1. Prove the identity. (3 marks)

$$\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$$

Solution ①

Left side	Right side
$\frac{\sin^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{1}\cdot\frac{\cos^2\theta}{\cos^2\theta}$	$\frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{\sin^2\theta}{1}$
$\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$	$\frac{\sin^2\theta\cdot\sin^2\theta}{\cos^2\theta}$
$\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$	
$\frac{\sin^2\theta\cdot\sin^2\theta}{\cos^2\theta}$	

1. Prove the identity. (3 marks)

$$\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$$

Solution ②

Left side	Right side
	$(\sec^2\theta - 1)\sin^2\theta$
	$\sec^2\theta\sin^2\theta - \sin^2\theta$
	$\frac{1}{\cos^2\theta}\cdot\frac{\sin^2\theta}{1} - \sin^2\theta$
	$\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$
	$\tan^2\theta - \sin^2\theta$

2. Prove the identity. (2 marks)

$$\frac{\sec\theta - \cos\theta}{\tan\theta} = \sin\theta$$

Method ①

Left side	Right side
$\frac{\cos\theta}{1}\cdot\frac{\cos\theta}{\cos\theta}$	

Method (1)

Left side	Right side
$\dfrac{\frac{1}{\cos\theta} - \frac{\cos\theta}{1}\cdot\frac{\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$	
$\dfrac{1-\cos^2\theta}{\cos\theta}\cdot\dfrac{\cos\theta}{\sin\theta}$	
$\dfrac{\sin^2\theta}{1}\cdot\dfrac{1}{\sin\theta}$	
$\dfrac{\sin\theta\cdot\sin\theta}{\sin\theta}$	
$\sin\theta$	

2. Prove the identity. (2 marks)

$$\frac{\sec\theta - \cos\theta}{\tan\theta} = \sin\theta$$

Method (2)

Left side	Right side
$\dfrac{\frac{1}{\cos\theta} - \cos\theta}{\frac{\sin\theta}{\cos\theta}}$	
$\left(\dfrac{1}{\cos\theta} - \cos\theta\right)\cdot\dfrac{\cos\theta}{\sin\theta}$	
$\dfrac{\cos\theta}{\cos\theta\sin\theta} - \dfrac{\cos^2\theta}{\sin\theta}$	
$\dfrac{1}{\sin\theta} - \dfrac{\cos^2\theta}{\sin\theta}$	
$\dfrac{1-\cos^2\theta}{\sin\theta}$	
$\dfrac{\sin^2\theta}{\sin\theta}$	
$\sin\theta$	

3. Prove the identity (2 marks)

$$\frac{\cos\theta + \sin\theta\tan\theta}{\sin\theta\sec\theta} = \csc\theta$$

Method ①

Left side	Right side
$\frac{\cos\theta}{1} + \frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\cos\theta}$ $\frac{\sin\theta}{1}\cdot\frac{1}{\cos\theta}$ $\frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$ $\frac{\sin\theta}{\cos\theta}$ $\frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}$ $\frac{1}{\sin\theta}$ $\csc\theta$	

3. Prove the identity (2 marks)

$$\frac{\cos\theta + \sin\theta\tan\theta}{\sin\theta\sec\theta} = \csc\theta$$

Method ②

Left side	Right side
$\cos\theta + \frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\cos\theta}$ $\frac{\sin\theta}{1}\cdot\frac{1}{\cos\theta}$ $\left(\frac{\cos\theta + \sin^2\theta}{\cos\theta}\right)\cdot\frac{\cos\theta}{\sin\theta}$ $\frac{\cos^2\theta}{\sin\theta} + \frac{\sin^2\theta}{\sin\theta}$ $\frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$ $\frac{1}{\sin\theta}$ $\csc\theta$	

4. Prove the identity (2 marks)

$$\frac{1}{1+\sin\theta} = \sec^2\theta - \frac{\tan\theta}{\cos\theta}$$

Method ①

Left side	Right side
	$\frac{1}{\cos^2\theta} - \frac{\frac{\sin\theta}{\cos\theta}}{\cos\theta}$
	$\frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$
	$\frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos^2\theta}$
	$\frac{1-\sin\theta}{\cos^2\theta}$
	$\frac{1-\sin\theta}{1-\sin^2\theta}$
	$\frac{1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$
	$\frac{1}{1+\sin\theta}$

