SHOW ALL WORK FOR FULL MARKS

1. Use long division to determine the remainder and the quotient for:

$$\left(3x^3 - 2x^2 - 20x - 4\right) \div (x + 3)$$

$$\begin{array}{r|l} & 3x^2 - 11x + 13 \\ \hline x+3 & 3x^3 - 2x^2 - 20x - 4 \\ & 3x^3 + 9x^2 \\ \hline & -11x^2 - 20x \\ & -11x^2 - 33x \\ \hline & 13x - 4 \\ & 13x + 39 \\ \hline & -43 \end{array}$$

$R = -43$

$Q = 3x^2 - 11x + 13$

1) ______________________

3 marks

2. Use synthetic division to determine remainder and quotient for:

$$\left(x^4 + 2x^3 - 3x^2 + x - 1\right) \div (x - 2)$$

$$\begin{array}{r|rrrrr} 2 & 1 & 2 & -3 & 1 & -1 \\ & & 2 & 8 & 10 & 22 \\ \hline & 1 & 4 & 5 & 11 & 21 \end{array}$$

$Q = x^3 + 4x^2 + 5x + 11$

$R = 21$

2) ______________

3 marks

3. For $P(x) = 3x^3 + 6x^2 - 5x + 4$ determine $P(-1)$

$P(-1) = 3(-1)^3 + 6(-1)^2 - 5(-1) + 4$
$= -3 + 6 + 5 + 4$
$= 12$

3) $P(-1) = 12$

2 marks

$x = -2$

4. Is $P(-2)$ a root of $P(x) = x^3 - 2x^2 - 5x + 6$? Explain why or why not. (SHOW ALL WORK)

$P(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$
$= -8 - 8 + 10 + 6$
$= -16 + 16$
$= 0$

only if $P(-2) = 0$

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -5 & 6 \\ & \downarrow & -2 & 8 & -6 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

Remainder is zero
∴ -2 is a root

yes since $P(-2)=0$ it means the remainder is zero when $P(x)$ is divided by $x+2$ ∴ $x=-2$

4) ______________

2 marks

5a. If $x + 8$ is a factor of the polynomial $P(x)$, which of the following must be true?

A. $P(-8) = 0$

B. $P(8) = 0$

C. $P(x) = 8$

D. $P(x) = -8$

5a) ______________

1 mark

5b. If $4x - 1$ is a factor of $P(x)$, which of the following must have a value of 0?

A. $P(-1)$

B. $P(1)$

C. $P\left(\frac{1}{4}\right)$

D. $P\left(-\frac{1}{4}\right)$

5b) ______________________

1 mark

5c. If a polynomial $P(x)$ is divided by $x-7$, which of the following represents the remainder?

A. $P(0)$

B. $P(x+7)$

C. $P(-7)$

D. $P(7)$

5c) ______________________

1 mark

5d. Which of the following is not a polynomial?

$a)$ $-3x^7-2x^5-6$

$b)$ $-4x^3-3x^2-3$

$c)$ -8

$d)$ $5x^{-4}+3\sqrt{x}+2$

5d) ______________________

1 mark

6. Find the remainder when $3x^{45}+4x^8-5x^3+2$ is divided by $x+1$.

$3(-1)^{45}+4(-1)^8-5(-1)^3+2$

$-3+4+5+2$

8

6) 12

3 marks

7. Determine the quotient when $5x^3-6x^2+64$ is divided by $x+2$.

$$\begin{array}{r|rrrr} -2 & 5 & -6 & 0 & 64 \\ & & -10 & 32 & -64 \\ \hline & 5 & -16 & 32 & 0 \end{array}$$

$5x^2-16x+32$

7) ______________________

3 marks

8. When $x^3 + x^2 - kx - 5$ is divided by $x - 2$ the remainder is 1. Find the value of k.

$2^3 + 2^2 - 2k - 5 = 1$

$8 + 4 - 2k - 5 = 1$

$7 - 2k = 1$

$6 = 2k$

$3 = k$

8) ______________________

3 marks

9. When the polynomial $mx^3 - nx^2 + 5x - 1$ is divided by $x + 2$ the remainder is -39. When the polynomial is divided by $x - 1$ the remainder is 3. Find the values of m and n.

