down

2.2 Square Root of a Function

KEY IDEAS

Graphing $y = f(x)$ and $y = \sqrt{f(x)}$

- To graph $y = \sqrt{f(x)}$, you can set up a table of values for the graph of $y = f(x)$. Then, take the square root of the elements in the range, while keeping the elements in the domain the same.
- When graphing $y = \sqrt{f(x)}$, pay special attention to the invariant points, which are points that are the same for $y = f(x)$ as they are for $y = \sqrt{f(x)}$. The invariant points are $(x, 0)$ and $(x, 1)$ because when $f(x) = 0$, $\sqrt{f(x)} = 0$, and when $f(x) = 1$, $\sqrt{f(x)} = 1$.



Domain and Range of $y = \sqrt{f(x)}$

- You cannot take the square root of a negative number, so the domain of $y = \sqrt{f(x)}$ is any value for which $f(x) \geq 0$.
- The range is the square root of any value in $y = f(x)$ for which $y = \sqrt{f(x)}$ is defined.

The Graph of $y = \sqrt{f(x)}$

$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
$y = \sqrt{f(x)}$ is *undefined* because you cannot take the square root of a negative number.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ *intersect* at $x = 0$.	The graph of $y = \sqrt{f(x)}$ is *above* the graph of $y = f(x)$.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ *intersect* at $x = 1$.	The graph of $y = \sqrt{f(x)}$ is *below* the graph of $y = f(x)$.

2.3 Solving Radical Equations Graphically

KEY IDEAS

Strategy for Solving Algebraically

Step 1: List any restrictions for the variable. You cannot take the square root of a negative number, so the value of the variable must be such that any operations under the radical sign result in a positive value.

Step 2: Isolate the radical and square both sides of the equation to eliminate the radical. Then, solve for x.

Step 3: Find the roots of the equation (that is, the value(s) of x that make the equation have a value of zero).

Step 4: Check the solution, ensuring that it does not contain *extraneous roots* (solutions that do not satisfy the original equation or restrictions when substituted in the original equation).

Example:

$7 = \sqrt{12 - x} + 4, x \leq 12$	Identify restrictions.	Check:
$3 = \sqrt{12 - x}$	Isolate the radical.	Solution meets the restrictions.
$3^2 = (\sqrt{12 - x})^2$	Square both sides.	$7 = \sqrt{12 - 3} + 4$
$9 = 12 - x$	Solve for x.	$7 = \sqrt{9} + 4$
$3 = x$		$7 = 7$

Strategies for Solving Graphically

- **Method 1: Graph a Single Equation**

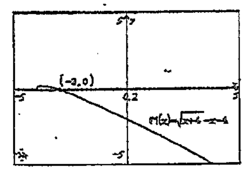
  Graph the corresponding function and find the zero(s) of the function.

  Example:

  $2 + \sqrt{x + 4} = x + 6$

  $\sqrt{x + 4} - x - 4 = 0$

  Graph $y = \sqrt{x + 4} - x - 4$.



  $x = -3$

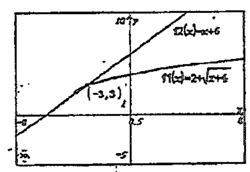
- **Method 2: Graph Two Equations**

  Graph each side of the equation on the same grid, and find the point(s) of intersection.

  Example:

  $2 + \sqrt{x + 4} = x + 6$

  Graph $y = 2 + \sqrt{x + 4}$ and $y = x + 6$.



  $x = -3$

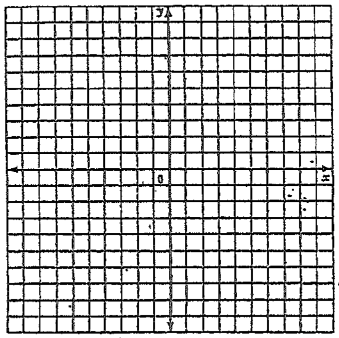
Chapter 2 Review

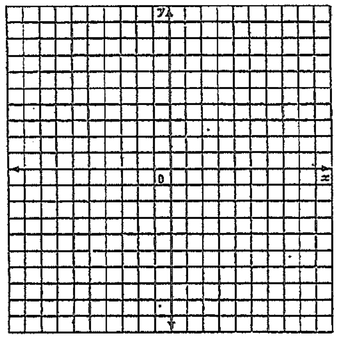
2.1 Radical Functions and Transformations, pages 39–46

1. Explain how to transform the graph of $y = \sqrt{x}$ to obtain the graph of each transformed function. Then, draw a sketch of the new function.

   a) $y = 4\sqrt{-(x-5)} + 1$

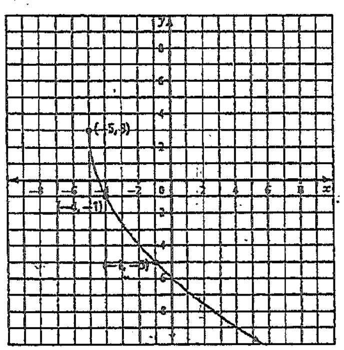
   b) $y = -3\sqrt{2(x+1)} - 3$



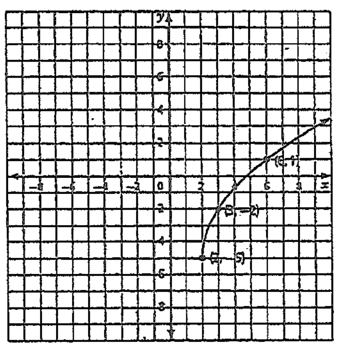


2. For each graph, write the equation of a radical function in the form $y = a\sqrt{b(x-h)} + k$. State the domain and range.

   a)



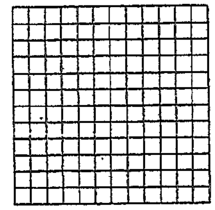
   b)



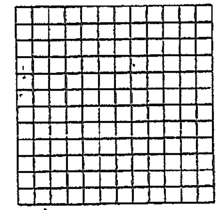
2.2 Square Root of a Function, pages 47–54

3. Use technology to graph $y = \sqrt{f(x)}$ given the following functions. Sketch the graph on the grid. State the domain and range.

   a) $f(x) = 4x - 1$



   b) $f(x) = x^2 - 9$



2.3 Solving Radical Equations Graphically, pages 55–62

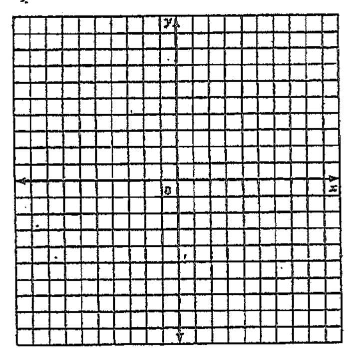
4. Determine the root(s) of each radical equation algebraically.

   a) $0 = \sqrt{x-2} - 3$

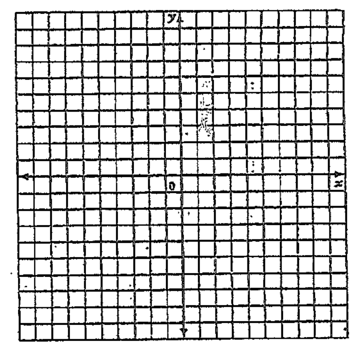
   b) $x = \sqrt{x-2} + 4$

5. Identify any restrictions on the variables. Then, solve each radical equation graphically.

   a) $\sqrt{x-1} - 5 = -2$



   b) $\sqrt{x+3} = -1$



Chapter 3 Polynomial Functions

3.1 Characteristics of Polynomial Functions

KEY IDEAS

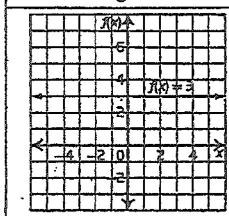
What Is a Polynomial Function?

A polynomial function has the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$, where

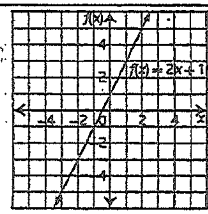
- n is a whole number
- x is a variable
- the coefficients a_n to a_0 are real numbers
- the degree of the polynomial function is n, the exponent of the greatest power of x
- the leading coefficient is a_n, the coefficient of the greatest power of x
- the constant term is a_0

Types of Polynomial Functions

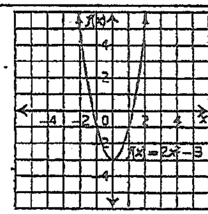
Constant Function — Degree 0



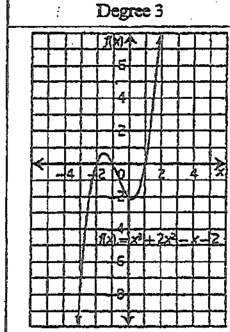
Linear Function — Degree 1



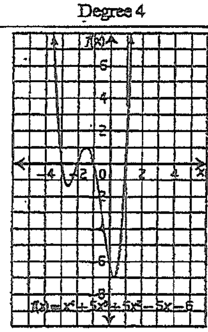
Quadratic Function — Degree 2



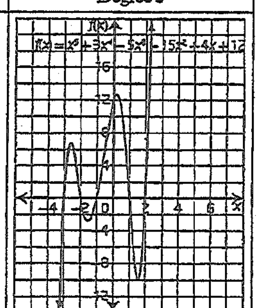
Cubic Function — Degree 3



Quartic Function — Degree 4



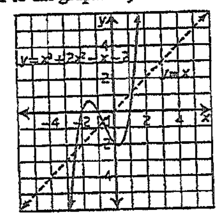
Quintic Function — Degree 5



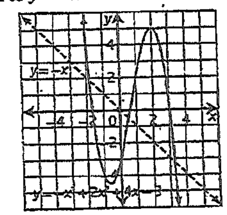
Characteristics of Polynomial Functions

Graphs of Odd-Degree Polynomial Functions

- extend from quadrant III to quadrant I when the leading coefficient is positive, similar to the graph of $y = x$



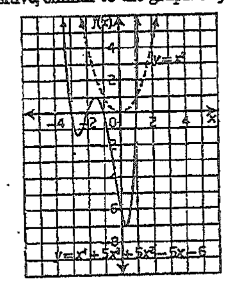
- extend from quadrant II to IV when the leading coefficient is negative, similar to the graph of $y = -x$



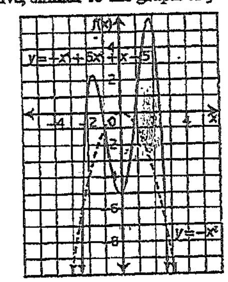
- have at least one x-intercept to a maximum of n x-intercepts, where n is the degree of the function
- have y-intercept a_0, the constant term of the function
- have domain $\{x \mid x \in R\}$ and range $\{y \mid y \in R\}$
- have no maximum or minimum values

Graphs of Even-Degree Polynomial Functions

- open upward and extend from quadrant II to quadrant I when the leading coefficient is positive, similar to the graph of $y = x^2$



- open downward and extend from quadrant III to IV when the leading coefficient is negative, similar to the graph of $y = -x^2$



- have from 0 to a maximum of n x-intercepts, where n is the degree of the function
- have y-intercept a_0, the constant term of the function
- have domain $\{x \mid x \in R\}$; the range depends on the maximum or minimum value of the function
- have a maximum or minimum value

KEY IDEAS

Long Division
You can use long division to divide a polynomial by a binomial: $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$

The components of long division are
- the dividend, $P(x)$, which is the polynomial that is being divided
- the divisor, $x-a$, which is the binomial that the polynomial is divided by
- the quotient, $Q(x)$, which is the expression that results from the division
- the remainder, R, which is the value or expression that is left over after dividing

To check the division of a polynomial, verify the statement $P(x) = (x-a)Q(x) + R$.

