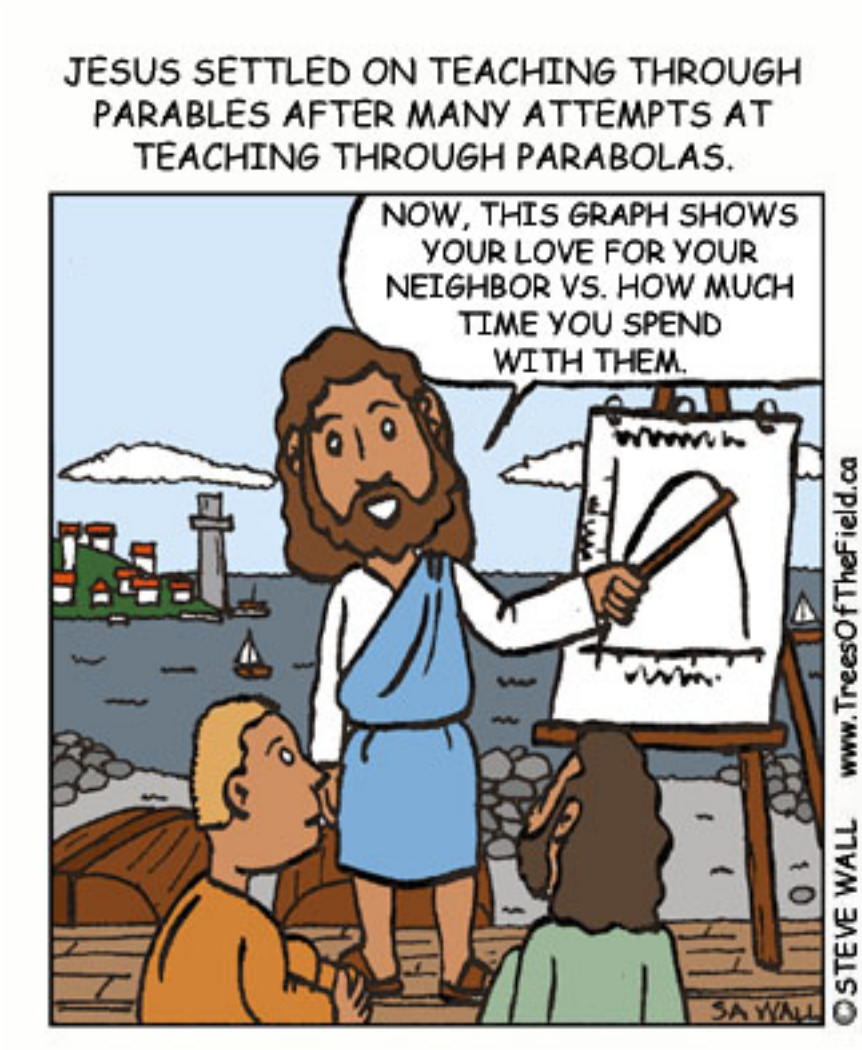


MPM2DI

Unit 4: Graphing Quadratics

Unit 4 Overview and Homework Tracker

Date	***Lesson – Topic***	☑ ***Homework to be Completed {No calculator!!}***	***Teacher's Initials***
	4.0 – Introduction to Graphing Quadratic Relations	☐ Complete the "Introduction to Graphing Quadratic Relations" MSIP Assignment {no calculator!}	
	4.1 – Vertex Form of a Quadratic Relation	☐ p. 351 #1, 2ce; WORKSHEET "Vertex Form of a Quadratic Relation" {no calculator!}	
	4.2 – Describing Transformations on a Quadratic Relation	☐ p. 363-366 #4, 7defg, 8abde, 11 {no calculator!}	
	4.3 – Finding the Equation of a Quadratic Relation	☐ p. 351-352 #5, 6be, 7, 10; p. 363-367 #1, 2, 9abd, 13abcd, 16abcd {no calculator!}	
	4.4 – Standard Form of a Quadratic Relation	☐ WORKSHEET "Standard Form of a Quadratic Relation" {no calculator!}	
	4.5 – Completing the Square	☐ p. 390-391 #4beg, 8abcdgij, 9ace, 10, 21 {no calculator!}	
	4.6 – Factored Form of a Quadratic Relation – I	☐ p. 280-282 #1, 2, 3acf, 5afgh (use fractions reduced to lowest terms instead of decimals for all answers), 7ceghi {no calculator!}	
	4.7 – Factored Form of a Quadratic Relation – II	☐ p. 308 #8abce; p. 282-283 #8, 9 {no calculator!}	
	Review I – Exploring and Comparing Forms of Quadratic Relations	☐ WORKSHEET "Graphing a Quadratic Relation Given Any Form" {no calculator!}	
	Review II – Unit 4	☐ p. 326 #6; p. 328 #7ac, 9; p. 418 #1, 2, 3; p. 421 #6, 7; p. 424 #12 {no calculator!}	
	Unit 4 Test	***Mark:***	***______%***

Homework Procedures:

1. Write the lesson number, the assigned questions, and your name at the top of the first page; for each page afterwards, include just your name (or initials) and the lesson number (*i.e.* "4.3" is Unit 4 Lesson 3)
2. Each question must be checked against the answers in the back of the textbook – place a checkmark next to each correct answer, and circle/star/highlight any questions you have problems with. Address these questions immediately, as you are likely to see them again on a test or quiz!
3. Homework will be checked daily and will be submitted prior to each unit test; it will be checked thoroughly by your teacher if the mark earned on the test is below 70%.

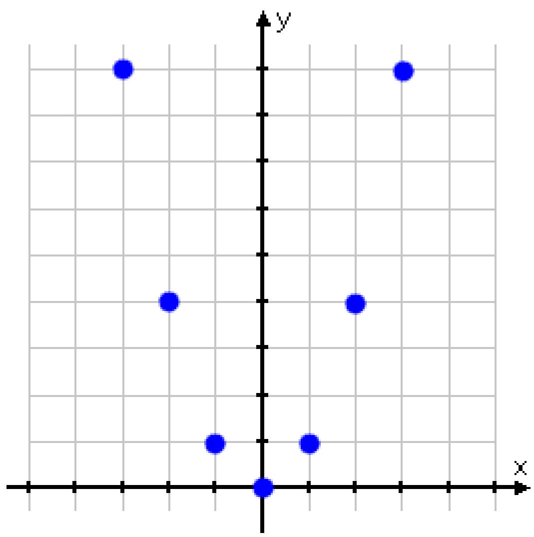
Introduction to Graphing Parabolas

In this assignment, you are going to graph parabolas! Parabolas are curves that result from equations that have the highest exponent value of 2, generally seen in the form $y = x^2$.

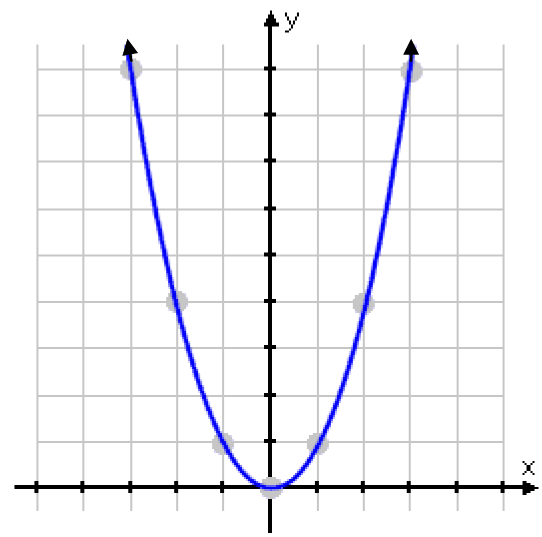
Steps for graphing a parabola using a table of values:

1. Complete the table of values by substituting x-values into the equation and evaluating for y.

2. Plot the key points from the table of values.



3. Join the points using a smooth curve, adding arrows to the ends of the parabola. Do NOT use straight line segments between points!



4. Label the appropriate equation with the corresponding parabola.

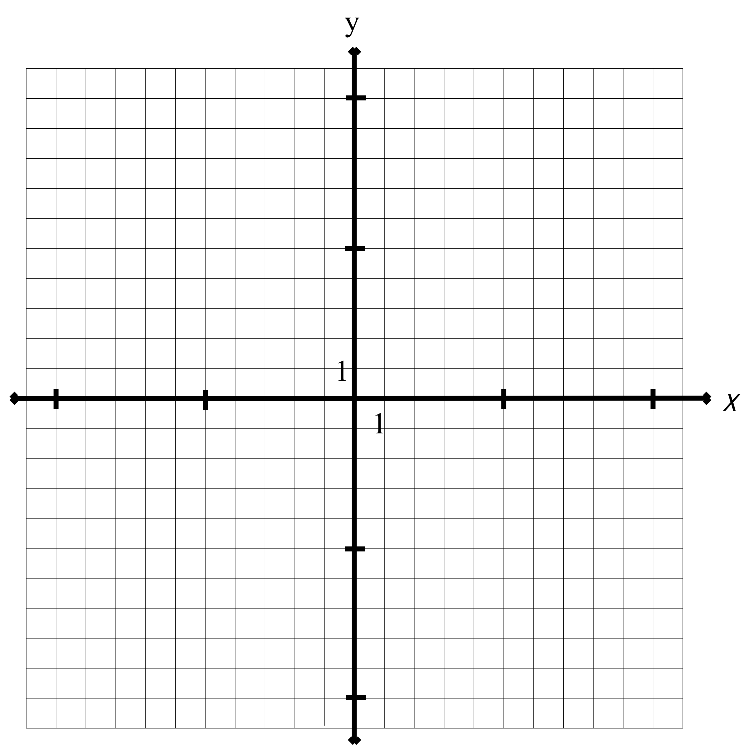
Graph the following relations on the grid to the right:

a) $y = x^2$

x	*y*
− 3	9 → $(-3)^2$
− 2	
− 1	
0	
1	
2	
3	

b) $y = -x^2$

x	*y*
− 3	− 9 → $-(-3)^2$
− 2	
− 1	
0	
1	
2	
3	



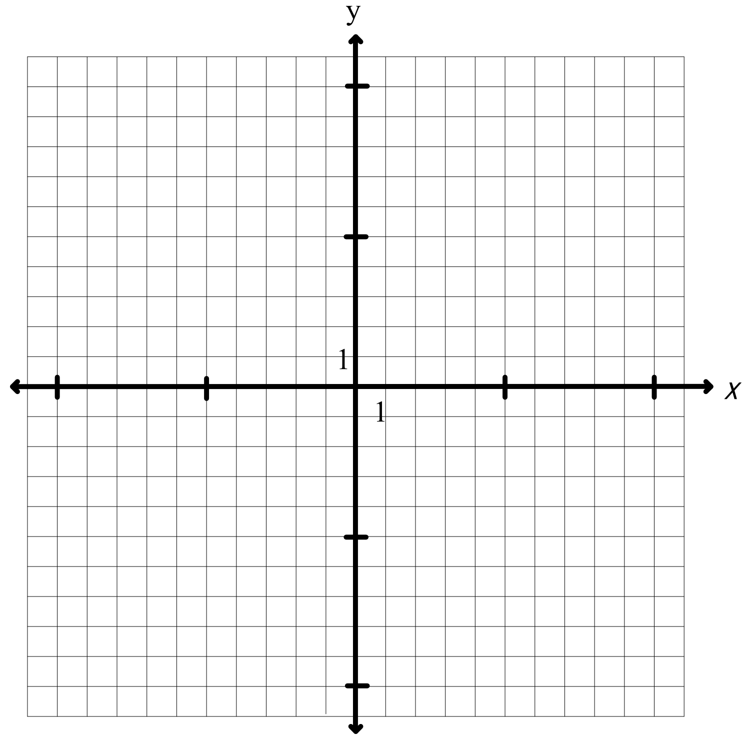
Graph the following relations on the grid to the right:

c) $y = x^2 - 5$

x	y	
− 3	4	→ $(-3)^2 - 5$
− 2		
− 1		
0		
1		
2		
3		

d) $y = -x^2 + 3$

x	y	
− 3	− 6	→ $-(-3)^2 + 3$
− 2		
− 1		
0		
1		
2		
3		



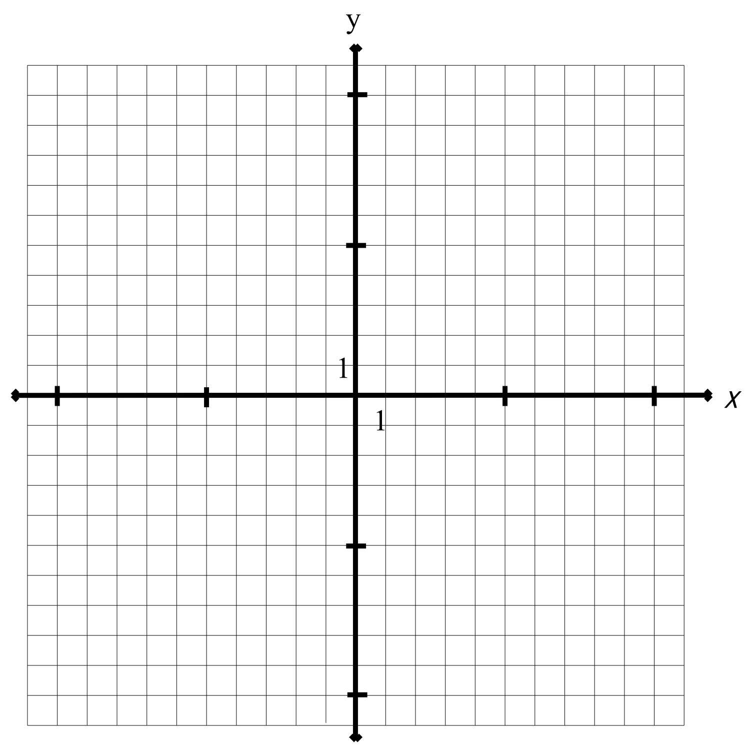
Graph the following relations on the grid to the right:

e) $y = (x+7)^2$

x	y	
− 10	9	→ $(-10 + 7)^2$
− 9		
− 8		
− 7		
− 6		
− 5		
− 4		

f) $y = (x-5)^2$

x	y	
8	9	→ $(8 - 5)^2$
7		
6		
5		
4		
3		
2		



Vertex Form of a Quadratic Relation

A quadratic relation is in vertex form if it is in the form ______________________________________.