$m(-2)^3 - n(-2)^2 + 5(-2) - 1 = -39$

$-8m - 4n - 11 = -39$

$-8m - 4n = -28$

$2m + n = 7$

$m(1)^3 - n(1)^2 + 5(1) - 1 = 3$

$m - n + 4 = 3$

$m - n = -1$

$2m + n = 7$

$m - n = -1$

$3m = 6$

$m = 2$

$m - n = -1$

$2 - n = -1$

$3 = n$

9) ______________________

4 marks

10. When a $x^3 + ax^2 + 2x + 9$ is divided by $x - 1$ the remainder is 7. What is the remainder when $x^3 + ax^2 + 2x + 9$ is divided by $x + 1$?

$1^3 + a(1)^2 + 2(1) + 9 = 7$

$a + 12 = 7$

$a = -5$

$P(-1) = (-1)^3 - 5(-1)^2 + 2(-1) + 9$

$= -1 - 5 - 2 + 9$

$= 1$

10) $R = 1$

3 marks

11. According to the Rational Zero Theorem, list all possible rational roots of $P(x) = 8x^4 - 3x^2 + 4x - 1$

$p = \pm 1$

$q = \pm 1, \pm 2, \pm 4, \pm 8$

$\frac{p}{q} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}$

11) ______________________

2 marks

12. Use any method to factor each of the following completely.

a) $3x^4 - 45x^2 - 48$

$3(x^4 - 15x^2 - 16)$

$3(x^2 - 16)(x^2 + 1)$

$3(x - 4)(x + 4)(x^2 + 1)$

a) ______________________

3 marks

b) $2x^4 - 62x^2 + 60x$

$$2x(x^3 - 31x + 30)$$

$$\begin{array}{r|rrrr} +1 & 1 & 0 & -31 & 30 \\ & & 1 & 1 & -30 \\ \hline & 1 & 1 & -30 & 0 \end{array}$$

$$2x(x-1)(x^2+x-30)$$

$$2x(x-1)(x+6)(x-5)$$

$\pm 1, \pm 2, \pm 5, \pm 10$

$\pm 1, \pm 2, \pm 3, \pm 6$

b) ______________________

3 marks

c) $10x^3 - 21x^2 - x + 6$

$$\begin{array}{r|rrrr} 2 & 10 & -21 & -1 & 6 \\ & & 20 & -2 & -6 \\ \hline & 10 & -1 & -3 & 0 \end{array}$$

$$\frac{p}{q} = \pm 1, \pm 2 \pm 3, \pm 6$$

$$\pm \frac{1}{2}, \pm \frac{1}{5}, \pm \frac{1}{10}$$

$$\pm \frac{2}{5}, \pm \frac{3}{2}, \pm \frac{3}{5}, \pm \frac{3}{10}$$

$$\pm \frac{6}{5}$$

$$(x-2)(10x^2 - x - 3)$$

$$(x-2)(5x-3)(2x+1)$$

c) ______________________

3 marks

$q = \pm 1$

$p = \pm 1, \pm 2, \pm 3 \pm 4 \pm 6, \pm 12$

d) $x^4 - 3x^3 + 8x^2 - 18x + 12$

$\frac{p}{q} = p$

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & 8 & -18 & 12 \\ & & 1 & -2 & 6 & 12 \\ \hline & 1 & -2 & 6 & -12 & 0 \end{array}$$

$$(x-1)(x^3 - 2x^2 + 6x - 12)$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 6 & -12 \\ & & 2 & 0 & 12 \\ \hline & 1 & 0 & 6 & 0 \end{array}$$

$$(x-1)(x-2)(x^2+6)$$

d) ______________________

3 marks

13. The volume in cubic meters of water in an aquarium is given by the polynomial
    a). $V(x) = x^3 - 16x^2 + 79x - 120$. If the depth in feet can be represented by x - 3, what are the possible dimensions of the rectangular aquarium in terms of x?.

