RADICAL AND RATIONAL FUNCTIONS REVIEW

Name: Ms. Hubbard

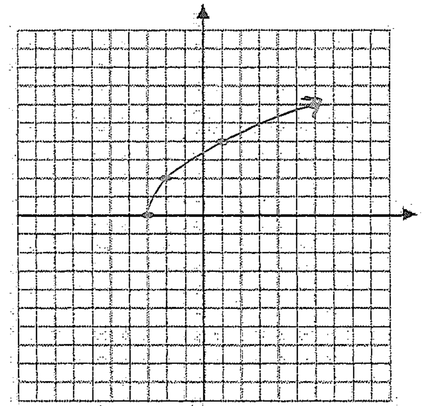
Block: Key

Date: ____________________

Total $\frac{\_\_\_\_}{112}$ = ________%

1. Sketch the graph of the following functions. State the domain and range.

$y = 2\sqrt{x+3}$ Vertical expansion by 2, HT 3 left



2 marks

$x+3 \geq 0 \quad x \geq -3$

$(m, n) \rightarrow (m-3, 2n)$

$(0,0) \rightarrow (-3, 0)$

$(1,1) \rightarrow (-2, 2)$

$(4,2) \rightarrow (1, 4)$

$(9,3) \rightarrow (6, 6)$

Domain: $x \geq -3$ *1 mark*

Range: $y \geq 0$ *1 mark*

2. Identify the transformations for each of the following from $y = \sqrt{x}$

a. $y = \sqrt{9(x-3)}$

$x \rightarrow 9x$ Horizontal Compression by $\frac{1}{9}$

$x \rightarrow x-3$ Horizontal Translation 3 Right

2 marks

b. $y = -\sqrt{4x+8} - 1$ $\quad y = -\sqrt{4(x+2)} - 1$

$x \rightarrow 4x$ H.C by $\frac{1}{4}$, $x \rightarrow x+2$ HT 2 left

$y \rightarrow -y$ Reflection in x-axis $y \rightarrow y+1$ VT 2 down

2 marks

c. $y+3 = \frac{1}{3}\sqrt{(-x-1)}$ $\quad y+3 = \frac{1}{3}\sqrt{-(x+1)}$

$x \rightarrow -x$ Reflection in y-axis $x \rightarrow x+1$ H.T 1 left

$y \rightarrow 3y$ V.C by 1/3, $y \rightarrow y+3$ VT 3 down

2 marks

0	0
1	1
4	2
9	3

3. Write the equation of the radical function that results from the following transformations on the graph of $y=\sqrt{x}$ in the order presented.

a. Horizontal expansion by a factor of 5 and a vertical translation down 3 units.

$x \to \frac{1}{5}x$ $\quad y+3=\sqrt{\frac{1}{5}x}$

$y \to y+3$

a. $y=\sqrt{\frac{1}{5}x}-3$ *2 marks*

b. Vertical compression by a factor of one third, reflection in the y axis, horizontal translation 4 units left.

$x \to -x$ $\quad y \to 3y$

$x \to x+4$

$3y=\sqrt{-(x+4)}$

b. $y=\frac{1}{3}\sqrt{-(x+4)}$ *2 marks*

c. Vertical expansion by a factor of four, reflection in the x axis, horizontal compression by a factor of one-sixth and a translation 5 units left and 1 unit up.

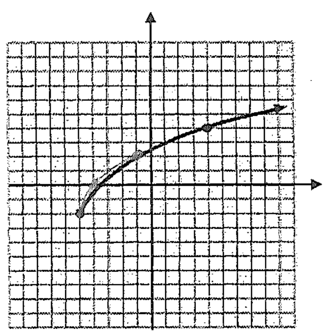
$x \to 6x$ $\quad y \to -\frac{1}{4}y$ $\quad -\frac{1}{4}y=\sqrt{6(x+5)}$

$x \to x+5$ $\quad y \to y-1$ $\quad y-1=-4\sqrt{6(x+5)}$

c. $y=-4\sqrt{6(x+5)}+1$ *2 marks*

4. Write the equation of each of the following. Use the form $y=a\sqrt{b(x-h)}+k$

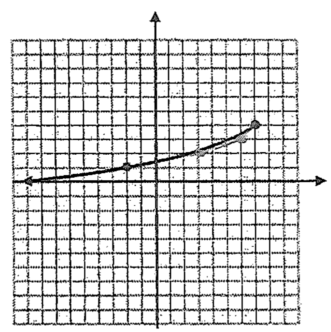
a.