A. Key Features of a Parabola

- The graph of a quadratic relation, generally seen in the form $y = x^2$ is called a _________________________.
- The turning point of a parabola is called the _________________________, also known as point (h, k).
  - The vertex will be a _________________________ value if the graph opens ____________ .
  - The vertex will be a _________________________ value if the graph opens _____________.
  - Given an equation, if $a > 0$, the parabola opens ______; if $a < 0$, the parabola opens ______________.
- The vertical line that runs through the vertex is the line of symmetry for the parabola, and it is referred to as the ____.
  - The equation of the axis of symmetry is given as _____________, where h is the x-value of the vertex.
- The point(s) where the parabola intersects the x-axis are referred to as the ______________________________ or the ________________ of the quadratic relation. There can be 0, 1, or 2 x-intercepts for a parabola.
- The point where the parabola intersects the y-axis is referred to as the __________________________________.

B. Graphing $y = a(x - h)^2 + k$ when a = 1 or –1

This form is called ***vertex form*** because the vertex is clearly visible as the point (h, k).

For the following relations		a) $y=-(x-5)^2+3$	b) $y=(x+4)^2-2$
state:	The vertex	____________________	____________________
	Direction of opening	____________________	____________________
	Equation of the A.O.S.	____________________	____________________
	Maximum/minimum?	____________________	____________________

Steps for graphing a quadratic given vertex form where a = 1 or –1:

1. Plot the vertex (h, k).
2. From the vertex, use the ***pattern*** and the ***direction of opening*** to plot the key points congruent to $y = x^2$.
   Pattern when a = 1 or –1**: **"x over, x^2 up/down".
   Example: 1 over, 1 up/down; 2 over, 4 up/down; 3 over, 9 up/down.
3. Join the points with a smooth curve, including arrows at the ends.
4. Label the vertex, sketch and label the equation of the A.O.S., and write the equation of the parabola.

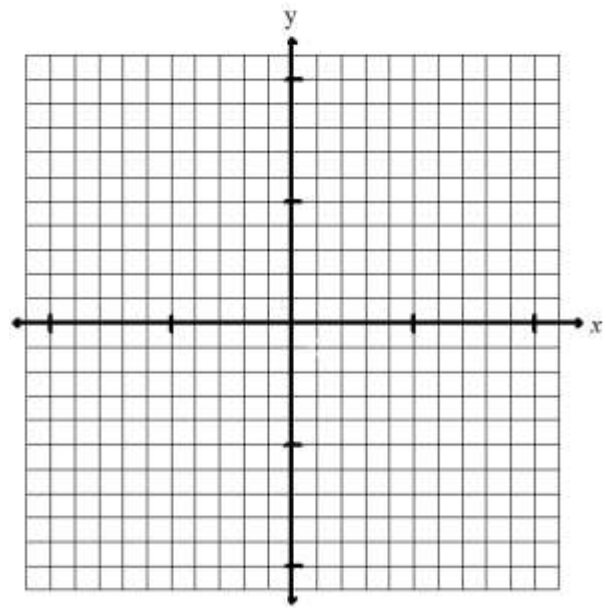
Example 1: *Graph the quadratic relation* $y = (x+3)^2 - 5$, *and state:*

The vertex ___________________

Direction of opening ___________________

Equation of the A.O.S. ___________________

Maximum/minimum? ___________________



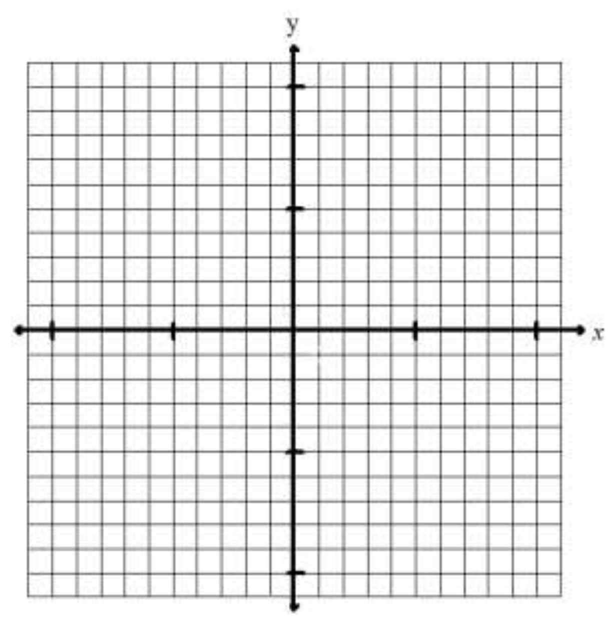
Example 2: *Graph the quadratic relation* $y = -(x-2)^2 + 6$, *and state:*

The vertex ___________________

Direction of opening ___________________

Equation of the A.O.S. ___________________

Maximum/minimum? ___________________



C. Graphing $y = a(x - h)^2 + k$ when $a \neq 1$ or -1

We have already seen that when $a > 0$, the parabola opens up and when $a < 0$, the parabola opens down. The a-value can also affect the shape of the parabola, expanding or compressing it vertically (in the y-direction).

When $a > 1$, the parabola is expanded by a factor of a; when $0 < a < 1$, the parabola is compressed by a factor of a.

Steps for graphing a quadratic given vertex form where $a \neq 1$ or -1:

1. Plot the vertex (h, k).
2. From the vertex, use the ***pattern*** and the ***direction of opening*** to plot the key points.
   Pattern when a ≠ 1 or –1: "x over, ax^2 up/down".
   Example: 1 over, $a \times 1$ up/down; 2 over, $a \times 4$ up/down; 3 over, $a \times 9$ up/down.
3. Join the points with a smooth curve, including arrows at the ends.
4. Label the vertex, sketch and label the equation of the A.O.S., and write the equation of the parabola.

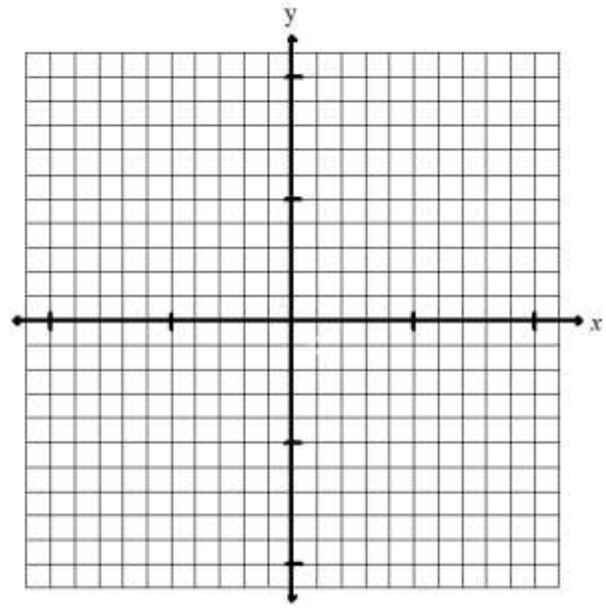
Example 3: *Graph the quadratic relation* $y = -3(x-7)^2 + 9$, *and state:*

The vertex ____________________

Direction of opening ____________________

Equation of the A.O.S. ____________________

Maximum/minimum? ____________________



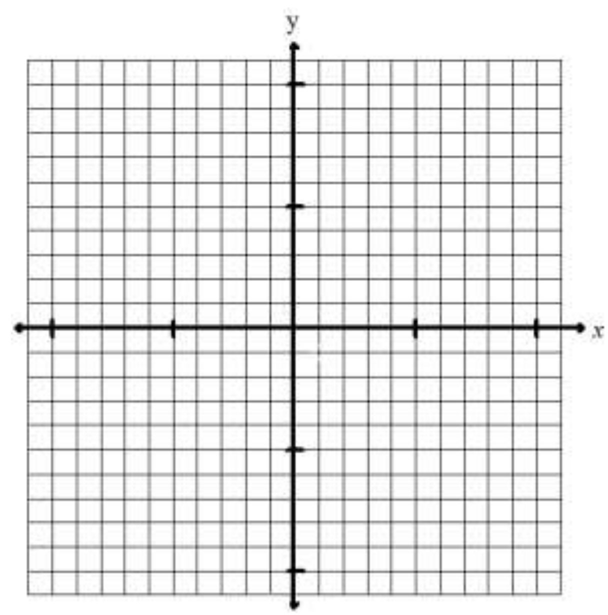
Example 4: *Graph the quadratic relation* $y = \frac{1}{2}x^2 + 5$, *and state:*

The vertex ____________________

Direction of opening ____________________

Equation of the A.O.S. ____________________

Maximum/minimum? ____________________



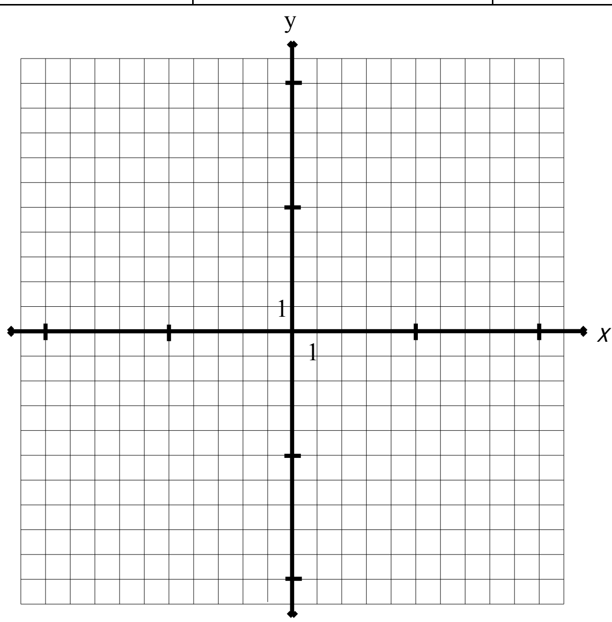
Note: The graph of $y = a(x - h)^2 + k$ is the graph of $y = ax^2$ translated h units *horizontally* and k units *vertically.*

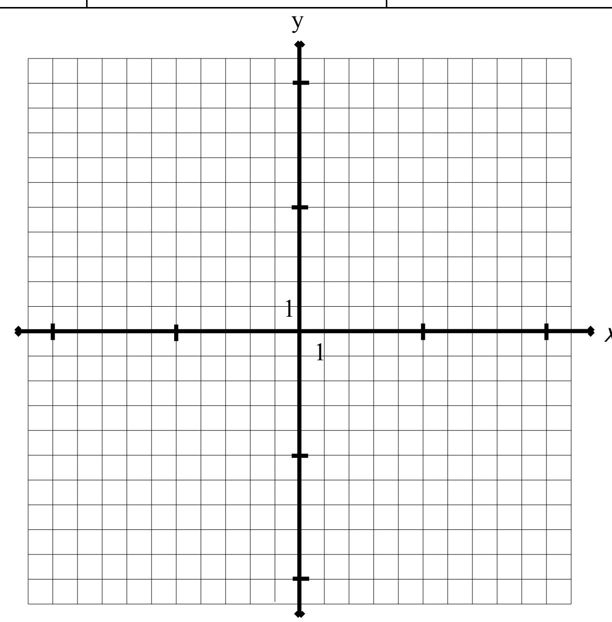
HW: p. 351 #1, 2ce; WORKSHEET "Vertex Form of a Quadratic Relation"

