Function: Operations, Compositions, Inverses Review

Name: Key

Block: _____

Date: _____

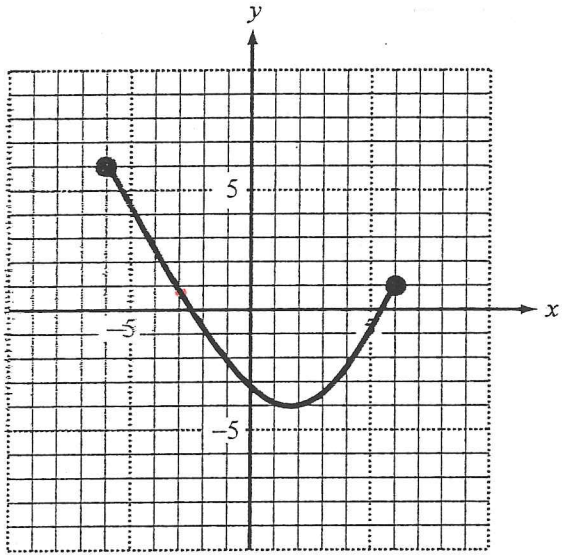
Total _____ = _____%
65

SHOW ALL WORK WHERE APPROPRIATE OR PROVIDE RATIONALE FOR YOUR ANSWERS FOR FULL MARKS

1. Each of the following graphs represent $y = f(x)$ Find the indicated value for each of the following from the graph.

a) *ai)* $f(-3)$ ≈ 0.8

aii) $f(6)$ $= 1$

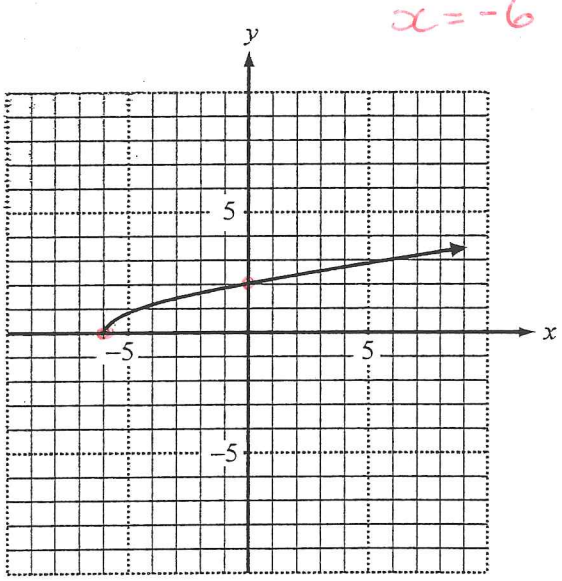


a) _____
2 marks

b) *bi)* $f(0)$ $= 2$

bii) $f(x) = 0$ ← Find x when $y = 0$

$x = -6$

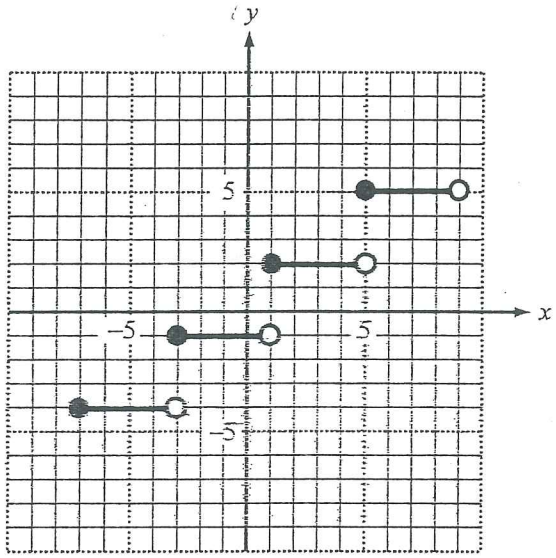


b) _____

2 marks

c) *ci)* $f(-3) = -1$

cii) $f(9)$ undefined



c) _____

2 marks

2. Given $f(x) = 3x - 1$ and $g(x) = 2x + 5$, find $(f + g)(x + 1)$. State any restrictions.

$= f(x+1) + g(x+1)$

$= 3(x+1) - 1 + 2(x+1) + 5$

$= 3x + 3 - 1 + 2x + 2 + 5$

$= 5x + 9$

2) $5x+9$, $x \in R$

3 marks

3. Given $f(x) = x^2 - 2x - 8$ and $g(x) = x^2 - 4$, find $\left(\frac{f}{g}\right)(-1)$. State any restrictions. for $\frac{f}{g}(x)$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^2-2x-8}{x^2-4} = \frac{(x-4)(x+2)}{(x-2)(x+2)}$