V.E by 2
$y \to \frac{1}{2}y$
5 left
$x \to x+5$
2 down
$y+2$

Equation: $y=2\sqrt{x+5}-2$ *2 marks*

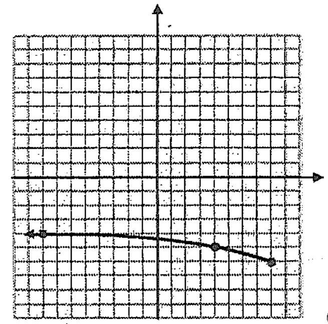
b.



Reflection in y-axis $x \to -x$
Reflection in x-axis $y \to -y$
7 Right
$x \to x-7$
V.T 4 up
$y \to y-4$
$-y=\sqrt{-x}$
$y-4=-\sqrt{-(x-7)}$

Equation: $y=-\sqrt{-(x-7)}+4$ *2 marks*

c.



V.C by $\frac{1}{2}$ $\quad y \to 2y$
Reflection in y-axis $x \to -x$
HT 8 Right $x \to x-8$
VT 6 down $y \to y+6$

$2y=\sqrt{-x}$

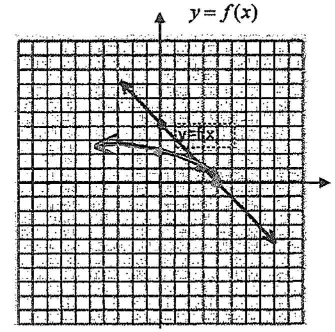
$y=\frac{1}{2}\sqrt{-x}$

$y+6=\frac{1}{2}\sqrt{-(x-8)}$

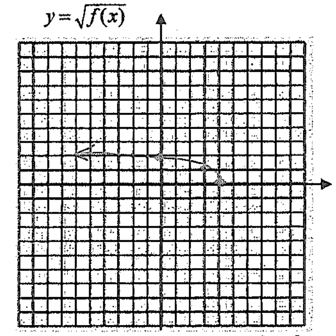
Equation: $y=\frac{1}{2}\sqrt{-(x-8)}-6$ *2 marks*

5. Given the graph of $y=f(x)$ graph the function that would represent $y=\sqrt{f(x)}$
List the domain and range of each graph, list the **EXACT VALUES** of any invariant points.

$y=f(x)$



$y=\sqrt{f(x)}$



2 marks

Domain $f(x)$: $\{x \mid x \in R\}$ *1 mark*

Domain $\sqrt{f(x)}$: $\{x \mid x \leq 4, x \in R\}$ *1 mark*

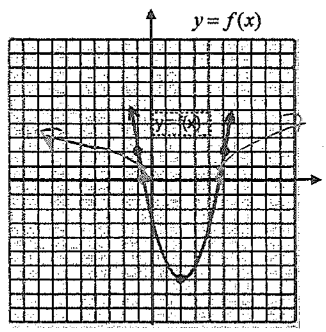
Range $f(x)$: $\{y \mid y \in R\}$ *1 mark*

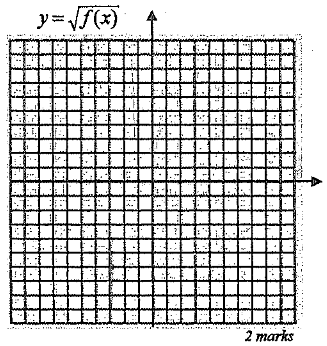
Range $\sqrt{f(x)}$: $\{y \mid y \geq 0, y \in R\}$ *1 mark*

invariant points
$(4,0)$ + $(3,1)$

5. Invariant Points: $(4,0)$ and $(3,1)$ *2 marks*

6. Given the graph of $y = f(x)$ on the left, and $f(x) = (x-2)^2 - 7$ graph the function that would represent $y = \sqrt{f(x)}$. List the domain and range of each graph, list the **EXACT VALUES** of any invariant points.