WORKSHEET: Vertex Form of a Quadratic Relation

1. Fill in the following table and graph the relations given on the axes below:

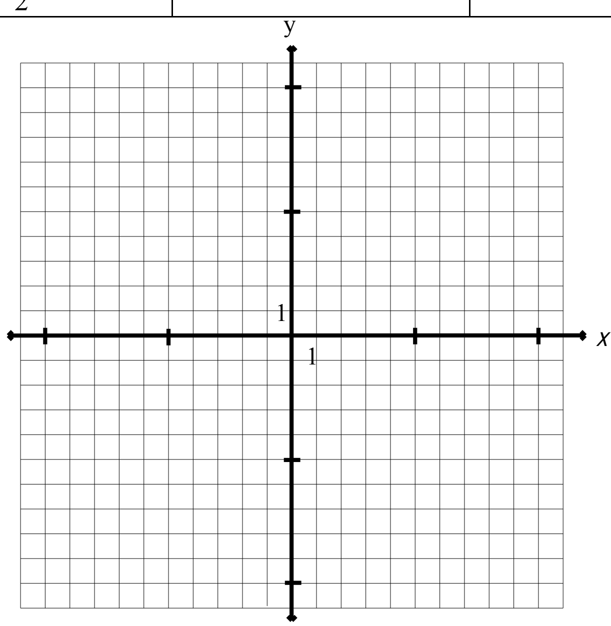
Relation	***Vertex***	***Direction of Opening***	***Equation of A.O.S.***	***Max/Min?***
$y = x^2$				
$y = x^2 - 3$				
$y = -\frac{1}{2}x^2$				
$y = -2x^2 + 5$				

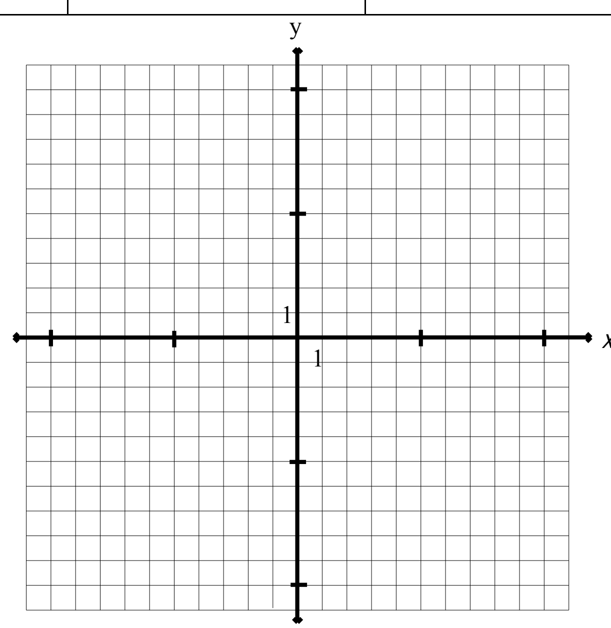




2. Fill in the following table and graph the relations given on the axes below:

Relation	***Vertex***	***Direction of Opening***	***Equation of A.O.S.***	***Max/Min?***
$y = 3(x - 3)^2$				
$y = -(x + 5)^2$				
$y = -(x - 2)^2$				
$y = \frac{1}{2}(x+4)^2 - 3$				





Practise, Apply, Solve 4.2

A

1. For the relation $y = -3(x + 5)^2 - 4$,
   (a) find the coordinates of the vertex
   (b) find the equation of the axis of symmetry
   (c) find the direction of opening
   (d) sketch the graph

2. For each quadratic relation, find
   i. the coordinates of the vertex
   ii. the equation of the axis of symmetry 351
   iii. the direction of opening

   (a) $y = (x - 2)^2 + 5$ (b) $y = -4(x + 3)^2 - 2$ (c) $y = 2(x - 4)^2$
   (d) $y = x^2$ (e) $y = -3x^2 + 2$ (f) $y = -(x + 7)^2 + 4$

4. Sketch the graph of the relation by hand. Start with the graph of $y = x^2$ and use the appropriate transformations.
   (a) $y = x^2 - 4$ (b) $y = (x - 3)^2$ (c) $y = x^2 + 2$ 364
   (d) $y = (x + 5)^2$ (e) $y = (x + 1)^2 - 2$ (f) $y = (x - 5)^2 + 3$

B

7. What transformations must you apply to $y = x^2$ to create the new graph? List the transformations in the order you would apply them.
   (a) $y = -x^2 + 9$ (b) $y = (x - 3)^2$
   (c) $y = (x + 2)^2 - 1$ (d) $y = -x^2 - 6$
   (e) $y = -2(x - 4)^2 + 16$ (f) $y = \frac{1}{2}(x + 6)^2 + 12$
   (g) $y = -\frac{1}{2}(x + 4)^2 - 7$ (h) $y = 5(x - 4)^2 - 12$
   (i) $y = \frac{3}{4}(x - 1)^2 + 5$

8. Sketch the graph of the relation by hand. Start with the graph of $y = x^2$ and use the appropriate transformations.
   (a) $y = -x^2 + 4$ (b) $y = -2(x + 3)^2$
   (c) $y = \frac{3}{4}x^2 - 7$ (d) $y = \frac{1}{2}(x + 4)^2 - 5$
   (e) $y = -3(x - 2)^2 + 12$ (f) $y = -1.5x^2 + 10$

11. **Communication:** Without graphing, tell how many zeros (x-intercepts) the quadratic relation has. Explain your answer.
    (a) $y = x^2 - 6$ (b) $y = (x + 2)^2 + 4$
    (c) $y = -4(x - 3)^2$ (d) $y = \frac{2}{5}(x - 1)^2 - 3$

Describing Transformations on a Quadratic Relation

A. What are the Roles of *a, h,* and *k* in the Quadratic Relation $y = a(x - h)^2 + k$?

$$y = \pm\, a(x - h)^2 + k$$

1. Reflections on the Function $y = x^2$

Reflection	Mathematical Form	Effect
Vertical	$y = -x^2$	Compared to $y = x^2$, the graph of $y = -x^2$ is a vertical reflection across the x-axis. The point (x, y) on $y = x^2$ becomes the point (x, – y) on $y = -x^2$.

2. Stretches on the Function $y = x^2$

Stretch	Mathematical Form	Effect
Vertical	$y = ax^2$	If $a > 1$, the graph is **vertically expanded** by a factor of a. If $0 < a < 1$, the graph is **vertically compressed** by a factor of a. The point (x, y) on $y = x^2$ becomes the point (x, ay) on $y = ax^2$

3. Translations on the Function $y = x^2$

Translation	Mathematical Form	Effect
Horizontal	$y = (x - h)^2$	Compared to the graph of $y = x^2$, the graph of $y = (x - h)^2$ is a horizontal translation of h units. When **h > 0** the graph is **horizontally translated to the RIGHT h units**. When **h < 0** the graph is **horizontally translated to the LEFT h units**. The point (x, y) on $y = x^2$ becomes the point (x + h, y) on $y = (x - h)^2$.
Vertical	$y = x^2 + k$	Compared to the graph of $y = x^2$, the graph of $y = x^2 + k$ is a vertical translation of k units. When **k > 0** the graph is **vertically translated up k units**. When **k < 0** the graph is **vertically translated down k units**. The point (x, y) on $y = x^2$ becomes the point (x, y + k) on $y = x^2 + k$.

You must describe combinations of transformations in the following order:

1. **Reflections** and **Stretches** (since both involve *multiplication* of the *y*-value, they must be done first)
2. **Translations** (just like in BEDMAS, *addition* and *subtraction* on the *x*- and *y*-values must be done last)

You can't go wrong if you describe the transformations as they occur from <u>left to right</u> across the equation!

B. Steps to Graphing and Describing Transformations

a) Describe in words the transformations relative to the graph of $y = x^2$ using the *correct order*!
b) Graph the following quadratic functions using the vertex, the pattern, and the direction of opening.
c) Label the vertex.
d) Sketch the axis of symmetry and label with its equation.

Ex. 1: $y = 2(x-3)^2 - 2$

Description:

Transformations on the graph of $y = x^2$ *are :*

____

____

____

____

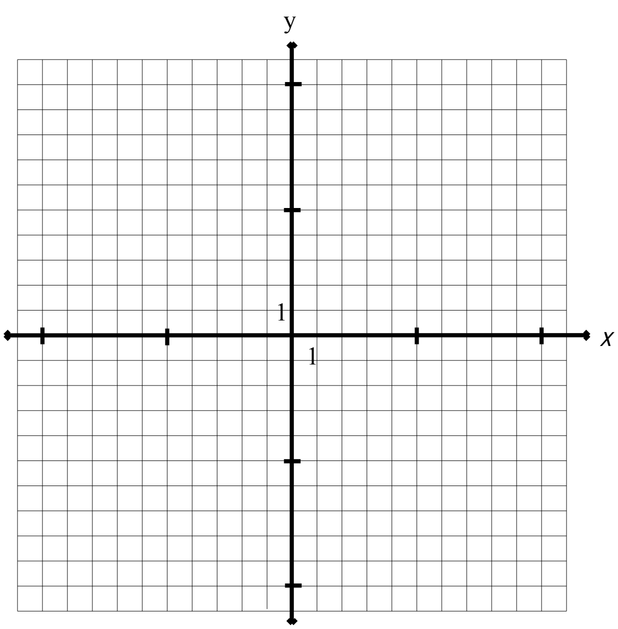
____

____

Vertex: ______________________

Direction of opening ______________________

Equation of A.O.S.: ______________________



Ex. 2: $y = -(x+1)^2 + 5$

Description:

Transformations on the graph of $y = x^2$ *are :*

____

____

____

____

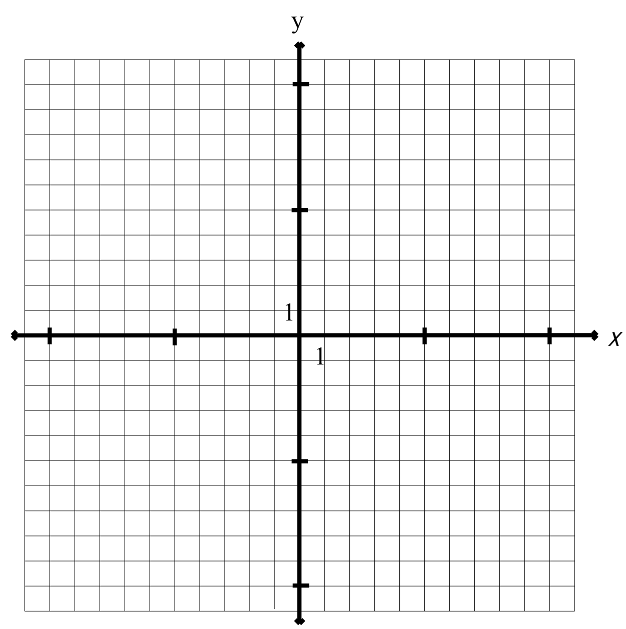
____

____

Vertex: ______________________

Direction of opening ______________________

Equation of A.O.S.: ______________________



HW: p. 363-366 #4, 7defg, 8abde, 11.

Finding the Equation of a Quadratic Relation

To write an equation in ***vertex form***, we must know the values of ***a, h, and k.***

Note: The vertex form of a quadratic relation includes five variables in total:

- the **vertex**, (h, k) comprises two of the variables;
- any **other point** on the curve, (x, y) comprises two more variables;
- the **vertical stretch factor**, a, is another variable.

$$y = a(x - h)^2 + k$$

Given any four variables, we can solve for the unknown variable by rearranging the equation and isolating the required variable!

Ex. 1: Rearrange the following equations to solve for the indicated variable.

a) $6 = a(1 - 2)^2 + 4$

b) $-8 = -3(-1 + 3)^2 + k$

c) $0 = a(5 - 1)^2 + 8$

Ex. 2: Use the information provided to determine the equation for each quadratic relation described.

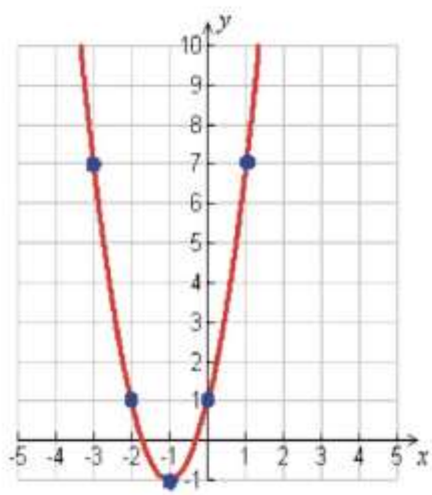
a) In vertex form, find the equation of the parabola with $a = -\frac{2}{3}$ and vertex at (– 2, 3)

b) In vertex form, find the parabola with vertex (5, – 8) and passing through (1, – 2).

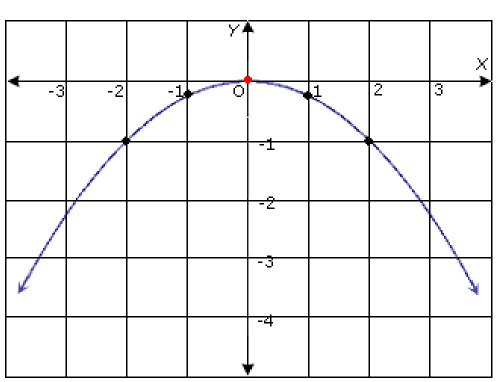
c) Using a formal check, determine if the point (4, – 11) lies on the parabola $y=-3(x-2)^2+1$

Ex. 3: Use the following graphs to determine an equation for each quadratic relation.

a)



b)



HW: p. 351-352 #5, 6be, 7, 10; p. 363-367 #1, 2, 9abd, 13abcd, 16abcd.

