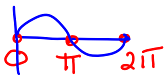
Review of Trigonometry II

1. State the restriction(s) for each in the domain $0 \le x \le 2\pi$.

a. $\dfrac{1}{\sin x}$ b. $\sec x$ c. $\cot x$ d. $\dfrac{\csc x \cot x}{\sin x \tan x}$

Restrictions
① $\csc x \neq \phi$
② $\cot x \neq \phi$
③ $\sin x \neq 0$
④ $\tan x \neq 0$
⑤ $\tan x \neq \phi$

a. $\sin x \neq 0$

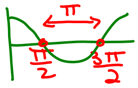


$x \neq 0, \pi, 2\pi$

b. $\sec x \neq \phi$

$\sec x = \dfrac{1}{\cos\theta}$

$\therefore$ $\sec x$ is undefined when $\cos\theta = 0$

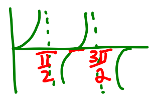


$\therefore x \neq \frac{\pi}{2}, \frac{3\pi}{2}$

c. $\cot x \neq \phi$

$\cot x = \dfrac{1}{\tan x}$

$\therefore$ $\cot x$ is undefined when $\tan x = 0$



$x \neq \frac{\pi}{2}, \frac{3\pi}{2}$

- since $\csc x = \dfrac{1}{\sin x}$, $\csc x = \phi$ when $\sin x = 0$

so $x = 0, \pi, 2\pi$ which satisfies restrictions ① and ③

- since $\cot x = \dfrac{1}{\tan x}$, $\cot x$ is undefined when $\tan x = 0$, so $x \neq 0, \pi, 2\pi$ which satisfies restrictions ② and ④

- $\tan x$ is undefined at $x = \frac{\pi}{2}, \frac{3\pi}{2}$

Combining all the answers

$x \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

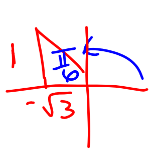
2. Find the restrictions for $\dfrac{\sin x \cos x}{3\tan x + \sqrt{3}}$, $x \in \Re$.

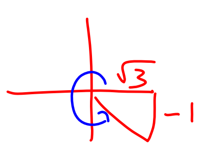
$\tan x \neq \phi$ at the asymptotes $x \neq \frac{\pi}{2} + \pi n$

$3\tan x + \sqrt{3} \neq 0$

$\tan x \neq \dfrac{-\sqrt{3}}{3} \times \dfrac{\sqrt{3}}{\sqrt{3}}$ unrationalize to get special Δ

$\tan x \neq -\dfrac{1}{\sqrt{3}}$





$2\pi - \frac{\pi}{\ } = \frac{11\pi}{\ }$

$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\qquad 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ $\qquad \tan x \neq -\frac{1}{\sqrt{3}}$

don't need to include this answer since you can get it from the first answer $\frac{5\pi}{6} + \pi = \frac{11\pi}{6}$

$\therefore x \neq \frac{5\pi}{6} + \pi n$

Combining the answers $\quad X \neq \frac{5\pi}{6} + \pi n$

$X \neq \frac{\pi}{2} + \pi n$

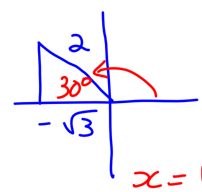
3. Solve.

a. $10 \cos x = 6 \cos x - \sqrt{12}$, $0 \le x \le 360^\circ$

$10 \cos x - 6\cos x + \sqrt{12} = 0$

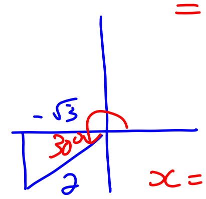
$4\cos x = -\sqrt{12}$

$\cos x = \frac{-\sqrt{12}}{4}$



$x = 180^\circ - 30^\circ$
$= 150^\circ$

$\cos x = -\frac{2\sqrt{3}}{4}$



$\cos x = -\frac{\sqrt{3}}{2}$

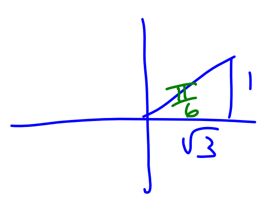
$x = 180^\circ + 30^\circ$
$= 210^\circ$

