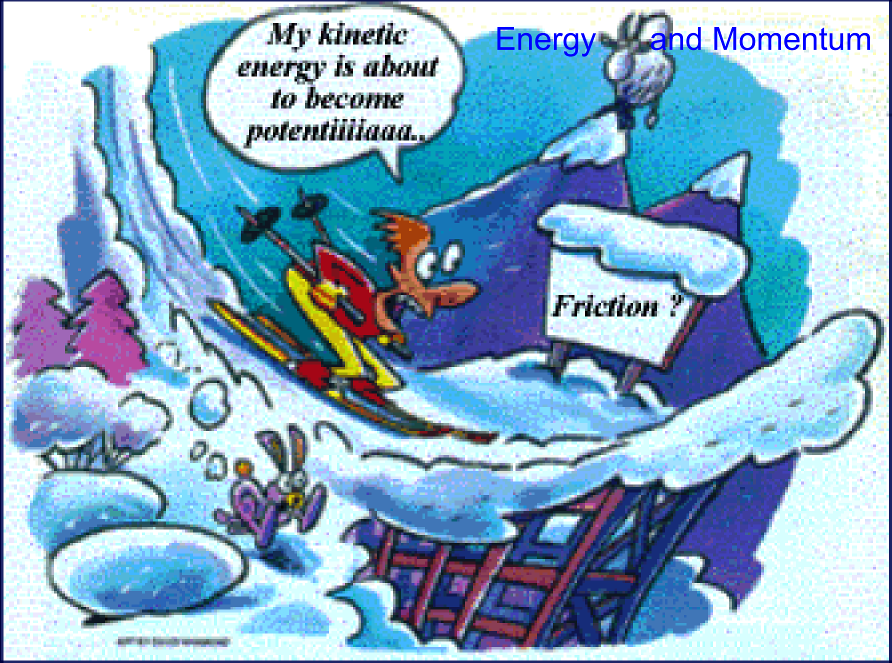
Energy and Momentum



Lesson 1

Physics 12 – **Work and Energy** Name:

WORK is defined as the **transfer** of energy from one body to another.

We can calculate the work done on an object with:

Units -

Note that these are the same units as <u>*torque*</u> *yet they are used to describe* ***very*** *difference quantities.*

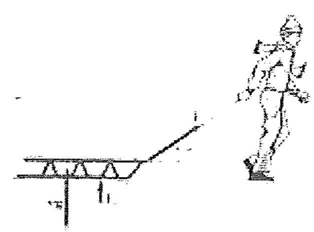
<u>Work against Gravity:</u>

How much work is required to lift a 2.0 kg textbook from the floor to a height of 1.5 m at a constant velocity?

We know that W = F•d, so we need to determine the force needed to lift the book at a constant velocity.

<u>Forces at an angle:</u>

A boy is pulling his sled at a constant velocity of 1.2 m/s. He pulls the 15 kg sled with a force of 35 N at an angle of 40° to the horizontal. How much work does he do in pulling the sled 20 m?



Rule: When finding the work done on an object we only consider =

<u>Most of the time the angle is 180° and cos180 = 1, so the formula becomes W = F• d</u>

Net Force vs. Applied Force – Which one do we use when calculating work?

A physics student is pushing a rope 15 m across a flat surface. The student pushes the rope with a force of 220 N, while the force of friction is 120 N. How much work is the student doing?

To find the amount of work done by the student should we use ΣF or F_A?

So when would we use net force?

Rule: When finding the **total work** done on an object, we always use the **applied force.**

Why can work be positive or negative when it is a scalar quantity?

Work is the product of a scalar and a vector, but work is ***scalar***. However, work can be positive or negative...

The concept of work plays an important role in physics since it connects Newton's second law of motion to the important scalar quantities of kinetic energy and potential energy (through the work-energy theorem).

Example 1

A constant force of 40.0 N is needed to accelerate a car as it moves 5.0 km down the road. How much work is done? Does the energy of the car increase or decrease?

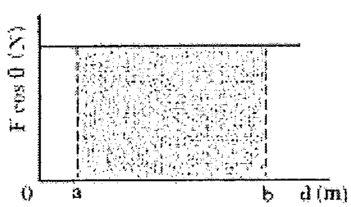
Example 2

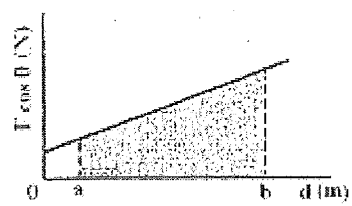
In reality, we know there will also be a force of friction acting between the surface of the road and the tires. How much work does the force do? Is this work causing an increase or decrease in the energy of the car?

<u>Using a Force vs. Displacement Graph –</u>

The graph below shows the component F of the net force that acts on a 5.0kg object as it moves along a flat horizontal surface. This information is graphed against the displacement of the object.

The work done by a force between two points equals the area under the curve of force vs. distance between two points.

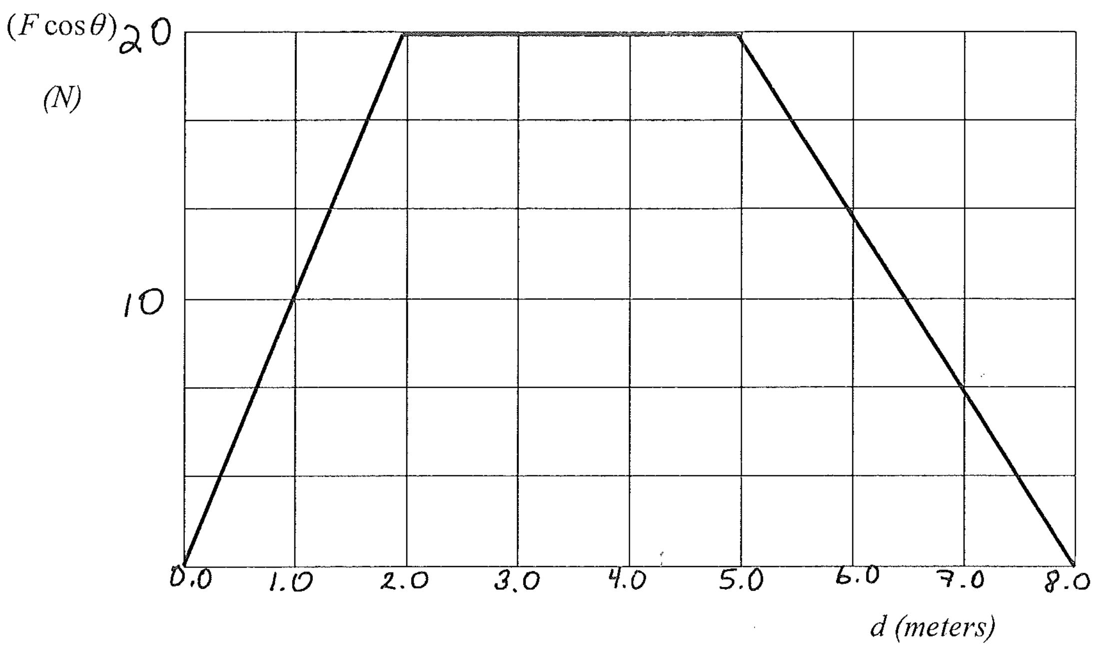




<u>Work Done by a Variable Force</u>

The work done by a variable force (a force that changes throughout the motion) is equal to the area under the F vs.d curve.