5. Find the quadratic relation in vertex form $y = a(x - h)^2 + k$.

(a) $a = 2$, vertex at $(0, 3)$
(b) $a = -3$, vertex at $(2, 0)$
(c) $a = -1$, vertex at $(3, -2)$
(d) $a = 0.5$, vertex at $(-3.5, 18.3)$

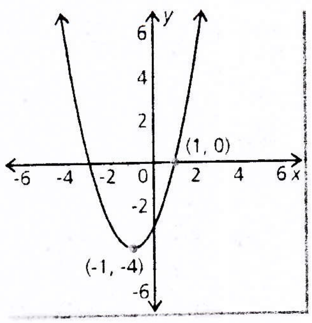
6. Which of these points are on the parabola $y = 2(x - 1)^2 + 5$?

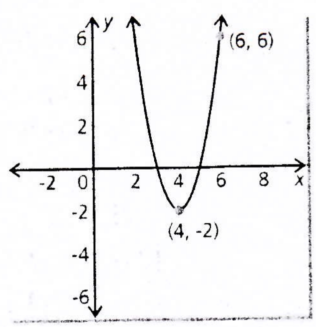
(a) $(2, 7)$
(b) $(4, 13)$
(c) $(0, 5)$
(d) $(-2, 23)$
(e) $(-1, 13)$

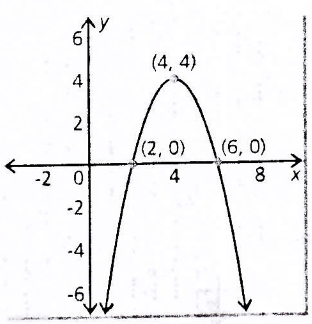
B

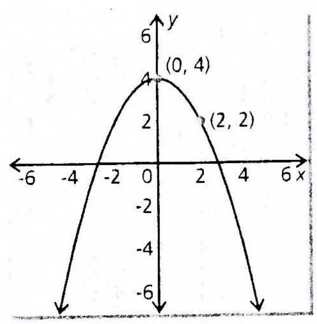
7. Find, in vertex form, the equation of the quadratic relation
(a) with vertex at $(0, 3)$, passing through $(2, -5)$
(b) with vertex at $(2, 0)$, passing through $(5, 9)$
(c) with vertex at $(-3, 2)$, passing through $(-1, 14)$
(d) with vertex at $(5, -3)$, passing through $(1, -8)$

10. Find the equation of the parabola, in vertex form.

(a)


(b)


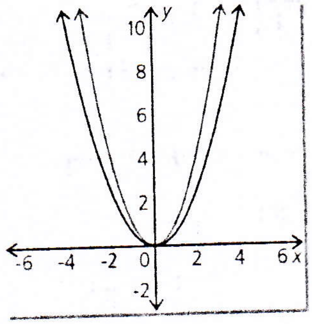
(c)


(d)


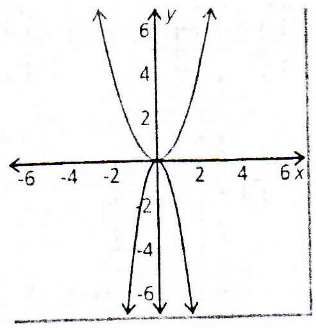
1. Match each graph with the correct equation. The graph of $y = x^2$ is also in each diagram in red.

(a) $y = 4x^2$ (b) $y = -3x^2$ (c) $y = \frac{2}{3}x^2$ (d) $y = -0.4x^2$

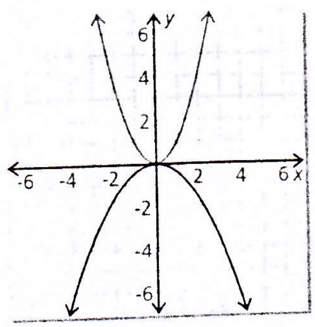
i.



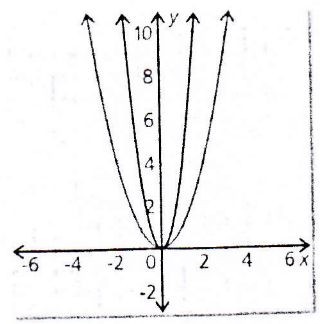
ii.



iii.



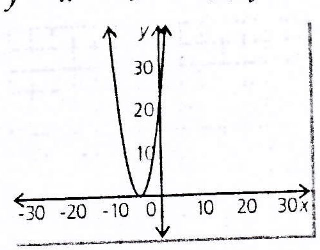
iv.



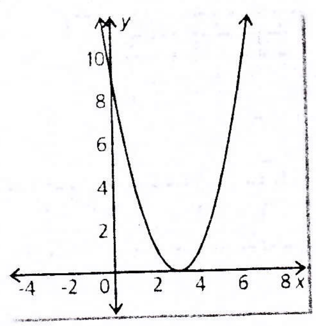
2. Match each graph with the correct equation.

(a) $y = x^2 + 5$ (b) $y = (x + 5)^2$ (c) $y = x^2 - 3$ (d) $y = (x - 3)^2$

i.



ii.



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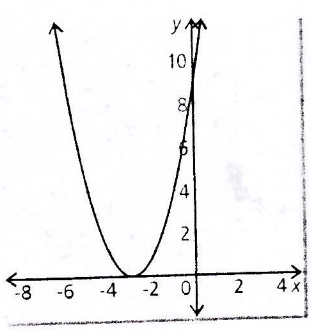

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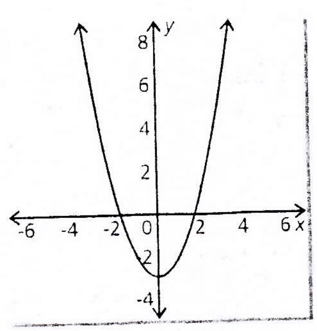

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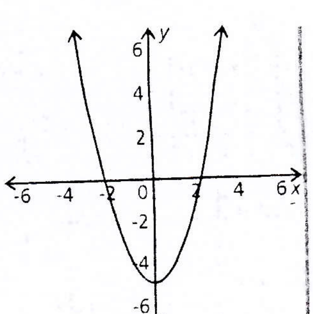
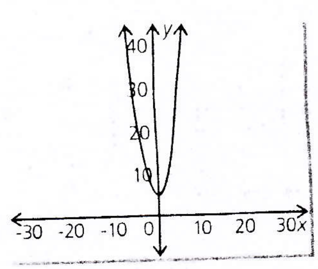
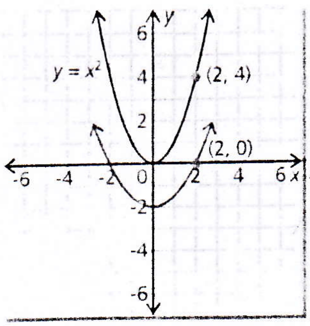


vi.

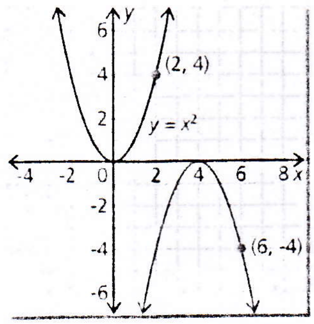


9. Transformations were applied to the graph of $y = x^2$ to obtain the coloured parabolas. Describe the transformations that were applied and use them to write a relation for each coloured parabola.

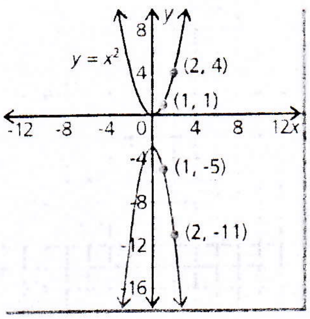
(a)



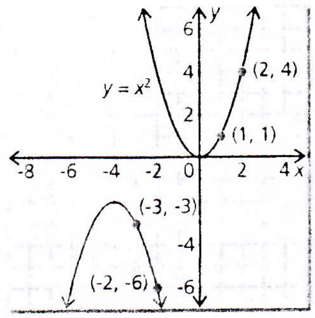
(b)



(c)



(d)



13. Write the relation for a parabola that satisfies each set of conditions.

(a) vertex at (0, 4); opens upward; the same shape as $y = x^2$

(b) vertex at (5, 0); opens downward; the same shape as $y = x^2$

(c) vertex at (2, -3); opens upward; narrower than $y = x^2$

(d) vertex at (-3, 5); opens downward; wider than $y = x^2$

(e) axis of symmetry $x = 4$; opens upward; two distinct zeros; narrower than $y = x^2$

(f) vertex at (3, -4); no zeros; wider than $y = x^2$

16. Write the relation for a parabola that satisfies each condition.

(a) The graph of $y = x^2$ is reflected about the x-axis, then translated down 7 units.

(b) The graph of $y = x^2$ is stretched vertically by a factor of $\frac{3}{2}$, then translated left 4 units.

(c) The graph of $y = x^2$ is compressed vertically by a factor of 3, then translated up 10 units.

(d) The graph of $y = x^2$ is reflected about the x-axis, stretched vertically by a factor of 2, then translated to the right 5 units and down 8 units.

Standard Form of a Quadratic Relation

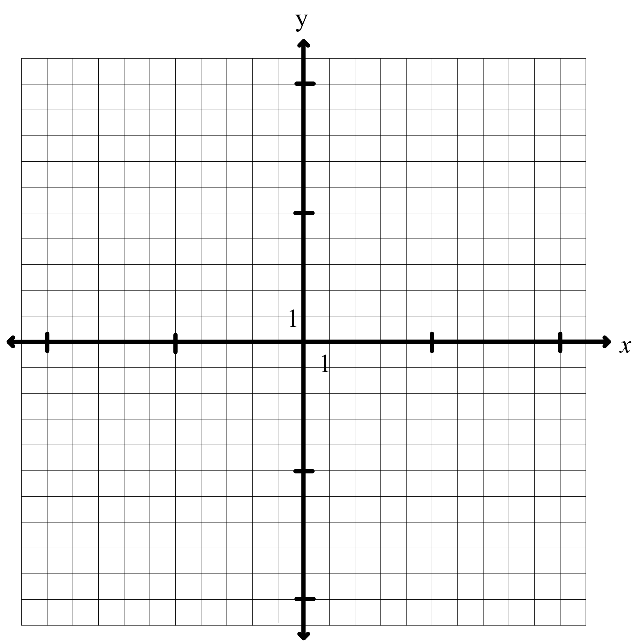
A quadratic relation is in standard form if it is in the form ____.

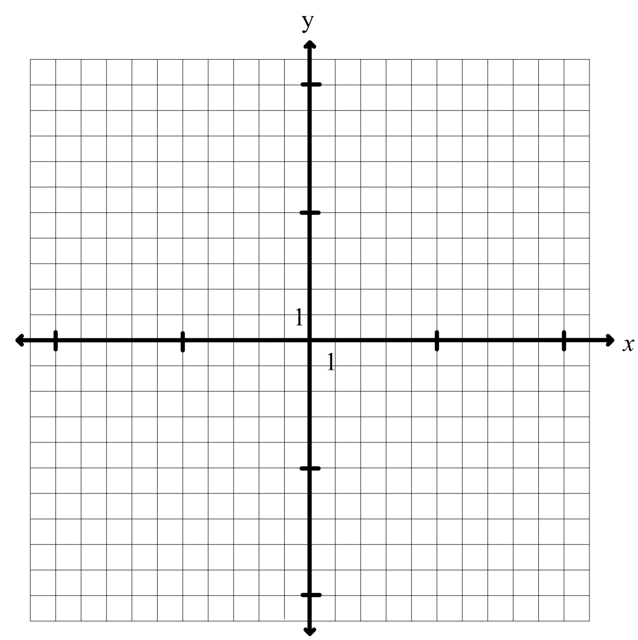
Fill in the table of values and graph the given (standard form) relations on the axes below:

a) $y = 2x^2 + 8x + 5$ *, where A.O.S is x = –2*

b) $y = -2x^2 + 12x - 11$ *, where A.O.S is x = 3*

x	*y*	





Summary for Graphing Standard Form of a Quadratic Relation:

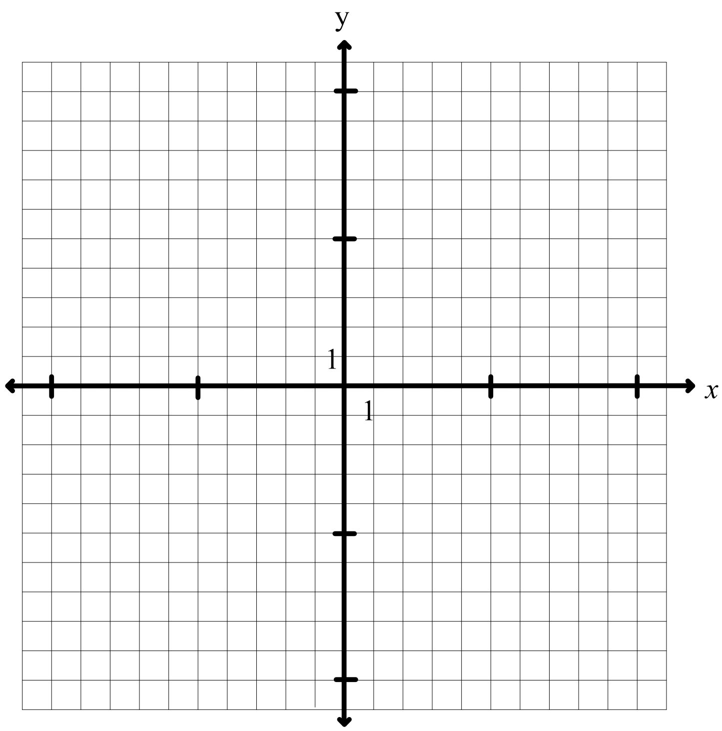
- To graph a quadratic relation in standard form, given its axis of symmetry:
  a) Choose strategic x-values around the A.O.S. to complete a table of values; or
  b) Find the vertex through substitution and graph using the "*a*-value" to complete the pattern.
- The standard form of a quadratic relation is useful to determine the ***y*-intercept (*c*-value)** and the **direction of opening and stretch factor (*a*-value)**.

