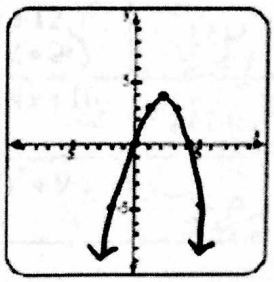
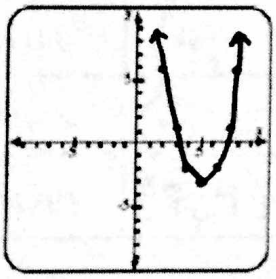
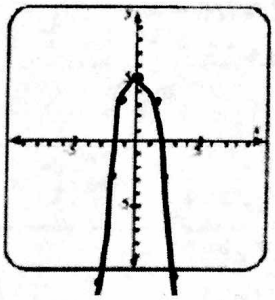
Quadratic Functions and Equations Review

KEY

1. For each of the following graphs, state:







Vertex:	(2,4)	(5,-3)	(0,5)
Line of symmetry:	x=2	x=5	x=0
X – intercept(s):	x=0, x=4	x=3, x=7	x=-2, x=2
Y-Intercept:	y=0	can't see	y=5
Min/max:	max	min	max
Domain:	x∈R	x∈R	x∈R
Range:	y≤4	y≥-3	y≤5

2. Graph the following equations

a) $y = x^2 + 7x + 10$ (up) y-int 10 → (x+5)(x+2) x=-5, x=-2

$y = -2(x+4)(x+1)$ (down) x=-4 x=-1 → $\frac{-4-1}{2} = -2.5$

vertex x=-3.5

$-3.5^2 + 7(-3.5) + 10$

-2.25

y int $-2(-4)(-1)$

vertex x=-2.5

$-2(-2.5-4)(-2.5-1)$

4.5

b) $y = (x+5)^2 - 4$ (opens up) Vertex (-5,-4) Symm → x=-5

y int → x=0

$y=(0+5)^2-4$

25-4

21

$y = -1(x-3)^2 + 2$ (down) Vertex (3,2)

y int x=0

$-1(0-3)^2+2$

$-1(-3)^2+2$

$-1(9)+2$

$-9+2$

-7

$\frac{6+2}{2}$ $\frac{-10+4}{2}$

3. Describe each graph

	Opens up/down	Min/max	Vertex	Line of symmetry	Intercepts
$y = x^2 + 8x + 12$ $(x+6)(x+2)$	up $a=1$	min	$(4,$	$x = 4$	y-int = 12 $x=-6$, $x=-2$
$y = -(x-4)(x+10)$	down $a=-1$	max	$(-3,$	$x = -3$	$x=4$ $x=-10$ $y=40$
$y = 2(x-7)^2 + 9$	up $a=2$	min	$(7, 9)$	$x = 7$	$y=107$

$2(0-7)^2+9$
$2(49)+9$
$98+9$

4. Solve using factoring:

a) $(x-7)(x+8)=0$

$x-7=0$ $x+8=0$
$x=7$ and $x=-8$

b) $x(x+10)=0$

$x=0$ and $x+10=0$, $x=-10$

c) $x^2-5x-14=0$

$(x-7)(x+2)=0$
$x-7=0$ $x+2=0$
$x=7$ and $x=-2$

$x^2+0x-16=0$

d) $10x^2-90x=0$

$10x(x-9)=0$
$10x=0$ $x-9=0$
$x=0$ and $x=9$

e) $x^2-16=0$

$(x-4)(x+4)=0$
$x-4=0$ $x+4=0$
$x=4$ and $x=-4$

f) $(3x-5)(x+2)=0$

$3x-5=0$ $x+2=0$
$3x=5$ $x=-2$
$x=\frac{5}{3}$ and

g) $2(x-9)(4x+1)=0$

$x-9=0$ $4x+1=0$
$x=9$ $4x=-1$
and $x=-\frac{1}{4}$

h) $3x^2-10x+3=0$

$(3x-1)(x-3)$

$3x-1=0$ $x-3=0$
$3x=1$ $x=3$
$x=\frac{1}{3}$ and

i) $x(3x+4)=0$

$x=0$ $3x+4=0$
$3x=-4$
$x=-\frac{4}{3}$

j) $2x^2+5x=7$

$2x^2+5x-7=0$
$(2x+7)(x-1)=0$
$2x+7=0$ $x-1=0$
$2x=-7$ $x=1$
$x=-7/2$

k) $6x^2=7x+3$

$\frac{6x}{2x}=\frac{7x}{x}$

$6x^2-7x-3=0$
$(2x-3)(3x+1)=0$
$2x-3=0$ $3x+1=0$
$2x=3$ $3x=-1$
$x=\frac{3}{2}$ and $x=-\frac{1}{3}$

l) $-5(2x-3)(4x+5)=0$

$2x-3=0$ $4x+5=0$
$2x=3$ $4x=-5$
$x=\frac{3}{2}$ and $x=-\frac{5}{4}$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