The graph shows a variable force acting on a 15.0 kg mass on a level surface which is initially at rest. Find the total work done on the block. Find the final speed of the mass assuming friction is negligible.



WORK-ENERGY THEOREM –

Work-Energy Relationships so far:

With Kinetic Energy:

When we **accelerate** an object we **do work by changing the object's kinetic energy**.

We can use the work-energy theorem to find the speed of an object and this gives us another framework to solve problems involving motion.

Example 1

If an 85.0 kg has a Net Work of 600 J done on it in order to accelerate it across a level horizontal floor starting from rest, what is the final velocity.

Another common use of the work-energy theorem is finding information about the forces acting on the object.

Example 2

A baseball pitcher can throw a 90.0g baseball with speed measured by a radar gun of 130 km/h. Assuming that the force exerted by the pitcher on the ball acts over a distance of 0.90m, what is the applied force exerted by the pitcher on the ball (with no friction)?

With Gravitational Potential Energy -

The gravitational force is one of a class of "special" forces called conservative forces. What makes the gravitational force special is that the work done on an object by gravity ONLY depends on its initial and final position. The work does NOT depend the path taken between the initial and final position.

When an object is lifted, work is done by gravity on the object over a distance. The height is changed as an object is lifted. **A change in height produces a change in potential energy.**

Example 3

A 3.00 kg model rocket is launched vertically upward with sufficient initial speed to reach a height of 100m. However, air resistance (a non-conservative force), performed 8.00×10^2 of work on the rocket. If we were to ignore air resistance, how high would the rocket have gone?

Lesson 1

Assignment – Work and Energy Problems

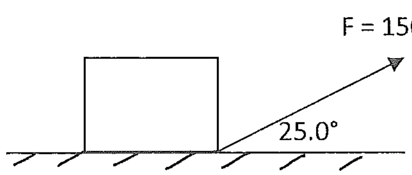
1. A 10.0 kg object is accelerated horizontally from rest to a velocity of 11.0 m/s in 5.00s by a horizontal force.

a) How much work is done on this object if the object is on a frictionless surface? (605J)

b) How much work is done in slowing the car if there is a coefficient of kinetic friction of 0.115 between the object and the horizontal surface? (-310J)

2. How much work is required to accelerate a 1.10 x 10^3 kg car from rest to 5.00 km/h along a level, frictionless surface?

3. A 150 N force is pulling a 50.0 kg box along a horizontal surface. The force acts at an angle of 25.0º as shown in the diagram. If this force acts through a displacement of 12.0 m, and the coefficient of kinetic friction is 0.250, what is the speed of the box, assuming it started from rest? What if its initial velocity was 2.00 m/s? (3.75 m/s, 4.25 m/s)



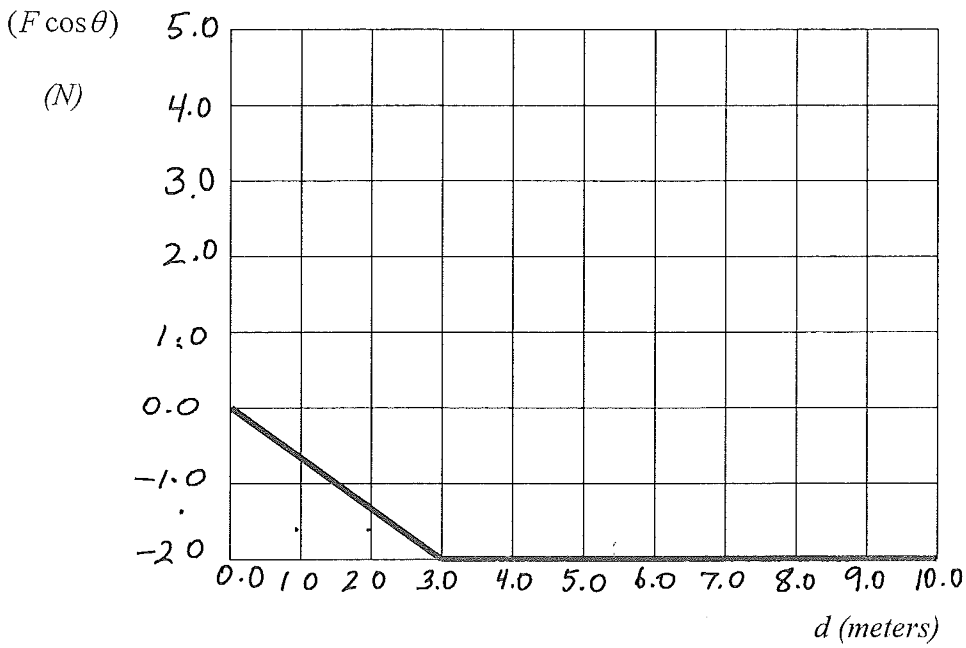
4. A 2000 kg truck descending on a 5.0º hill is brought to a stop in 250 m. The driver applies the brakes so that the wheels lock.

a) If the coefficient of kinetic friction between the truck tires and the road is 0.60, how much work is done by friction in stopping the truck? (-2.93 x 10^6 J)

b) How much work is done on the truck by gravity? (4.27 x 10^5 J)

c) How fast was the truck travelling immediately before the brakes were applied? (54.1m/s)

5. Given the following force-displacement graph of an object being pulled along a level surface, what is the work done in moving the object 8.0 m? Which force is responsible for the work on the object shown here?



6. A student wants to lift a 20 kg crate off a floor.

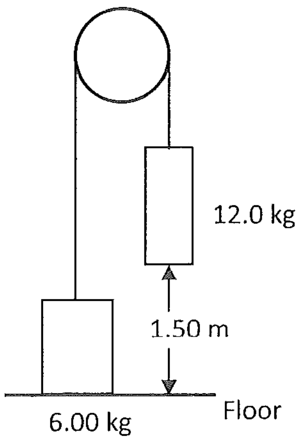
a) If the crate is lifted a height of 0.80 m at a constant speed, what is the work done by the student? (157 J)

b) If the crate is lifted to a height of 0.80 m at a constant speed using a frictionless ramp with an angle of 60° above the horizontal, what is the required force and the work done by the student? (170 N, 157 J)

c) If the crate is lifted to a height of 0.80 m at a constant speed using a frictionless ramp with an angle of 30° above the horizontal, what is the required force and the work done by the student? (98 N, 157 J)

d) If the crate is lifted to a height of 0.80 m at a constant speed using a ramp with an angle of 30° above the horizontal, and the coefficient of kinetic friction between the crate and the ramp is 0.30, what is the required force and the work done by the student? (149N, 238 J)

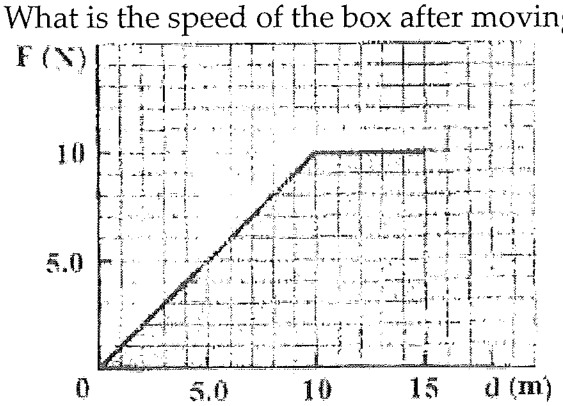
7. A system containing a frictionless pulley is illustrated below. If this system is released, at what speed does the 12.0 kg object hit the floor? (3.14 m/s)