Synthetic Division
- a short form of division that uses only the coefficients of the terms
- it involves fewer calculations

Remainder Theorem
- When a polynomial $P(x)$ is divided by a binomial $x-a$, the remainder is $P(a)$.
- If the remainder is 0, then the binomial $x-a$ is a factor of $P(x)$.
- If the remainder is *not* 0, then the binomial $x-a$ is *not* a factor of $P(x)$.

Working Example 1: Divide a Polynomial by a Binomial of the Form $x - a$

a) Divide $P(x) = 9x + 4x^3 - 12$ by $x + 2$. Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$.
b) Identify any restrictions on the variable.
c) Write the corresponding statement that can be used to check the division.

Solution

a) $x + 2\overline{)4x^3 + 0x^2 + 9x - 12}$

Why is the order of the terms different? Why is it necessary to include the term $0x^2$?

See Example 1 on page 120 of *Pre-Calculus 12* for help with long division.

$\frac{4x^3 + 9x - 12}{x + 2} =$ ____________

KEY IDEAS

Factor Theorem
The factor theorem states that $x-a$ is a factor of a polynomial $P(x)$ if and only if $P(a) = 0$.
If and only if means that the result works both ways. That is,
- if $x-a$ is a factor then, $P(a) = 0$
- if $P(a) = 0$, then $x-a$ is a factor of a polynomial $P(x)$

Integral Zero Theorem
- The integral zero theorem describes the relationship between the factors and the constant term of a polynomial. The theorem states that if $x-a$ is a factor of a polynomial $P(x)$ with integral coefficients, then a is a factor of the constant term of $P(x)$ and $x = a$ is an integral zero of $P(x)$.

Factor by Grouping
- If a polynomial $P(x)$ has an even number of terms, it may be possible to group two terms at a time and remove a common factor. If the binomial that results from common factoring is the same for each pair of terms, then $P(x)$ may be factored by grouping.

Steps for Factoring Polynomial Functions
To factor polynomial functions using the factor theorem and the integral zero theorem,
- use the integral zero theorem to list possible integer values for the zeros
- next, apply the factor theorem to determine one factor
- then, use division to determine the remaining factor
- repeat the above steps until all factors are found

Working Example 1: Use the Factor Theorem to Test for Factors of a Polynomial

Which binomials are factors of the polynomial $P(x) = x^3 + 4x^2 + x - 6$? Justify your answers.

a) $x - 1$ b) $x - 2$ c) $x + 2$ d) $x + 3$

Solution

Use the factor theorem to evaluate $P(a)$ given $x - a$.

a) For $x - 1$, substitute $x =$ ______ into the polynomial expression.

$P($______$) =$

Since the remainder is ______, $x - 1$ ____________ a factor of $P(x)$.
(*is* or *is not*)

3.4 Equations and Graphs of Polynomial Functions

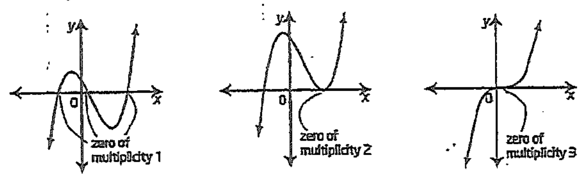
KEY IDEAS

Sketching Graphs of Polynomial Functions

- To sketch the graph of a polynomial function, use the x-intercepts, the y-intercept, the degree of the function, and the sign of the leading coefficient.
- The x-intercepts of the graph of a polynomial function are the roots of the corresponding polynomial equation.
- Determine the zeros of a polynomial function from the factors.
- Use the factor theorem to express a polynomial function in factored form.

Multiplicity of a Zero

- If a polynomial has a factor $x - a$ that is repeated n times, then $x = a$ is a zero of multiplicity n.
- The multiplicity of a zero or root can also be referred to as the *order* of the zero or root.
- The shape of a graph of a polynomial function close to a zero of $x = a$ (multiplicity n) is similar to the shape of the graph of a function with degree equal to n of the form $y = (x - a)^n$.
- Polynomial functions change sign at x-intercepts that correspond to *odd* multiplicity. The graph crosses over the x-axis at these intercepts.
- Polynomial functions do not change sign at x-intercepts of *even* multiplicity. The graph touches, but does not cross, the x-axis at these intercepts.



Transformation of Polynomial Functions

To sketch the graph of a polynomial function of the form $y = a[b(x - h)]^n + k$ or $y - k = a[b(x - h)]^n$, where $n \in N$, apply the following transformations to the graph of $y = x^n$.

Note: You may apply the transformations represented by a and b in any order before the transformations represented by h and k.

Parameter	Transformation
k	• Vertical translation up or down • $(x, y) \rightarrow (x, y + k)$
h	• Horizontal translation left or right • $(x, y) \rightarrow (x + h, y)$
a	• Vertical stretch about the x-axis by a factor of $\lvert a \rvert$ • For $a < 0$, the graph is also reflected in the x-axis • $(x, y) \rightarrow (x, ay)$

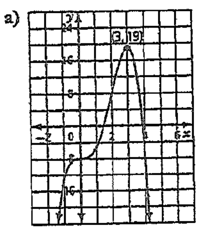
Chapter 3 Review

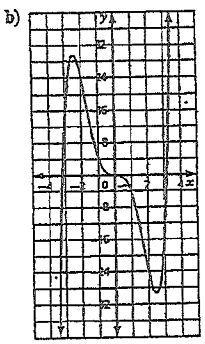
3.1 Characteristics of Polynomial Functions, pages 66–77

1. Complete the chart for each polynomial function.

Polynomial Function	Degree	Type	Leading Coefficient	Constant Term
a) $f(x) = -2x^4 - x^3 + 3x - 7$				
b) $y = 3x^5 + 2x^4 - x^3 + 3$				
c) $g(x) = 0.5x^3 - 8x^2$				
d) $p(x) = 10$				

2. For each of the following,
   - determine whether the graph represents an odd-degree or an even-degree polynomial function
   - determine whether the leading coefficient of the corresponding function is positive or negative
   - state the number of x-intercepts
   - state the domain and range

a) 

b) 

3. The [illegible]ce, d, in metres, travelled by a boat from the moment it leaves shore can be mo[illegible]y the function $d(t) = 0.002t^3 + 0.05t^2 + 0.3t$, where t is the time, in seconds.

a) What is the degree of the function $d(t)$?

b) What are the leading coefficient and constant of this function? What does the constant represent?

c) Describe the end behaviour of the graph of this function.

d) What are the restrictions on the domain of this function? Explain why you selected those restrictions.

e) What distance has the boat travelled after 15 s?

f) Make a sketch of what you think the function will look like. Then, graph the function using technology. How does it compare to your sketch?

3.2 The Remainder Theorem, pages 78–83

4. a) Use long division to divide $5x^3 - 7x^2 - x + 6$ by $x - 1$. Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$.

b) Identify any restrictions on the variable.

c) Write the corresponding statement that can be used to check the division. Then, [illegible]y your answer.

5. Determine the remainder resulting from each division.

a) $(x^3 + 2x^2 - 3x + 9) \div (x + 3)$

b) $(2x^3 + 7x^2 - x + 1) \div (x + 2)$

c) $(x^3 + 2x^2 - 3x + 5) \div (x - 3)$

d) $(2x^4 + 7x^2 - 8x + 3) \div (x - 4)$

6. a) Determine the value of m such that when $f(x) = x^4 - mx^3 + 7x - 6$ is divided by $x - 2$, the remainder is -8.

b) Use the value of m from part a) to determine the remainder when $f(x)$ is divided by $x + 2$.

7. When a polynomial $P(x)$ is divided by $x - 2$, the quotient is $x^2 + 4x - 7$ and the remainder is -4. What is the polynomial?

3.3 The Factor Theorem, pages 84–90

8. What is the corresponding binomial factor of a polynomial, $P(x)$, given the value of the zero?

a) $P(7) = 0$

b) $P(-6) = 0$

c) $P(c) = 0$

9. Determine whether $x + 2$ is a factor of each polynomial.

a) $x^3 + 2x^2 - x - 2$

b) $x^4 + 2x^3 - 4x^2 + x + 10$

10. What are the possible integral zeros of each polynomial?

a) $x^3 - 5x^2 + 3x - 27$

b) $x^3 + 6x^2 + 2x + 36$

11. Factor fully.

a) $x^3 - 4x^2 + x + 6$

b) $3x^3 - 5x^2 - 26x - 8$

c) $5x^4 + 12x^3 - 101x^2 + 48x + 36$

d) $2x^4 + 5x^3 - 8x^2 - 20x$

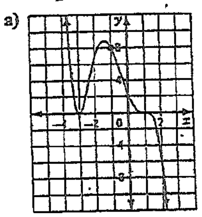
12. Rectangular blocks of ice are cut up and used to build the front entrance of an ice castle. The volume, in cubic feet, of each block is represented by $V(x) = 5x^3 + 7x^2 - 8x - 4$, where x is a positive real number. What are the factors that represent possible dimensions, in terms of x, of the blocks?

3.4 Equations and Graphs of Polynomial Functions, pages 91–102

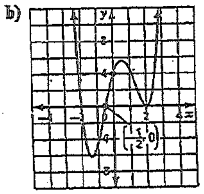
13. For each graph of a polynomial function, determine

- the least possible degree
- the sign of the leading coefficient
- the x-intercepts and their multiplicity
- the intervals where the function is positive and the intervals where it is negative
- the equation for the polynomial function

a)



b)



14. a) Given the function $y = x^5$, list the parameters of the transformed polynomial function $y = -2\left(\frac{1}{3}(x - 1)\right)^5 + 4$ and describe how each parameter transforms the graph of the function $y = x^5$.

b) Determine the domain and range for the transformed function.

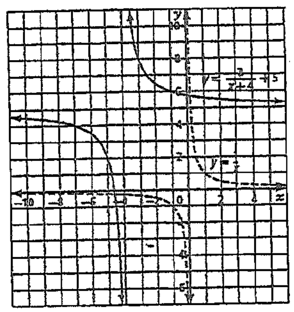
15. Determine the equation with least degree for a cubic function with zeros −2 (multiplicity 2) and 3 (multiplicity 1), and y-intercept 36.