HW: WORKSHEET "Standard Form of a Quadratic Relation"

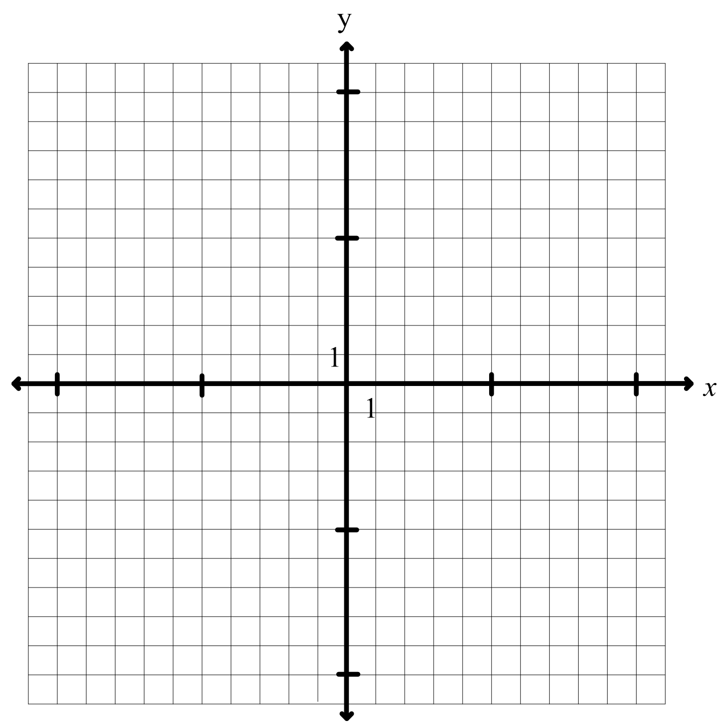
WORKSHEET: Standard Form of a Quadratic Relation

*1. For each quadratic relation given, create a **table of values** in your notebook and then graph on the axes below. Label all key features.*

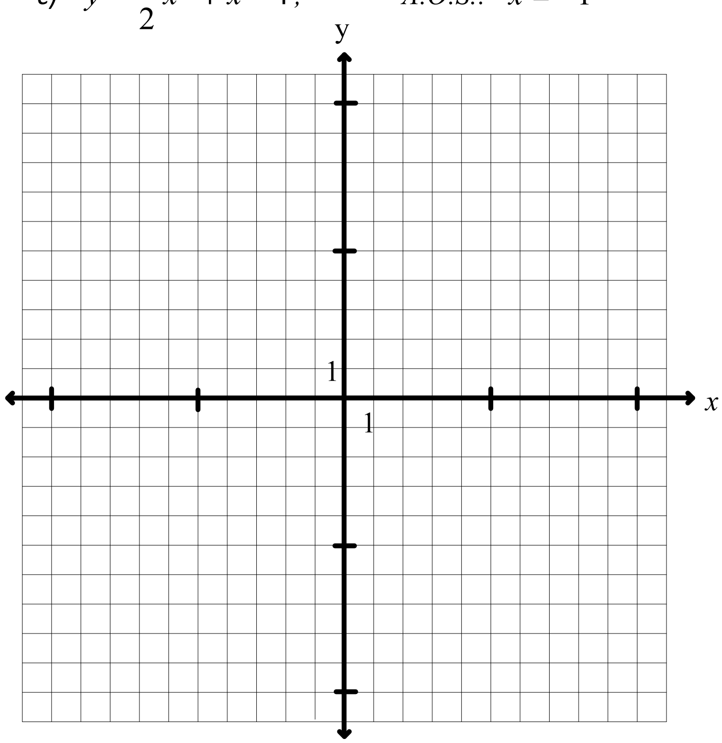
a) $y = x^2 - 6x + 5$; *A.O.S.:* $x = 3$



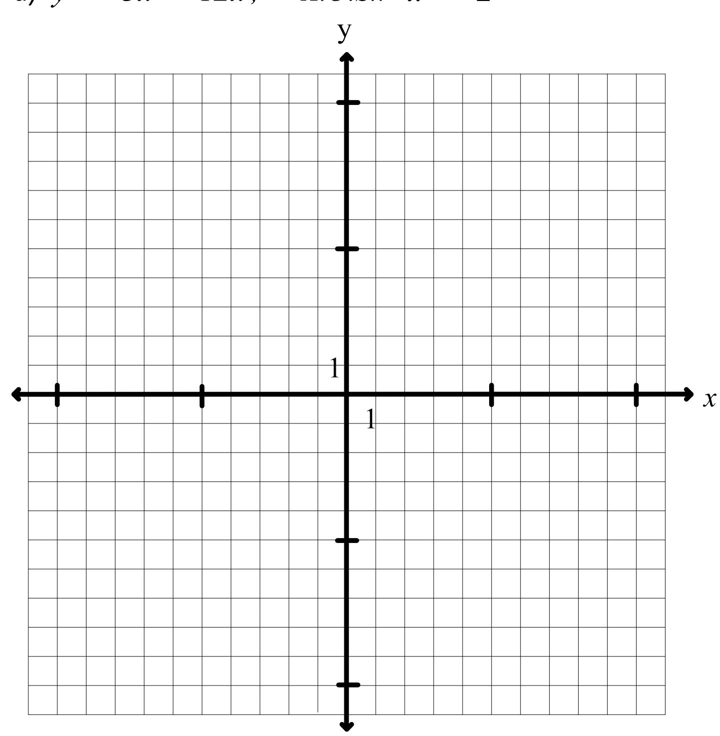
b) $y = -2x^2 + 12x - 16$; *A.O.S.:* $x = 3$



c) $y = \frac{1}{2}x^2 + x - 4$; *A.O.S.:* $x = -1$



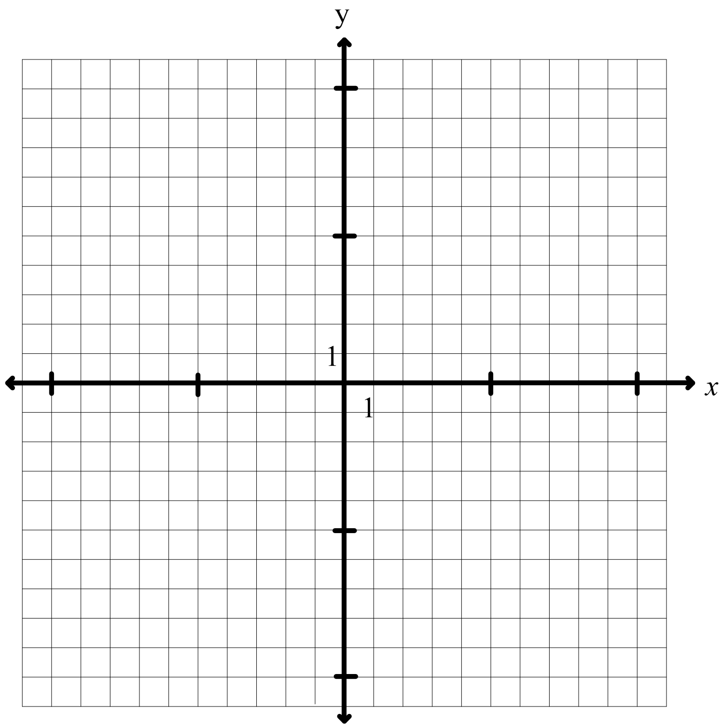
d) $y = -3x^2 - 12x$; *A.O.S.:* $x = -2$



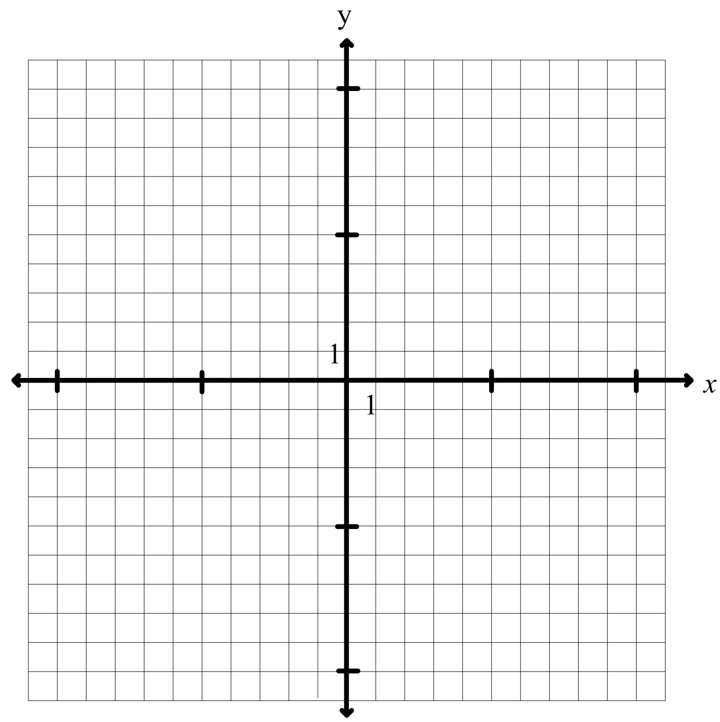
2. *For each quadratic relation given, use the given equation of the A.O.S. to* ***determine the coordinates of the vertex****.*

Then use the ***a-value and the pattern*** *to graph the relation on the axes below. Label all key features.*

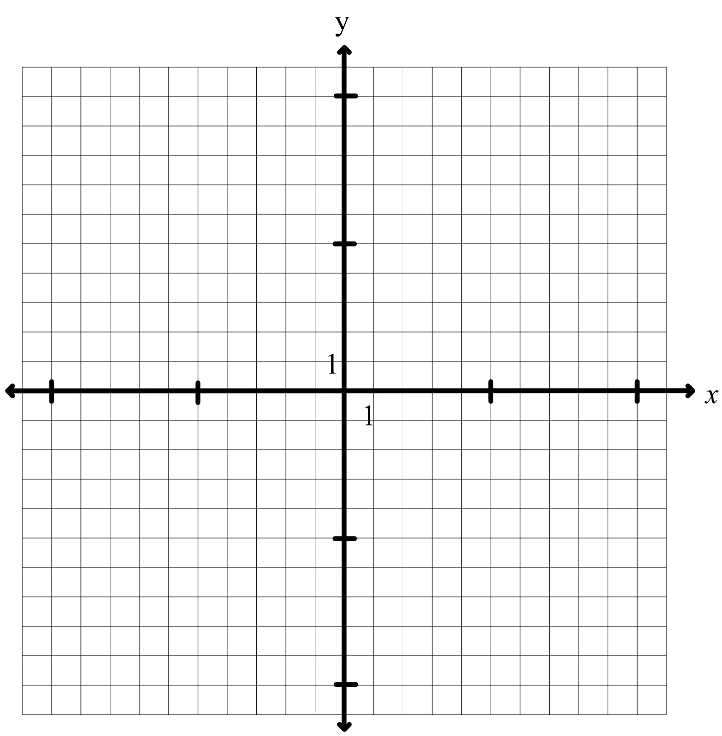
a) $y = -x^2 - 6x - 13$; A.O.S.: $x = -3$



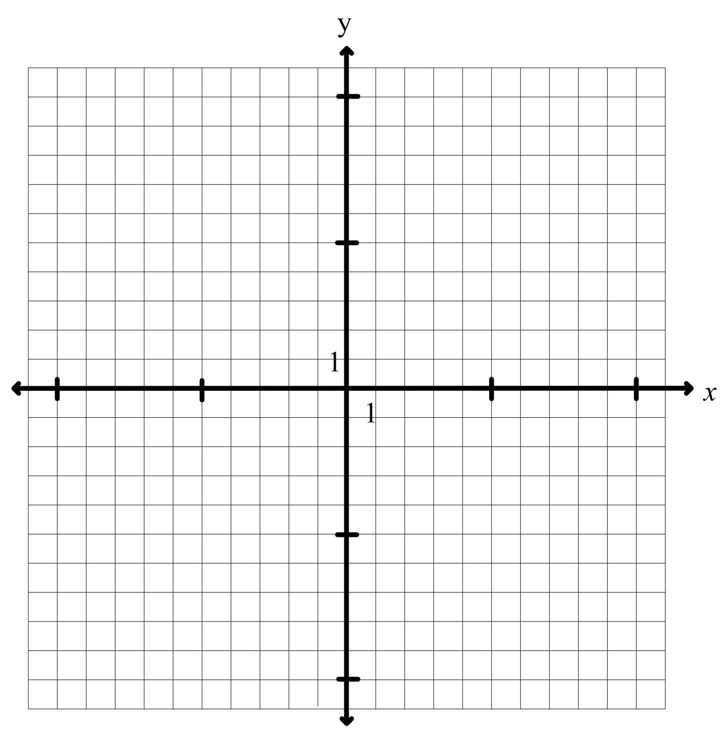
b) $y = 2x^2 + 4x - 6$; A.O.S.: $x = -1$



c) $y = -\frac{1}{2}x^2 + 2x + 3$; A.O.S.: $x = 2$



d) $y = -3x^2 + 12x - 20$; A.O.S.: $x = 2$



Completing the Square

A. When To Complete the Square?

Quadratic relations written in expanded, or standard, form (______________________________) can also be written in vertex form (________________________________). We can change from expanded form to vertex form via a procedure called "***completing the square***."