8. A 45.0 kg box is pulled across a horizontal surface by a constant horizontal force of 192 N. If the box starts from rest, and the coefficient of kinetic friction is 0.35, what is the final speed of the box when it has travelled 8.0 m? (3.68 m/s)

9. How much energy is needed to accelerate a 1.1×10^3 kg object along a horizontal frictionless surface from 15 km/h to 25 km/h in 5.0 s? (1.7×10^4 J)

10. A 4.0 kg box moves on a floor by a force that varies with distance as shown in graph. What is the speed of the box after moving 15 m starting from rest? (100 J)



11. A 1200 kg car is travelling at 10 m/s.

a) If the car is accelerated from 10 m/s to 15 m/s, how much work is required? (7.5×10^4J)

b) If the car is accelerated again from 15 m/s to 20 m/s, how much work is required? (1.1×10^5 J)

c) The car is traveling at 10 m/s comes to a stop in 12 m when its brakes are applied. If the speed of the car is doubled to 20 m/s and all else is the same, how far will the car move before coming to a stop? (48 m)

12. A 0.08 kg arrow is drawn back from a bow whose string exerts an average net force of 120 N on the arrow over a distance of 0.90 m.

a) How much energy is stored in the bow? (108 J)

b) What is the speed of the arrow when it leaves the bow? (52.0 m/s)

c) When the arrow hits a wooden target, an average force of 4500 N brings the arrow to rest. What distance does the arrow penetrate the wooden target? Ignore air resistance. (0.024m)

Lesson 2

Physics 12 - **Conservation of Energy**

Energy is neither created nor destroyed. It may only be ___________________ from one type to another. This means that if we are looking at a **closed system** (a situation that has no outside sources of energy), the **total change in energy** is always _________.

There are many forms of energy: mechanical (potential and kinetic), thermal, electrical, nuclear, chemical etc. **One form of energy can be converted into another form by** __________________________.

We will be mainly focusing on potential and kinetic (and a little bit of thermal) energy.

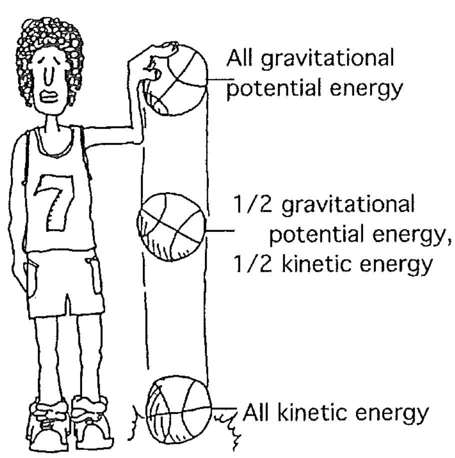
Potential Energy (PE):

Remember:
Potential Energy is always…

Kinetic Energy (KE):

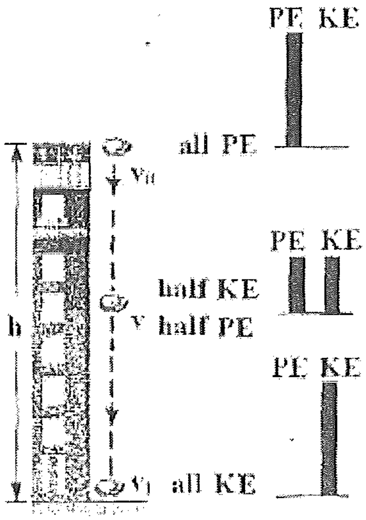
Total Initial Energy = Total Final Energy

Total Change in Energy = 0



But at all times the **SUM** of the gravitational potential energy and the kinetic energy is constant.

The principle of the conservation of energy states that the energy can be transformed into another form, but the total energy remains the same. This is true even when there is friction.



Some terms to know:

<u>**Conservative Force**</u> – A force which does work on an object that is independent of the path taken by the object between its starting point and ending point.

Both gravitational and elastic forces are considered conservative forces. This means that work done against the force (energy) can be recovered = it is stored.

<u>**Non-Conservative Force**</u> – A force whose work on an object <u>IS DEPENDENT</u> on the path taken by the object from starting point to ending point.

An example of a **non-conservative force is friction**. When work is done against friction, the energy cannot be recovered as it is converted to another form = mainly thermal energy.

When work is done by a non-conservative force, it produces a change in the total mechanical energy of the object.

Last class we learned through the work-energy theorem that:

The change in *Potential Energy* of an object is equal to the work done on the object.

The change in *Kinetic Energy* of an object is equal to the work done on the object.

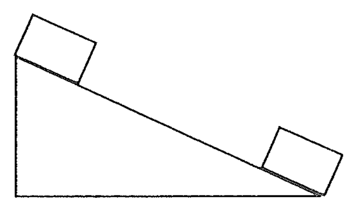
But what if the Potential Energy <u>*and*</u> the Kinetic Energy change?????

Example: The first peak of a roller coaster is 55 m above the ground. The 1200 kg car starts from rest and goes down the hill and up a second hill which is 30 m high. How fast is the car traveling at the top of the second hill? (assume no friction)

When we are dealing with **non-conservative forces** (such as *friction*) acting on an object, not all energy will be transferred between KE and PE.

The "work" done by friction on an object will change some of the mechanical energy into HEAT (thermal energy). This energy is quickly conducted or radiated in all directions and is effectively dispersed.

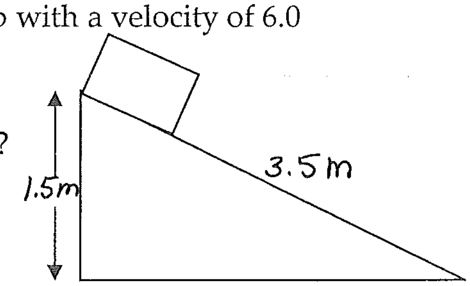
If we observe a block of wood sliding down a ramp with a small amount of friction, how would the blocks kinetic energy at the bottom compare with its potential energy at the top?



This does **<u>not</u>** change the fact that the total energy in the system is **CONSTANT.** Now…

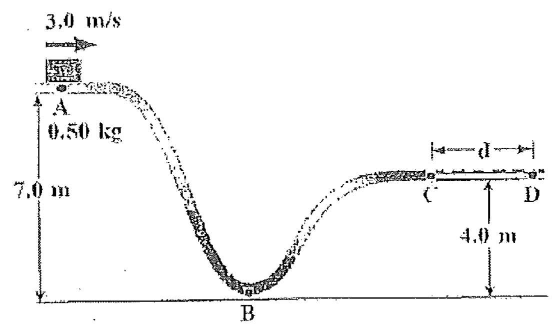
Example: A 5.0 kg block of wood is now pushed down a ramp with a velocity of 6.0 m/s. At the bottom of the ramp it is traveling at 7.5 m/s.

a. How much thermal energy is generated due to friction?
b. Determine the force of friction.



Example: A 0.50 kg block moving at 3.0 m/s slides from A to C along a frictionless surface, and then passes through the horizontal surface CD, where a frictional force acts on it. As a result, the block slows down and comes to a stop at point D. The coefficient of kinetic friction between the block and the surface in the region CD is 0.40. Ignore air resistance.

a) What is the total energy of the block at point A?
b) What is the speed of the block at point B?
c) What are the speed and the kinetic energy of the block when it reaches C?
d) How far will the block move before coming to a stop at point D?