4. Prove the identity (2 marks)

$$\frac{1}{1+\sin\theta} = \sec^2\theta - \frac{\tan\theta}{\cos\theta}$$

Method ②

Left side	Right side
	$\frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos\theta} \div \cos\theta$
	$\frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$
	$\frac{1-\sin\theta}{\cos^2\theta} \quad \frac{(1+\sin\theta)}{(1+\sin\theta)}$
	$\frac{1-\sin^2\theta}{\cos^2\theta\,(1+\sin\theta)}$
	$\frac{\cos^2\theta}{\cos^2\theta\,(1+\sin\theta)}$
	$\frac{1}{1+\sin\theta}$

5. Prove the following identity. (2 marks)

$$\sin\theta + \cos\theta\cot\theta = \csc\theta$$

Left side	Right side
$\sin\theta + \cos\theta\left(\frac{\cos\theta}{\sin\theta}\right)$	
$\frac{(\sin\theta)\sin\theta}{(\sin\theta)1} + \frac{\cos^2\theta}{\sin\theta}$	
$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$	
$\frac{1}{\sin\theta}$	
$\csc\theta$	

6. Prove the following identity (3 marks)

$$\frac{\cot\theta}{\csc\theta - 1} = \frac{\csc\theta + 1}{\cot\theta}$$

Method ①

Left side	Right side
	$\frac{\csc\theta + 1}{\cot\theta}\cdot\frac{(\csc\theta - 1)}{(\csc\theta - 1)}$
	$\frac{\csc^2\theta - 1}{\cot\theta(\csc\theta - 1)}$
	$\frac{\cot^2\theta}{\cot\theta(\csc\theta - 1)}$
	$\frac{\cot\theta}{\csc\theta - 1}$

6. Prove the following identity (3 marks)

$$\frac{\cot\theta}{\csc\theta-1} = \frac{\csc\theta+1}{\cot\theta}$$

Method (2)

Left side	Right side
$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}-1}\left(\frac{\sin\theta}{\sin\theta}\right)$	$\frac{\frac{1}{\sin\theta}+1}{\frac{\cos\theta}{\sin\theta}}$
$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1-\sin\theta}{\sin\theta}}$	$\left(\frac{1}{\sin\theta}+1\right)\frac{\sin\theta}{\cos\theta}$
$\frac{\cos\theta}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{1-\sin\theta}$	$\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$
$\frac{\cos\theta}{1-\sin\theta}$	$\frac{1+\sin\theta}{\cos\theta}\frac{(1-\sin\theta)}{(1-\sin\theta)}$
	$\frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}$
	$\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$
	$\frac{\cos\theta}{1-\sin\theta}$

7. Prove the identity. (3 marks)

$$\frac{\sin\theta+\tan\theta}{1+\cos\theta} = \frac{\sin 2\theta}{2\cos^2\theta}$$

Left side	Right side
$\frac{(\cos\theta)}{(\cos\theta)}\frac{\sin\theta+\frac{\sin\theta}{\cos\theta}}{1+\cos\theta}$	$\frac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos^{\cancel{2}}\theta}$
$\frac{\cos\theta\sin\theta+\sin\theta}{\frac{\cos\theta}{1+\cos\theta}}$	$\frac{\sin\theta}{\cos\theta}$
$\frac{\sin\theta(\cos\theta+1)}{\cos\theta}\cdot\frac{1}{\cancel{1+\cos\theta}}$	
$\frac{\sin\theta}{\cos\theta}$	

$\frac{}{c} \cdot 1-$

$\frac{\sin\theta}{\cos\theta}$

8. Prove the identity (2 marks)

$$\frac{\csc\theta}{\tan\theta+\cot\theta} = \cos\theta$$

Left side	Right side
$\frac{1}{\sin\theta}$	
$\frac{1}{\frac{(\sin\theta)}{(\sin\theta)}\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\frac{(\cos\theta)}{(\cos\theta)}}$	
$\frac{1}{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}$	
$\frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta\cos\theta}}$	
$\frac{1}{\sin\theta} \cdot \frac{\sin\theta\cos\theta}{1}$	
$\cos\theta$	