B. The Procedure

Here is the procedure for completing the square, as a means of changing expanded form to vertex form:

Steps: ***Worked Example, using*** $y = 2x^2 + 12x - 3$

1) *Common factor out the a-value from* $\boldsymbol{ax^2 + bx}$ *terms only*

2) *Create a perfect square by finding* $\left(\frac{b}{2}\right)^2$

   Add, then subtract, the new "c" term

3) *Keep the balance by adding and subtracting new "c" term*

4) *Factor the perfect square and simplify*

For this example, what is the vertex? ____________ *Equation of the axis of symmetry?* _____________

What is the optimal value? ____________________ *Maximum or minimum?* _____________________

C. Practice Makes Perfect!

Let's try some examples together, to work on mastering the procedure for completing the square:

$y = x^2 + 14x - 1$

$y = 3x^2 - 12x + 8$

Vertex ____________

Vertex ____________

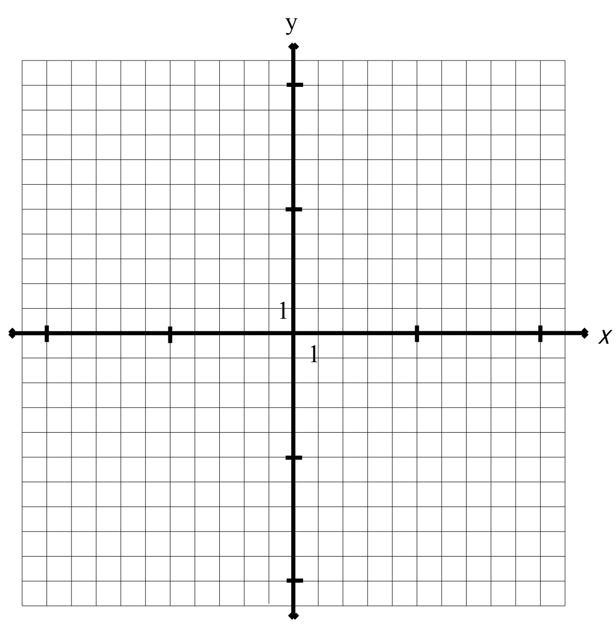
$y = -\frac{1}{2}x^2 + 5x - 4$

$y = -3x^2 - 9x - 25$

Vertex _____________

Vertex _____________

Sketch the following quadratic relation: $y = -x^2 - 6x - 5$



HW: p. 390–391 #4beg, 8abcdgij, 9ace, 10, 21

4. Write the equation in vertex form $y = a(x - h)^2 + k$ by completing the square.

(a) $y = x^2 + 4x$ (b) $y = x^2 - 8x$ (c) $y = x^2 + 6x + 2$

(d) $y = x^2 + 10x - 12$ (e) $y = x^2 + 12x - 15$ (f) $y = x^2 - 14x + 20$

(g) $y = x^2 - 6x - 8$ (h) $y = x^2 - 10x - 5$ (i) $y = x^2 + 20x - 20$

8. Express the equation in vertex form by completing the square.

(a) $y = 2x^2 + 4x$ (b) $y = 3x^2 - 9x$

(c) $y = -x^2 + 6x$ (d) $y = -4x^2 + 8x + 9$

(e) $y = 2x^2 - 4x + 5$ (f) $y = -3x^2 + 6x - 7$

(g) $y = -3x^2 - 12x + 5$ (h) $y = 5x^2 + 10x - 11$

(i) $y = -\frac{1}{2}x^2 + 6x + 5$ (j) $y = 0.2x^2 + 2x + 9$

(k) $y = 0.5x^2 - 4x + 6$ (l) $y = -0.1x^2 - 0.6x - 0.4$

9. For each quadratic relation,

i. complete the square to express the relation in vertex form

ii. graph the relation

(a) $y = x^2 - 4x + 7$ (b) $y = x^2 + 8x + 6$

(c) $y = \frac{1}{2}x^2 - 2x + 5$ (d) $y = -x^2 + 6x - 11$

(e) $y = -3x^2 - 18x + 13$ (f) $y = 2x^2 + 20x + 43$

10. Communication: What transformations must be applied to the graph of $y = x^2$ to produce the graph of $y = 2x^2 - 12x + 7$? Justify your reasoning.

21. Hassan used the method of completing the square to express $y = 2x^2 + 3x + 4$ in vertex form. If Hassan's solution is correct, write "Correct." If not, identify the errors and show the correct solution.

(1) $y = 2(x^2 + \frac{3}{2}x) + 4$

(2) $y = 2(x^2 + \frac{3}{2}x + \frac{9}{4} - \frac{9}{4}) + 4$

(3) $y = 2(x + \frac{3}{4})^2 - \frac{9}{2} + 4$

(4) $y = 2(x + \frac{3}{4})^2 - \frac{1}{2}$

Factored Form of a Quadratic Relation I

A. Quadratic Relations in Factored Form

A quadratic relation is in factored form if it is in the form ______________________________________.

There is information visible in this form that tells us about the quadratic relation:

a) If a > 0, the parabola opens ___________ . If a < 0, the parabola opens ___________ .

b) The values of ***s*** and ***t*** are the ___________ /____________________________ of the quadratic relation.

c) If s ≠ t, there are two _____________________ zeros/x-intercepts. (e.g. _________________________________)

d) If s = t, there are two _____________________ zeros/x-intercepts. (e.g. _________________________________)

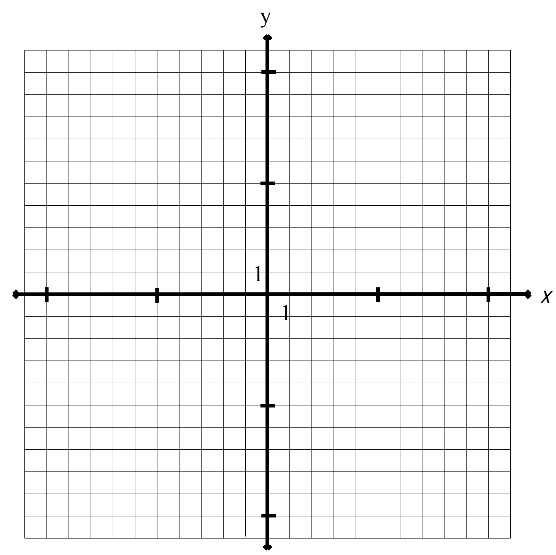
Example 1: For ***y = (x + 2)(x – 4)****, state* ***a*** *_________,* ***s*** *_________, and* ***t*** *_________ ; find the following, and sketch:*

a) the x- intercepts (the zeros)

b) the direction of opening

c) the equation of the A.O.S.

d) the coordinates of the vertex



Example 2: For ***y = – 2(x – 3)(x – 1)****, state* ***a*** *_______,* ***s*** *_______, and* ***t*** *_______ ; find the following, and sketch:*

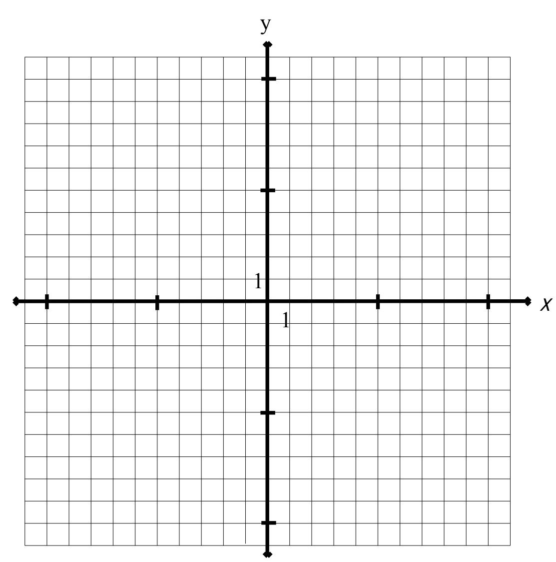
a) the x- intercepts (the zeros)

b) the direction of opening

c) the equation of the A.O.S.

d) the coordinates of the vertex

e) find the y-intercept for the parabola



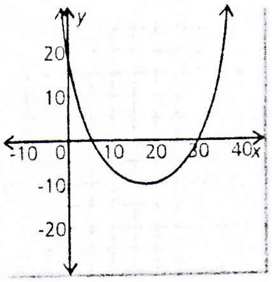
HW: p. 280-282 #1, 2, 3acf, 5afgh (use fractions reduced to lowest terms instead of decimals for all answers), 7ceghi

Practise, Apply, Solve 3.4

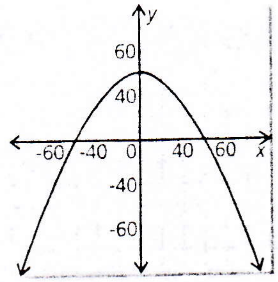
A

1. Examine each parabola. What are the zeros of the quadratic relation?

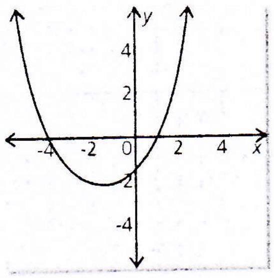
(a)



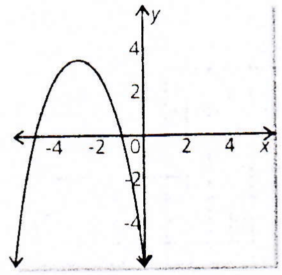
(b)



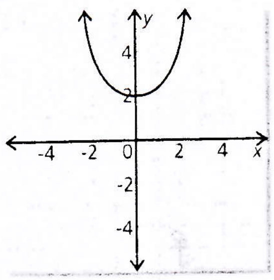
(c)



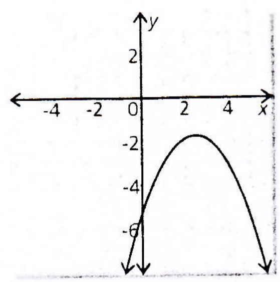
(d)



(e)



(f)



2. Find the equation of the axis of symmetry for each parabola in question 1.

3. Match each factored form equation to the appropriate graph.

(a) $y = (x - 2)(x + 3)$

(b) $y = (x - 3)(x + 2)$

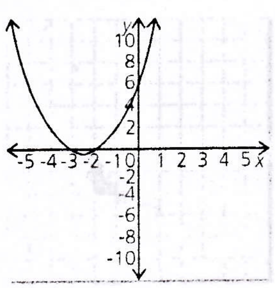
(c) $y = (x + 2)(x + 3)$

(d) $y = (3 - x)(2 + x)$

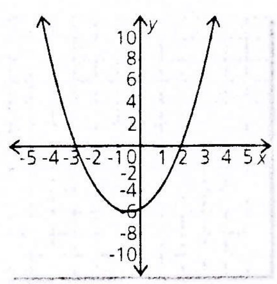
(e) $y = (3 + x)(2 - x)$

(f) $y = (x - 2)(x - 3)$

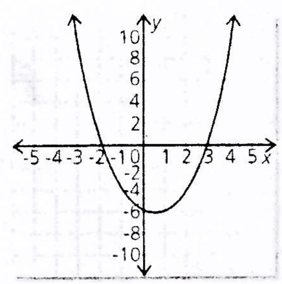
i.



ii.



iii.



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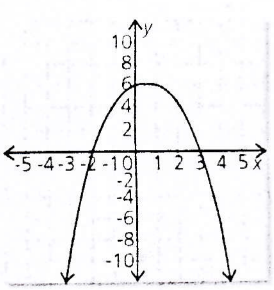

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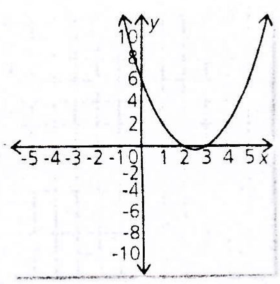
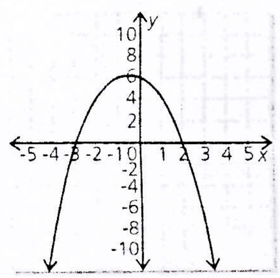


vi.