$= \frac{x-4}{x-2}$, $x \neq 2$ asymptote, $x \neq -2$ hole

$\frac{f}{g}(-1) = \frac{-1-4}{-1-2} = \frac{5}{3}$

3) _____
3 marks

4. Given $f(x) = 2\sqrt{3x}$ and $g(x) = 4\sqrt{12x}$, find $(f - g)(9)$. State any restrictions. for $(f-g)(x)$

$(f-g)(x) = f(x) - g(x)$
$= 2\sqrt{3x} - 4\sqrt{12x}$, $x \geq 0$

$(f-g)(9) = 2\sqrt{3(9)} - 4\sqrt{12(9)}$
$= 2 \times 3\sqrt{3} - 4 \times 6\sqrt{3}$
$= 6\sqrt{3} - 24\sqrt{3}$
$= -18\sqrt{3}$

4) _____
3 marks

5. Given $f(x) = \dfrac{2}{x-6}$ and $g(x) = \dfrac{1}{x+1}$, find $(f + g)(x)$. State any restrictions.

$(f+g)(x) = f(x) + g(x)$
$= \dfrac{2}{x-6} + \dfrac{1}{x+1}$
$= \dfrac{2(x+1) + 1(x-6)}{(x-6)(x+1)}$
$= \dfrac{2x+2+x-6}{(x-6)(x+1)}$
$= \dfrac{3x-4}{(x-6)(x+1)}$

$x \neq 6$, $x \neq -1$
both asymptotes

5) _____
3 marks

6. Given $f(x) = x^2 - 9$ and $g(x) = x^2 + 3x - 1$, find $(fg)(x)$.

$$(fg)(x) = f(x) \cdot g(x)$$
$$= (x^2 - 9)(x^2 + 3x - 1)$$
$$= x^4 + 3x^3 - x^2 - 9x^2 - 27x + 9$$
$$= x^4 + 3x^3 - 10x^2 - 27x + 9, \quad x \in R$$

6) _____
3 marks

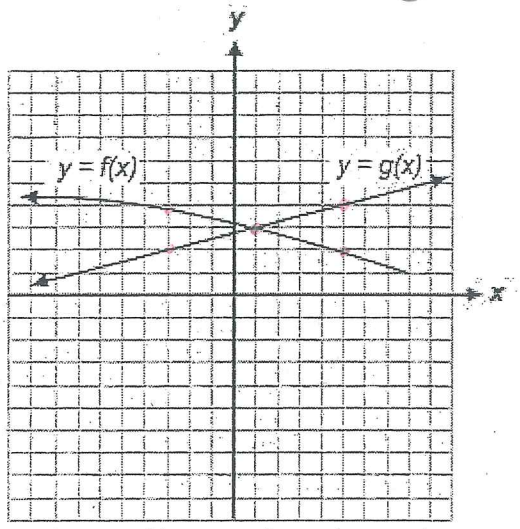
7. Use the graphs of $f(x)$ and $g(x)$ to evaluate the following: *(1 mark each)*

a) $(f+g)(1) = f(1) + g(1) = 3 + 3 = 6$

b) $(f-g)(5) = f(5) - g(5) = 2 - 4 = -2$

c) $\left(\frac{f}{g}\right)(-3) = \frac{f(-3)}{g(-3)} \approx \frac{3.8}{2} \approx 1.9$