9. Prove the identity. (2 marks)

$$\frac{1-\cos\theta}{\sin^2\theta} = \frac{1}{1+\cos\theta}$$

Method (1)

Left side	Right side
$\frac{1-\cos\theta}{1-\cos^2\theta}$	
$\frac{1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$	
$\frac{1}{1+\cos\theta}$	

9. Prove the identity. (2 marks)

$$\frac{1-\cos\theta}{\sin^2\theta} = \frac{1}{1+\cos\theta}$$

Method (2)

Left side	Right side
	$\frac{1}{(1+\cos\theta)}\frac{(1-\cos\theta)}{(1-\cos\theta)}$
	$\frac{1-\cos\theta}{1-\cos^2\theta}$
	$\frac{1-\cos\theta}{\sin^2\theta}$

10. Prove the identity. (3 marks)

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

Left side	Right side
$\frac{2\sin\theta\cos\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta}$	$\frac{1}{\sin\theta}$
$\frac{2\sin\theta(\sin\theta)}{1(\sin\theta)} + \frac{\cos 2\theta}{\sin\theta}$	
$\frac{2\sin^2\theta + \cos 2\theta}{\sin\theta}$	
$\frac{2\sin^2\theta + 1 - 2\sin^2\theta}{\sin\theta}$	
$\frac{1}{\sin\theta}$	

10. Prove the identity. (3 marks)

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

Method 2

Left side	Right side
$\frac{2\sin\theta\cos\theta}{\cos\theta} + \frac{\cos^2\theta - \sin^2\theta}{\sin\theta}$	
$\frac{(\sin\theta)}{\sin\theta}\frac{2\sin\theta}{1} + \frac{\cos^2\theta - \sin^2\theta}{\sin\theta}$	
$\frac{2\sin^2\theta + \cos^2\theta - \sin^2\theta}{\sin\theta}$	
$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$	
$\frac{1}{\sin\theta}$	
$\csc\theta$	

10. Prove the identity. (3 marks)

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

Method 3

Left side	Right side
$\frac{2\sin\theta\cos\theta}{\cos\theta} + \frac{2\cos^2\theta - 1}{\sin\theta}$	
$\frac{(\sin\theta)2\sin\theta}{(\sin\theta)1} + \frac{2\cos^2\theta - 1}{\sin\theta}$	
$\frac{2\sin^2\theta + 2\cos^2\theta - 1}{\sin\theta}$	
$\frac{2(\sin^2\theta + \cos^2\theta) - 1}{\sin\theta}$	
$\frac{2(1) - 1}{\sin\theta}$	
$\frac{1}{\sin\theta}$	
$\csc\theta$	

11. Prove: (3 marks)

$$\frac{\sin 2\theta}{2 - 2\cos^2\theta} = \cot\theta$$

Left side	Right side
$\frac{2\sin\theta\cos\theta}{2(1 - \cos^2\theta)}$	
$\frac{\sin\theta\cos\theta}{\sin^2\theta}$	
$\frac{\cos\theta}{\sin\theta}$	
$\cot\theta$	

12. Prove: (4 marks)

$$\frac{\sin\theta\cos\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\tan\theta}$$

Left side	Right side

$$\frac{1-\cos\theta}{\tan\theta}\cdot\frac{(1+\cos\theta)}{(1+\cos\theta)}$$

$$\frac{1-\cos^2\theta}{\tan\theta\,(1+\cos\theta)}$$

$$\frac{\sin^2\theta}{\frac{\sin\theta}{\cos\theta}(1+\cos\theta)}$$

$$\frac{\sin^2\theta}{1+\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin\theta\cos\theta}{1+\cos\theta}$$

