Absolute Value and Radicals

Part 1: Evaluate:

1. $|-6|$ 2. $|9|-|-8|$ 3. $|-3(2-4)^2+8|$ 4. $|4x^2+3x-7|$ for x= -2

Part 2: Entire Radical and Mixed Radicals

Convert the following into entire radicals: 1. $4\sqrt{3}$ 2. $a\sqrt[3]{a}$ 3. $2b\sqrt[3]{5b^2}$ 4. $-\frac{1}{3}\sqrt[3]{\frac{2}{3}}$

Convert the following into mixed radicals: 5. $\sqrt{24}$ 6. $\sqrt[3]{c^5}$ 7. $\sqrt{32y^7}$ 8. $\sqrt[3]{\frac{-125}{72}}$

Part 3: Simplify and identify the values for which the variables of each radical is defined.

1. $\sqrt{18}+4\sqrt{2}$ 2. $-\sqrt{8}+4\sqrt{7}-\sqrt{28}+\sqrt{18}$ 3. $\sqrt{4d}-\sqrt{32d}$

4. $\sqrt{48ab^4}+b^2\sqrt{36a}+\sqrt{75ab^4}-6a^2\sqrt{18b}$ 5. $\sqrt[3]{27b}+\sqrt[3]{16b^4}-\sqrt[3]{-64b}$

6. $(-2\sqrt{3x})(5\sqrt{8})$ 7. $4\sqrt{3}(6\sqrt{5}-5\sqrt{3})$ 8. $(6\sqrt{2}-7)(9\sqrt{8}+6)$

9. $2\sqrt[3]{16k}(\sqrt[3]{4k}+5\sqrt[3]{24})$ 10. $\frac{4\sqrt{125}}{\sqrt{5}}$ 11. $\frac{-4}{2\sqrt[3]{7p}}$ 12. $\frac{2}{-5+4\sqrt{8}}$ 13. $\frac{2\sqrt{8}+3\sqrt{5}}{\sqrt{2}+\sqrt{20}}$

14. $\frac{5\sqrt{3}-\sqrt{11}}{\sqrt{7}}$ 15. $\frac{6-3\sqrt{5}}{4\sqrt{3}+\sqrt{6}}$ 16. $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{3}-\sqrt{7}}$

Part 4: Solve and verify:

1. $4+\sqrt{2x-1}=15$ 2. $x-\sqrt{4-x}=-8$ 3. $4+\sqrt{3x}=\sqrt{6x+1}+3$

4. $\sqrt{-4x+7}=\sqrt{-3x+8}$ 5. $3\sqrt{x-6}=4\sqrt{2x-3}$ 6. $4\sqrt{x+2}-7=1$

Answers:

Part 1: 1. 6, 2. 1, 3. 4, 4. 3. Part 2: 1.$\sqrt{48}$, 2.$\sqrt[3]{a^4}$, 3. $\sqrt[3]{40b^5}$, 4. $-\sqrt[3]{\frac{2}{81}}$, 5. $2\sqrt{6}$, 6.$c\sqrt[3]{c^2}$, 7. $4y^3\sqrt{2y}$, 8.$-\frac{5}{2}\sqrt[3]{\frac{1}{9}}$ Part 3:1. $7\sqrt{2}$, 2. $2\sqrt{7}+\sqrt{2}$, 3. $2\sqrt{d}-4\sqrt{d}$, d≥ 0, 4. $9b^2\sqrt{3a}-6b^2\sqrt{a}-18a^2\sqrt{2b}$, a≥0,b≥0, 5 .$7\sqrt[3]{b}+2b\sqrt[3]{2b}; b\geq 0$, 6. $-20\sqrt{6x}, x\geq 0$, 7. $24\sqrt{15}-60$, 8. 174-90$\sqrt{2}$, 9. $8\sqrt[3]{k^2}+40\sqrt[3]{6k}, k\geq 0$, 10. 20, 11. $-2\frac{\sqrt[3]{49p^2}}{7p}$, p> 0, 12. $\frac{15+24\sqrt{2}}{103}$, 13. $\frac{22+5\sqrt{10}}{18}$, 14. $\frac{5\sqrt{21}-\sqrt{77}}{7}$, 15. $\frac{24\sqrt{3}-6\sqrt{6}-12\sqrt{15}+3\sqrt{30}}{42}$, 16. $\frac{-5-2\sqrt{21}}{2}$, Part 4: 1. 61, 2. -12,-5, 3. 0, $\frac{4}{3}$, 4. -1 , 5. No real numbers , 6. 2