Chapter 9 Rational Functions

9.1 Exploring Rational Functions Using Transformations

KEY IDEAS

- Rational functions are functions of the form $y = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$.
- You can graph a rational function by creating a table of values and then graphing the points in the table. To create a table of values,
  - identify the non-permissible value(s)
  - write the non-permissible value in the middle row of the table
  - enter positive values above the non-permissible value and negative values below the non-permissible value
  - choose small and large values of x to give you a spread of values
- You can use what you know about the base function $y = \frac{1}{x}$ and transformations to graph equations of the form $y = \frac{a}{x-h} + k$.



  Example:

  For $y = \frac{3}{x+4} + 5$, the values of the parameters are

  $a = 3$, representing a vertical stretch by a factor of 3
  $h = 4$, representing a horizontal translation 4 units to the left
  $k = 5$, representing a vertical translation 5 units up
  vertical asymptote: $x = -4$
  horizontal asymptotes: $y = 5$

- Some equations of rational functions can be manipulated algebraically into the form $y = \frac{a}{x-h} + k$ by creating a common factor in the numerator and the denominator.

  Example:

  $y = \frac{3x+6}{x-4}$

  $y = \frac{3x-12+12+6}{x-4}$

  $y = \frac{3x-12+18}{x-4}$

  $y = \frac{3(x-4)}{x-4} + \frac{18}{x-4}$

  $y = \frac{18}{x-4} + 3$

9.2 Analysing Rational Functions

KEY IDEAS

Determining Asymptotes and Points of Discontinuity
The graph of a rational function may have an asymptote, a point of discontinuity, or both. To establish these important characteristics of a graph, begin by factoring the numerator and denominator fully.

• **Asymptotes: No Common Factors** If the numerator and denominator do not have a common factor, the function has an asymptote. – The vertical asymptotes are identified by the non-permissible values of the function. – For a function that can be rewritten in the form $y = \frac{a}{x-h} + k$, the k parameter identifies the horizontal asymptote.	Example: $y = \frac{x+4}{x-3}$ Since the non-permissible value is $x = 3$, the vertical asymptote is at $x = 3$. $y = \frac{x+4}{x-3}$ $y = \frac{x-3+3+4}{x-3}$ $y = \frac{x-3}{x-3} + \frac{7}{x-3}$ $y = \frac{7}{x-3} + 1$ Since $k = 1$, the horizontal asymptote is at $y = 1$.
• **Points of Discontinuity: At Least One Common Factor** If the numerator and denominator have at least one common factor, there is at least one point of discontinuity in the graph. – Equate the common factor(s) to zero and solve for x to determine the x-coordinate of the point of discontinuity. – Substitute the x-value in the simplified expression to find the y-coordinate of the point of discontinuity.	Example: $y = \frac{(x-4)(x+2)}{x+2}$ $x + 2 = 0$: the x-coordinate of the point of discontinuity is -2. Substitute $x = -2$ into the simplified equation: $y = x - 4$ $y = -2 - 4$ $y = -6$ point of discontinuity: $(-2, -6)$
• **Both Asymptote(s) and Point(s) of Discontinuity** If a rational expression remains after removing the common factor(s), there may be both a point of discontinuity and asymptotes.	Example: $y = \frac{(x-4)(x+2)}{(x+2)(x-1)}$ $y = \frac{(x-4)}{(x-1)}$ – common factor: $x + 2$, so there is a point of discontinuity at $(-2, 2)$ – non-permissible value: $x = 1$, so the vertical asymptote is at $x = 1$. – simplified function can be rewritten as $y = \frac{-3}{x-1} + 1$, so the horizontal asymptote is at $y = 1$

KEY IDEAS

Solving Rational Equations
You can solve rational equations algebraically or graphically.

- **Algebraically**
  Solving algebraically determines the exact solution and any extraneous roots. To solve algebraically,
  - Equate to zero and list the restrictions.
  - Factor the numerator and denominator fully (if possible).
  - Multiply each term by the lowest common denominator to eliminate the fractions.
  - Solve for x.
  - Check the solution(s) against the restrictions.
  - Check the solution(s) in the original equation.

Example:

$$\frac{16}{x+6} = 4 - x$$

$$x + \frac{16}{x+6} - 4 = 0;\ x \neq -6$$

$$(x+6)\left(x + \frac{16}{x+6} - 4\right) = (x+6)(0)$$

$$(x+6)(x) + \left(\cancel{x+6}\right)\left(\frac{16}{\cancel{x+6}}\right) - (x+6)(4) = 0$$

$$x^2 + 6x + 16 - 4x - 24 = 0$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

roots: $x = -4$ and $x = 2$

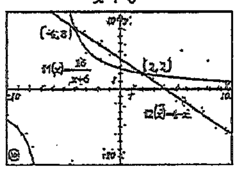
- **Graphically**
  There are two methods for solving equations graphically.

  Method 1: Use a System of Two Functions
  - Graph each side of the equation on the same set of axes.
  - The solution(s) will be the x-coordinate(s) of any point(s) of intersection.

  Method 2: Use a Single Function
  - Rearrange the equation so that one side is equal to zero.
  - Graph the corresponding function.
  - The solution(s) will be the x-intercept(s).

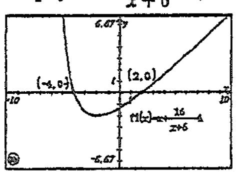
Example: $\frac{16}{x+6} = 4 - x$

Graph $y = \frac{16}{x+6}$ and $y = 4$ on the same axes.



The points of intersection are $(-4, 8)$ and $(2, 2)$, so the roots are $x = -4$ and $x = 2$.

Graph $y = x + \frac{16}{x+6} - 4$.

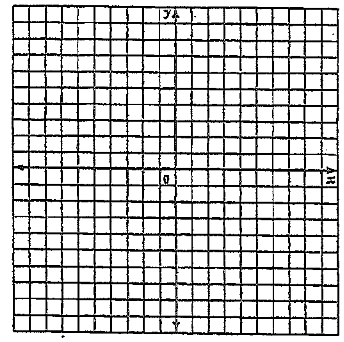


x-intercepts: $x = -4$ and $x = 2$

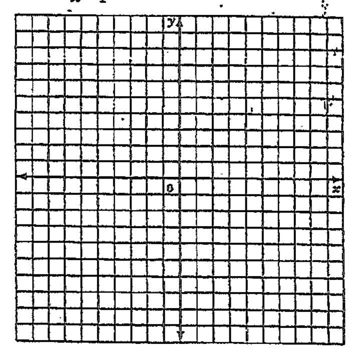
9.1 Exploring Rational Functions Using Transformations, pages 297–304

1. Graph each function using transformations. Label the important parts of the graph.

a) $y = \frac{3}{x-4} + 2$

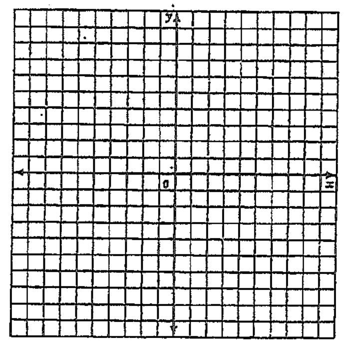


b) $y = \frac{7}{x-1} - 2$

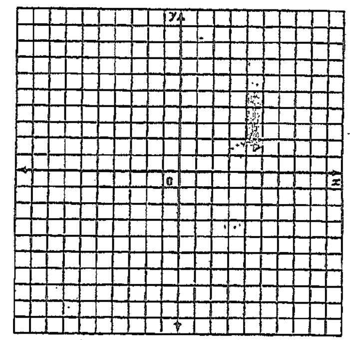


2. Graph the following functions without technology. Label all the important parts.

a) $f(x) = \frac{4x+5}{x-3}$

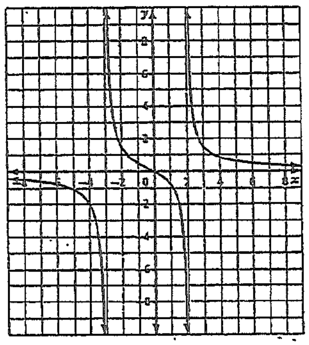


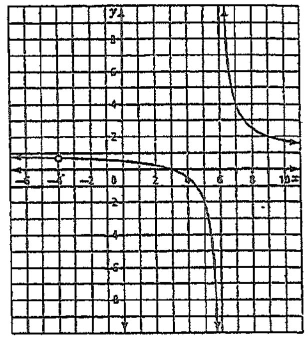
b) $f(x) = \frac{-2x+5}{x-3}$

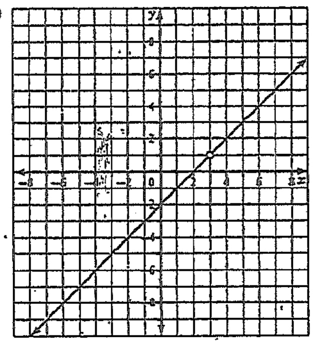


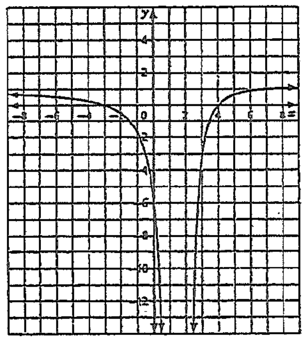
9.2 Ana[illegible]g Rational Functions, pages 305–313

3. Match the graph of each rational function with the most appropriate equation. Give reasons for each choice.

a) 

b) 

c) 

d) 

A $f(x)=\frac{x^2+x-12}{x^2-2x-24}$

B $g(x)=\frac{x^2-x-12}{x^2-3x+2}$

C $h(x)=\frac{x^2-5x+6}{x-3}$

D $j(x)=\frac{3x}{x^2+x-6}$

4. For each function, predict the location of any points of discontinuity, vertical asymp[illegible], and intercepts.

a) $f(x)=\frac{2x+1}{x+5}$

b) $f(x)=\frac{x^2-8x+12}{x-2}$

9.3 Connecting Graphs and Rational Equations, pages 314–320

5. Solve each rational equation algebraically.

a) $\frac{3}{x}-\frac{6}{x-2}=\frac{1}{4}$

b) $\frac{x-2}{3}=\frac{2x-4}{x}$

c) $\frac{x+1}{x+3}=\frac{x+4}{x+5}$

b) $\frac{x+2}{x-2}=\frac{2x+4}{x+1}$

6. Use technology to solve each rational equation graphically. Sketch and label a graph of the solution. Provide answers to the nearest tenth.

a) $\frac{4}{x}+\frac{3}{x+1}=\frac{1}{2}$

$x=$ ______________

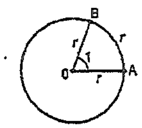
b) $\frac{3x-1}{x+4}+3=\frac{6}{x-4}$

$x=$ ______________

Chapter 4 Trigonometry and the Unit Circle

4.1 Angles and Angle Measure

KEY IDEAS

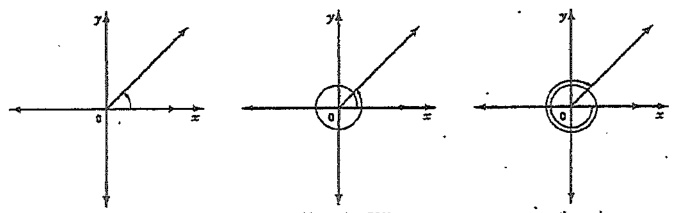


- One radian is the measure of the central angle subtended in a circle by an arc equal in length to the radius of the circle.
- Travelling one rotation around the circumference of a circle causes the terminal arm to turn $2\pi r$. Since $r = 1$ on the unit circle, $2\pi r$ can be expressed as 2π, or 2π radians.