13. Prove the identity. (3 marks)

$$\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$$

Method ①

Left side	Right side

$$(1+\cot^2 x)(1+\tan^2 x)$$

$$1+\tan^2 x+\cot^2 x+\cot^2 x\tan^2 x$$

$$1+\sec^2 x-1+\csc^2 x-1+\frac{1}{\tan^2 x}\cdot\frac{\tan^2 x}{1}$$

$$1+\sec^2 x-1+\csc^2 x-1+1$$

$$\sec^2 x+\csc^2 x$$

13. Prove the identity. (3 marks)

$$\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$$

Method ②

Left side	Right side
$\frac{\cos^2 x}{\cos^2 x}\cdot\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}\cdot\frac{\sin^2 x}{\sin^2 x}$	$\frac{1}{\sin^2 x}\cdot\frac{1}{\cos^2 x}$
$\frac{\cos^2 x + \sin^2 x}{\cos^2 x \sin^2 x}$	$\frac{1}{\cos^2 x \sin^2 x}$
$\frac{1}{\cos^2 x \sin^2 x}$	

14. Prove the identity: (3 marks)

$$\frac{1}{\sec\theta + \tan\theta} = \frac{1-\sin\theta}{\cos\theta}$$

Left side	Right side
$\frac{1}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$	
$\frac{1}{\frac{1+\sin\theta}{\cos\theta}}$	
$1 \cdot \frac{\cos\theta}{1+\sin\theta}$	
$\frac{\cos\theta}{(1+\sin\theta)}\cdot\frac{(1-\sin\theta)}{(1-\sin\theta)}$	
$\frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$	
$\frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$	

$1-\sin^2\theta$

$$\frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$$

$$\frac{1-\sin\theta}{\cos\theta}$$

15. Prove the identity: (3 marks)

$$\frac{\cos 2\theta}{\sin\theta} = \frac{\cot^2\theta - 1}{\csc\theta}$$

Method ①

Left side	Right side
	$\dfrac{\dfrac{\cos^2\theta}{\sin^2\theta} - 1}{\dfrac{1}{\sin\theta}}$
	$\left(\dfrac{\cos^2\theta}{\sin^2\theta} - 1\right)\dfrac{\sin\theta}{1}$
	$\dfrac{\cos^2\theta}{\sin\theta} - \dfrac{\sin\theta}{1}\dfrac{\sin\theta}{\sin\theta}$
	$\dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta}$
	$\dfrac{\cos 2\theta}{\sin\theta}$

15. Prove the identity: (3 marks)

$$\frac{\cos 2\theta}{\sin\theta} = \frac{\cot^2\theta - 1}{\csc\theta}$$

Method ②

Left side	Right side
	$\dfrac{\csc^2\theta - 1 - 1}{\dfrac{1}{\sin\theta}}$
	$(\csc^2\theta - 2)\cdot\dfrac{\sin\theta}{1}$
	$\csc^2\theta\sin\theta - 2\sin\theta$
	$\dfrac{1}{\sin\theta}\cdot\sin\theta - \dfrac{2\sin\theta}{1}\dfrac{(\sin\theta)}{(\sin\theta)}$

$\frac{1}{\sin^2\theta} \cdot \sin\theta - \frac{2\sin\theta}{1}\frac{(\sin\theta)}{(\sin\theta)}$

$\frac{1 - 2\sin^2\theta}{\sin\theta}$

$\frac{\cos 2\theta}{\sin\theta}$

16. Prove the identity: (4 marks)

$$\frac{\cot\theta - 1}{1 - \tan\theta} = \frac{\csc\theta}{\sec\theta}$$

Method ①

Left side	Right side
$\frac{\frac{1}{\tan\theta} - \frac{1}{1}\frac{(\tan\theta)}{(\tan\theta)}}{1 - \tan\theta}$	$\frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}}$
$\frac{1-\tan\theta}{\tan\theta} \cdot \frac{1}{1-\tan\theta}$	$\frac{1}{\sin\theta} \cdot \frac{\cos\theta}{1}$
$\frac{1}{\tan\theta}$	$\frac{\cos\theta}{\sin\theta}$
$\cot\theta$	$\cot\theta$

16. Prove the identity: (4 marks)

$$\frac{\cot\theta - 1}{1 - \tan\theta} = \frac{\csc\theta}{\sec\theta}$$

Method ②

Left side	Right side
$\frac{\frac{\cos\theta}{\sin\theta} - \frac{1}{1}\frac{(\sin\theta)}{(\sin\theta)}}{\frac{(\cos\theta)}{(\cos\theta)}1 - \frac{\sin\theta}{\cos\theta}}$	
$\frac{\cos\theta - \sin\theta}{\sin\theta}$	