Solving Quadratic Equations

Factor the following:

1. $81x^2 - 49y^2$ 2. $2x^2 - 15x + 18$ 3. $9y^2 - 24y + 16$ 4. $1.5x^2 + 5.5x - 2$ 5. $\frac{5}{3} - \frac{7}{3}x - 2x^2$

6. $16(2x-5)^2 - 25(y+3)^2$ 7. $8(3x-1)^2 + 10(3x-1) - 3$ 8. $4x^2 + 12x + 9 - 16y^2$

9. $64x^2 - 16a^2 + 24a - 9$ 10. $(4a-3)^2 - (3a+1)^2$ 11. $3a^2(3y-1) - 13a(3y-1) + 10(1-4y)$

Solve the following using any method:

12. $x^2 - 7x + 10 = 0$ 13. $3x^2 - 17x = 6$ 14. $2x(x-4) + 9x = 4x + 35$ 15. $\sqrt{2x^2+9} + 3 = 2x$

16. $\frac{x^2}{2} + \frac{17x}{6} = 1$ 17. $3x^2 - 4x - 1 = 0$ 18. $(2x+3)^2 + 2 = 0$ 19. $-2x^2 + \frac{3}{2}x - \frac{5}{6} = 0$

20. $1.5x^2 = 1.2x + 4$ 21. $3x = \sqrt{9x+28} - 4$

Solve by completing the square:

22. $x^2 - 3x + 1 = 0$ 23. $\frac{1}{4}x^2 + x - 3 = 0$

Determine whether each of the following have one, two, or no real roots

24. $2x^2 - 3.6x - 1.5 = 0$ 25. $-2x^2 + 4x - 5 = 0$

26. Consider the equation: $6x^2 - 5x + k = 0$. Determine the values of k if the equation has

a) no real roots b) one real root or c) two real roots

27. The product of two consecutive even integers is 24 more than 4 times the smaller integer. Determine the integers.

28. A model rocket is launched from a platform. Its trajectory can be approximated by the function: $h(d) = -0.01d^2 + 4d + 1$, where h is the height , in metres, of the rocket, and d is the horizontal distance, in metres, the rocket travels. How far does the rocket land from its launch position? (Round to one decimal place)

Answers:

1. $(9x+7y)(9x-7y)$, 2. $(2x-3)(x-6)$, 3.$(3y-4)^2$, 4. $0.5(3x-1)(x+4)$, 5. $\frac{-1}{3}(2x-1)(3x+5)$, 6. $(8x-5y-35)(8x+5y-5)$, 7. $(6x+1)(12x-5)$, 8. $(2x+3-4y)(2x+3+4y)$, 9. $(8x+4a-3)(8x-4a+3)$, 10. $(a-4)(7a-2)$, 11. $(3y-1)(a-5)(3a+2)$, 12. 2, 5, 13. $\frac{-1}{3}$, 6 , 14. $\frac{-7}{2}$, 5, 15. 0, 6, 16. $\frac{1}{3}$, -6, 17. $\frac{2\pm\sqrt{7}}{3}$, 18. No real roots, 19. No real roots, 20. $\frac{6\pm2\sqrt{159}}{15}$, 21. $\frac{-5\pm\sqrt{73}}{6}$, 22. $\frac{3\pm\sqrt{5}}{2}$, 23. 2,-6, 24. 2 real roots, 25. no real roots, 26.a) $k > \frac{25}{24}$,b) $k = \frac{25}{24}$,c) $k < \frac{25}{24}$, 27. -4,-2 or 6,8 , 28. 400.2m

Analyzing Quadratic Functions

Graph each of the following functions. State the x and y intercepts, whether the quadratic has a maximum or a minimum, what the max or min value is and when it occurs, the vertex, the equation of the axis of symmetry, the direction of opening , the domain and the range of each of the following:

1. $y = 2(x-3)^2 - 4$ 2. $y = -\frac{1}{2}(x+3)^2 + 2$ 3. $y = -2x^2 + 16x + 9$ 4. $y = 3x^2 - 12x + 5$

Determine the zeros of: 5. $y = x^2 - 2x - 15$ 6. $y = 5x^2 - 7x - 6$

Determine the roots of: 7. 16= $10c^2$-36c 8. $7x + 4 = 2x^2$

Write an equation of the quadratic function for:

9. A quadratic function that has a vertex (-2, 6) that opens up and is congruent to $y = 4x^2$

10. A quadratic function that has a vertex (3, -8) and x-intercepts: 1 and 5.

11. A quadratic function that has a vertex (6, 3) and a y-intercept: -8.

12. A quadratic function with zeros -4 and 6, passing through (2, 3).

13. Two numbers have a difference of 10. Their product is a minimum. Determine the numbers.

14. 120m of fencing is used to enclose a rectangular playground. What is the maximum area that can be enclosed. What are the dimensions that produce the maximum area.

15. A student is charged $10 for a gym membership . 2000 students purchase the membership. Each increase by $1 in membership, 100 fewer students signed up. What membership fee would provide maximum revenue for the gym?

16. A doorway is a parabolic arch. The base of the doorway is 3m wide. At a point 1.0m from one side of the doorway, it is 3m high. Determine an equation to represent the function. Could an item 3.5m high fit through the doorway.

Answers: 1. x- int;1,5 y-int;14, min value of -4 occurs when x=3, vertex:(3,-4), AOS: x=3,opens up, D $\{x|x \in R\}$, R$\{y|y \geq -4, y \in R\}$, 2. x-int.-1,-5 y- int:$\frac{-5}{2}$, max value of 2 occurs when x=-3, vertex: (-3,2), AOS: x=-3, opens down, D $\{x|x \in R\}$, R$\{y|\ y \leq 2, y \in R\}$, 3. x-int:$\frac{8 \pm \sqrt{82}}{2}$, y-int: -23, max value of 9 occurs when x=4, vertex: (4,9), AOS: x=4, opens down, D $\{x|x \in R\}$, R$\{y \leq 9, y \in R\}$, 4. X-int: $\frac{6 \pm \sqrt{21}}{3}$, y-int: 5, min value of -7 occurs when x=2, vertex: (2,-7), AOS: x=2, opens up, , D $\{x|x \in R\}$, R$\{y|y \geq -7\}, y \in R$, 5. 5,-3 , 6. $\frac{-3}{5}$, 2, 7. $-\frac{2}{5}$, 4, 8. $-\frac{1}{2}$, 4, 9. $y = 4(x+2)^2 + 6$ 10. $y = 2(x-3)^2$-8, 11. $y = \frac{-11}{36}(x-6)^2 + 3$, 12. $y = \frac{-1}{8}(x+4)(x-6)$, 13. 5, -5, 14. $20m^2$, 20mx20m, 15. Max Rev. $22500 when the membership price is $15, 16. $y = -1.5x^2 + 4.5x$, No, it could not fit through the doorway

Graphing Inequalities and Systems of Equations

Solve each one variable inequality. Represent each solution on a number line and in set bracket notation:

1. $(x-5)(x-8)>0$ 2. $(x+3)(x-4)\leq 0$ 3. $(6x+5)(4x-1)\geq 0$

4. $x^2+x-12>0$ 5. $2x^2-15x+28<0$ 6. $7x^2\leq 35x$

Create an inequality that has each solution:

7. $\{x|-9<x<3, x\in R\}$ 8. $\{x|x\leq -4 \text{ or } x\geq 3, x\in R\}$

Graph each linear inequality. Give the coordinates of 2 points that satisfy the inequality:

9. $5x+3y>12$ 10. $3x-2y\leq -9$ 11. $y<4x-3$ 12. $5x-4y\geq 24$

Graph each quadratic inequality. Give the coordinates of 2 points that satisfy the inequality:

13. $y\leq -(x-2)^2+4$ 14. $y>(x+3)^2$ 15. $y\geq x^2+8x+19$ 16. $y<2x^2-12x+13$

Solve each system of equations graphically:

17. $y=x^2+1$
$y=3x+1$

18. $y=-2(x-3)^2+5$
$y=(x-3)^2+2$

Solve each system of equations algebraically:

19. $y=x^2-16$
$x+y=4$

20. $y=x-16$
$y=x^2+x$

21. $y=3(x-3)^2+1$
$y=-(x-3)^2+5$

22. $y=x^2+2x-6$
$y=-2x^2-4x+3$

Answers: Use desmos or wolframalpha to check solutions

Rational Expressions

Part A: Simplify each expression, identify all non-permissible values.

1) $\frac{48x^3y^5}{-8x^2y}\cdot\frac{-10xy^2}{12x^2y}$ 2) $\frac{27x}{63x^2+54x}$ 3) $\frac{x^2+6x}{x^2+7x+6}$ 4) $\frac{x^2-4x-45}{x^2-25}$

5) $\frac{9x^3y^2}{5(a+1)}\cdot\frac{20(a+1)}{3xy}$ 6) $\frac{6x^2+11x-35}{6x^2+12x-40}$ 7) $\frac{3x-6}{4x^2+8x}\div\frac{6}{8x^2+16}$ 8) $\frac{x^2-64}{x^2-8x+16}\cdot\frac{7x^2-28x}{5x^2+40x}$

9) $\frac{5x^2-7x+2}{x^3-3x^2+2x}\cdot\frac{x^2-2x}{3x^2+2x-1}$

10) $\frac{4x^2+26x-14}{16x^2-25}\div\frac{12x^3-10x^2+2x}{4x^2-29x+30}\cdot\frac{12x^2+11x-5}{2x^2-5x-42}$

Part B: Simplify each sum or difference, identify all non-permissible values.

1) $\frac{x-5}{7x}+\frac{x+3}{7x}$ 2) $\frac{1}{a+1}+\frac{a}{a+1}$ 3) $\frac{2x-4}{x-1}-\frac{x+7}{x-1}$ 4) $\frac{3x-1}{3}+\frac{x+4}{5}$ 5) $\frac{8}{7x}-\frac{3}{2x}$

6) $\frac{2}{5}-\frac{8}{3x^2}+\frac{7}{6x}$ 7) $\frac{5}{x+4}-\frac{6}{x-2}$ 8) $\frac{6x-1}{x-4}+\frac{2x-5}{x-2}$ 9) $4-\frac{3x}{x-7}$

10) $\frac{8y}{5y-2}+\frac{3y}{3y+2}$ 11) $\frac{7x}{2x-14}+\frac{2x}{3x-21}$ 12) $\frac{7}{x^2-7x+12}+\frac{5}{x-4}-\frac{9}{x-3}$

13) $\frac{2}{x^2-7x-12}-\frac{3}{x^2+3x-18}+\frac{6}{x^2+2x-24}$ 14) $\frac{4x^2-9x+2}{4x^2+3x-1}-\frac{3x^2+4x-4}{3x^2-8x+4}$

Part C: Solve each of the following rational equations

1) $\frac{3x}{5}=\frac{2}{x}+\frac{3x+2}{5}$ 2) $\frac{3x-2}{2}+4=\frac{13}{x}=\frac{1-6x}{4}$ 3) $\frac{2x+1}{3x-2}=\frac{4x+3}{6x-5}$