You can use this information to translate rotations into radian measures. For example,

1 full rotation (360°) is 2π radians	$\frac{1}{6}$ rotation (60°) is $\frac{\pi}{3}$ radians
$\frac{1}{2}$ rotation (180°) is π radians	$\frac{1}{8}$ rotation (45°) is $\frac{\pi}{4}$ radians
$\frac{1}{4}$ rotation (90°) is $\frac{\pi}{2}$ radians	$\frac{1}{12}$ rotation (30°) is $\frac{\pi}{6}$ radians

- Angles in standard position with the same terminal arms are coterminal. For an angle in standard position, an infinite number of angles coterminal with it can be determined by adding or subtracting any number of full rotations.
- Counterclockwise rotations are associated with positive angles. Clockwise rotations are associated with negative angles.

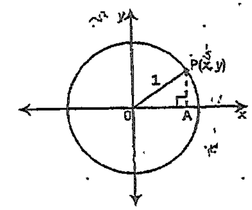


quadrant I angle — positive angle > 360° — negative angle

- The general form of a coterminal angle (in degrees) is $\theta \pm 360°n$, where n is a natural number (0, 1, 2, 3, ...) and represents the number of revolutions. The general form (in radians) is $\theta \pm 2\pi n$, $n \in \mathrm{N}$.
- Radians are especially useful for describing circular motion. Arc length, a, means the distance travelled along the circumference of a circle of radius r. For a central angle θ, in radians, $a = \theta r$.

4.2 The Unit Circle

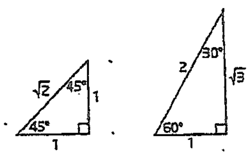
KEY IDEAS



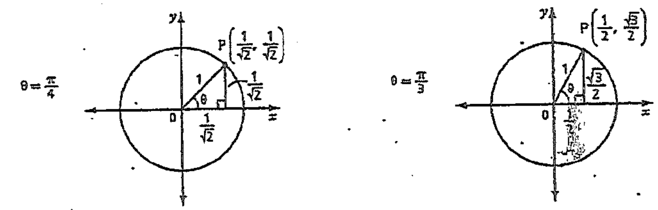
- In general, a circle of radius r centred at the origin has equation $x^2 + y^2 = r^2$.
- The unit circle has radius 1 and is centred at the origin. The equation of the unit circle is $x^2 + y^2 = 1$. All points $P(x, y)$ on the unit circle satisfy this equation.
- An arc length measured along the unit circle equals the measure of the central angle (in radians).

In other words, when $r = 1$, the formula $a = \theta r$ simplifies to $a = \theta$.

- Recall the special right triangles you learned about previously.



These special triangles can be scaled to fit within the unit circle ($r = 1$).



4.3 Trigonometric Ratios

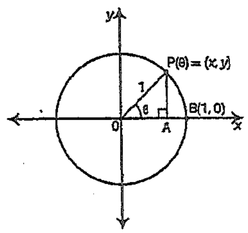
KEY IDEAS

- These are the primary trigonometric ratios:

sine	cosine	tangent
$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x}$

- For points on the unit circle, $r = 1$. Therefore, the primary trigonometric ratios can be expressed as:

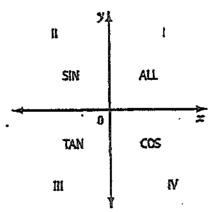
  $\sin\theta = \frac{y}{1} = y \quad \cos\theta = \frac{x}{1} = x \quad \tan\theta = \frac{y}{x}$



- Since $\cos\theta$ simplifies to x and $\sin\theta$ simplifies to y, you can write the coordinates of $P(\theta)$ as $P(\theta) = (\cos\theta, \sin\theta)$ for any point $P(\theta)$ at the intersection of the terminal arm of θ and the unit circle.
- These are the reciprocal trigonometric ratios:

cosecant	secant	cotangent
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
$\csc\theta = \frac{r}{y}$	$\sec\theta = \frac{r}{x}$	$\cot\theta = \frac{x}{y}$

- Recall from the CAST rule that
  - $\sin\theta$ and $\csc\theta$ are positive in quadrants I and II
  - $\cos\theta$ and $\sec\theta$ are positive in quadrants I and IV
  - $\tan\theta$ and $\cot\theta$ are positive in quadrants I and III

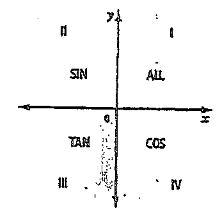


4.4 Introduction to Trigonometric Equations

KEY IDEAS

- Solving an equation means to determine the value (or values) of a variable that make an equation true (Left Side = Right Side).
  For example, $\sin\theta = \frac{1}{2}$ is true when $\theta = 30°$ or $\theta = 150°$, and for every angle coterminal with 30° or 150°. These angles are solutions to a very simple trigonometric equation.
- The variable θ is often used to represent the unknown angle, but any other variable is allowed.
- In general, solve for the trigonometric ratio, and then determine
  - all solutions within a given domain, such as $0 \le \theta < 2\pi$

    or
  - all possible solutions, expressed in general form, $\theta + 2\pi n$, $n \in I$
- Unless the angle is a multiple of 90° or $\frac{\pi}{2}$, there will be two angles per solution of the equation within each full rotation of 360° or 2π. As well, there will be two expressions in general form per solution, one for each angle. It is sometimes possible to write a combined expression representing both angles in general form.
- If the angle is a multiple of 90° or $\frac{\pi}{2}$ (that is, the terminal arm coincides with an axis), then there will be at least one angle within each full rotation that is a correct solution to the equation.
- Note that $\sin^2\theta = (\sin\theta)^2$. Also, recall that
  - $\sin\theta$ and $\csc\theta$ are positive in quadrants I and II
  - $\cos\theta$ and $\sec\theta$ are positive in quadrants I and IV
  - $\tan\theta$ and $\cot\theta$ are positive in quadrants I and III



Chapter 4 Review

4.1 Angles and Angle Measure, pages 109–119

1. Convert each degree measure to radian measure and each radian measure to degree measure. Give exact values.

a) 270°

b) $\frac{5\pi}{3}$

c) 300°

d) −4

e) 495°

f) $\frac{13\pi}{4}$

2. Identify one positive and one negative angle measure that is coterminal with each angle. Then, write a general expression for all the coterminal angles in each case.

a) $\frac{11\pi}{6}$

b) −375°

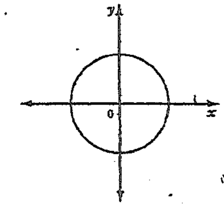
3. Determine the measure of the central angle subtended by each arc to one decimal place.

a) arc length 31.4 cm, radius 5.0 cm, in radians

b) arc length 11.3 m, radius 22.6 m, in degrees

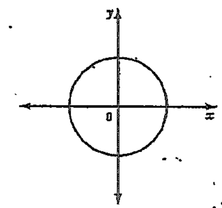
4.2 The Unit Circle, pages 120–128

4. Determine the missing coordinate for point $P\left(x, -\frac{2}{3}\right)$ in quadrant III on the unit circle.

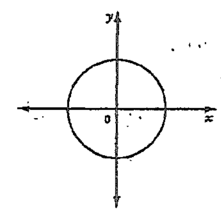


5. Determine the value of angle θ in standard position, $0 \leq \theta < 2\pi$, given the coordinates of P(θ), the point at which the terminal arm of θ intersects the unit circle.

a) $P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ θ =

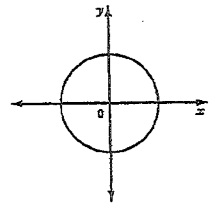


b) $P\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ θ =



4.3 Trigonometric Ratios, pages 129–137

6. Determine the measure of all angles that satisfy $\sec\theta = 1.788$, $0° \leq \theta < 720°$. Round your answers to the nearest degree.



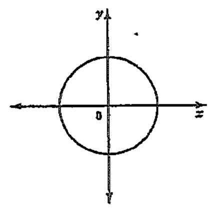
7. Determine the exact value of

a) $\cot\left(\frac{5\pi}{6}\right)$

b) $\csc\left(\frac{5\pi}{3}\right)$

4.4 Introduction to Trigonometric Equations, pages 138–144

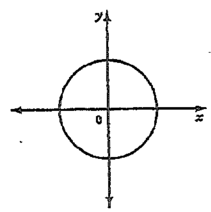
8. Write the general form of the solutions to $\sec\theta + 10 = 2 - 4\sec\theta$ (in degrees).



$\theta_1 \approx$ ____________, $n \in I$

$\theta_2 \approx$ ____________, $n \in I$

9. Solve $2\sin^2\theta + \sin\theta = 1$, $0 \leq \theta < 2\pi$. Give exact solutions.

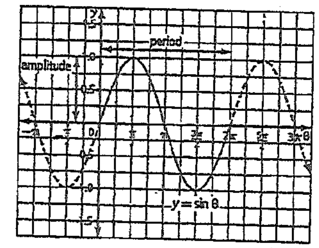


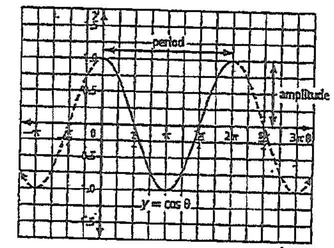
Chapter 5 Trigonometric Functions and Graphs

5.1 Graphing Sine and Cosine Functions

KEY IDEAS

- Sine and cosine functions are *periodic* or *sinusoidal functions*. The values of these functions repeat in a regular pattern. These functions are based on the unit circle.
- Consider the graphs of $y = \sin\theta$ and $y = \cos\theta$.





– The maximum value is +1.
– The minimum value is –1.
– The amplitude is 1.
– The period is 2π.
– The y-intercept is 0.
– The θ-intercepts on the given domain are $-\pi$, 0, π, 2π, and 3π.
– The domain of $y = \sin\theta$ is $\{\theta \mid \theta \in R\}$.
– The range of $y = \sin\theta$ is $\{y \mid -1 \leq y \leq 1, y \in R\}$.