2 marks

Domain $f(x)$: $\{x \mid x \in R\}$ *1 mark*

Domain $\sqrt{f(x)}$: $x \le 2 - \sqrt{7}$ or $x \ge 2 + \sqrt{7}$ *1 mark*

Range $f(x)$: $y \ge 7$ *1 mark*

Range $\sqrt{f(x)}$: $y \ge 0$ *1 mark*

$(x-2)^2 - 7 = 0$
$(x-2)^2 = 7$
$x - 2 = \pm\sqrt{7}$
$x = 2 \pm \sqrt{7}$

$(x-2)^2 - 7 = 1$
$(x-2)^2 = 8$
$(x-2) = \pm\sqrt{8}$
$x = 2 \pm 2\sqrt{2}$

6. Invariant Points: $(2-\sqrt{7}, 0)$, $(2+\sqrt{7}, 0)$ $(2+2\sqrt{2}, 1)$ $(2-2\sqrt{2}, 1)$ *2 marks*

7. Compare the domains and ranges of the following two functions. Using graphing calculator. *(1 mark each)*

a. $f(x) = 8 - 2x^2$ *and* $f(x) = \sqrt{8 - 2x^2}$

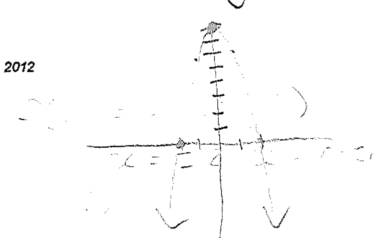
8 up
Reflection in x-axis

Domain $f(x)$: $x \in R$

Domain $\sqrt{f(x)}$: $-2 \le x \le 2$

Range $f(x)$: $y \le 8$

Range $\sqrt{f(x)}$: $0 \le y \le 2\sqrt{2}$



$8 - 2x^2 \ge 0$
$2(4 - x^2) \ge 0$

8. Solve each of the following algebraically for their exact values.

a. $\sqrt{3x-5} = 3$

D: $x \ge \frac{5}{3}$

$3x - 5 = 9$
$3x = 14$
$x = \frac{14}{3}$

a. ____________ *2 marks*

b. $\frac{1}{3}\sqrt{x+5} - 3 = 1$

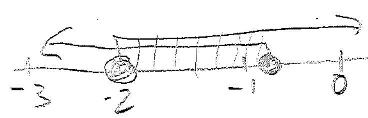
D: $x \ge -5$

$\frac{1}{3}\sqrt{x+5} = 4$
$\sqrt{x+5} = 12$
$x + 5 = 144$
$x = 139$

b. ____________ *2 marks*

c. $\sqrt{-12-13x} - 2 = x$

D: $-12 - 13x \ge 0$
$-12 \ge 13x$
$-\frac{12}{13} \ge x$



$(\sqrt{-12-13x})^2 = (x+2)^2 \leftarrow x \ge -2$
$-12 - 13x = x^2 + 4x + 4$
$0 = x^2 + 17x + 16$
$0 = (x+16)(x+1)$
$x = -16$ $x = -1$
reject

c. $x = -1$ *3 marks*

9. ***Bonus: Graphing Calculator question! Solve the following graphically for a decimal approximation to the nearest 100th.