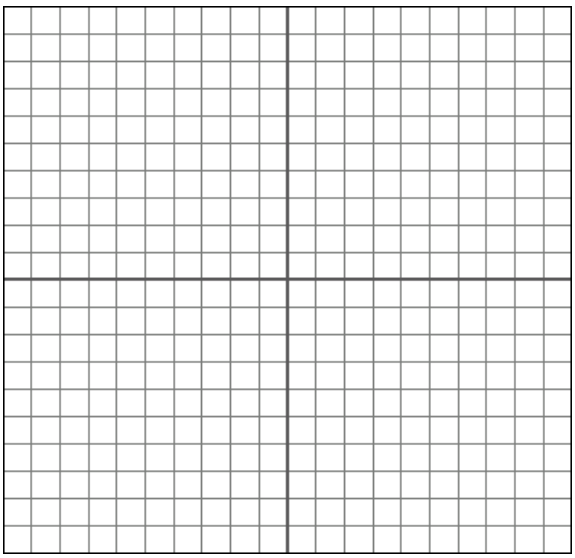
4) $\frac{1}{2+x}+\frac{4}{2x-1}=1$ 5) $x+\frac{6}{x+5}=2$ 6) $\frac{2x}{2x-1}-\frac{x+1}{x+3}=\frac{3x+1}{2x-1}$

Answers: use wolfram alpha or mathpapa.com to check answers

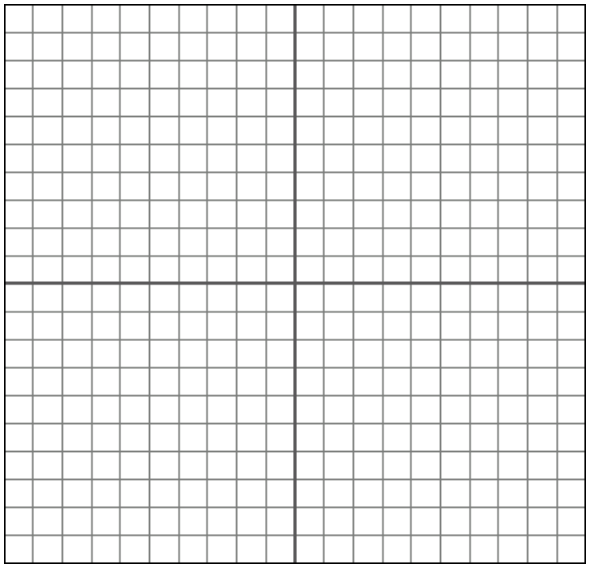
Absolute Value Functions

1. Sketch a graph of

a) $y = |4x + 2|$



b) $y = |(x-2)(x+3)|$



2. Write each absolute value function in piecewise notation:

a) $y = |-x - 5|$

b) $y = |(x-2)(x+3)|$

3. Solve by graphing:

a) $3 = |2x - 3|$

b) $|x^2 - x| = 6$

4. Use algebra to solve each equation.

a) $y = |(x-1)^2 - 2|$

b) $y = \frac{1}{2}|3x - 5|$

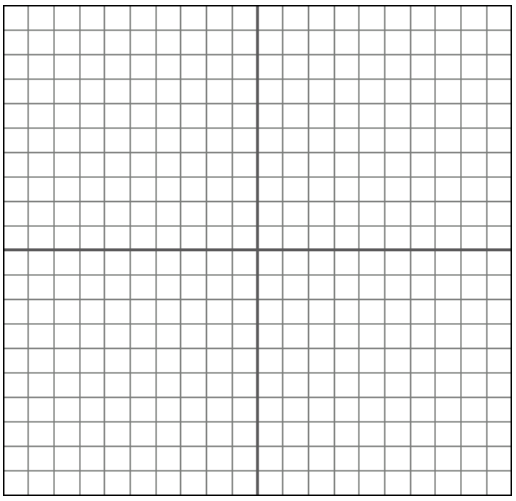
Reciprocal Functions

1. Given: $y = -x - 4$

   a) Graph this function.

   b) Graph the reciprocal of this function.

   Clearly draw in any asymptotes with dashed lines, include invariant lines and accurately locate the key points.