Lesson 2

Conservation of Energy Problems

1. A physics student lifts his lifts his pet rock 2.8m straight up. He then lets it drop to the ground. Use the Law of Conservation of Energy to calculate how fast the rock will be moving (a) half way down and (b) just before it hits the ground (ignore air resistance). (5.2 m/s, 7.4 m/s)

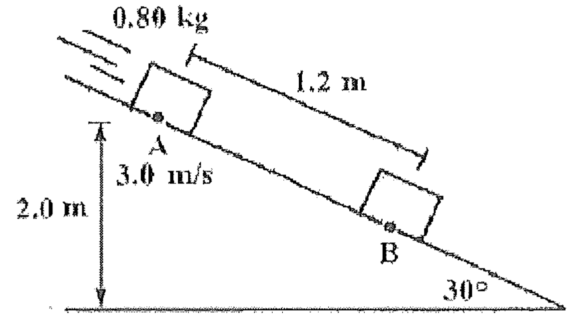
2. A 65 kg girl is running with a speed of 2.5 m/s. How much kinetic energy does she have? She grabs on to a rope which is hanging from the ceiling, and swings from the end of the rope. How high off the ground will she swing? (ignore air resistance) (203 J, 0.32m)

3. How much kinetic energy will an 80.0 kg skier sliding down a frictionless slope have when he is two-thirds of the way down the slope? The vertical height of the slope is 60.0m (3.14×10^4J)

4. A golfer wishes to hit his drives further by increasing the kinetic energy of the golf club when it strikes the ball. Which would have the greater effect on the energy transferred to the ball by the driver – doubling the mass of the club head or doubling the speed of club head? (double speed)

5. A rubber ball falls from a height of 2.0m, bounces off the floor and goes back up to a height of 1.6m. What percentage of its initial gravitational potential energy has been lost? Where does this energy go? Has the Law of Conservation of Energy been broken? (20% lost)

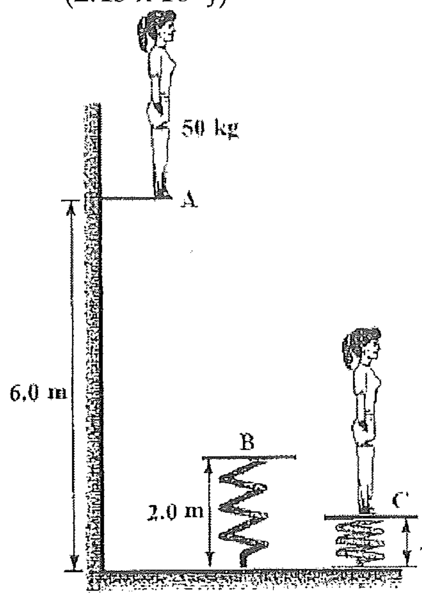
6. A 0.80 kg block slides along a frictionless surface of a 30.0° incline. When the block passes point A, the velocity of the block is 3.0 m/s. After the block moves 1.2 m from point A, it reaches point B as shown in the diagram.



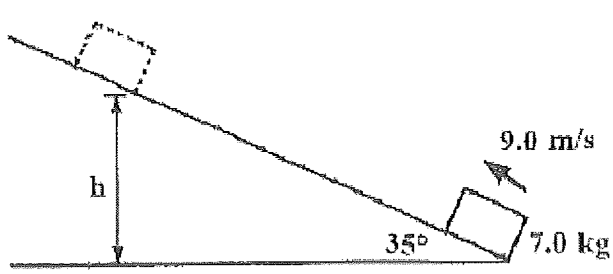
a) Find the kinetic energy, potential energy, and total energy at point A.
b) Find the kinetic energy, potential energy, and total energy at point B.
c) Find the changes in kinetic energy, potential energy, and total energy between point A and B.

7. A 3.0 kg stone is projected directly upward with an initial speed of 12 m/s. This stone experiences an average air resistance force of 20 N.
a) What maximum height does the stone reach? (4.37 m)
b) What is the speed of the stone when it lands on the ground? (5.2 m/s)

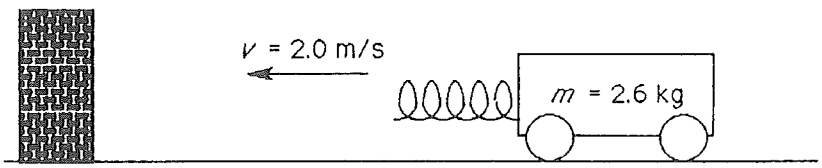
8. A 50 kg student steps off a 6.0 m high platform and drops onto a 2.0 m tall spring-loaded board. As the spring-loaded board is compressed, it brings her to a stop 1.0 m above the ground. Ignore air resistance.
a) What is the speed of the student when she hits the board? (8.85 m/s)
b) How much energy is momentarily stored in the spring when she comes to rest? (2.45×10^3 J)



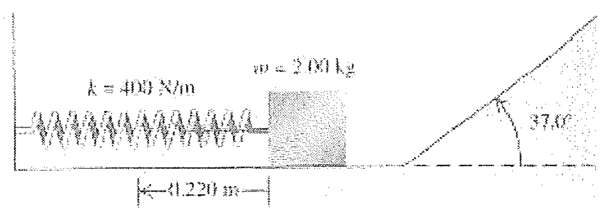
9. A 7.0 kg block is fired up a 35° incline with an initial speed of 9.0 m/s as shown by the diagram. If a frictional force of 55 N acts on the block as it moves up the incline, what maximum vertical height will the block reach? (1.7 m)



10. A 2.6kg laboratory cart is given a push and moves with a speed of 2.0 m/s toward a solid barrier, where it is momentarily brought to a rest by its spring bumper. How much elastic potential energy will be stored in the spring at the moment when the spring is fully compressed? (5.2J)



11. A 2.00-kg block is pushed against a spring with negligible mass and force constant k = 400 Nm, compressing it 0.220m. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope 37.0°
<u>Spring forces are conservative</u>. **(PE of an ideal spring that is completely compressed can be found using $PE_{compressed\ spring} = 1/2kx^2$)**
A) What is the speed of the block as it slides along the horizontal surface after having left the spring? (3.11 m/s)
B) How far does the block travel up the incline before starting to slide back down? (0.82m)



Lesson 3 (part 1)

Physics 12 – **Power and Efficiency**

POWER is the *rate* of doing work or the *rate* of using energy. In other words, power is concerned with the amount of time it takes to do a certain amount of work.

Mathematically we define power as:

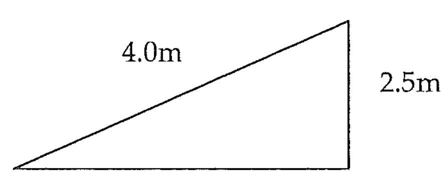
The unit of power is J/s or Watts (W)

Example: A wrestler is setting up a body slam and lifts his 80 kg opponent clear over his head to a height of 2.2 m in 0.675 s. How much power did the wrestler generate?

Example: While cruising along level ground in a sleigh at 4.0 m/s, the driver cracks the whip and speeds up to 12.0 m/s in 4.5 s. If the sleigh and driver have a combined mass of 850 kg, how much power did it generate? Ignore friction.

Another useful equation for power can be derived:

Example: A student pushes 14 kg of their physics homework up a 40° ramp at a constant velocity of 3.2 m/s. The friction force is 26 N. How much power must the student exert?

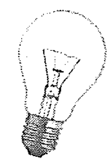


Whenever we use a machine to do work, some of the energy we put into the machine is always lost, mainly due to **friction.**

For example: An electric heater is approximately 95% efficient.

A car is approximately 30% efficient.

A light bulb is approximately 3% efficient.