d) $(fg)(5) = f(5) \cdot g(5) = (2)(4) = 8$



7a) 6

7b) −2

7c) ≈ 1.9

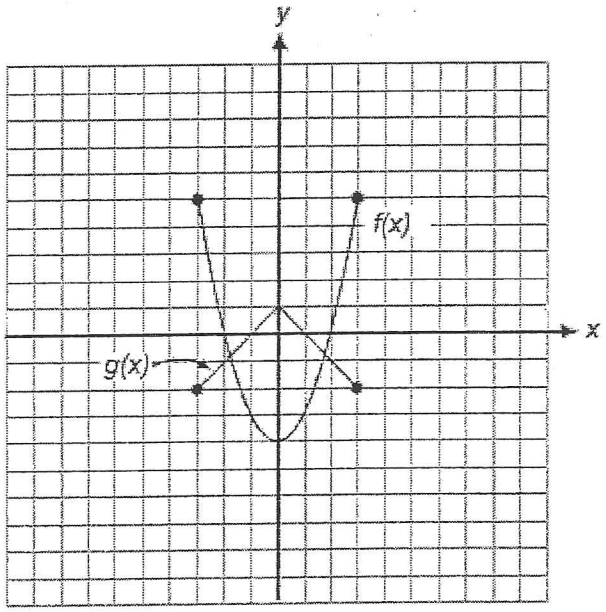
7d) 8

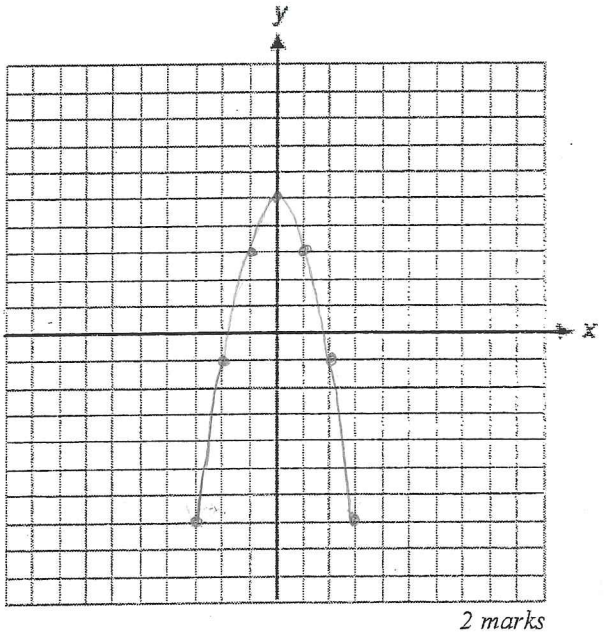
x	g	f	$g-f$
-3	-2	5	-7
-2	-1	0	-1
-1	0	-3	3
0	1	-4	5

7d) _____

4 marks

8. Sketch the graph of the combined function of $y = (g - f)(x)$ for the following functions of $f(x)$ and $g(x)$ on the grid provided.