5. For each relation, state

i. the x-intercepts

ii. the equation of the axis of symmetry

iii. the coordinates of the vertex

(a) $y = (x + 4)(x + 2)$

(b) $y = (x + 5)(2 - x)$

(c) $y = (4 + x)(1 + x)$

(d) $y = (1 - x)(3 + x)$

(e) $y = (x - 3)(2 - x)$

(e) $y = (x + 1)(x - 4)$

(g) $y = 3(x + 1)(x - 3)$

(h) $y = -2(x + 3)(x - 3)$

7. Sketch a graph for each relation. Do not make a table of values or use graphing technology.

(a) $y = (x + 3)(x + 5)$

(b) $y = (x - 3)(x - 5)$

(c) $y = (x - 6)(x - 2)$

(d) $y = -(x - 1)(x - 2)$

(e) $y = 3(x - 5)(x + 1)$

(f) $y = -2(x + 2)(x + 1)$

(g) $y = \frac{1}{2}(x - 4)(x - 2)$

(h) $y = -2(3 - x)(5 - x)$

(i) $y = 10(x - 1)(x + 6)$

Factored Form of a Quadratic Relation II

A. Converting From Standard Form to Factored Form and Graphing

Quadratic relations written in expanded, or standard, form (_______________________________) can also be written in factored form (________________________________). We can change from expanded form to factored form by factoring the quadratic relation. ***Recall***: always common factor first, if possible!

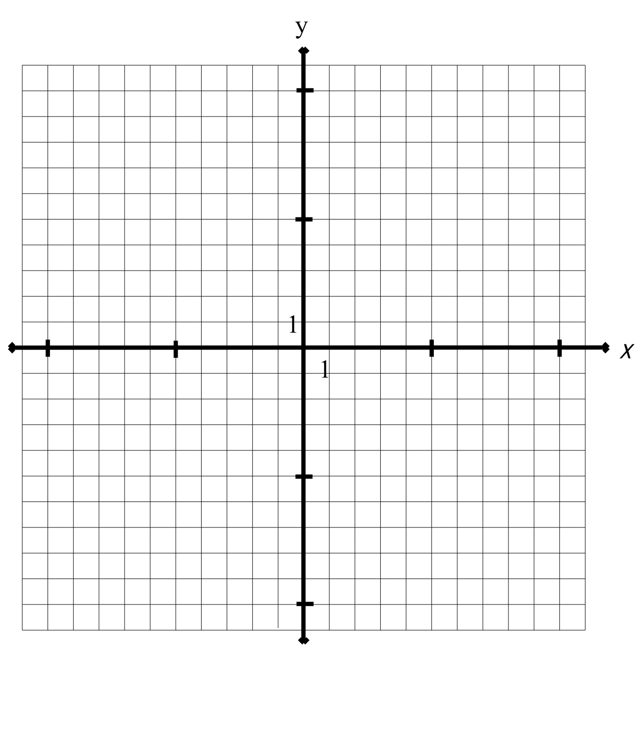
Example 1: For the relation $y = -2x^2 - 4x + 6$

i) express it in factored form

ii) determine its zeros

iii) determine the coordinates of its vertex

iv) graph and label all key features



B. Finding the Equation of Quadratic Relation in Factored Form

To write an equation in ***factored form***, we must know the values of ***a, s, and t.***

$$y = a(x - s)(x - t)$$

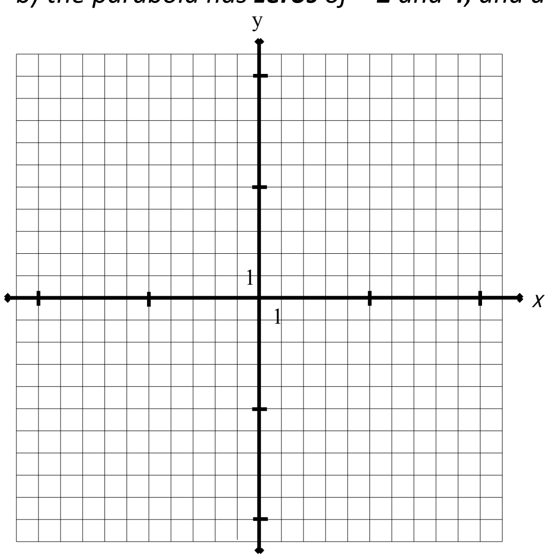
Note: The factored form of a quadratic relation includes five variables in total:

- the **zeros**, ***s*** and ***t***, comprise two of the variables;
- any **other point** on the curve, (*x*, *y*) comprises two more variables;
- the **vertical stretch factor**, *a*, is another variable.

Example 2: Find the equation of the quadratic relation in factored form for each of the following:

a) ***y = a(x – 3)(x + 5)*** *and* ***(– 2, 30)*** *is a point on the parabola*

b) the parabola has ***zeros*** *of* ***– 2*** *and* ***4****, and a* ***y-intercept*** *of* ***– 4*** *(let's sketch this one!)*



c) the parabola has ***zeros*** *of* ***– 5*** *and* ***3****, and a* ***maximum/optimum*** *value of* ***6***

HW: p. 308 #8abce; p. 282-283 #8, 9

8. Knowledge and Understanding: For each relation

i. express it in factored form
ii. determine its zeros
iii. determine the coordinates of its vertex
iv. graph the relation

308

(a) $y = x^2 - 4$
(b) $y = x^2 + 6x + 8$
(c) $y = x^2 - 6x + 5$
(d) $y = -x^2 + 2x + 24$
(e) $y = x^2 + 2x + 1$
(f) $y = -x^2 + 3x + 18$

8. A quadratic relation has the equation $y = a(x - s)(x - t)$.
Find the value of a when
(a) $y = a(x - 2)(x + 6)$ and (3, 5) is a point on the graph
(b) the parabola has zeros of 4 and -2 and a y-intercept of 1
(c) the parabola has x-intercepts of 4 and -2 and a y-intercept of -1
(d) the parabola has zeros of 5 and 0 and a minimum value of -10
(e) the parabola has x-intercepts of 5 and -3 and a maximum value of 6

9. Determine the equation (in factored form) of the quadratic relation and the direction of opening of the parabola.

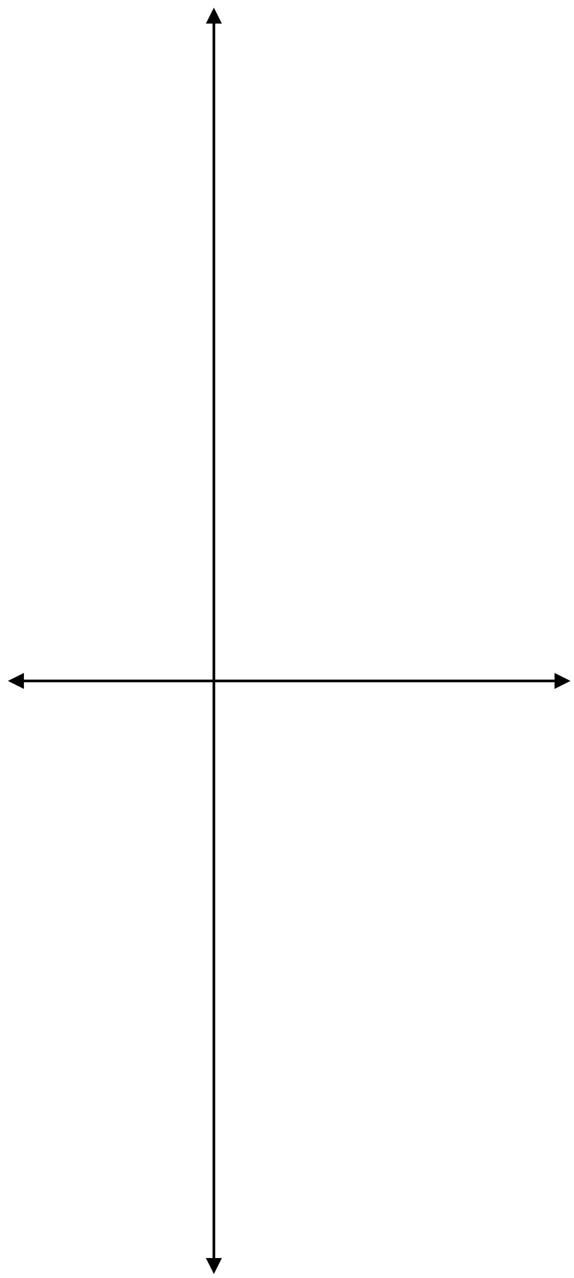
	x-Intercepts	y-Intercepts
(a)	-2 and 4	5
(b)	-2 and 4	-5
(c)	-5 and -2	4
(d)	-5 and -2	-4
(e)	3 and 8	6
(f)	3 and 8	-6

10. Sketch the graphs in question 9. Put any graphs that have the same axis of symmetry on the same axes.

Exploring and Comparing Forms of Quadratic Relations

Each form of a quadratic relation provides different kinds of information about the key features of the graph!

***Any* Form:**	• the *a*-value gives the ***direction of opening*** and the ***pattern*** to use for plotting points • any uncertain points on a graph can be checked using the equation and substitution



Vertex Form: $y = a(x - h)^2 + k$

Characteristics	*How to Graph…*
• In this form, the only point evident is the vertex, (h, k) • The *a*-value gives the ***direction of opening*** and the ***pattern***	• Plot and label the vertex, (h, k) • Use the *a*-value to plot the remaining points • Sketch and label all key features, including the A.O.S., $x = h$

Factored Form: $y = a(x - s)(x - t)$

Characteristics	*How to Graph…*
• In this form, the only points evident are the x-intercepts, $(s, 0)$ and $(t, 0)$ • The *a*-value gives the ***direction of opening*** and the ***pattern***	• Plot and label the x-intercepts, $(s, 0)$ and $(t, 0)$ • Find the axis of symmetry, $x = h$, using: $h = \frac{s+t}{2}$ • Find the y-value of the vertex, k, by substituting $x = h$ • Plot the vertex, (h, k) • Use the *a*-value to plot the remaining points • Sketch and label all key features, including the A.O.S., $x = h$

Standard Form: $y = ax^2 + bx + c$

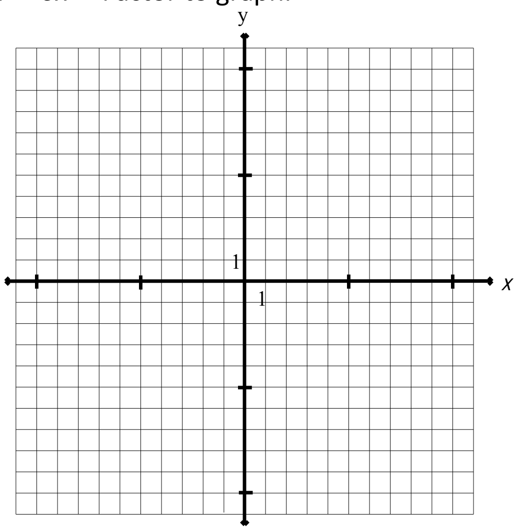
Characteristics	*How to Graph…*
• In this form, the only point evident is the y-intercept, $(0, c)$ • The *a*-value gives the ***direction of opening*** and the ***pattern***	• Plot and label the y-intercept, $(0, c)$ • Given the A.O.S., $x = h$: ✓ Find the y-value of the vertex, k, by substituting $x = h$ ✓ Plot the vertex, (h, k) ✓ Use the *a*-value to plot the remaining points ***OR*** ✓ Make a table of values using points on either side of the A.O.S. • Sketch and label all key features, including the A.O.S., $x = h$

HW: WORKSHEET "Graphing a Quadratic Relation Given Any Form" ; p. 326 #6; p. 328 #7ac, 9; p. 418 #1, 2, 3; p. 421 #6, 7; p. 424 #12

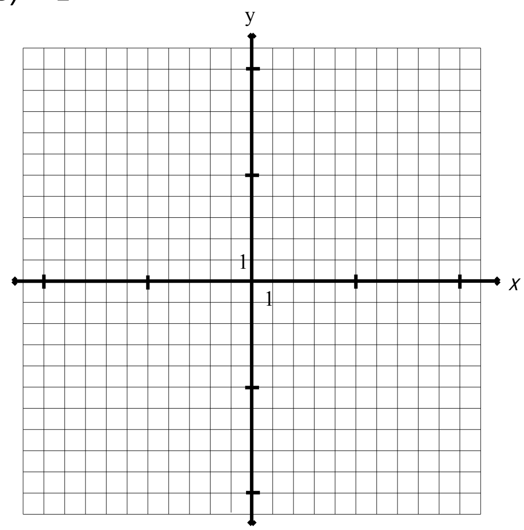
WORKSHEET: Graphing a Quadratic Relation Given Any Form

Graph the following relations. Complete any additional work on a separate sheet of paper. Label all key features!

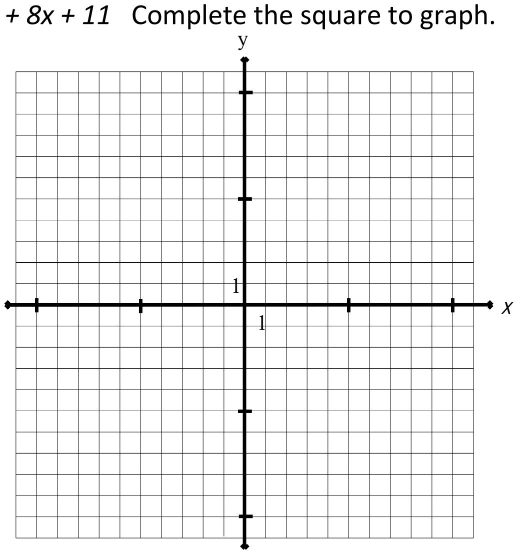
$y = -x^2 + 6x$ Factor to graph.