$\left(2\sqrt{3x^2-4}\right)^2 = (1-x)^2$

$1 - x \geq 0$ $\quad 3x^2 - 4 \geq 0$

$1 \geq x$ $\quad x^2 \geq \frac{4}{3}$

and $-\frac{2}{\sqrt{3}} \geq x$ or $x \geq \frac{2}{\sqrt{3}}$

$4(3x^2-4) = 1 - 2x + x^2$

$12x^2 - 16 = 1 - 2x + x^2$

$11x^2 + 2x - 17 = 0$

$\frac{-2 \pm \sqrt{2^2 - 4(11)(-17)}}{2(11)}$

$= \frac{-2 \pm \sqrt{4 + 748}}{22}$

$= \frac{-2 \pm \sqrt{752}}{22}$

$= \frac{-2 \pm 4\sqrt{47}}{22}$

$= \frac{-1 \pm 2\sqrt{47}}{11}$

$x \approx 1.15 \leftarrow$ reject

$x \approx -1.34$

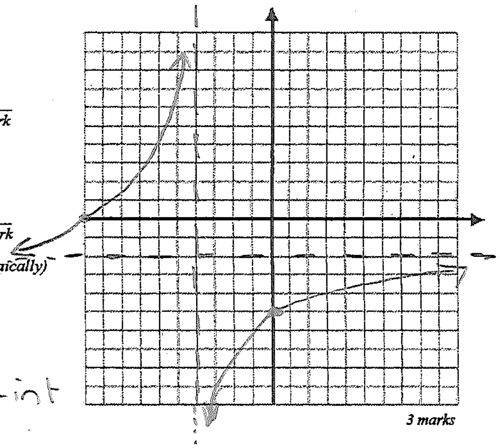
9. $x \approx -1.34$

3 marks

$\frac{3x}{x+4} - \frac{5(x+4)}{x+4} = 3x - 5x - 20$

10. Sketch the graphs of the following functions and show all asymptotes with a dotted line. (*3 marks for graph*)

a $y = \frac{3x}{x+4} - 5$ $\quad y = \frac{-2x - 20}{x+4}$



3 marks

i) Equation of any vertical asymptote(s)

$x = -4$

1 mark

ii) State any restrictions or non-permissible value(s)

$x \neq -4$

1 mark

iii) Determine coordinates of any intercept(s). *(algebraically)*

$-2x - 20 = 0$

$x = -10$ x-int

$y = \frac{-2(0) - 20}{0 + 4} = -5$ y-int

2 marks

iv) Describe the behavior of the function as it approaches and leaves vertical asymptotes and/or point of discontinuity.

as $x \to -4$ from the right $y \to -\infty$

as $x \to -4$ from the left $y \to +\infty$

as $x \to +\infty$ $y \to -2$

as $x \to -\infty$ $y \to -2$

1 mark

v) State the horizontal asymptote.

$\frac{-2x}{x} = -2$ $\quad y = -2$

1 mark

vi) State the Domain and Range

$x \neq -4$

$y \neq -2$

1 mark

b. $y = \frac{2x-6}{x^2-5x+4} = \frac{2(x-3)}{(x-4)(x-1)}$

i) Equation of any vertical asymptote(s)

$x=1,\ x=4$

1 mark

ii) State any restrictions or non-permissible value(s)

$x\neq 1 \quad x\neq 4$

1 mark

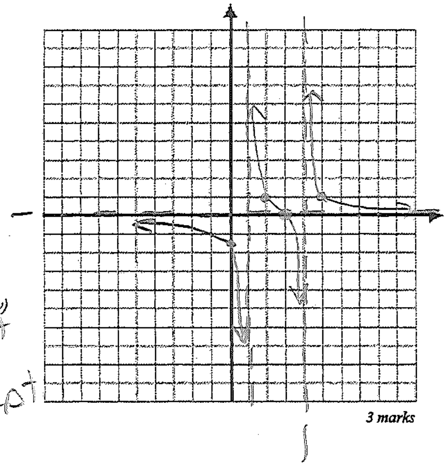
iii) Determine coordinates of any intercept(s). *(algebraically)*

$x-3=0 \quad x=3$ x-intercept

$y=\frac{2(0)-6}{0^2-5(0)+4} = -1.5$ y-intercept

$(3,0),\ (0,-1.5)$

2 marks



3 marks

iv) Describe the behavior of the function as it approaches and leaves vertical asymptotes and/or point of discontinuity.

as $x\to-\infty,\ y\to0$ as $x\to+\infty\ y\to0$

as $x\to1$ from the left $y\to-\infty$

as $x\to1$ from the right $y\to+\infty$

as $x\to4$ from the left $y\to-\infty$

as $x\to4$ from the right $y\to+\infty$

1 mark

v) State the horizontal asymptote

Degree numerator is less than denominator

$\therefore\ y=0$

1 mark

vi) State the Domain and Range

$\{x \mid x\neq1,\ x\neq4,\ x\in R\}$

$\{y \mid y\in R\}$

1 mark

$y=\frac{2(5-3)}{(5-4)(5-1)} = \frac{4}{(1)(4)} = 1 \quad (5,1)$

Test Points

$y=\frac{2(2-3)}{(2-4)(2-1)} = \frac{-2}{(-2)(1)} = 1 \quad (2,1)$

note the function crosses the HA at $x=3$ therefore the range is $y\in R$

c. $y=\frac{3x^2-18x+24}{x^2-2x-8} = \frac{3(x^2-6x+8)}{(x-4)(x+2)} = \frac{3(x-4)(x-2)}{(x-4)(x+2)}$

$y=\frac{3(x-2)}{x+2}$

i) Equation of any vertical asymptote(s)