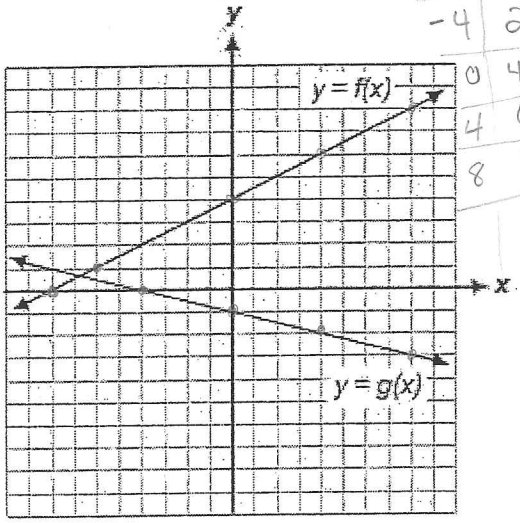
2 marks

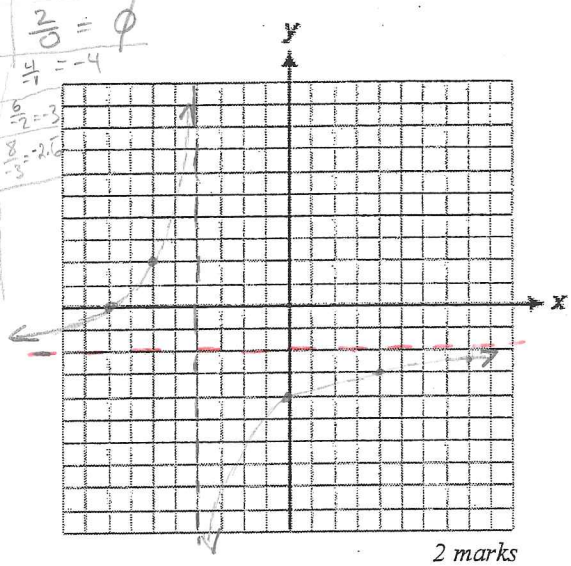
Bonus

9. Given the graphs of $y = f(x)$ and $y = g(x)$ sketch the graph of $k(x)$ given

$$k(x) = \left(\frac{f}{g}\right)(x)$$

x	f	g	K
-8	0	1	$\frac{0}{1}=0$
-6	1	0.5	$\frac{1}{0.5}=2$
-4	2	0	$\frac{2}{0}=\phi$
0	4	-1	$\frac{4}{-1}=-4$
4	6	-2	$\frac{6}{-2}=-3$
8	8	-3	$\frac{8}{-3}=-2.5$





2 marks

10. Given $(f+g)(x) = 10-3x$ and $(f-g)(x) = 5x-14$, find $f(x)$ and $g(x)$.

$$f(x) + g(x) = 10 - 3x$$
$$+ \quad f(x) - g(x) = 5x - 14$$
$$2f(x) = 2x - 4$$
$$f(x) = x - 2$$

$$f(x) + g(x) = 10 - 3x$$
$$- \quad f(x) - g(x) = 5x - 14$$
$$2g(x) = -8x + 24$$
$$g(x) = -4x + 12$$

10) _____

3 marks

11. Given $f(x) = 3x + 2$ and $g(x) = 2x^2 - 1$ find the following and state if the composite function exists. Explain why or why not.

a) $(f \circ g)(x)$ $= f(g(x))$

$= f(2x^2-1)$

$= 3(2x^2-1) + 2$

$= 6x^2 - 3 + 2$

$= 6x^2 - 1$, yes the domain $x \in R$

Range $y \geq -1$

a) _____
3 marks

b) $(g \circ f)(x)$

$g(f(x))$

$g(3x+2) = 2(3x+2)^2 - 1$

$= 2(9x^2 + 12x + 4) - 1$

$= 18x^2 + 24x + 8 - 1$

$= 18x^2 + 24x + 7$

yes the domain is $x \in R$

b) _____
3 marks

12. Given $f(x) = \sqrt{x+4}$ and $g(x) = -2x^2 - 5$ find the following and state if the composite function exists. Explain why or why not.

a) $(f \circ g)(x) = f(-2x^2-5) = \sqrt{-2x^2-5+4}$

$= \sqrt{-2x^2-1}$ ← this function does not exist because $-2x^2-1 \geq 0$ has no solution. so there are no x-values for the domain

a) _____
3 marks

b) $(g \circ f)(x)$ $g(\sqrt{x+4}) = -2(\sqrt{x+4})^2 - 5$

$x \geq -4$

$= -2(x+4) - 5$

$= -2x - 8 - 5$

$= -2x - 13, \quad x \geq -4$

b) _____

3 marks

13. Given $f(x) = \frac{3}{5-x}$ and $g(x) = 2x - 1$ find the following and state any restrictions.

a) $(f \circ g)(x)$ $= f(2x-1) = \frac{3}{5-(2x-1)}$

$= \frac{3}{-2x+6}$, $x \neq 3$

a) _____

3 marks

b) $(g \circ f)(\sqrt{2})$ *(Rationalize denominator in answer)*

$f\sqrt{2} = \frac{3}{(5-\sqrt{2})} \cdot \frac{(5+\sqrt{2})}{(5+\sqrt{2})}$

$= \frac{15+3\sqrt{2}}{25-2}$

$= \frac{15+3\sqrt{2}}{23}$

$g\left(\frac{15+3\sqrt{2}}{23}\right) = 2\left(\frac{15+3\sqrt{2}}{23}\right) - 1$