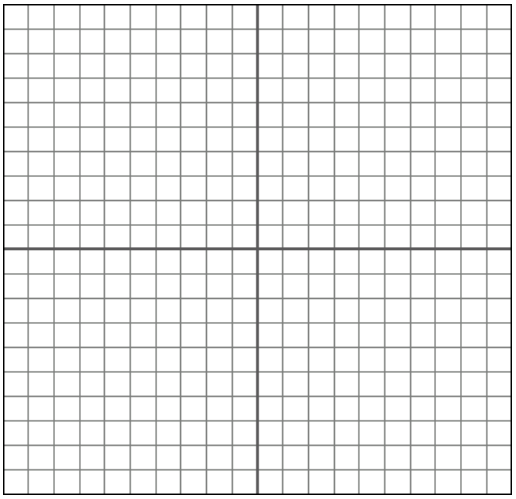


2. Given: $y = x^2 - 9$

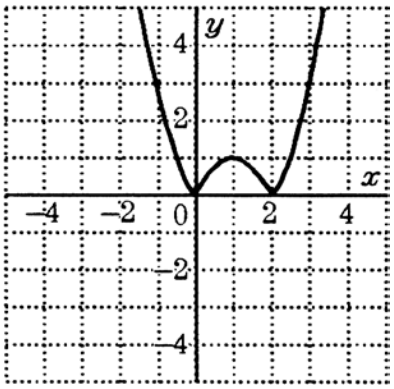
   a) Graph this function.

   b) Graph the reciprocal of this function.

   Clearly draw in any asymptotes with dashed lines, include invariant lines and accurately locate the key points.



3. What is the equation of the graph shown?



   a) $y = |x^2 - 2x|$

   b) $y = \dfrac{1}{x^2 - 2x}$

   c) $y = x^2 - 2x$

   d) $y = |x^2 + 2x|$

   e) $y = x^2 + 2x$

4. Given the functions, what is (are) the equations of the asymptote(s) of the reciprocal function?

   a) $y = 3x^2 - 75$

   b) $y = (x-2)^2 - 9$

   c) $y = (x+4)(x-2)$

Answers: 1

Trigonometry

1. In ΔXYZ, YZ = 5, XZ = $\sqrt{50}$, YZ = $\sqrt{75}$, ∠X=90°. Find *cos* Y.
2. In ΔABC, AB = 17 cm, BC = 8 cm, AC = 15cm and ∠C = 90°. Only one of the following ratios is correct, the correct ratio is: a) sin A = $\frac{8}{15}$ b) cos A = $\frac{8}{17}$ c) sin B = $\frac{15}{17}$ d) tan B = $\frac{8}{15}$ e) tan A = $\frac{8}{17}$
3. If *tan* θ = $\frac{5}{12}$, then *cos* θ =
4. Solve the triangle ΔDEF if ∠E=90°, EF=36, and DF=45.
5. Find other trigonometric ratios for θ if :

a) *cos* θ = $\frac{12}{37}$ b) *cos* θ = $\frac{a}{a+7}$

6. Find the angle of inclination of a ladder that reaches 19 metres up the wall of a house and whose foot is 7 metres from the wall. (nearest degree)
7. Two buildings are 42 metres apart and one is shorter than the other. From a window on the shorter building, the angle of elevation to the top of the taller building is 38° and the angle of depression to the base of the taller building is 53°. How tall is the taller building? (to 1 decimal place)
8. The goal posts of an indoor soccer field are 3.3 m wide. A ball is 15.8 m from one post and 17.1 m from the other post. Through what angle must the ball be kicked to score a goal?
9. In which quadrants is the tangent function negative?
10. What is the reference angle of –235°?
11. In what quadrant does the terminal side of angle 835° lie?
12. What is the next positive coterminal angle to –24°?
13. Given point P(-3, 5) is on the terminal arm of angle θ.

a) draw a diagram showing θ in standard position
b) find sin θ, cos θ, and tan θ. (leave answers in simplest radical form)
c) Find θ. Express θ to the nearest degree.

14. Solve for θ to the nearest degree if $0 \le \theta < 360°$.

a) sin θ = 0.570 b) tan θ = - 0.782 c) tan θ = 4.5257 d) cos θ = - 0.3553

15. Solve 2*cos* θ - 1 = 0 to the nearest degree for $0° \le \theta \le 180°$.
16. Solve *sin* θ = -0.723 for θ to the nearest degree; $0° \le \theta \le 360°$
17. Solve 8 *cos* θ + 7 = 0 for θ to the nearest degree; $0° \le \theta \le 360°$
18. Solve 6 $sin^2\theta$ - 7 *sin* θ + 2 = 0 for θ to the nearest degree; $0° \le \theta \le 360°$
19. In ΔABC, AB = 9 cm, AC = 13 cm, ∠ABC = 113°. To calculate the measure of ∠BCA, what is the most appropriate method for solving the problem?
20. What values must be known to use the Sine Law in ΔDEF?