We can define **efficiency** in one of two ways:

The most common confusion when calculating efficiency is in understanding which values apply to work/power IN and which applies to work/power OUT.

Work/Power In: Work/Power Out:

Remember that energy is always LOST somewhere in using the machine, so…

Example: The Top Thrill Dragster is one of the tallest roller coasters in the world. The car is accelerated along a level track until they take a 90° vertical turn and travel to the peak, 120 m high. A typical fully loaded car has a mass of 2800 kg.

a) Calculate the minimum amount of work done on the car
in order for it to reach the peak?

b) In reality, the roller coaster is accelerated from 0 to 193 km/h in 3.8 s. Find the actual power input of the ride.

c) Determine the efficiency of the ride from start to peak.

Example: On the Incredible Hulk roller coaster the car is initially launched up a hill 34 m high, travelling from 0 to 64 km/h in 2.0 s. A full car has a mass of 4500 kg.

a) Find the power output of the ride.

b) The power consumption during the initial launch is actually 1.45 MW. Determine the efficiency of the ride during the initial launch.

c) If the car pulls into the station at 8.0 m/s. How much heat has been generated since the first peak?

d) The car is finally brought to rest over a distance of 2.0 m. How much force is required?

Lesson 3 (part 1)

Power and Efficiency Problems:

1. A 20.0 kg object is lifted vertically at a constant velocity 2.50 m in 2.00 s by a student. Calculate the power output of the student. (245 W)

2. A 2.00 kg object is accelerated uniformly from rest to 3.00 m/s while moving 1.5 m across a level frictionless surface. Calculate the power output. (9.0 W)

3. An 8.5×10^2 kg elevator is pulled up at a constant velocity of 1.00 m/s by a 10.0 kW electric motor. Calculate the efficiency of the motor. (83%)

4. A 5.0 kg object is accelerated uniformly from rest to 6.0 m/s while moving 2.0 m across a level surface. If the force of friction is 4.0 N, what is the power output? (135 W)

5. If a 100 kW motor has an efficiency of 82%, how long will it take to lift a 50.0 kg object to a height of 8.00m? (0.048 s)

6. A 2.10×10^4 N airplane requires a power of 7.46×10^4 W at the propeller to climb at an angle of 20.0° to the horizontal at a constant speed. How much altitude could it gain in 10.0 minutes if air resistance is ignored? (2133 m)

7. A 1500 kg car accelerates from rest to 75 km/h in 45 s. How much power is supplied by the engine to accelerate the car? (7.2×10^3 W)

8. A locomotive engine can supply 1.49×10^6 W to accelerate a train car from rest to 20.0 m/s in 9.0 min. Find the mass of the train. (Ignore friction) (4.02×10^6 kg)

9. A skateboarder increases his kinetic energy from 800 J to 1600 J in 20 s. He expends 1500 J of energy during this activity.

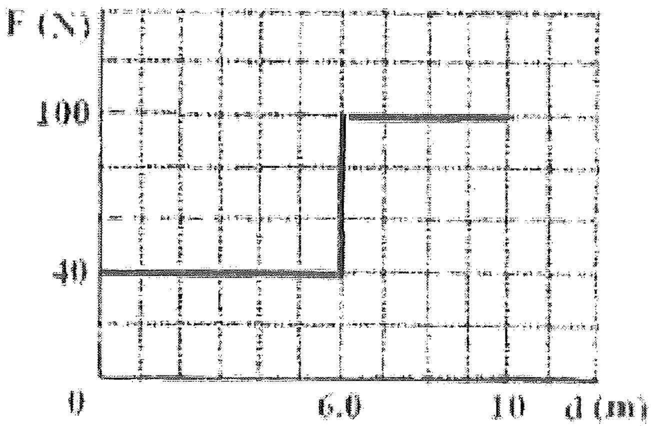
a) What is his power output? (40 W)

b) What is the efficiency of this process? (53%)

10. A 1000 kg automobile starts from rest and accelerates along a road to 30 m/s in 15s. Assume that the air resistance and frictional force remain constant at 500 N during this time?

What is the power developed by the engine?

11. A student pushes a lawn mower 10 m from rest. The graph shows the applied force versus distance.

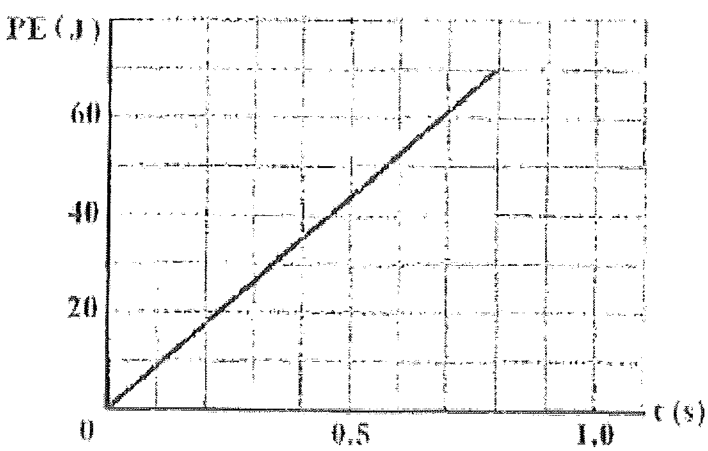


a) How much work does she do moving the lawn mower 10 m? (640 J)

b) After she pushes the 30 kg lawn mower 10 m, it moves at 5.0 m/s. What is the kinetic energy of the lawn mower? (375 J)

c) What is the efficiency of this process? (59%)

12. The graph shows the potential energy of a model rocket versus time.



a) Find the power output of the model rocket. (87.5 W)

b) If the model rocket is 60% efficient, find the power delivered to the rocket by the engine? (146 W)

13. A force acting upon an object to cause a displacement is known as _____.
a. energy b. potential c. kinetic d. work

14. Two acceptable units for work are ________. Choose two.
a. joule b. newton c. watt d. N•m

15. Power is defined as the _______ is done.
a. amount of work which b. direction at which work c. angle at which work d. the rate at which work

16. Two machines (e.g., elevators) might do identical jobs (e.g., lift 10 passengers three floors) and yet the machines might have different power outputs. Explain how this can be so.

17. There are a variety of units for power. Which of the following would be *fitting* units of power (though perhaps not standard)? Include all that apply.
a. Watt b. Joule c. Joule / second d. horsepower

18. Two physics students, Will and Ben, are in the weightlifting room. Will lifts the 100-pound barbell over his head 10 times in one minute; Ben lifts the 100-pound barbell over his head 10 times in 10 seconds.
Which student does the most work? ______________

Which student delivers the most power? ______________ Explain your answers.

19. Jack and Jill ran up the hill. Jack is twice as massive as Jill; yet Jill ascended the same distance in half the time. Who did the most work? ______________

Who delivered the most power? ______________ Explain your answers.

Conservation of Energy Lab

Names:

In this lab, you will investigate the law of conservation of energy. **You will be demonstrating how to maximize the conversion of potential energy into kinetic energy.** In other words, in what way will you lose the least amount of mechanical energy to other forms?

You will be provided with 1 meter stick and1 cart and you may request additional materials. Your team must develop and carry out a lab procedure to achieve the stated goal of finding the maximum conversion of potential energy to kinetic energy.

Procedure:

Part One: Using a ruler and car, your team needs to design a lab that can exploit the Law of Conservation of Energy to change one form of energy into another. Once your team is given the "go," you will have 7 minutes to outline the procedure for such a lab. You will not be able to use the materials yet, so you must sketch out a design to show me. Once that is complete we will move onto part 2."