$= \frac{30+6\sqrt{2}}{23} - \frac{23}{23}$

$= \frac{7+6\sqrt{2}}{23}$

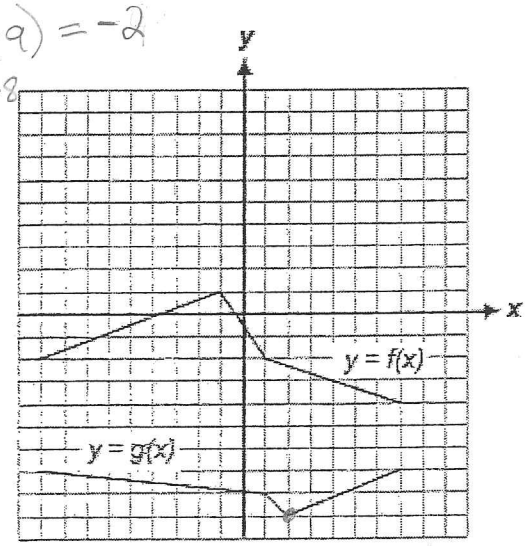
b) _____

3 marks

14. Use the graphs of $f(x)$ and $g(x)$ to evaluate the following:

a. $f(g(2)) = f(-9) = -2$

b. $g(f(-1)) = g(1) = -8$



a) -2

2 marks

b) -8

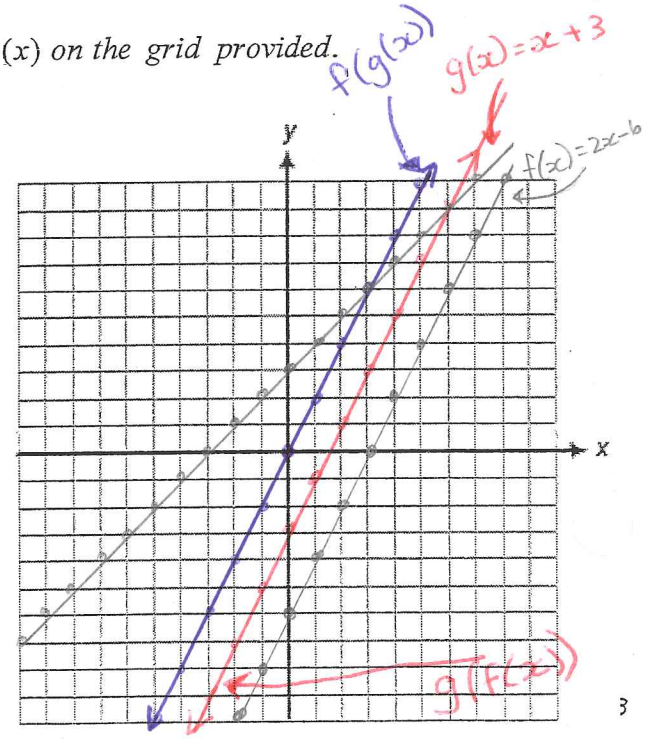
2 marks

15. Given: $f(x) = 2x - 6$ and $g(x) = x + 3$

Sketch the graph of $f(x)$, $g(x)$, $(f \circ g)(x)$, *and* $(g \circ f)(x)$ *on the grid provided.*

$f(g(x)) = f(x+3)$
$= 2(x+3) - 6$
$= 2x + 6 - 6$
$= 2x$

$g(f(x)) = g(2x-6)$
$= 2x - 6 + 3$
$= 2x - 3$



4 marks

16. Tracey lives in Saskatoon, Saskatchewan and wants to buy a new road bike at a local bike shop. All bikes are marked as 25% off. Saskatchewan has a provincial sales tax of 5% which along with the federal GST of 5% is added to the selling price.

a) Write a function $s(p)$, that relates the regular price, p, in dollars, to the sale price, s, in dollars. $S(p) = 0.75p$

b) Write the function $t(s)$, that relates the sales price, s, in dollars, to the total cost including tax, t, in dollars. $t(s) = 1.1s$

c) Write a composite function that expresses ~~the~~ cost ~~in~~ terms of the regular price. Calculate how much ~~Brent would pay~~ for a new road bike that normally sold for $1550.

$t(s(p)) = t(0.75p) = 1.1(0.75p)$

$= 0.825p$

Brent $= 0.825(1550)$

$\$1278.75$

16) _____

4 marks

17. Prove that $f(x)$ and $g(x)$ are inverses ~~using composite functions~~ if:

$f(x) = 3^{2-x}$ *and* $g(x) = -\log_3 x + 2$ (omit)