– The maximum value is +1.
– The minimum value is –1.
– The amplitude is 1.
– The period is 2π.
– The y-intercept is 1.
– The θ-intercepts on the given domain are $-\frac{\pi}{2}$, $\frac{\pi}{2}$, $\frac{3\pi}{2}$, and $\frac{5\pi}{2}$.
– The domain of $y = \cos\theta$ is $\{\theta \mid \theta \in R\}$.
– The range of $y = \cos\theta$ is $\{y \mid -1 \leq y \leq 1, y \in R\}$.

- For sinusoidal functions of the form $y = a\sin bx$ or $y = a\cos bx$, a represents a vertical stretch of factor $|a|$ and b represents a horizontal stretch of factor $\frac{1}{|b|}$. Use the following key features to sketch the graph of a sinusoidal function.
  – the maximum and minimum values
  – the amplitude, which is one half the total height of the function

  $$\text{Amplitude} = \frac{\text{maximum value} - \text{minimum value}}{2}$$

  The amplitude is given by $|a|$.
  – the period, which is the horizontal length of one cycle on the graph of a function

  $$\text{Period} = \frac{2\pi}{|b|} \text{ or } \frac{360°}{|b|}$$

  Changing the value of b changes the period of the function.
  – the coordinates of the horizontal intercepts

5.2 Transformations of Sinusoidal Functions

KEY IDEAS

- You can apply the same transformation rules to sinusoidal functions of the form $y = a \sin b(\theta - c) + d$ or $y = a \cos b(\theta - c) + d$.
  - A vertical stretch by a factor of $|a|$ changes the amplitude to $|a|$.

    $y = a \sin \theta$ $\qquad$ $y = a \cos \theta$

    If $a < 0$, the function is reflected through the horizontal mid-line of the function.
  - A horizontal stretch by a factor of $\frac{1}{|b|}$ changes the period to $\frac{360°}{|b|}$ or $\frac{2\pi}{|b|}$ radians.

    $y = \sin (b\theta)$ $\qquad$ $y = \cos (b\theta)$

    If $b < 0$, the function is reflected in the y-axis.
  - For sinusoidal functions, a horizontal translation is called the *phase shift*.

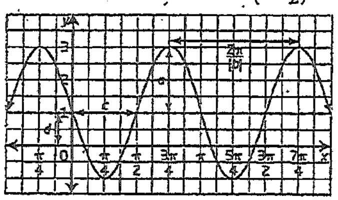
    $y = \sin (\theta - c)$ $\qquad$ $y = \cos (\theta - c)$

    If $c > 0$, the function shifts c units to the right.
    If $c < 0$, the function shifts c units to the left.
  - The *vertical displacement* is a vertical translation.

    $y = \sin \theta + d$ $\qquad$ $y = \cos \theta + d$
  - If $d > 0$, the function shifts d units up.
  - If $d < 0$, the function shifts d units down.

    $d = \frac{\text{maximum value} + \text{minimum value}}{2}$
  - The *sinusoidal axis* is defined by the line $y = d$. It represents the mid-line of the function.
- Apply transformations of sinusoidal functions in the same order as for any other functions:
  i) horizontal stretches and reflections, $\frac{1}{|b|}$
  ii) vertical stretches and reflections, $|a|$
  iii) translations, c and d
- The domain of a sinusoidal function is not affected by transformations.
  The range of a sinusoidal function, normally $\{y \mid -1 \le y \le 1, y \in R\}$, is affected by changes to the amplitude and vertical displacement.

  Consider the graph of $y = 2 \sin 2\left(x - \frac{\pi}{2}\right) + 1$.



$a = 2$, so the amplitude is 2

$b = 2$, so the period is $\frac{2\pi}{2}$, or π

$c = \frac{\pi}{2}$, so the graph is shifted $\frac{\pi}{2}$ units right

$d = 1$, so the graph is shifted 1 unit up

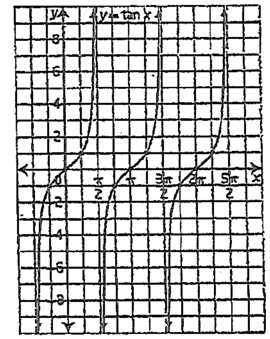
domain: $\{x \mid x \in R\}$

range: $\{y \mid -1 \le y \le 3, y \in R\}$

5.3 The Tangent Function

KEY IDEAS

- The graph of the tangent function, $y = \tan x$, is periodic, but it is *not* sinusoidal.



- These are the characteristics of the tangent function graph, $y = \tan x$:
  - It has period π or 180°.
  - It is discontinuous where $\tan x$ is undefined, that is, when $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots, \frac{\pi}{2} + n\pi$, $n \in I$. The discontinuity is represented on the graph of $y = \tan x$ as *vertical asymptotes*.
  - The domain is $\left\{x \mid x \ne \frac{\pi}{2} + n\pi, x \in R, n \in I\right\}$.
  - It has no maximum or minimum values.
  - The range is $\{y \mid y \in R\}$.
  - It has x-intercepts at every multiple of π: $0, \pi, 2\pi, \ldots, n\pi, n \in I$. Each of the x-intercepts is a turning point, where the slope changes from decreasing to increasing.
- On the unit circle, you can express the coordinates of the point P on the terminal arm of angle θ as (x, y) or $(\sin \theta, \cos \theta)$. The slope of the terminal arm is represented by the tangent function:

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{y-0}{x-0} = \frac{y}{x} = \tan \theta$$

OR

$$\text{slope} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

Therefore, you can use the tangent function to model the slope of a line from a fixed point to a moving object as the object moves through a range of angles.

Chapter 10 Function Operations

10.1 Sums and Differences of Functions

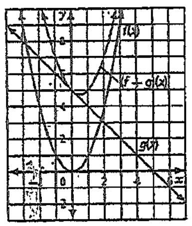
KEY IDEAS

- You can form new functions by performing operations with functions.

Sum of Functions

$h(x) = f(x) + g(x)$
or
$h(x) = (f + g)(x)$

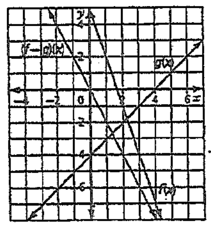
Example
$f(x) = x^2$ and $g(x) = -x + 5$
$h(x) = f(x) + g(x)$
$h(x) = x^2 + (-x + 5)$
$h(x) = x^2 - x + 5$



Difference of Functions

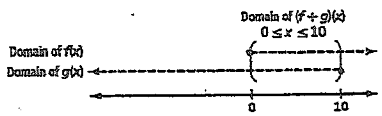
$h(x) = f(x) - g(x)$
or
$h(x) = (f - g)(x)$

Example
$f(x) = -2x$ and $g(x) = x - 4$
$h(x) = f(x) - g(x)$
$h(x) = -2x - (x - 4)$
$h(x) = -3x + 4$



- The domain of the combined function formed by the sum or difference of two functions is the domain common to the individual functions.

  Example
  If the domain of $f(x)$ is $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and the domain of $g(x)$ is $\{x \mid x \leq 10, x \in \mathrm{R}\}$, the domain of $(f + g)(x)$ is $\{x \mid 0 \leq x \leq 10, x \in \mathrm{R}\}$.



- The range of a combined function can be determined using its graph.

10.2 Products and Quotients of Functions

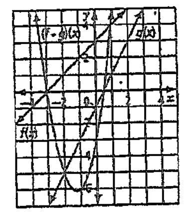
KEY IDEAS

- New functions can be formed by performing the operations of multiplication and division with functions.

Product of Functions

$h(x) = f(x) \cdot g(x)$
or
$h(x) = (f \cdot g)(x)$

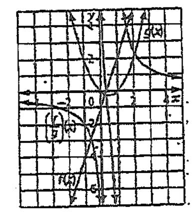
Example
$f(x) = x + 3$ and $g(x) = 2x - 1$
$h(x) = f(x) \cdot g(x)$
$h(x) = (x + 3)(2x - 1)$
$h(x) = 2x^2 + 5x - 3$



Quotient of Functions

$h(x) = \dfrac{f(x)}{g(x)}$, where $g(x) \neq 0$
or
$h(x) = \left(\dfrac{f}{g}\right)(x)$, where $g(x) \neq 0$

Example
$f(x) = 3x - 1$ and $g(x) = x^2 - x$
$h(x) = \dfrac{f(x)}{g(x)}$
$h(x) = \dfrac{3x - 1}{x^2 - x}$, where $x \neq 1, 0$

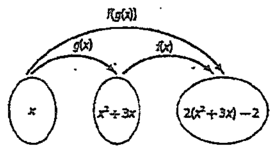


- The domain of a product or a quotient of functions is the domain common to the original functions. The domain of a quotient of functions must have the restriction that the divisor cannot equal zero. That is, for $h(x) = \dfrac{f(x)}{g(x)}$, the values of x are such that $g(x) \neq 0$.
- The range of a combined function can be determined using its graph.

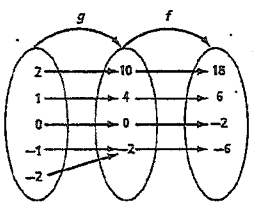
KEY IDEAS

- Composite functions are functions that are formed from two functions, $f(x)$ and $g(x)$, in which the output of one of the functions is used as the input for the other function.
  - $f(g(x))$ is read as "f of g of x"
  - $(f \circ g)(x)$ is another way of writing $f(g(x))$ and is read the same way

For example, if $f(x) = 2x - 2$ and $g(x) = x^2 + 3x$, then $f(g(x))$ is shown in the mapping diagram.



The output for $g(x)$ is the input for $f(x)$.



To determine the equation of a composite function, substitute the second function into the first. To determine $f(g(x))$,

$f(g(x)) = f(x^2 + 3x)$	Substitute $x^2 + 3x$ for $g(x)$.
$f(g(x)) = 2(x^2 + 3x) - 2$	Substitute $x^2 + 3x$ into $f(x) = 2x - 2$.
$f(g(x)) = 2x^2 + 6x - 2$	Simplify.

To determine $g(f(x))$,

$g(f(x)) = g(2x - 2)$	Substitute $2x - 2$ for $f(x)$.
$g(f(x)) = (2x - 2)^2 + 3(2x - 2)$	Substitute $2x - 2$ into $g(x) = x^2 + 3x$.
$g(f(x)) = 4x^2 - 8x + 4 + 6x - 6$	Simplify.
$g(f(x)) = 4x^2 - 2x - 2$	

Note that $f(g(x)) \neq g(f(x))$.

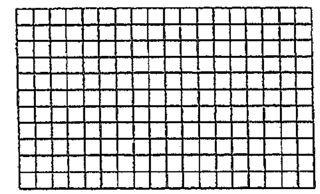
- The domain of $f(g(x))$ is the set of all values of x in the domain of g for which $g(x)$ is the domain of f. Restrictions must be considered.