$x=-2$

$y=\frac{3(4-2)}{4+2}=1$

$(4,1)$ hole

1 mark

ii) State any restrictions or non-permissible value(s)

$x\neq4,\ x\neq-2$

1 mark

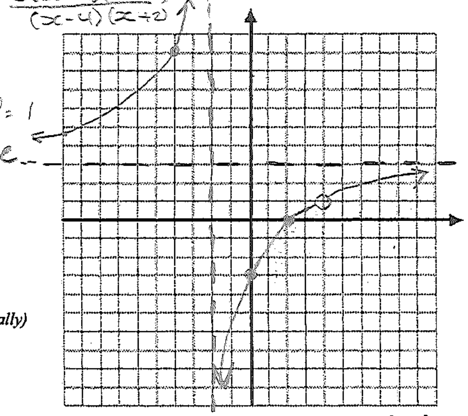
iii) Determine coordinates of any intercept(s). *(algebraically)*

$y=\frac{3(0-2)}{0+2} = \frac{-6}{2} = -3$

$x-2=0 \quad x=2$

$(2,0) \quad (0,-3)$

2 marks



3 marks

Test points:

$y=\frac{3(-10-2)}{-10+2} = \frac{-36}{-8} = +4.5$

$y=\frac{3(-3-2)}{-3+2} = \frac{-15}{-1} = +15$

$y=\frac{3(-4-2)}{-4+2} = \frac{-18}{-2} = 9$

iv) Describe the behavior of the function as it approaches and leaves vertical asymptotes and/or point of discontinuity.

as $x\to-\infty$ $y\to3$

as $x\to+\infty$ $y\to3$

as $x\to-2$ from the left $y\to+\infty$

as $x\to-2$ from the right $y\to-\infty$

1 mark

v) State the horizontal asymptote.

$y=\frac{3x^2}{x^2} \quad y=3$

1 mark

vi) State the Domain and Range

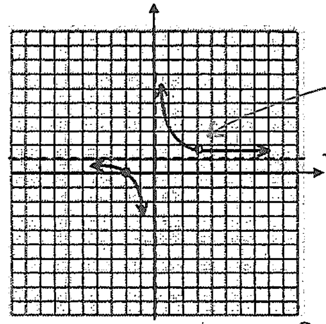
$x\neq4,\ x\neq-2$

$y\neq3 \quad y\neq1$

1 mark

11. Write a possible equation for each of the following graphs. Explain your reasoning.

a.



hole at $x=3$ ∴ $x-3$ is a factor of numerator and denominator

— H. A at $x=1$ ∴ $\frac{ax^n}{bx^n}=1$

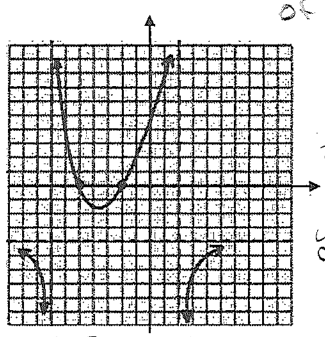
x-int at $x=-2$ ∴ $x+2$ is factor of numerator

VA at $x=0$ ∴ x is factor of denominator

$$r(x)=\frac{(x-3)(x+2)}{x(x-3)}$$

a. or $r(x)=\frac{x^2-x-6}{x^2-3x}$ *3 marks*

b.



x-int $x=-2$, $x=-5$ ∴ $x+2$, $x+5$ factors of numerator

$y\neq -4$ HA at $y=-4$ ∴ $\frac{ax^n}{bx^n}=-4$

$x\neq -7$ $x+7$ factor of denominator

$x\neq 2$ ∴ $x-2$ is a factor of denominator

b. $r(x)=\frac{-4(x+2)(x+5)}{(x-2)(x+7)}$ *3 marks*

$$=\frac{-4x^2-28x-40}{x^2+5x-14}$$

12. Write the equation of a possible rational function with the following characteristics. Explain your reasoning.

$(x-3)(x+3)$ factors of denominator $x-5$ and $x+1$ factors of numerator

a) Vertical asymptotes at $x=\pm 3$, x intercepts of $x=5$ *and* $x=-1$, and a horizontal asymptote of $y=\frac{1}{2}$ ∴ $\frac{ax^n}{bx^n}=\frac{1}{2}$