$$\begin{array}{r|rrrr} 3 & 1 & -16 & 79 & -120 \\ & & 3 & -39 & +120 \\ \hline & 1 & -13 & 40 & 0 \end{array}$$

$(x-3)(x^2-13x+40)$

$(x-3)(x-8)(x-5)$

13a) $x-3$ by $x-8$ by $x-5$

1 marks

b). If the aquarium holds 70 cubic metres, what are the dimensions of the aquarium?

$70 = x^3 - 16x^2 + 79x - 120$

$0 = x^3 - 16x^2 + 79x - 190$

$\frac{p}{q} = \pm 1, \pm 2, \pm 5, \pm 10, \pm 19, \pm 190$

$$\begin{array}{r|rrrr} 10 & 1 & -16 & 79 & -190 \\ & & 10 & -60 & 190 \\ \hline & 1 & -6 & 19 & 0 \end{array}$$

$\therefore x = 10$ and the dimensions are $10-3$ by $10-8$ by $10-5$

$(x-10)(x^2-6x+19) = 0$

$\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(19)}}{2(1)}$ $\mathbb{R}$ undefined

13b) 7 by 2 by 5

2 marks

14. Solve each of the following polynomial equations algebraically. Answer in exact form.

a) $6x^3 + 7x^2 - 5x = 0$

$x(6x^2 + 7x - 5) = 0$

$x(2x-1)(3x+5) = 0$

$x = 0 \quad x = \frac{1}{2} \quad x = -\frac{5}{3}$

a) $x = -\frac{5}{3}, 0, \frac{1}{2}$

3 marks

b) $x^3 - 3x^2 + 16x - 48 = 0$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 18, \pm 24, \pm 48$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 16 & -48 \\ & & 3 & 0 & 48 \\ \hline & 1 & 0 & 16 & 0 \end{array}$$

$(x-3)(x^2+16) = 0$

$x = 3$

b) ______________ 3 marks

c) $x^4 + 4x^3 + x^2 - 6x = 0$

$\pm 1, \pm 2, \pm 3, \pm 6$

$x(x^3 + 4x^2 + x - 6) = 0$

$x(x-1)(x^2 + 5x + 6) = 0$

$x(x-1)(x+3)(x+2) = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 1 & -6 \\ & & 1 & 5 & 6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

c) $x = 0, 1, -3, -2$ 3 marks

d) $4x^3 - 4x^2 - 21x - 9 = 0$

$\pm 1, \pm 2, \pm 4$

$\pm 1, \pm 3 \pm 9$

$\frac{p}{q} = \pm 1, \pm 3, \pm 9, \pm \frac{1}{2} \pm \frac{1}{4} \pm \frac{3}{2} \pm \frac{3}{4} \pm \frac{9}{2} \pm \frac{9}{4}$

$$\begin{array}{r|rrrr} 3 & 4 & -4 & -21 & -9 \\ & & 12 & 24 & 9 \\ \hline & 4 & 8 & 3 & 0 \end{array}$$

$(x-3)(4x^2 + 8x + 3) = 0$

$(x-3)(2x+1)(2x+3) = 0$

d) $x = 3, -\frac{1}{2}, -\frac{3}{2}$ 3 marks

e) $x^5 + 4x^4 - 8x^3 - 10x^2 + 23x - 10 = 0$

$q = \pm 1$

$\frac{p}{q} = p = \pm 1, \pm 2, \pm 5, \pm 10$

$$\begin{array}{r|rrrrrr} 1 & 1 & 4 & -8 & -10 & 23 & -10 \\ & & 1 & 5 & -3 & -13 & 10 \\ \hline & 1 & 5 & -3 & -13 & 10 & 0 \end{array}$$