Chapter 10 Review

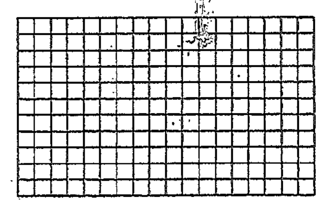
10.1 Sums and Differences of Functions, pages 325–334

1. Given $f(x) = 2x - 1$ and $g(x) = x^2 + 4$, determine each of the following.
   a) $(f + g)(-3) =$
   b) $(f - g)(4)$

2. Let $f(x) = \sqrt{x + 6}$ and $g(x) = 4x^2 - 1$.
   a) Determine $h(x) = f(x) + g(x)$.
   b) Use graphing technology to graph $y = h(x)$. Sketch the graph on the grid.



   c) State the domain of $h(x)$. Use the graph to approximate the range of $h(x)$.
   d) Determine $k(x) = f(x) - g(x)$.
   e) Use graphing technology to graph $y = k(x)$. Sketch the graph on the grid.



   f) State the domain of $k(x)$. Use the graph to approximate the range of $k(x)$.

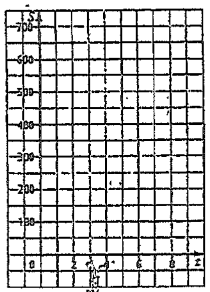
3. If $h(x) = (f - g)(x)$ and $f(x) = -x + 6$, determine $g(x)$.

a) $h(x) = 4x^2 - 12x + 9$

b) $h(x) = \sqrt{x} + x - 6$

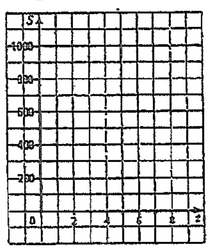
4. Extreme Sports has two store locations. Between the years 2007–2012, the sales, S_1, in thousands of dollars, at the first location decreased according to the function $S_1(t) = 750 - 0.6t^2$, where t represents the number of years after the year 2000. During the same six-year period, the sales in the second store, S_2, in thousands of dollars, increased according to the function $S_2(t) = 335 + 0.8t$, where t represents the number of years after the year 2000.

a) Graph $S_1(t)$ and $S_2(t)$ on the same set of axes.



b) Write a combined function that represents the total sales of the two stores.

c) Graph the combined function.



d) Have the total sales been increasing or decreasing? Explain.

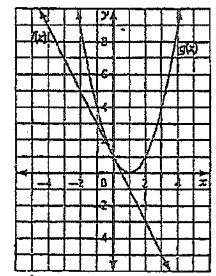
10.2 Products and Quotients of Functions, pages 335–344

5. Let $f(x) = 1 - 2x$ and $g(x) = x^2 + 3$. Determine each combined function and state any restrictions on x.

a) $h(x) = f(x) \cdot g(x)$

b) $k(x) = \dfrac{g(x)}{f(x)}$

6. Use the graphs of $f(x)$ and $g(x)$ to determine the following.



a) $(f \cdot g)(0)$

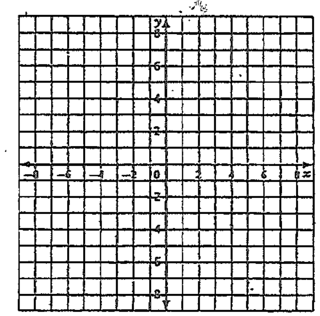
b) $(f \cdot g)(-1)$

c) $\left(\frac{f}{g}\right)(2)$

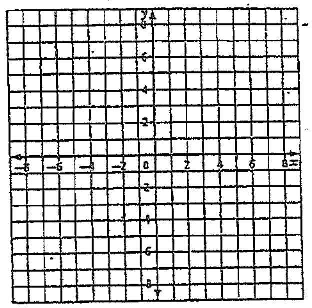
d) $\left(\frac{f}{g}\right)(-2)$

7. Consider $f(x) = \frac{1}{x - 1}$ and $g(x) = x$.

a) Determine $h(x) = (f \cdot g)(x)$. Then, sketch the graph of $y = h(x)$ and state its domain.



b) Determine $k(x) = \left(\frac{f}{g}\right)(x)$. Then, sketch the graph of $y = k(x)$ and state its domain.



8. If $h(x) = f(x) \cdot g(x)$ and $f(x) = 2x - 3$, determine $g(x)$.

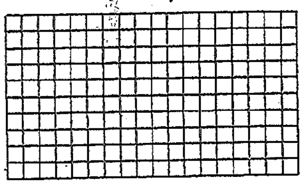
a) $h(x) = 2x^2 - 5x + 3$

b) $h(x) = 2x(\sin x) - 3(\sin x)$

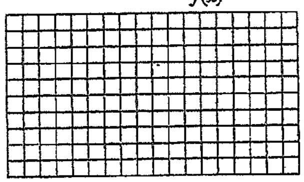
c) $h(x) = -2x^3 + 3x^2$

9. Let $f(x) = \sin x$ and $g(x) = \cos x$.

a) Sketch the graphs of $f(x)$ and $g(x)$.



b) Sketch the graph of $y = \frac{g(x)}{f(x)}$.



c) State the domain and range of the combined function.

d) Use your knowledge of trigonometric identities to state the equation of the function $y = \frac{g(x)}{f(x)}$ as a single trigonometric function.

10.3 Composite Functions, pages 345–355

10. Let $f(x) = x - 3$ and $g(x) = 1 - x^2$. Determine each of the following.

a) $(f \circ g)(x)$

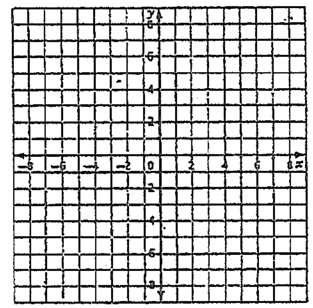
b) $(g \circ g)(x)$

c) $(f \circ g)(-3)$

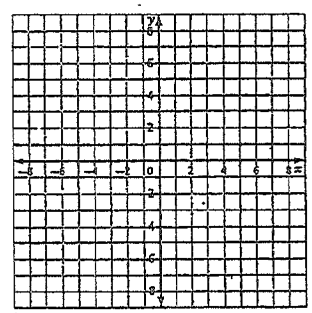
d) $(g \circ g)(2)$

11. Let $f(x) = x^2 - 9$ and $g(x) = \sqrt{x}$.

a) Sketch the graph of $y = f(g(x))$ and state its domain and range.



b) Sketch the graph of $y = g(f(x))$ and state its domain and range.



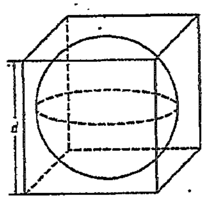
12. Given that $h(x) = (f \circ g)(x)$, determine $g(x)$.

a) $h(x) = \sqrt{9 - x}$ and $f(x) = \sqrt{x}$

b) $h(x) = \dfrac{12}{(7x - 2)^2}$ and $f(x) = \dfrac{12}{x^2}$

c) $h(x) = 4x^2 - 20x + 25$ and $f(x) = x^2$

13. The side length, d, of a cube that contains a sphere depends on the radius, r, of the sphere. Assume that the faces of the cube are tangent to the sphere.



a) Write the side length of the cube as a function of the radius of the sphere.

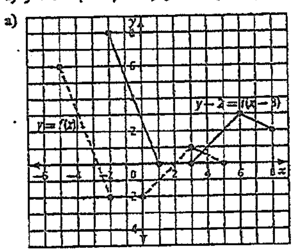
b) Write the volume of the cube as a function of the radius of the sphere.

c) What is the volume of a cube that contains a sphere of radius 7.5 cm?

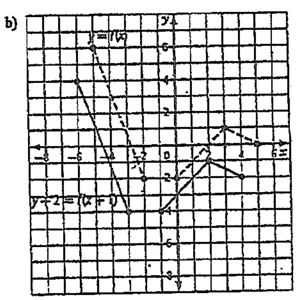
2. a) $f^{-1}(x) = x + 4$ b) $f^{-1}(x) = -\frac{1}{6}x - \frac{1}{3}$
   c) $f^{-1}(x) = \frac{5}{3}x + 5$ d) $f^{-1}(x) = 2x - 6$
3. Examples: a) $\{x \mid x \geq 2, x \in \mathrm{R}\}$ or $\{x \mid x \leq 2, x \in \mathrm{R}\}$
   b) $\{x \mid x \geq -4, x \in \mathrm{R}\}$ or $\{x \mid x \leq -4, x \in \mathrm{R}\}$
4. a) For $f(x) = -x^2 + 6, x \geq 0$, the inverse is $f^{-1}(x) = \sqrt{-(x-6)}$. For $f(x) = -x^2 + 6, x \leq 0$, the inverse is $f^{-1}(x) = -\sqrt{-(x-6)}$.
   b) For $f(x) = \frac{1}{2}x^2 + 4, x \geq 0$, the inverse is $f^{-1}(x) = \sqrt{2(x-4)}$. For $f(x) = \frac{1}{2}x^2 + 4, x \leq 0$, the inverse is $f^{-1}(x) = -\sqrt{2(x-4)}$.
5. $y = \pm\sqrt{x+2} - 3$
6. a) $42 < x < 105$
   b) $f^{-1}(x) = \sqrt{\dfrac{x}{0.01634}} + 26.643$, where x = CRL, in millimetres
   c) 14.3 weeks
7. Answers may vary.