$x = 150^\circ, 210^\circ$

b. $3 \sin x + 2 - \sqrt{3} \cos x = 2$, $0 \le x \le 2\pi$

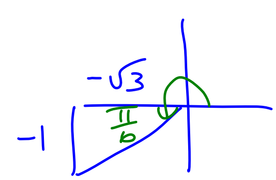
$3\sin x = \sqrt{3} \cos x$

$\frac{\sin x}{\cos x} = \frac{\sqrt{3}}{3}$



$x = \frac{\pi}{6}$

$\tan x = \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{\sqrt{3}}$



$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\tan x = \frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$

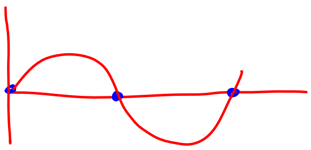
4. Solve $6 \sin x \cos^2 x + 11 \sin x \cos x + 4 \sin x = 0$, $0 \le x \le 360^\circ$.

$\sin x \cdot (6\cos^2 x + 11\cos x + 4) = 0$

$$\sin x\,(6\cos^2 x + 11\cos x + 4) = 0$$

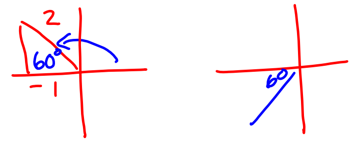
$$\sin x\,(3\cos x + 4)(2\cos x + 1) = 0$$

$\sin x = 0$ $\qquad$ $\cos x = -\frac{4}{3}$ $\qquad$ $\cos x = \frac{-1}{2}$



No solution

$-1 \le \cos x \le 1$



$x = 0, 180°, 360°$

$180° - 60°$ $\qquad$ $180 + 60°$

$x = 120°$ $\qquad$ $x = 240°$

$$x = 0, 120°, 180°, 240°, 360°$$

5. Solve $\sec\frac{1}{4}x + \sqrt{2} = 0$, $0 \le x \le 12\pi$.

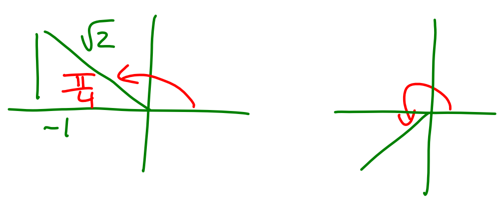
HE by 4 $\qquad$ $\sec\frac{1}{4}x = -\sqrt{2}$

$\therefore$ Period $= 4 \cdot 2\pi = 8\pi$

First find $\sec\theta = -\sqrt{2}$ and then multiply answers by 4 and then add the period

$\sec\theta = -\sqrt{2}$

$\cos\theta = \frac{-1}{\sqrt{2}}$



$\pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\qquad$ $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$\frac{3\pi}{4}$ and $\frac{5\pi}{4}$ are solutions to $\sec\theta = -\sqrt{2}$

$X_1 = \frac{3\pi}{4} \times 4 = 3\pi$ and $X_2 = \frac{5\pi}{4} \times 4 = 5\pi$ are solutions to $\sec\frac{1}{4}x = -\sqrt{2}$, Add the period to find remaining answers.

$3\pi + 8\pi = 11\pi$

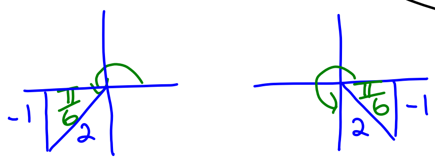
$5\pi + 8\pi = 13\pi \leftarrow$ Don't include since it is more than 12π

$\therefore x = 3\pi, 5\pi, 11\pi$

6. Find the general solution for:

a. $2\sin\theta + 1 = 0$ | **b.** $\tan^2\theta - 1 = 0$

$\sin\theta = \frac{-1}{2}$



$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$ $\qquad$ $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

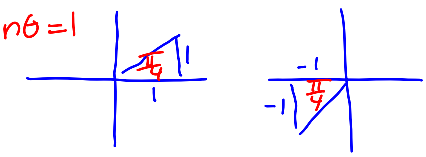
$$\theta = \left.\begin{array}{l} \frac{7\pi}{6} + 2\pi n \\ \frac{11\pi}{6} + 2\pi n \end{array}\right\} n \in I$$

b) $\tan^2\theta = 1$

$\tan\theta = \pm\sqrt{1}$

$\tan\theta = \pm 1$

$\tan\theta = 1$

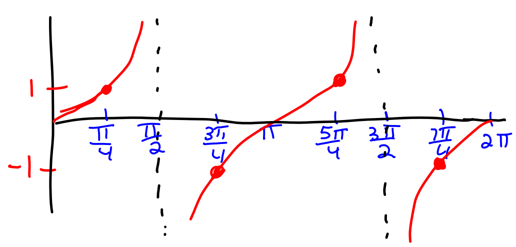


$\theta = \frac{\pi}{4}$ $\qquad$ $\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$\tan\theta = -1$