$(x-1)(x^4 + 5x^3 - 3x^2 - 13x + 10) = 0$

$$\begin{array}{r|rrrrr} 1 & 1 & 5 & -3 & -13 & 10 \\ & & 1 & 6 & 3 & -10 \\ \hline & 1 & 6 & 3 & -10 & 0 \end{array}$$

$(x-1)(x-1)(x^3 + 6x + 3x - 10) = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 6 & 3 & -10 \\ & & 1 & 7 & 10 \\ \hline & 1 & 7 & 10 & 0 \end{array}$$

$(x-1)(x-1)(x-1)(x^2 + 7x + 10) = 0$

$(x-1)^3(x+5)(x+2) = 0$

$x = 1, -2, -5$

e) ______________________

3 marks

4^{th}

15. Write a polynomial equation with the following roots. A quartic function with roots of -3, -1 and 4 {multiplicity of 2} and passes through the point $(5,16)$.

$p(x) = a(x+1)(x+3)(x-4)^2$

$p(5) = 16 = a(5+1)(5+3)(5-4)^2$

$16 = a(6)(8)(1)$

$\frac{16}{48} = a$

$\frac{1}{3} = a$

15) $P(x) = \frac{1}{3}(x+1)(x+3)(x-4)^2$

3 marks

16. Sketch the graph of each polynomial function without a calculator. Clearly show the zeros and the y-intercept. **Show algebraically how you determined the zeros and y-intercept.**

a) $f(x) = 2x^3 - x^2 - 2x + 1$ $\quad \pm 1, \pm 2 \quad \pm 1 \quad \frac{p}{q} = \pm 1, \pm \frac{1}{2}$

$$\begin{array}{r|rrrr} 1 & 2 & -1 & -2 & +1 \\ & & 2 & 1 & -1 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$

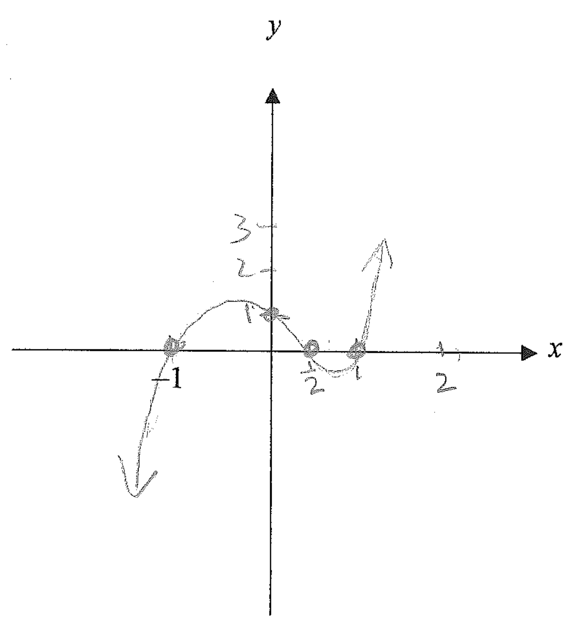
$f(x) = (x-1)(2x^2+x-1)$

$f(x) = (x-1)(2x-1)(x+1)$

$\underbrace{x = 1, \quad x = \frac{1}{2} \quad x = -1}_{x\text{-intercepts}}$

$f(0) = 2(0)^3 - 0^2 - 2(0) + 1$

$= 1$ ← y-intercept



3 marks

opens down

$q = \pm 1$

$p = \pm 1, \pm 2, \pm 3, \pm 4, \pm 12$

b) $f(x) = -x^4 + 4x^3 + x^2 - 16x + 12$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 12$