Part Two: Carry out your procedures. Make your measurements are as accurate as you can. Once the lab is complete, you will turn in your data and conclusions.

Data: Include all measurements. Clearly label all data and use data tables where appropriate.

Calculations: Show all calculations. For repetitive calculations you only need to show one sample calculation.

Conclusion:

Write a conclusion which states your results. In what way did you maximize the energy conversion? What was the efficiency of your process? (Energy Out/Energy In x 100%) Completely explain what caused the "loss" of energy which caused the efficiency to be less than 100%.

The lab (following lab procedure guidelines) should be completed neatly on lined paper or typed and turned in next class. One per group.

Conservation of Energy Lab Write-Up Instructions

Purpose: Clearly state the purpose of the lab.

Equipment: List necessary equipment

Procedure: Clearly state the steps that would need to be taken to replicate the experiment. Include any diagrams that clarify your procedure.

Data: Include all measurements. Clearly label all data and use data tables where appropriate.

Calculations: Show all calculations. For repetitive calculations you only need to show one sample calculation.

Discussion: No measurement can be perfect. Measurements always have some uncertainty. Due to the presence of measurement uncertainty, measured values will never be equal to predicted values. So the question is not: "are the values equal to each other" but instead "do the values agree with each other within acceptable uncertainty (±5.00%). The values should agree within this margin, i.e. the percent difference should be less than the percent uncertainty. If the values are in agreement, we will conclude that the data has supported the predictions of the theory. No data can ever prove a theory, only support or disprove.

Conclusion:

i) Restate the purpose: what were you trying to measure? What was the hypothesis?
ii) State the measured (and % uncertainty) and predicted values
iii) State the percent difference between these values
iv) State whether the theory is supported by your data
v) Discuss the largest source of uncertainty - the main reason that the predicted values (based on theory) are different from the measured values (collected in the experiment).

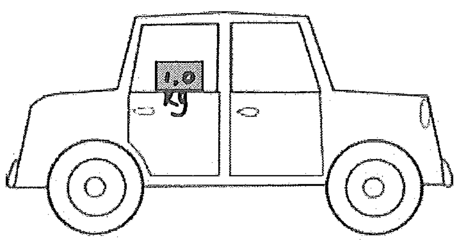
Lesson 3 (part 2)

Physics 12 - **Impulse – Momentum Theorem**

What is **Momentum** and how is it different from **Kinetic Energy?** In physics 11, you learned that momentum is equal to the *mass* of an object multiplied by the *velocity* of an object. The larger the mass a moving object has, the larger the momentum, and the faster an object moves, the larger the momentum. ***But how is this different from kinetic energy???***

MOMENTUM HAS DIRECTION WHILE ENERGY IS ONLY A QUANTITY!!!

Both energy and momentum are ***relative*** quantities. If you are driving in your car down the highway holding a brick in your hand, relative to the ground the brick has both energy and momentum, yet relative to the car the brick has zero energy and zero momentum.



Momentum can also be thought of in terms of force. A force applied for a certain *time* on an object will change the velocity of the object (therefore changing its momentum).

We have a special name for this *amount* of force given to the object. We call this *amount* of force **IMPULSE.**

Momentum is a quantity of motion that depends on both mass and velocity of the object in question.

The units of momentum are → kg•m/s or N•s

Example: A baseball pitcher hurls a ball (mass = 0.100 kg) at 32 m/s. The batter crushes it and the ball leaves the bat at 48 m/s. What was the ball's change in momentum?

Impulse:

Since momentum is the product of mass and velocity. Since we will not be dealing with changing masses, we can define an object's change in momentum as:

Whenever a net force acts on an object, an acceleration results and so its momentum must change.

How do **forces** relate to **changes in momentum?**

A student jumps off a desk. When they land they bend their knees on impact. Why does this help prevent damage to their knees?	Coaches for many sports such as baseball, tennis and golf can often be heard telling athletes to "follow through" with their swing. Why is this so important?

Conventional wisdom suggests that cars should be made tough and rigid to prevent injury during a collision. However, newer vehicles are all built with large crumple zones. Why?

Example: A 115 kg fullback running at 4.0 m/s east is stopped in 0.75 s by a head-on tackle. Calculate:

a) The impulse felt by the fullback
b) The impulse felt by the tackler
c) The average net force exerted on the tackler

Example: A 1250 kg car traveling east at 25 m/s turns due north and continues on at 15 m/s. What was the impulse of the car exerted while turning the corner?

The Law of Conservation of Momentum

Momentum is a useful quantity because in a closed system it is always conserved. This means that in any collision, the total momentum before the collision must equal the total momentum after the collision.

Collisions can be grouped into two categories:

Elastics Collisions:

Inelastic Collisions:

In reality, collisions are generally somewhere in between perfectly elastic and perfectly inelastic. It is actually impossible for a *macroscopic* collision to ever be perfectly elastic. Perfectly elastic collisions can only occur at the *atomic* or *subatomic level.*

INELASTIC:

When two or more objects collide and *stick* together. Is momentum conserved? Is energy conserved? Is kinetic energy conserved?	A 9500 kg caboose is at rest on some tracks. An 11000 kg engine moving east at 12.0 m/s collides with it and they stick together. What is the velocity of the train cars after the collision?

ELASTIC:

When two or more objects collide and *bounce* off each other (do not stick). Is momentum conserved? Is energy conserved? Is kinetic energy conserved?	An alpha particle has a mass approximately 4 times larger than a proton. A proton travelling to the right at 3200 m/s strikes a stationary alpha particle and it rebounds at 1920 m/s. What is the final speed of the alpha particle?

EXPLOSION:

Initial momentum is zero and therefore the sum of momentums after the explosion must also equal zero. Is momentum conserved? Is energy conserved? Is kinetic energy conserved?	A firecracker is placed in a pumpkin which explodes in into exactly two pieces. The first piece has a mass of 2.2 kg and flies due east at 26 m/s. The second chunk heads due west at 34 m/s. What was the initial mass of the pumpkin?

Lesson 3 (part 2)

Conservation of Momentum Problems – Inelastic Collisions and Explosions

1. A 54 kg women dives straight down into the water. Just before she strikes the water, her speed is 4.7 m/s. At a time of 2.1 s after she enters the water, her speed is 0.80 m/s.

a) What is the impulse given to the woman by the water? (+211 kg•m/s)

b) What is the net force exerted on her by the water? (+100 N)

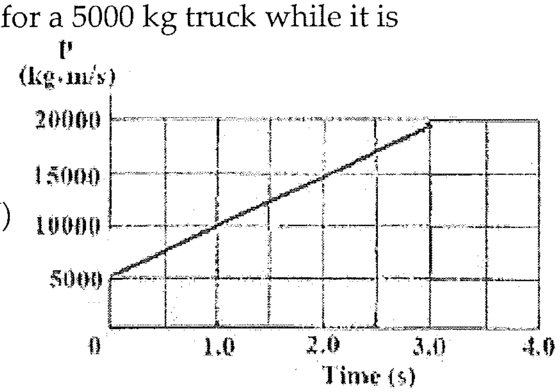
c) What is her acceleration while she is entering the water? (+1.86 m/s^2)

2. A 0.06 kg tennis ball travels east at 15 m/s.

a) If a net force of 12 N is exerted on the ball for 0.030 s in the same direction, what is the final velocity of the ball? (+21 m/s)

b) If a net force of 18 N is now exerted on the ball (travelling at the final velocity from part a) in the opposite direction, how long should the impact last to stop the ball? (0.070s)

3. The graph below shows momentum versus time for a 5000 kg truck while it is accelerating uniformly.



a) What is the initial speed of the truck? (1.0 m/s)

b) What is the net force acting on the truck? (5000N)

c) What is the acceleration of the truck? (1.0 m/s^2)

part 1
Lesson 4 (homework).