Chapter 1 Review, pages 35–37

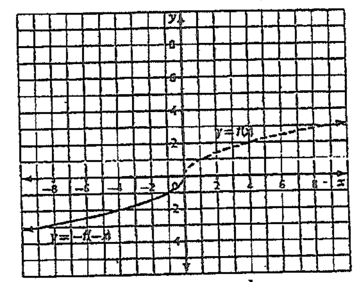
1. a) $y + 3 = |x - 5|$ b) $y - 1 = |x + 4|$
2. a)



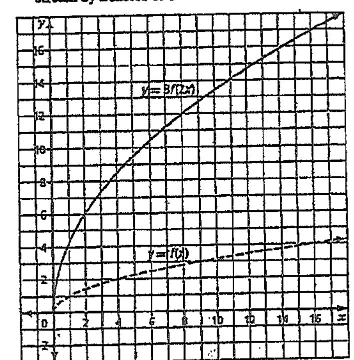
b)



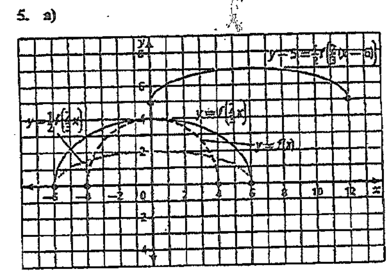
3. a) (12, 5) b) (−3, −5) c) (36, −10)
4. a) reflection in the y-axis and reflection in the x-axis

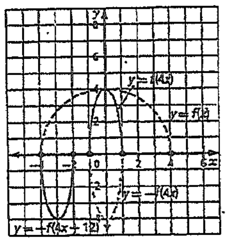


b) horizontal stretch by a factor of $\frac{1}{2}$, vertical stretch by a factor of 3



5. a)



b) 

6. a) $f^{-1}(x) = -2x + 10$

b) Example: restricted domain of $f(x)$: $\{x \mid x \geq 1, x \in R\}, f^{-1}(x) = \sqrt{\frac{1}{2}x} + 1$

Chapter 2

2.1 Radical Functions and Transformations, pages 39–46

1. a) vertical stretch by a factor of 3, reflection in the y-axis, translation 4 units left and 2 units down; domain: $\{x \mid x \leq -4, x \in R\}$; range: $\{y \mid y \geq -2, y \in R\}$

b) vertical stretch by a factor of 2, reflection in the x-axis, horizontal stretch by a factor of $\frac{1}{4}$, translation of 3 units right and 5 units up; domain: $\{x \mid x \geq 3, x \in R\}$; range: $\{y \mid y \leq 5, y \in R\}$

c) vertical stretch by a factor of 4, horizontal stretch by a factor of $\frac{1}{5}$, translation of 1 unit left and 4 units down; domain: $\{x \mid x \geq -1, x \in R\}$; range: $\{y \mid y \geq -4, y \in R\}$

d) horizontal stretch by a factor of $\frac{1}{3}$, reflection in the x-axis and y-axis, translation 2 units left; domain: $\{x \mid x \leq -2, x \in R\}$; range: $\{y \mid y \leq 0, y \in R\}$

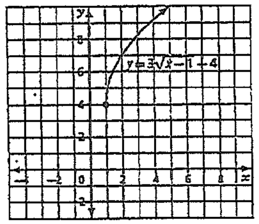
2. a) $y = -3\sqrt{x-4} - 2$

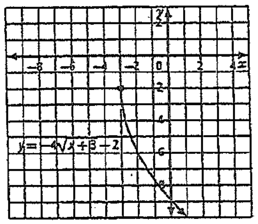
b) $y = \sqrt{-4(x+5)} + 3$

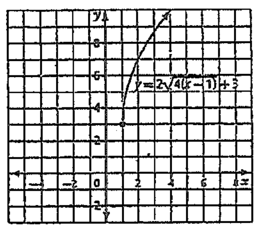
c) $y = 2\sqrt{\frac{1}{3}(x+4)} + 1$

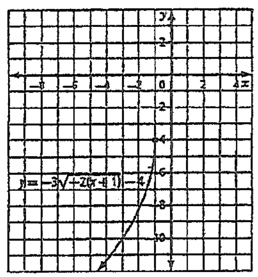
d) $y = -3\sqrt{-2(x+6)}$

3. a) B b) C c) D d) A

4. a) 

b) 

c) 

d) 

3.4 Equations and Graphs of Polynomials Functions, pages 91–102

1. a) $x = 0, -2, \frac{1}{2}$

b) $x = -1, 3, 5$

c) $x = 2$

2. a) $f(x) = -2(x-1)(x+1)(x-3); -1, 1, 3$

b) $f(x) = 0.5(x-2)^2(x+1)(x+3); -1, -3, 2$

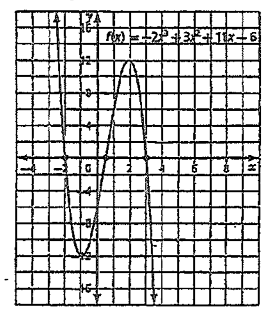
c) $f(x) = -0.2(x-2)^3(x+4)^2; -4, 2$

3. a) -4 and 5; positive for $-4 < x < 5$; negative for $x < -4$ and $x > 5$; -4 (multiplicity 1) and 5 (multiplicity 3); the function changes sign at both, but is flatter at $x = 5$

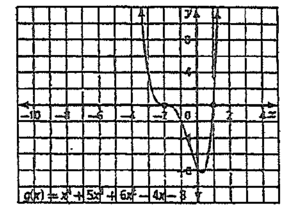
b) -6 and 3; positive for $-6 < x < 3$ and $x > 3$; negative for $x < -6$; -6 (multiplicity 3) and 3 (multiplicity 2); the function changes sign at $x = -6$, but not at $x = 3$

c) $-4, -1$, and 3; positive for $x < -4, -4 < x < -1$, and $x > 3$; negative for $-1 < x < 3$; -4 (multiplicity 2), -1 (multiplicity 1), and 3 (multiplicity 1); the function changes sign at $x = -1$ and at $x = 3$, but not at $x = -4$

4. a) x-intercepts: $-2, 0.5, 3$ (all of multiplicity 1); y-intercept: -6



b) x-intercepts: -2 (multiplicity 3) and 1 (multiplicity 1); y-intercept: -8



5. a) $f(x) = (x+4)(x-1)(x+2)$

b) $f(x) = -(2x+1)(x-3)(x+2)$

c) $f(x) = -0.25(x+2)^2(x-3)^3$

6. a) $a = \frac{1}{2}$; vertical stretch by a factor of $\frac{1}{2}$

$b = 3$; horizontal stretch by a factor of $\frac{1}{3}$

$h = -4$; translation of 4 units left

$k = -5$; translation of 5 units down

b) domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in R\}$

7. a) $y = -\frac{3}{4}(x-2)^2(x+5)$

b) $y = (x+1)^2(x-3)(x+2)^2$

8. 26 ft by 46 ft

9. h and k; these parameters represent the horizontal translation and the vertical translation, respectively, of the graph and do not change its shape or orientation.

Chapter 3 Review, pages 103–107

1.

Polynomial Function	Degree	Type	Leading Coefficient	Constant Term
a) $f(x) = -2x^4 - x^3 + 3x - 7$	4	Quartic	-2	-7
b) $y = 3x^5 + 2x^4 - x^3 + 3$	5	Quintic	3	3
c) $g(x) = 0.5x^3 - 8x^2$	3	Cubic	0.5	0
d) $p(x) = 10$	0	Constant	0	10

2. a) even degree; negative leading coefficient; 2 x-intercepts; domain: $\{x \mid x \in R\}$; range: $\{y \mid y \leq 19, y \in R\}$

b) odd degree; positive leading coefficient; 3 x-intercepts; domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in R\}$

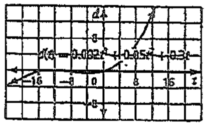
3. a)

b) leading coefficient: 0.002; constant: 0; The constant represents the distance that the boat is from the shore at time 0 s (the initial position of the boat).

c) degree: 3; positive leading coefficient; extends from quadrant III to I

d) domain: $\{t \mid t \geq 0, t \in R\}$; it is impossible to have negative time

e) When $t = 15$, $d(15) = 22.5$. After 15 s, the boat is 22.5 m from the shore.

f) 

4. a) $\frac{5x^3 - 7x^2 - x + 6}{x - 1} = 5x^2 - 2x - 3 + \frac{3}{x - 1}$

b) $x \neq 1$

c) $(x - 1)(5x^2 - 2x - 3) + 3 = 5x^3 - 7x^2 - x + 6$

5. a) $R = 9$ b) $R = 15$ c) $R = 41$ d) $R = 595$

6. a) $m = 4$ b) 28

7. $P(x) = x^3 + 2x^2 - 15x + 10$

8. a) $x - 7$ b) $x + 6$ c) $x - c$

9. a) Yes b) No

10. a) ±1, ±3, ±9, ±27

b) ±1, ±2, ±3, ±4, ±6, ±9, ±12, ±18, ±36

11. a) $(x - 3)(x - 2)(x + 1)$

b) $(x - 4)(x + 2)(3x + 1)$

c) $(x - 3)(x - 1)(x + 6)(5x + 2)$

d) $x(x - 2)(x + 2)(2x + 5)$

12. $x - 1$, $x + 2$, and $5x + 2$

13. a) degree 5; negative leading coefficient; −3 (multiplicity 2) and 1 (multiplicity 3); the function changes sign at $x = 1$, but not at $x = -3$; positive for $x < -3$ and $-3 < x < 1$; negative for $x > 1$; $f(x) = -0.25(x + 3)^2(x - 1)^3$

b) degree 4; positive leading coefficient; −2 (multiplicity 1), −0.5 (multiplicity 1), and 2 (multiplicity 2); the function changes sign at $x = -2$ and at $x = -0.5$, but not at $x = 2$; positive for $x < -2$, $-0.5 < x < 2$, and $x > 2$; negative for $-2 < x < -0.5$; $f(x) = 0.5(x + 2)(2x + 1)(x - 2)^2$

14. a) $a = -2$; vertical stretch by a factor of 2 and reflection in the x-axis

$b = \frac{1}{3}$; horizontal stretch by a factor of 3

$h = 1$; translation of 1 unit to the right

$k = 4$; translation of 4 units up

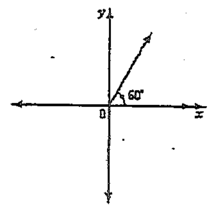
b) domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in R\}$

15. $y = -3(x + 2)^2(x - 3)$

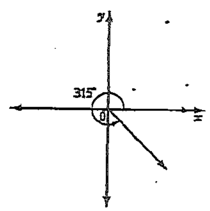
Chapter 4

4.1 Angles and Angle Measure, pages 109–119

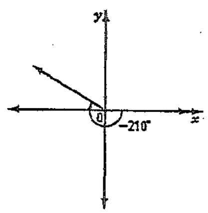
1. a) $\frac{\pi}{3}$

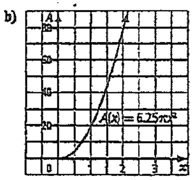


b) $\frac{7\pi}{4}$



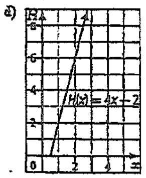
c) $-\frac{7\pi}{6}$



13. b) 

domain: $\{x \mid x \geq 0, x \in R\}$;
range: $\{A \mid A \geq 0, A \in R\}$

c) $H(x) = 4x - 2$

d) 

domain: $\{x \mid x \geq 0.5, x \in R\}$;
range: $\{H \mid H \geq 0, H \in R\}$

10.3 Composite Functions, pages 345–355

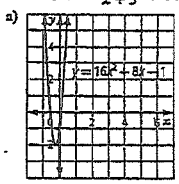
1. a) 0 b) 2 c) 6 d) 5

2. a) 23 b) −13 c) 62 d) 11

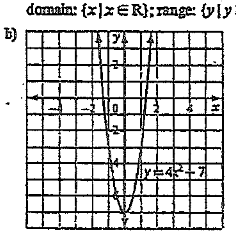
3. a) $(f \circ g)(x) = \sqrt{x^2 + 4}$; $(g \circ f)(x) = x + 4$

b) $(f \circ g)(x) = |-1 - x|$; $(g \circ f)(x) = 3 - |x - 4|$

c) $(f \circ g)(x) = \frac{1}{x + 3}$, $(g \circ f)(x) = \frac{1}{x} + 3$

4. a) 

domain: $\{x \mid x \in R\}$; range: $\{y \mid y \geq -2, y \in R\}$

b) 

domain: $\{x \mid x \in R\}$; range: $\{y \mid y \geq -7, y \in R\}$

5. a) $g(x) = x - 2$ b) $g(x) = x^2 - 2$ c) $g(x) = x - 5$

6. a) $(f \circ g)(x) = \sqrt{3x - 3}$; domain: $\{x \mid x \geq 1, x \in R\}$

b) $(g \circ f)(x) = 3\sqrt{x - 4} + 1$; domain: $\{x \mid x \geq 4, x \in R\}$

7. a) $f(g(x)) = x + 4$ b) $g(f(x)) = x + 4$

c) restriction on domain of $f(g(x))$: $x \neq 1$; restriction on domain of $g(f(x))$: $x \neq \pm 1$