$\underbrace{x = 3, -1, 2, -2}_{x\text{-intercepts}}$

$f(0) = 12 \leftarrow$ y-intercept

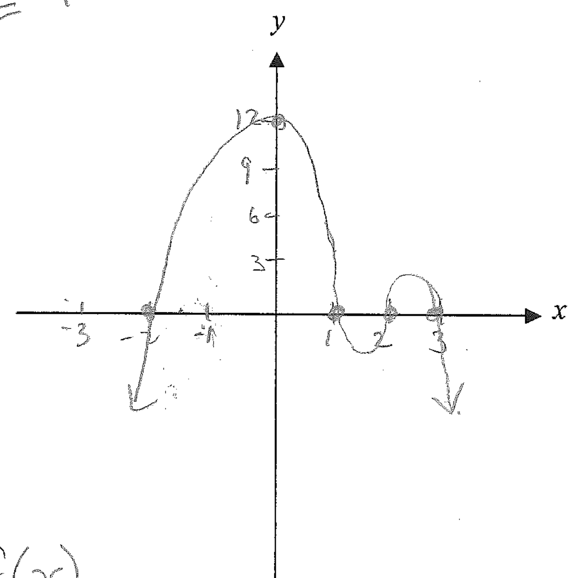
$$\begin{array}{r|rrrrr} 3 & -1 & 4 & 1 & -16 & 12 \\ & & -3 & 3 & 12 & -12 \\ \hline & -1 & 1 & 4 & -4 & 0 \end{array}$$

$(x-3)(-x^3 + x^2 + 4x - 4) = f(x)$

$$\begin{array}{r|rrrr} 1 & -1 & 1 & 4 & -4 \\ & & -1 & 0 & 4 \\ \hline & -1 & 0 & 4 & 0 \end{array}$$

$(x-3)(x-1)(-x^2 + 4)$

$(x-3)(x-1)(2-x)(2+x) = f(x)$



3 marks

c) $f(x) = x^4 - 4x^3 - 10x^2 + 28x - 15$ $\frac{p}{q} = \pm 1, \pm 3, \pm 5, \pm 15$

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & -10 & 28 & -15 \\ & & 1 & -3 & -13 & 15 \\ \hline & 1 & -3 & -13 & 15 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -13 & 15 \\ & & 1 & -2 & -15 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$(x-1)(x^3 - 3x^2 - 13x + 15) = f(x)$

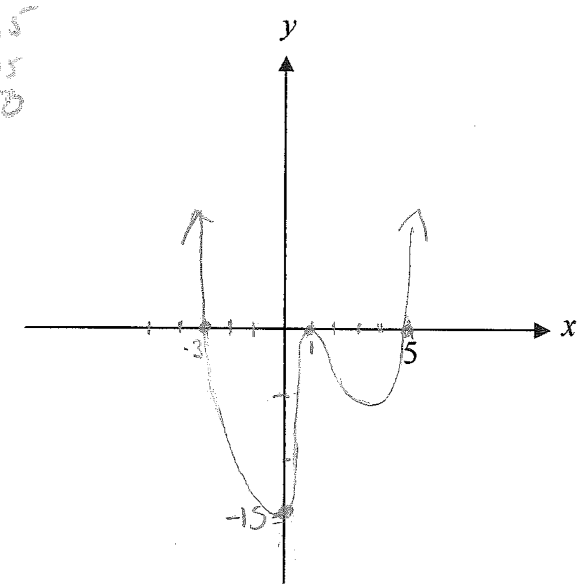
$(x-1)(x+1)(x^2 - 2x - 15) = f(x)$

$(x-1)^2(x-5)(x+3) = f(x)$

bounces at $x = 1$

$x = 5$ $x = -3$

x-intercepts



$f(0) = -15$ y-intercept

3 marks

skip

17. Use your graphing calculator to determine the following for the polynomial function:
$f(x) = x^4 - 4x^3 - 2x^2 + 5x + 9$ *(8 marks)*