4. Two objects collide, one of which was initially moving and the other initially at rest.

a) Is it possible for *both* particles to be at rest after the collision? Give an example in which this happens, or explain why it cannot happen.

b) Is it possible for *one* particle to be at rest **after** the collision? Give an example in which this happens, or explain why it cannot happen.

5. A 1.0 x 10^5 N truck moving at a velocity of 15 m/s north collides head on with a 1.0x10^4 N car moving at a velocity of 25 m/s south. If they stick together upon impact, what is the velocity of the combined masses immediately after the collision? (+11.4 m/s)

6. A 3.2 kg cart moving at 1.2 m/s collides with a 1.8 kg wooden box at rest. After the collision, the cart and the wooden box stick together.
a) Find the final speed immediately after the collision. (0.77 m/s)
b) Find the energy transformed from initial kinetic energy to other forms. (0.82 J)

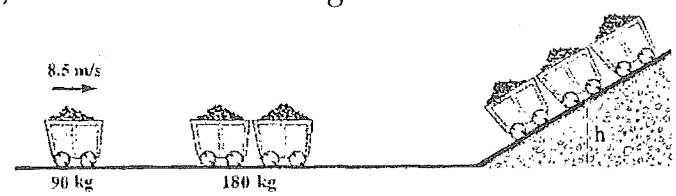
7. A 5.0 kg block moving at 3.0 m/s to the right collides with a 10 kg block moving at 2.0 m/s to the left. After the collision, the two blocks stick together.
a) Find the speed of the combined blocks after the collision. (-0.33 m/s)
b) Find how much energy is transformed from the initial kinetic energy to other forms. (41.7 J)

8. A 4.0 kg stone moving at 6.4 m/s overtakes and becomes embedded in a 2.8 kg lump of clay moving in the same direction. After the collision, the combined object moves at 4.2 m/s.
a) Find the initial speed of the lump of clay. (1.06 m/s)
b) Find the kinetic energy lost in the collision. (23.5 J)

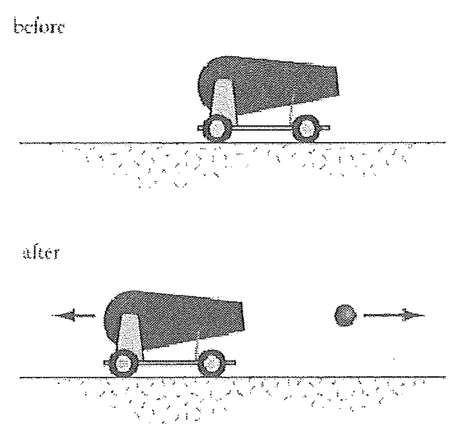
9. A 20 kg boy and his 40 kilogram sister are at rest on ice skates in the middle of a frozen lake. The boy pushes the girl and the boy moves to the left at 2 m/s. What is the velocity of the girl after the ***explosion***? (+1.0 m/s)

10. A 12.0 kg object splits into two parts. If part A has a mass of 5.0 kg and a velocity of 7.0 m/s right, what is the velocity of part B? (-5.0 m/s)

11. A 90 kg ore cart moving at 8.5 m/s collides with two carts of total mass 180 kg at rest on a frictionless horizontal track as shown in the figure. The three carts stick together and rise up the hill. If the hill has no friction, find the maximum height to which the combined ore cars rise up the hill. (0.41 m)

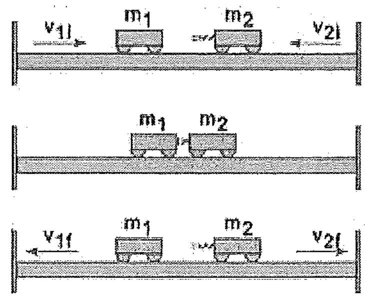


12. A 5.50 kg cannonball is fired from a 100 kg cannon. If the velocity of the cannonball is 300 m/s, what is the recoil velocity of the cannon? (-16.5 m/s)



Lesson 4

Physics 12- **Conservation of Momentum - Elastic Collisions**



Last class we dealt with problems involving **explosions and inelastic collisions**. We saw that in an inelastic collision, some of the initial kinetic energy of the system is transformed to other forms of energy resulted in less final kinetic energy for the system. We also saw that momentum is conserved in explosions and inelastic collisions.

Now, we are going to learn about **elastic collisions**. In an elastic collision, the objects 'bounce' off each other. They **do not 'stick together'** as they do during inelastic collisions.

In an elastic collision, **both momentum AND kinetic energy are conserved.**

There is a "spectrum" of collisions and explosions with the extremes being perfectly inelastic and elastic collisions.

INELASTIC--------------------ELASTIC------------------EXPLOSION

$P_o = P_f$	$P_o = P_f$	$P_o = P_f$
$KE_o > KE_f$	$KE_o = KE_f$	$KE_o < KE_f$

Elastic Collisions:

Example 1: A 30.0 kg object moving to the right at a speed of 1.00 m/s collides with a 20.0 kg object moving to the left at a velocity of 5.00 m/s. If the 20.0 kg object continues to move left at a velocity of 1.25 m/s, what is the velocity of the 30.0 kg object? Assume a perfectly elastic collision.

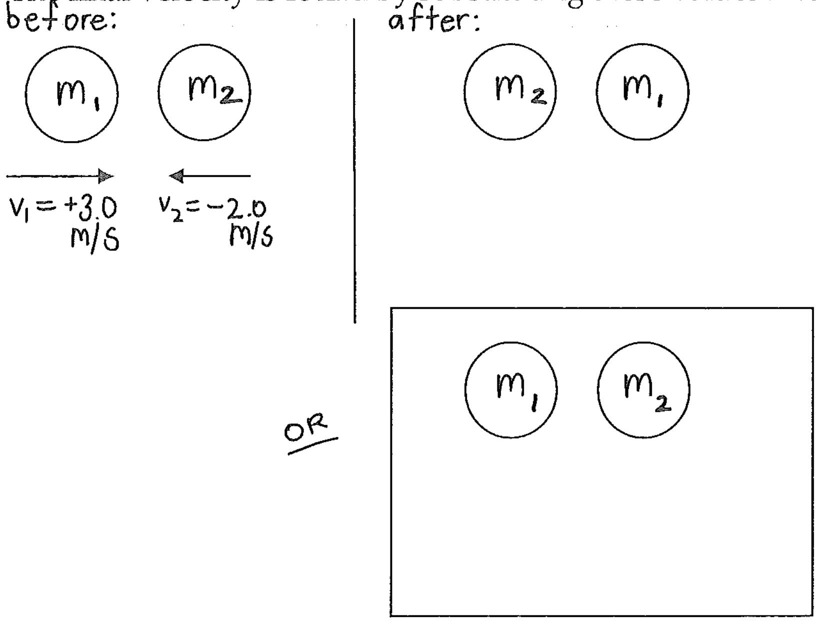
Example 2: When Kinetic Energy is conserved we can find the velocities of BOTH of the objects after the collision: Two identical pool balls make a perfectly elastic head-on collision on a frictionless table. The speeds of the balls for the collision are 2.0 m/s and 3.0 m/s. What are the speeds and direction of motion for the balls after the collision?