8. a) $h(k(x)) = k(h(x)) = \frac{1}{x^2}$ b) $x \neq 0$

9. a) $C(x(t)) = 787.5t + 900$

b) $C(x(t))$ represents the cost of manufacturing engines after t hours of production.

c) $7200 d) 42 h

10. a) $R = p - 1200$ b) $D = 0.9p$

c) $(R \circ D)(p) = 0.9p - 1200$; The composite function represents the cost of the snowmobile when the dealer discount is computed before the factory rebate.

d) $(D \circ R)(p) = 0.9p - 1080$; The composite function represents the cost of the snowmobile when the factory rebate is subtracted before the dealer discount.

e) $8475; $8595; The lower cost is given by the composite function $(R \circ D)(p)$. Example: The price is lower when the $1200 is subtracted after the dealer discount.

11. The sales representative's bonus is represented by $g(f(x)) = 0.5(x - 50\,000)$. Example: The bonus is computed after the $50 000 is subtracted.

12. a) $f(x) = x^2$; $g(x) = 2x + 1$

b) $f(x) = \sqrt{x}$; $g(x) = 9 - x$

13. a) Example: $f(x) = 2x$, $g(x) = x - 3$, $h(x) = 5x + 1$

b) $((f \circ g) \circ h)(x) = 10x - 4$; $(f \circ (g \circ h))(x) = 10x - 4$; Therefore, the composition of functions does follow the associative property.

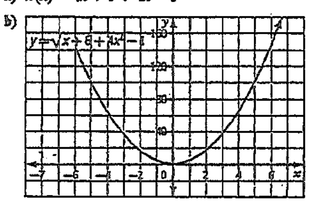
14. a) Example: $f(x) = x^2$, $g(x) = 3x - 2$

b) $(f \circ g)(x) = 9x^2 - 12x + 4$, $(g \circ f)(x) = 3x^2 - 2$; Therefore, the composition of functions does not follow the commutative property. There are no restrictions in this case.

Chapter 10 Review, pages 356–362

1. a) 6 b) −13

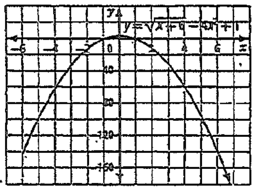
2. a) $h(x) = \sqrt{x + 6} + 4x^2 - 1$

b) 

c) domain: $\{x \mid x \geq -6, x \in R\}$; range: $\{y \mid y \geq 1.4, y \in R\}$

d) $k(x) = \sqrt{x+6} - 4x^2 + 1$

e)

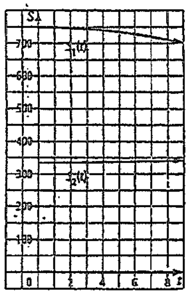


f) domain: $\{x \mid x \geq -6, x \in R\}$; range: $\{y \mid y \leq 3.5, y \in R\}$

3. a) $g(x) = -4x^2 + 11x - 3$

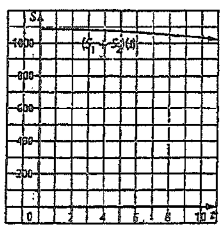
b) $g(x) = -2x - \sqrt{x} + 12$

4. a)



b) $(S_1 + S_2)(t) = 1085 - 0.6t^2 + 0.8t$

c)



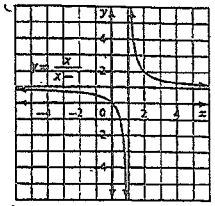
d) The total sales have been decreasing. Example: The combined sales in 2007 were $1 061 200. The combined sales in 2012 were $1 008 200.

5. a) $h(x) = -2x^3 + x^2 - 6x + 3$

b) $k(x) = \frac{x^2+3}{1-2x^2}, x \neq \frac{1}{2}$

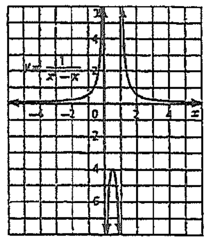
6. a) 1 b) 12 c) −3 d) $\frac{5}{9}$

7. a) $h(x) = \frac{x}{x-1}$



domain: $\{x \mid x \neq 1, x \in R\}$

b) $k(x) = \frac{1}{x^2 - x}$

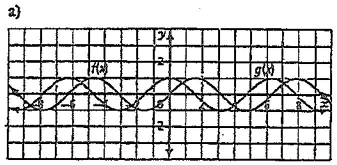


domain: $\{x \mid x \neq 0, 1, x \in R\}$

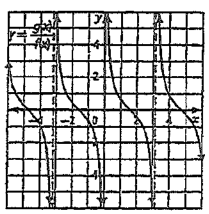
8. a) $g(x) = x - 1$ b) $g(x) = \sin x$

c) $g(x) = -x^2$

9. a)



b)



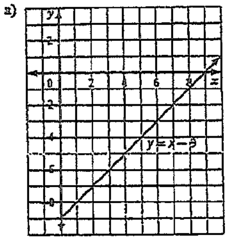
c) domain: $\{x \mid x \neq n\pi, n \in I, x \in R\}$; range: $\{y \mid y \in R\}$

d) $y = \cot x$

10. a) $(f \circ g)(x) = -x^2 - 2$ b) $(g \circ g)(x) = -x^4 + 2x^2$

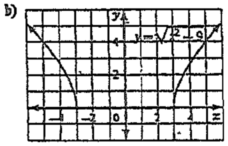
c) −11 d) −8

11. a)



domain: $\{x \mid x \geq 0, x \in R\}$; range: $\{y \mid y \geq -9, y \in R\}$

b)



domain: $\{x \mid x \leq -3 \text{ and } x \geq 3, x \in R\}$; range: $\{y \mid y \geq 0, y \in R\}$

12. a) $g(x) = 9 - x$

b) $g(x) = 7x - 2$

c) $g(x) = 2x - 5$

13. a) $d = 2r$ b) $V = 8r^3$ c) 3375 cm³

14. a) $(C \circ n)(t) = -15t^2 + 4125t + 1195$

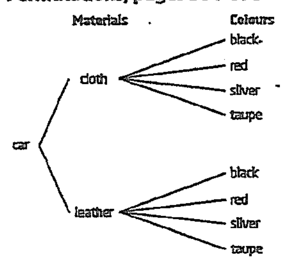
b) $70 585

c) 24 h

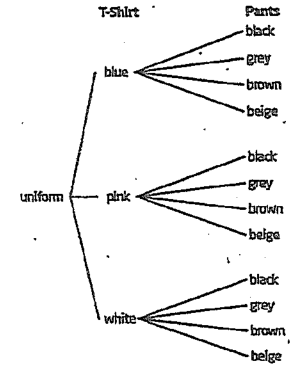
Chapter 11

11.1 Permutations, pages 364–373

1. a)



b)



2. a)

First	Second Digit
3	4
3	6
3	7
3	8
4	3
4	6
4	7
4	8
6	3
6	4
6	7
6	8
7	3
7	4
7	6
7	8
8	3
8	4
8	6
8	7

b)

Toronto to Calgary	Calgary to Vancouver
Plane	Bus
Plane	Plane
Plane	Train
Plane	Car
Train	Bus
Train	Plane
Train	Train
Train	Car

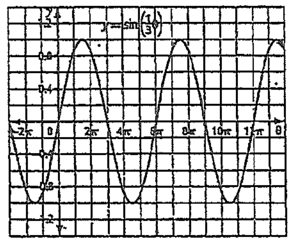
Chapter 4 Review, pages 145–147

1. a) $\frac{3\pi}{2}$ b) 300°
   c) $\frac{5\pi}{3}$ d) $\frac{-720°}{\pi}$
   e) $\frac{11\pi}{4}$ f) 585°
2. Examples:
   a) $\frac{23\pi}{6}, -\frac{\pi}{6}$,
   general form: $\frac{11\pi}{6} \pm 2\pi n, n \in N$
   b) 345°, −735°,
   general form: $-375° \pm (360°)n, n \in N$
3. a) 6.3 b) 28.6°
4. $-\frac{\sqrt{5}}{3}$
5. a) $\frac{\pi}{3}$ b) $\frac{7\pi}{4}$
6. 56°, 304°, 416°, 664°
7. a) $-\sqrt{3}$ b) $-\frac{2}{\sqrt{3}}$
8. $\theta_1 \approx 128.7° + 360°n, n \in I$;
   $\theta_2 \approx 231.3° + 360°n, n \in I$
9. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Chapter 5

5.1 Graphing Sine and Cosine Functions, pages 149–157

1. a) 2 b) $\frac{1}{4}$
   c) 5 d) 3
2. a) 360°, 2π b) 180°, π
   c) 1440°, 8π d) 240°, $\frac{4\pi}{3}$
3. a) $2\pi; \frac{1}{2}$ b) $\frac{2\pi}{3}; 1$
   c) $\frac{\pi}{2}; 2$ d) 6π; 1.5
4. a) For $y = \sin\theta$:
   amplitude: 1; maximum value: 1; minimum value: −1; period: 2π; θ-intercepts: $\pi n, n \in I$; y-intercept: 0
   For $y = \sin\left(\frac{1}{3}\theta\right)$:
   amplitude: 1; maximum value: 1; minimum value: −1; period: 6π; θ-intercepts: $3\pi n, n \in I$; y-intercept: 0



   b) For $y = \sin\theta$:
   amplitude: 1; maximum value: 1; minimum value: −1; period: 2π; θ-intercepts: $\pi n, n \in I$; y-intercept: 0
   For $y = 1.5\sin(2\theta)$:
   amplitude: 1.5; maximum value: 1.5; minimum value: −1.5; period: π; θ-intercepts: $\frac{\pi}{2}n, n \in I$; y-intercept: 0