a) Domain a) ______________

$x \in R$

b) Range b) ______________

c) The zeros c) ______________

d) Y-intercept d) ______________

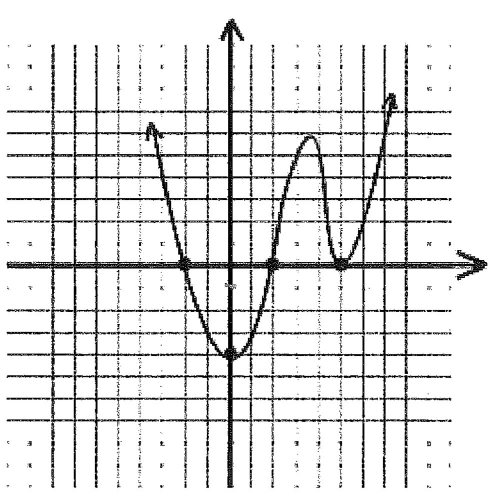
e) coordinates of relative maximum e) ______________

f) coordinates of relative minimum f) ______________

g) intervals where $f(x) \geq 0$ g) ______________

h) intervals where $f(x) < 0$ h) ______________

18. Write the polynomial equation given the following graph. Answer in factored form.



$f(x) = a(x+2)(x-2)(x-5)^2$

$f(0) = -4 = a(0+2)(0-2)(0-5)^2$

$-4 = a(-100)$

$\frac{1}{25} = a$

$f(x) = \frac{1}{25}(x+2)(x-2)(x-5)^2$

18) ______________________

4 marks

19. A solid block of yellow cedar used for carving is in the shape of a rectangular prism. It has dimensions of 20 cm long, 12 cm wide, and 10 cm in height. The carver wants to reduce the volume of the block to $768\ cm^3$ by removing the same amount off all three dimensions. Write a polynomial function to represent this situation. Calculate how much he should remove from each dimension algebraically.

Let x = amount removed

$(20-x)(12-x)(10-x) = 768$

$(240 - 32x + x^2)(10-x) = 768$

$2400 - 320x + 10x^2 - 240x + 32x^2 - x^3 - 768 = f(x)$

$f(x) = x^3 - 42x^2 + 560x - 1632$

$\frac{p}{q} = \pm1, \pm2, \pm3, \pm4, \pm6, \pm8 \ldots$

$f(4) = 4^3 - 42(4)^2 + 560(4) - 1632$
$= 64 - 672 + 2240 - 1632$
$= 0$

4	1	-42	560	-1632
		4	-152	1632
	1	-38	408	0

$(x-4)(x^2 - 38x + 408) = 0$

$\therefore x = 4$

$\frac{--38 \pm \sqrt{38^2 - 4(1)(408)}}{2(1)}$

No solution

19.) He should remove 4cm

3 marks

20. Four consecutive integers have a product of 360. Find the integers by writing a polynomial equation that represents the integers and then solving algebraically.

let x = 1st # 3
then $x+1$ = 2nd # 4
$x+2$ = 3rd # 5
$x+3$ = 4th # 6

$\pm\frac{p}{q} = \pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \ldots$

$2 \times 3 \times 4 \times 5 = 120$ ← too small

$3 \times 4 \times 5 \times 6 = 360$ ✓

$\therefore x = 3$

$x(x+1)(x+2)(x+3) = 360$

$(x^2+x)(x^2+5x+6) - 360 = P(x)$

$x^4 + 5x^3 + 6x^2 + x^3 + 5x^2 + 6x - 360$

$P(x) = x^4 + 6x^3 + 11x^2 + 6x - 360$

$0 = x^4 + 6x^3 + 11x^2 + 6x - 360$

$0 = (x-3)(x^3 - 9x^2 + 38x + 120)$

$\therefore x = 3$

3	1	6	11	6	-360
		3	27	114	360
	1	9	38	120	0

20.) 3, 4, 5, 6