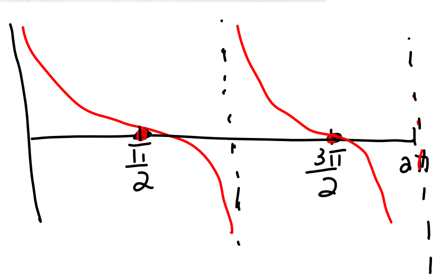
$\pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\qquad$ $2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$\therefore \theta = \frac{\pi}{4} + \frac{\pi}{2}n, \; n \in I$



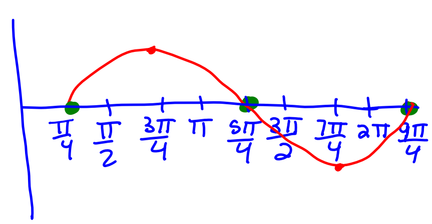
7. Find the general solution for:

a. $\cot\theta = 0$ $\qquad$ **b.** $\sin\left(\theta - \frac{\pi}{4}\right) = 0$



$\theta = \frac{\pi}{2} + \pi n, \; n \in I$

b) $\sin\left(\theta - \frac{\pi}{4}\right) = 0$



$\theta = \frac{\pi}{4} + \pi n, \; n \in I$

8. Prove $\cos^2 x\cos^2 y+\sin^2 x\sin^2 y+\sin^2 x\cos^2 y+\sin^2 y\cos^2 x=1$.

Factor out $\sin^2 x$

Factor out $\cos^2 x$

$$\cos^2 x\underbrace{(\cos^2 y+\sin^2 y)}_{1} + \sin^2 x\underbrace{(\sin^2 y+\cos^2 y)}_{1}$$

$$\cos^2 x+\sin^2 x$$

$$1$$

9. Prove $\sec^2 x-\sec^2 y=\tan^2 x-\tan^2 y$.

$$\tan^2 x-1 - (\tan^2 +1)$$

$$\tan^2 x+1-\tan^2 y-1$$

$$\tan^2 x-\tan^2 y$$

10. Show that $1+\tan^2 x=\sec^2 x$.

$$1+\frac{\sin^2 x}{\cos^2 x}$$

$$\frac{\cos^2 x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}$$

$$\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$$

$$\frac{1}{\cos^2 x}$$

$$\sec^2 x$$

11. prove $\dfrac{1+\sin x}{\tan x}=\cot x+\cos x$.

$$\frac{1+\sin x}{\frac{\sin x}{\cos x}}$$

$$(1+\sin x)\cdot\frac{\cos x}{\sin x}$$

$$\frac{\cos x}{\sin x}+\frac{\cos x(\sin x)}{1(\sin x}$$

$$\frac{\cos x+\cos x\sin x}{\sin x}$$

$$\frac{\cos x(1+\sin x)}{\sin x}$$

$$(1+\sin x)\cdot\frac{\cos x}{\sin x} \quad \Bigg| \quad \frac{\cos x\,(1+\sin x)}{\sin x}$$

$$\frac{\cos x\,(1+\sin x)}{\sin x}$$

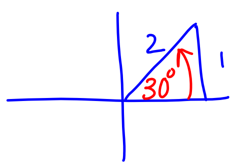
12. Solve $\sec x \cot x - 2 = 0.\ \ 0 \le x \le 810°$.

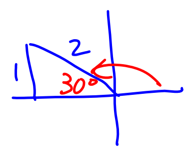
$$\frac{1}{\cancel{\cos x}}\cdot\frac{\cancel{\cos x}}{\sin x} - 2 = 0$$

$$\frac{1}{\sin x} = 2$$

$$\csc x = 2$$

$$\sin x = \frac{1}{2}$$





$x_1 = 30°$

$x_2 = 180° - 30°$
$= 150°$

$x_3 = 30° + 360°$
$= 390°$

$x_4 = 150° + 360°$
$= 510°$

$x_5 = 390° + 360°$

$x_5 = 750°$

$x_6 = 510° + 360°$
$= 870°$

Not less than 810° so don't count

$x = 30°, 150°, 390°, 510°, 750°$

13. Describe the features of the graph of $\sin^2 x + \cos^2 x$.

It is the horizontal line $y=1$

14. Prove $\cot x - \cot 2x = \csc 2x$.

$$\frac{\cos x}{\sin x} - \frac{\cos 2x}{\sin 2x} \quad \Bigg| \quad \frac{1}{\sin 2x}$$