$\frac{\cos\theta - \sin\theta}{\sin\theta}$

$\frac{\cos\theta - \sin\theta}{\cos\theta}$

$\frac{1}{\frac{\sin\theta}{\cos\theta}}$

$\frac{1}{\frac{\cos\theta}{\cos\theta}}$

$\frac{\csc\theta}{\sec\theta}$

17. Prove the identity: (4 marks)

$$(1-\sin\theta)(\sec\theta+\tan\theta) = \frac{1}{\sec\theta}$$

Method (1)

Left side	Right side
$(1-\sin\theta)\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)$	$\frac{1}{\frac{1}{\cos\theta}}$
$\frac{(1-\sin\theta)}{1}\left(\frac{1+\sin\theta}{\cos\theta}\right)$	$1 \cdot \frac{\cos\theta}{1}$
$\frac{1-\sin^2\theta}{\cos\theta}$	$\cos\theta$
$\frac{\cos^2\theta}{\cos\theta}$	
$\cos\theta$	

17. Prove the identity: (4 marks)

$$(1-\sin\theta)(\sec\theta+\tan\theta) = \frac{1}{\sec\theta}$$

Method (2)

Left side	Right side
$\sec\theta+\tan\theta-\sin\theta\sec\theta-\sin\theta\tan\theta$	
$\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta}$	
$\frac{1-\sin^2\theta}{\cos\theta}$	
$\frac{\cos^2\theta}{\cos\theta}$	

$\cos\theta$

$\frac{\cos^2\theta}{\cos\theta}$

$\cos\theta$

$\frac{1}{\sec\theta}$

18. Prove the identity: (5 marks)

$\sin 2x(\tan x + \cot x) = 2$

Left side	Right side
$\sin 2x \left(\frac{(\sin x)}{(\sin x)}\frac{\sin x}{\cos x} + \frac{\cos x(\cos x)}{\sin x(\cos x)}\right)$	
$\sin 2x \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)$	
$2\sin x\cos x \left(\frac{1}{\sin x\cos x}\right)$	
2	

19. Prove the identity: (5 marks)

$$\frac{\cot\theta}{\sin\theta - \csc\theta} = -\sec\theta$$

Left side	Right side
$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{(\sin\theta)}{(\sin\theta)}\frac{\sin\theta}{1} - \frac{1}{\sin\theta}}$	
$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin^2\theta - 1}{\sin\theta}}$	

$\frac{\sin^2\theta - 1}{\sin\theta}$

$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{-\cos^2\theta}{\sin\theta}}$

$\frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{-\cos^2\theta}$

$\frac{-1}{\cos\theta}$

$-\sec\theta$

20. Prove: (5 marks)

$$\frac{\sin 2x}{1+\cos 2x} = \frac{\sec^2 x - 1}{\tan x}$$

Left side	Right side
$\frac{2\sin x\cos x}{1+2\cos^2 x-1}$	$\frac{\tan^2 x + 1 - 1}{\tan x}$
$\frac{2\sin x\cos x}{2\cos^2 x}$	$\frac{\tan^2 x}{\tan x}$
$\frac{\sin x}{\cos x}$	$\tan x$
$\tan x$	

21. Prove: (5 marks)

$$\frac{2\cos x + +2\cos^2 x}{\sin 2x} = \frac{\sin x}{1-\cos x}$$

Left side	Right side
$\frac{2\cos x(1+\cos x)}{2\sin x\cos x}$	$\frac{(\sin x)(1+\cos x)}{(1-\cos x)(1+\cos x)}$
	$\sin x(1+\cos x)$

$\frac{2\sin x\cos x}{1+\cos x}$

$(1-\cos x)(1+\cos x)$

$\frac{\sin x(1+\cos x)}{1-\cos^2 x}$

$\frac{\sin x(1+\cos x)}{\sin^2 x}$

$\frac{1+\cos x}{\sin x}$