a) d, e, ∠F, b) e, f, ∠D c) ∠D, ∠E, ∠F d) d, e, f e) d, e, ∠D

21. A possible Cosine Law for ΔDEF is:

a) $e^2 = d^2 + f^2 - 2df \sin E$ b) $f^2 = d^2 + e^2 - 2ed \cos E$ c) $d^2 = e^2 + f^2 - 2ef \cos D$

22. In ΔMNO, find side m if ∠M = 46°, n = 48 cm, and o = 32 cm.
23. Find the measure of the smallest angle in ΔPQR if p = 101, q = 136, and r = 62.
24. In ΔYZA, ∠Y = 36°, y = 24, and z = 34, find a.

Answers:
1.

Sequences and Series

1. Which of the following is a geometric sequence?
   a) 2, 4, 6, …
   b) 8, 12, 18, …
   c) 10, 30, 90, …
   d) 12, 8, 4, …
2. Which of the above sequences is (are) arithmetic?
3. Find the 18th term of the sequence: 11, 8, 5, 2, …
4. Determine the geometric mean between 7 and 28.
5. A geometric sequence is formed by inserting two geometric means between m^2 and m^{12}. Determine the sum of the two means.
6. Determine the first four terms of the sequence defined by $t_n = \frac{(-1)^n}{n+2}$.
7. For each of the sequences, write an expression for the general term t_n:
   a) 4, 7, 10, 13, 16, …
   b) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$
8. Which term of the sequence 3, 7, 11, 15, … has a value of 111?
9. In a grocery store cans of tomato juice are displayed in a pyramid containing 15, 14, 13, …, 3 cans in each row. How many cans are displayed?
10. Determine the 9th term of the sequence: $-\frac{1}{8}, \frac{1}{2}, -2, 8, \ldots$
11. Find the first term if $S_n = -\frac{91}{2}$ and $r = -3$.
12. Find the sum of the series: $10 + 6 + 2 + \cdots + -34$.
13. The sum of the 4th and 9th terms of an arithmetic sequence is 89. The sum of the 2nd and 6th terms is 54. What is the 3rd term of the sequence?
14. Alison's new business has $10 000 in sales the first year. Sales are expected to increase by a constant amount each year. How much is this constant yearly increase in sales if the total sales for the first five years is $500 000?
15. Determine the common ratio 'r' for a geometric sequence of positive terms with $t_1 = 4$ and $t_3 = 9$.
16. How many terms are in the sequence: -16, -12, -8, … , 384?
17. Find the common ratio for the sequence: $pr, \frac{pr^3}{q^2}, \frac{pr^5}{q^4}, \ldots$
18. The second term of an arithmetic series is m and the third term is n. What is the fifth term?
19. Find the 3rd term of the geometric series with $t_2 = 30$ and $S_3 = -35$.
20. What value of x in $x + 1, 2x + 3, x^2 - 5, \ldots$ will form an arithmetic sequence?
21. Find the 7th term if $S_7 = 756$ and $S_6 = 457$.
22. A ball is dropped from a height of 16 meters. The ball rebounds to half of the height after each bounce. Calculate the total vertical distance the ball travels before coming to rest.

Answers: 1. b,c, 2. a,c, 3. -40, 4. ± 4. 5. $m^6 + m^{10}$, 6. $\frac{-1}{3}, \frac{1}{4}, \frac{-1}{5}, \frac{1}{6}$, 7a) $t_n = 3n + 1$, b) $t_n = \frac{1}{n}, n \neq 0$, 8. 28, 9. 117, 10. -8192, 11. $\frac{1}{4}$, 12. -144, 13. 20, 14. $45 000, 15. $\frac{3}{2}$, 16. 101, 17. $\frac{r^2}{q^2}$, 18. $3n - 2m$, 19. -20 or -45, 20. 5, 21. 293, 22. 48.