$$\frac{(2\cos x)}{(2\cos x)}\frac{\cos x}{\sin x} - \frac{\cos 2x}{2\sin x\cos x} \quad \Bigg| \quad \frac{1}{2\sin x\cos x}$$

$$\frac{2\cos^2 x - (2\cos^2 x - 1)}{2\sin x\cos x}$$

$$\frac{2\cos^2 x - 2\cos^2 x + 1}{2\sin x\cos x}$$

$$\frac{2\cos^2 x - 2\cos^2 x + 1}{2\sin x\cos x}$$

$$\frac{1}{2\sin x\cos x}$$

15. Use $x = \pi$ to show that $\sin 2x = 2\sin x\cos x$.

$$\sin 2\pi = 2\sin\pi\cos\pi$$

$$0 \quad \Big| \quad = 2(0)(-1)$$

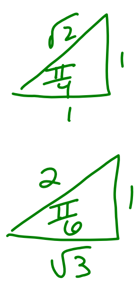
$$= 0$$

16. Find the exact value of $\sin\frac{\pi}{12}$.

$$\frac{\pi}{6} = \frac{2\pi}{12}$$

$$\frac{\pi}{4} = \frac{3\pi}{12}$$

$$\frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{12}$$



$$\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

$$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \cdot \frac{(\sqrt{2})}{(\sqrt{2})}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

17. If $\sin A = -\frac{2}{3}$ and $\cos B = \frac{1}{5}$, where $\frac{\pi}{2} \le A \le \frac{3\pi}{2}$ and $\pi \le B \le 2\pi$, then the exact value of $\sin(A + B)$ is

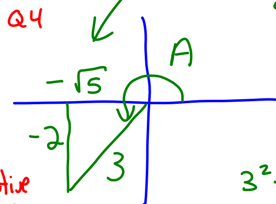
Q2 and Q3

Q3 and Q4

since $\sin A$ is negative $\angle A$ is in Q3

since $\cos B$ is postive $\angle B$ is in Q4

Don't forget negative



$3^2 - (-2)^2 = x^2$

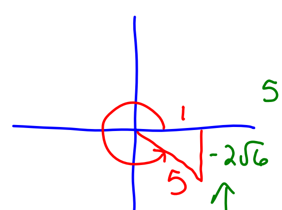
$\sqrt{5} = x$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$= \left(\frac{-2}{3}\right)\left(\frac{1}{5}\right) + \left(\frac{-\sqrt{5}}{3}\right)\left(\frac{-2\sqrt{6}}{5}\right)$$

$$= \frac{-2}{15} + \frac{2\sqrt{30}}{15}$$

$$= \frac{2\sqrt{30} - 2}{15}$$



$5^2 - 1^2 = y^2$

$\sqrt{24} = y$

$2\sqrt{6} = y$

Don't forget negative

15

Answers to Review of Trigonometry II

1a. $x = 0, \pi, 2\pi$ **b.** $x = \frac{\pi}{2}, \frac{3\pi}{2}$ **c.** $x = 0, \pi, 2\pi$ **d.** $x = 0, \pi, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2}$

2. $x = \frac{5\pi}{6} + n\pi, n \in I$ **3a.** $x = 150°, 210°$ **b.** $\frac{\pi}{6}, \frac{7\pi}{6}$ **4.** $x = 0, 120°, 180°, 240°, 360°$

5. $x = 3\pi, 5\pi, 11\pi$ **6a.** $\frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi, n \in I$ **b.** $\frac{\pi}{4} + \frac{n\pi}{2}, n \in I$

7a. $\frac{\pi}{2} + n\pi, n \in I$ **b.** $\frac{\pi}{4} + n\pi, n \in I$ **10.** $1 + \frac{\sin^2 x}{\cos^2 x}$, $\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$, $\frac{1}{\cos^2 x}$,

12. 30°, 150°, 390°, 510°, 750° **13.** a horizontal line at $y = 1$ ($\sin^2 x + \cos^2 x = 1$)

15. $\sin 2\pi = 2 \sin \pi = 0 = 0$ **16.** $\frac{\sqrt{6} - \sqrt{2}}{4}$ **17.** $\frac{2\sqrt{30} - 2}{15}$