$$r(x)=\frac{(x-5)(x+1)}{2(x-3)(x+3)}$$

a. ______ *2 marks*

$$r(x)=\frac{x^2-4x-5}{2x^2-18}$$

$4x-1$ denominator x numerator $x-5$ numerator + denominator

b) Vertical asymptotes at $x=\frac{1}{4}$, x intercept of $x=0$, and a discontinuous point at $\left(5,\frac{5}{19}\right)$

$$r(x)=\frac{x(x-5)}{(4x-1)(x-5)}$$

b. $r(x)=\frac{x^2-5x}{4x^2-21x+5}$ *2 marks*

c) Y-intercept at -5, no x-intercepts, discontinuous points at $(-1,-5)$ *and* $(3,-5)$

$x+1$, $x-3$ factors of numerator + denominator

$$r(x)=\frac{-5(x+1)(x-3)}{(x+1)(x-3)}$$

c. ______ *3 marks*

13. Solve each of the following rational functions algebraically for their exact values.

a) $\frac{8}{(x-1)^2}=2$ $x\neq 1$

$8=2(x-1)^2$

$8=2(x^2-2x+1)$

$8=2x^2-4x+2$

$0=2x^2-4x-6$

$0=2(x^2-2x-3)$

$0=2(x-3)(x+1)$

a. $x=3,-1$ *3 marks*

b) $\frac{1}{x}=\frac{x}{3x+18}$ $x(3x+18)$ $(x)(3x+18)$ $x\neq 0$ $x\neq -6$

$3x+18=x^2$

$0=x^2-3x-18$

$0=(x-6)(x+3)$

$x=6$

$x=-3$

b. $x=6,-3$ *3 marks*

$(4x+1)(2x-1)$ $\qquad x \neq \frac{1}{2}, \frac{-1}{4}$

c) $\frac{3x}{4x+1} - 1 = \frac{x}{2x-1}$

$(2x-1)(3x) - 1(4x+1)(2x-1) = x(4x+1)$

$6x^2 - 3x - 1(8x^2 - 2x - 1) = 4x^2 + x$

$6x^2 - 3x - 8x^2 + 2x + 1 = 4x^2 + x$

$0 = 6x^2 + 2x - 1$

$\frac{-2 \pm \sqrt{2^2 - 4(6)(-1)}}{2(6)} = \frac{-2 \pm \sqrt{28}}{12}$

$\frac{-2 \pm 2\sqrt{7}}{12}$

c. $x = \frac{-1 \pm \sqrt{7}}{6}$

3 marks

$x \neq 1$
$x \neq 3$

d) $\frac{2x}{x-1} - \frac{x}{x^2 - 4x + 3} = \frac{x+1}{x-3} - 2$ $\qquad (x-3)(x-1)$

$2x(x-3) - x = (x+1)(x-1) - 2(x-3)(x-1)$

$2x^2 - 6x - x = x^2 - 1 - 2(x^2 - 4x + 3)$

$2x^2 - 7x = x^2 - 1 - 2x^2 + 8x - 6$

$3x^2 - 15x + 7 = 0$

$\frac{--15 \pm \sqrt{(-15)^2 - 4(3)(7)}}{2(3)}$

$\frac{15 \pm \sqrt{225 - 84}}{6}$

$\frac{15 \pm \sqrt{141}}{6}$

d. $x = \frac{15 \pm \sqrt{141}}{6}$

3 marks

14. A river boat can travel at 20 km per hour in still water. The boat travels 30 km upstream against the current then turns around and travels the same distance back with the current. If the total trip took 7.5 hours, what is the speed of the current? Solve this question algebraically as well as graphically.

let x = speed of current

a) Algebra Solution $\qquad$ time $= \frac{d}{\text{speed}}$ $\qquad x \neq 20$

Boat time upstream $\frac{30}{20-x}$

$2(20-x)(20+x)\left(\frac{30}{20-x} + \frac{30}{20+x} = \frac{15}{2}\right)$

down stream $\frac{30}{20+x}$

$60(20+x) + 60(20-x) = 15(400-$

$1200 + 60x + 1200 - 60x = 6000 - 15x^2$

$15x^2 = 3600$

$x^2 = 240$

$x \approx \pm 4\sqrt{15}$

a. ______

3 marks

$\therefore$ speed of current ≈ 15.49 km/h

b) Bonus: graphing calculator! Graphical.

x min	x max	y min	y max
−30	30	−10	10

2 marks