5. Solve using the quadratic formula

a) $x^2 - x - 6 = 0$

$a=1$, $b=-1$, $c=-6$

$x = \frac{-(-1) \pm \sqrt{(-1)^2-4(1)(-6)}}{2(1)}$

$x = \frac{1 \pm \sqrt{1+24}}{2}$

$x = \frac{1+5}{2}$ and $x = \frac{1-5}{2}$

$x = 3$ and $x = -2$

b) $2x^2 - 7x + 3 = 0$

$a=2$, $b=-7$, $c=3$

$x = \frac{-(-7) \pm \sqrt{(-7)^2-4(2)(3)}}{2(3)}$

$x = \frac{7 \pm \sqrt{49-24}}{6}$

$x = \frac{7 \pm 5}{6}$ $\quad x=2$ and $x=1/3$

c) $3x(4x-9) = -10$

$12x^2 - 27x = -10$

$12x^2 - 27x + 10 = 0$

$a=12$, $b=-27$, $c=10$

$x = \frac{-(-27) \pm \sqrt{(-27)^2-4(12)(10)}}{2(12)}$

$x = \frac{27 \pm \sqrt{249}}{24}$

or $x \doteq 1.78$ and $x \doteq 0.47$

d) $8x^2 + 14x - 15 = 0$

$a=8$, $b=14$, $c=-15$

$x = \frac{-14 \pm \sqrt{14^2-4(8)(-15)}}{2(8)}$

$x = \frac{-14 \pm \sqrt{676}}{16}$

or $x \doteq 0.75$ and $x = -2.5$

e) $3x^2 - 11x = -20$

$3x^2 - 11x + 20 = 0$

$a=3$, $b=-11$, $c=+20$

$x = \frac{-(-11) \pm \sqrt{(-11)^2-4(3)(+20)}}{2(3)}$

$x = \frac{11 \pm \sqrt{-119}}{6}$

No solutions! can't take $\sqrt{}$ of negative #

f) $-0.5x^2 - 3x + 1.4 = 0$

$a=-0.5$, $b=-3$, $c=1.4$

$x = \frac{-(-3) \pm \sqrt{(-3)^2-4(-0.5)(1.4)}}{2(-0.5)}$

$x = \frac{3 \pm \sqrt{11.8}}{-1}$

or $x \doteq 6.4$ and $x \doteq 0.4$

6. Determine the equation of the parabola given the following clues.

a) The roots of the equation are 2 and -5

$x = 2$ $\quad x = -5$

$x - 2 = 0$ $\quad x + 5 = 0$

$(x-2)(x+5) = 0 \rightarrow y = (x-2)(x+5)$

b) The roots of the equation are $\frac{1}{2}$ and $\frac{3}{5}$

$x = \frac{1}{2}$ $\quad x = \frac{3}{5}$

remove fraction $\quad 2(x - \frac{1}{2} = 0)$ $\quad 3(x - \frac{3}{5} = 0)$

$2x - 1 = 0$ $\quad 3x - 5 = 0$

one possible Equation $(2x-1)(3x-5) = y$

c) The roots of the equation are 1 and 3 and go through the point (2,-4)

$x = 1$ $\quad x = 3$

$x - 1 = 0$ $\quad x - 3 = 0$

factored form $\quad a(x-1)(x-3) = y$

goes through $(2, -4)$

$a(2-1)(2-3) = -4$

$a(1)(-1) = -4$

$a = 4$

Eqn: $4(x-1)(x-3) = y$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

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$a(1)(-1) = -4$

$a = 4$

Eqn: $4(x-1)(x-3) = y$

vertex form

d) The vertex of the graph is at (6,1) and goes through the point (10,-31)

x y x y

$y = a(x-6)^2 + 1$

$-31 = a(10-6)^2 + 1$

$-31 = a(4)^2 + 1$

$-31 = 16a + 1$

$-32 = 16a$

$\frac{-32}{16} = \frac{16a}{16}$

$a = -2$

Eqn $-2(x-6)^2 + 1 = y$

e) The zeros are -5 and 9 and the graph has a y-intercept of -90

factored form

$y = -5$ $x = 9$

$x + 5 = 0$ $x - 9 = 0$

$x = 0 \rightarrow y = -90$

$a(x+5)(x-9) = y$

$a(0+5)(0-9) = -90$

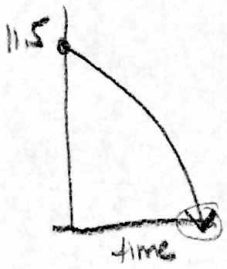
$a(5)(-9) = -90$

$-45a = -90$

$a = 2$

Eqn $2(x+5)(x-9) = y$

7. A rock is thrown vertically downwards from a height of 11.5 m above the ground with a velocity of 6.5 m/s. The height in metres, of the rock above the ground, is given by the function $h(t) = -4.9t^2 - 6.5t + 11.5$. Determine the time taken, in seconds, for the rock to hit the ground.



$a = -4.9$

$b = -6.5$

$c = 11.5$

$t = \frac{-(-6.5) \pm \sqrt{(-6.5)^2 - 4(-4.9)(11.5)}}{2(-4.9)}$

$t = \frac{6.5 \pm \sqrt{42.25 + 225.4}}{-9.8}$

$t \doteq -2.33$ sec and $t = 1.0$ sec

can't have negative time

General Form: $y = ax^2 + bx + c$ Vertex Form: $y = a(x-p)^2 + q$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Factored Form: $y = a(x-r)(x-s)$

Axis of Symmetry: $x = \frac{r+2}{2}$