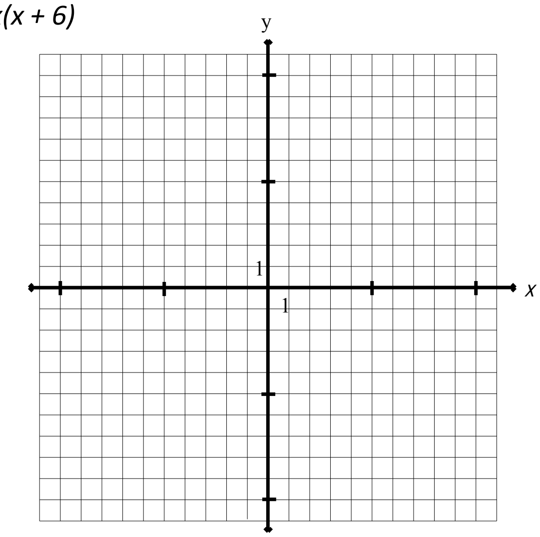
$y = (x - 3)^2 + 2$



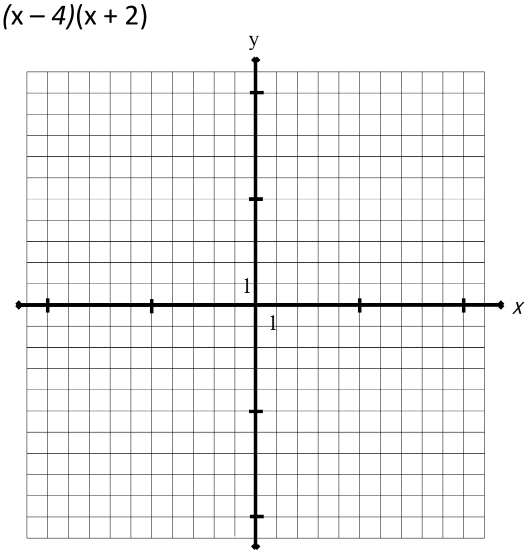
$y = x^2 + 8x + 11$ Complete the square to graph.



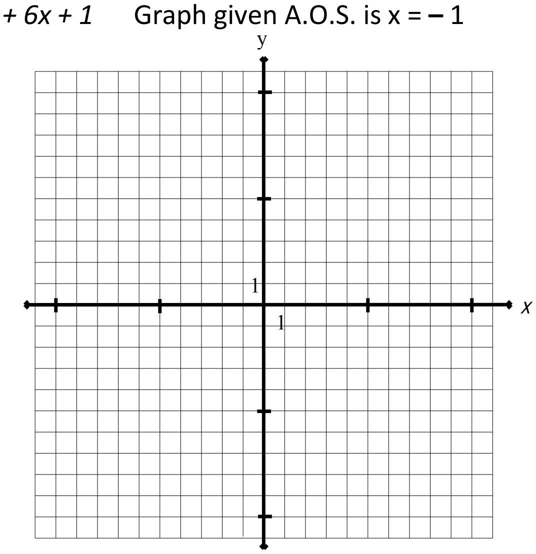
$y = \frac{1}{2}x(x + 6)$



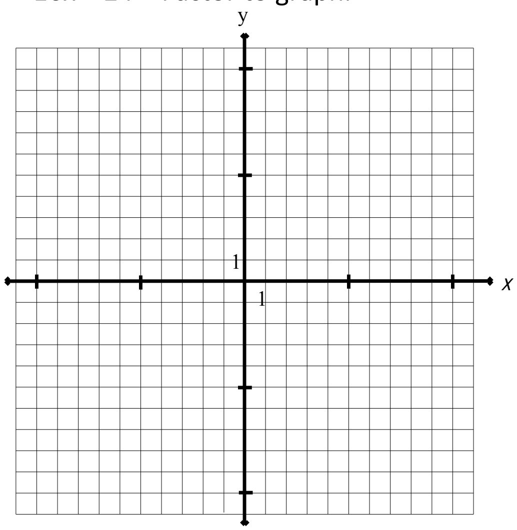
$y = -(x - 4)(x + 2)$



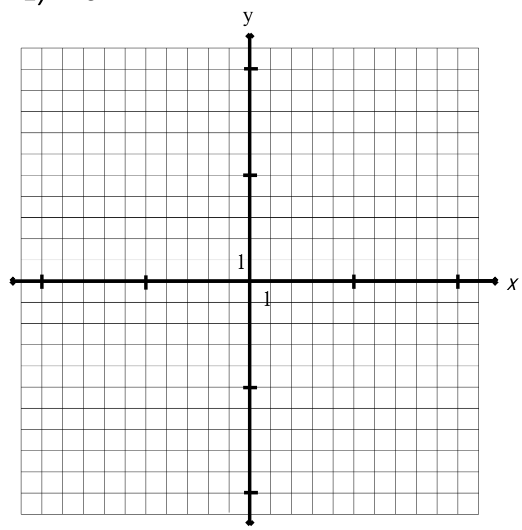
$y = 3x^2 + 6x + 1$ Graph given A.O.S. is $x = -1$



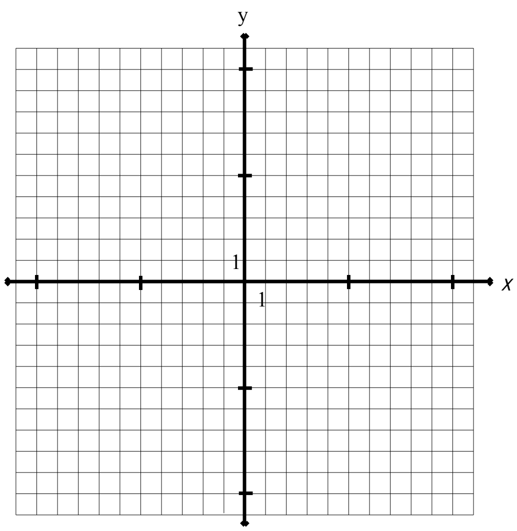
$y = 2x^2 - 16x + 24$ Factor to graph.



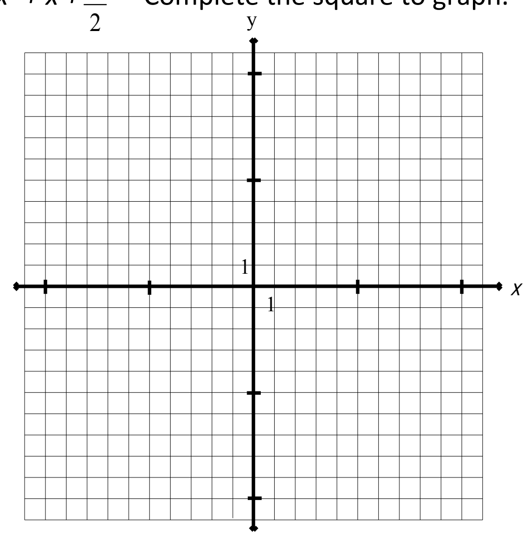
$y = -3(x + 1)^2 - 6$



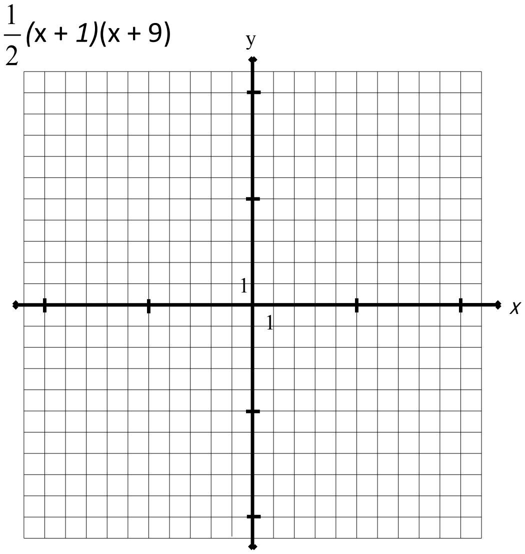
$y = x^2 - 2x + 5$ Graph given A.O.S. is x = 1



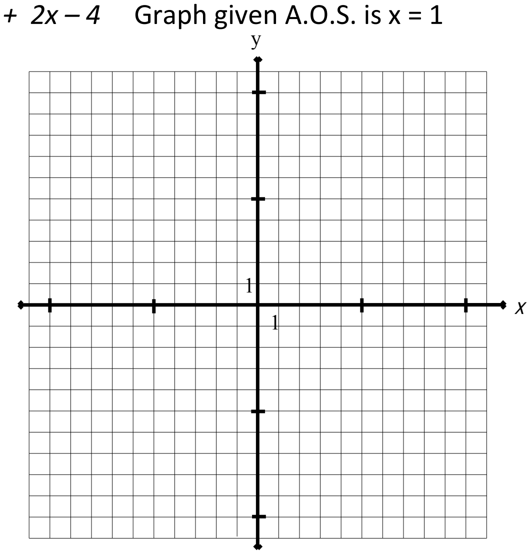
$y = -\frac{1}{2}x^2 + x + \frac{13}{2}$ Complete the square to graph.



$y = -\frac{1}{2}(x + 1)(x + 9)$



$y = -x^2 + 2x - 4$ Graph given A.O.S. is x = 1



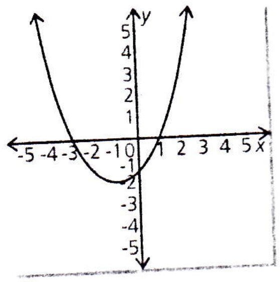
6. For each parabola,
   i. find where it crosses the x-axis
   ii. state the equation of the axis of symmetry
   iii. without graphing, determine whether the vertex represents a maximum or a minimum value
   iv. find the coordinates of the vertex

(a) $A = 18w - w^2$ (b) $A = -L^2 + 10L$ (c) $y = 4x - 16x^2$

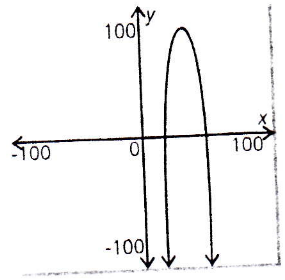
(d) $h = 25t - 5t^2$ (e) $y = 15x + 6x^2$ (f) $A = 42w - 6w^2$

7. Determine the equation of each parabola.

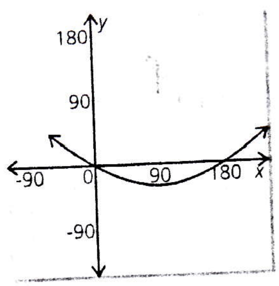
(a)



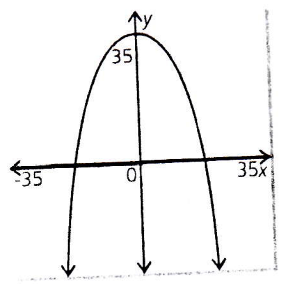
(b)



(c)



(d)



9. Determine the quadratic equation for a parabola with
   (a) zeros at 5 and 9, and an optimal value of -2
   (b) zeros at -3 and 7, and an optimal value of 4
   (c) zeros at -6 and 2, and a y-intercept of -9
   (d) zeros at -9 and -5, and a y-intercept of 8

Extra Practice

1. Find the vertex and direction of opening of the graph of the quadratic relation.

   (a) $y = 2(x - 4)^2$ (b) $y = (x + 2)^2 - 5$ (c) $y = -3x^2 + 6$

2. Find, in vertex form, the equation of the quadratic relation
   (a) with vertex at (0, 7), passing through $(-2, -3)$
   (b) with vertex at (3, 0), passing through $(1, -8)$
   (c) with vertex at $(-1, -4)$, passing through (3, 4)

3. For each quadratic relation you found in question 2,
   (a) is the point $(2, -3)$ on the graph?
   (b) find one other point on the graph

Extra Practice

6. Find the vertex, axis of symmetry, and direction of opening of the parabola. Use this information to sketch the graph.

   (a) $y = (x - 2)^2 + 1$ (b) $y = -\frac{1}{2}(x + 4)^2$ (c) $y = 2(x + 1)^2 - 8$

7. Describe, using transformations, how the graph of $y = x^2$ can be transformed into the graph of the quadratic relation. How many x-intercepts does the graph have?

Extra Practice

12. Express the quadratic relation in vertex form by completing the square.

    (a) $y = x^2 + 6x - 3$ (b) $y = -2x^2 - 8x - 11$

    (c) $y = \frac{1}{2}x^2 + 5x - 7$ (d) $y = 3x^2 + 6x - 5$

13. A baseball is hit from a height of 0.8 m. After t seconds, its height h, in metres, is $h = -5t^2 + 20t + 0.8$.

    (a) Use the method of completing the square to find the maximum height of the ball.

    (b) Describe a second method you could use to find the maximum height.

14. In the example above, the Snowflake Winter Carnival committee did not include the revenue from sales of Snowflake toques and scarves in their profit estimate. Taking the expected sales into account, the new estimated profit is given by $P = -37t^2 + 1110t - 4368$.