Because in an elastic collision, $KE_0 = KE_f$ and since we are only dealing with velocity, not the KE, **AND the masses are the same,** we can simplify the formula to just the 'velocity' portion of the equation.

$$ax^2 + bx + c = 0 \longrightarrow$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \longrightarrow$$

The final velocity is found by substituting these values into →



The only resulting velocities that make sense is for $v_1' = -2.0$ m/s and $v_2' = +3.0$ m/s. The other option shows velocities in which the objects pass right through each other which is not realistic.

The objects transferred their initial velocities to the other object. This is the case when the masses are the same.

Example 3: Different masses with an initial stationary object:

A 1 kg object travelling at 9 m/s to the right strikes a 2 kg stationary object. What are the velocities of the objects after the collision?

Derived from the conservation of momentum and conservation of kinetic energy:

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

Lesson 4 (part 2 homework)

Physics 12 – Conservation of Momentum Assignment – Elastic Collisions

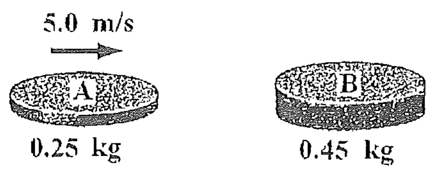
1. A 2.0 kg object travelling at 12 m/s to the right strikes a 2.0 kg object travelling to the left at 3 m/s. What are the velocities of the objects after the collision? (v_1'= -3.0m/s, v_2'= +12m/s)

2. A 2.0 kg object travelling at 12 m/s to the right strikes a stationary 4.0 kg object. What are the velocities of the objects after the collision? (v_1'= -4.0m/s, v_2'= +8.0 m/s)

3. A 2.0 kg object travelling at 12 m/s to the right strikes a 2.0 kg object travelling to the left at 12 m/s. What are the velocities of the objects after the collision? (v_1'= -12 m/s, v_2'= +12 m/s)

4. A pool ball moving with a speed of 2.5 m/s makes an elastic head-on collision with an identical ball travelling in the opposite direction with a speed of 5.9 m/s. Find the velocities of the balls after the collision. (v_1'= -2.5 m/s, v_2'= +5.9 m/s)

5. A 0.25 kg puck moving at 5.0 m/s due east collides head-on with a 0.45 kg puck at rest. If the collision is elastic, find the velocities of the pucks after the collision. (v_1'= -1.43m/s, v_2'=+3.57 m/s)



6. A 225 g ball moves with a velocity of 30.0 cm/s to the right. This ball collides with a 125 g ball moving in the same direction at a velocity of 10.0 cm/s. After the collision, the velocity of the 125 g ball is 24.0 cm/s to the right.

a) What is the velocity of the 225 g ball after the collision? (22.0 cm/s [right])

b) Is this an elastic or inelastic collision? Provide mathematical evidence for your answer. (NOTE: Calculate the kinetic energy of the objects before and after collision. Re-read your notes on inelastic and elastic collisions and determine the type of collision based on whether the energies are the same or not.)

c) If kinetic energy is lost, what happened to it?

7. A 30.0 kg object moving to the right at a velocity of 1.00 m/s collides with a 20.0 kg object moving to the left at a velocity of 5.00 m/s. If the 20.0 kg object continues to move left at a velocity of 1.25 m/s, what is the velocity of the 30.0 kg object? (-1.50 m/s)

8. A 10.0 g object is moving with a velocity of 20.0 cm/s to the right when it collides with a stationary 30.0 g object. After collision, the 10.0 g object is moving left at a velocity of 6.00 cm/s.

a) What is the velocity of the 30.0 g ball after the collision?

b) Is this an elastic or inelastic collision? Provide mathematical evidence for your answer.

c) If kinetic energy is lost, what happened to it?

Lesson 5

Momentum in 2-D

Always draw a triangle 1st and the use the **resultant** as **initial** or **final momentum.** (Motion in 2D Problems can also be solved using horizontal and vertical components of momentum vectors or using the cosine and/or sin laws-see Snap p.229 for examples)

Examples:

1. A 60.0 kg football player traveling at 2.00 m/s North collides and sticks to a 55.0 kg teammate traveling at 1.00 m/s East. Calculate their resulting velocity.

2. During supper preparation (spaghetti and meatballs) a 0.0200 kg meatball explodes into 3 pieces:

Piece 1 m = 0.0100 kg traveling 7.00m/s, N
Piece 2 m = 0.00500 kg traveling 5.00 m/s, E
Calculate the velocity of the remaining piece.

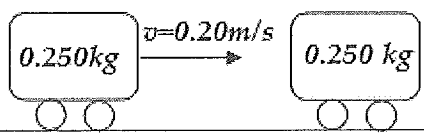
3. A 4.0 kg object traveling at 7.0 m/s 40^0 above the horizontal strikes a 2.0 kg object traveling from below the horizontal or reference line. They stick together and travel along the horizontal at 3.0 m/s. Find v_2.

THE LAW OF CONSERVATION OF MOMENTUM   Name:

In this activity, you will observe a series of events involving a system consisting of two carts. The carts will undergo inelastic and elastic collisions with various speeds and masses, as well as some explosions with varying masses. Prior to running each scenario, you will need to make a <u>prediction</u> as to what you think will happen <u>after</u> the collision or explosion in each scenario. Once you have completed your prediction, you will run the scenario and observe.

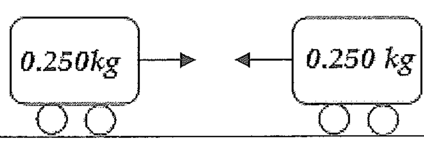
Indicate the magnitude and direction of the velocity of each cart by drawing an arrow that reflects the values.

Scenario #1 - A 0.250 kg cart is moving to the right at 0.20 m/s when it hits a stationary 0.250 kg cart. It is an **inelastic collision.**

BEFORE	AFTER	
	Prediction	**Actual**
		

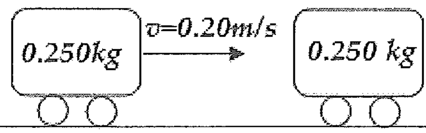
Calculate the **final velocity** of the two carts when they are stuck together.

Scenario #2 – A 0.250 kg cart is moving to the right at 0.20 m/s when it hits a cart of equal mass moving at 0.20 m/s to the left. It is an **inelastic collision.**

BEFORE	AFTER	
	Prediction	**Actual**
		

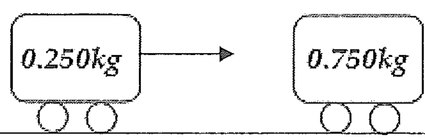
Calculate the **final velocity** of the two carts when they are stuck together.

Scenario #3 – A 0.250 kg cart is moving to the right at 0.20 m/s when it hits a stationary cart of equal mass. It is an **elastic collision.**

BEFORE	**AFTER**	
	Prediction	**Actual**
		

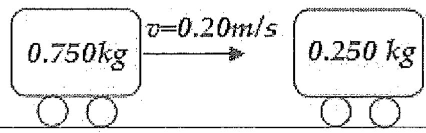
Calculate the **final velocity** of the second cart after the collision.

Scenario #4 – A 0.250 kg cart is moving to the right at 0.20 m/s when it hits a stationary 0.750 kg cart. It is an **elastic collision.**

BEFORE	**AFTER**	
	Prediction	**Actual**
		

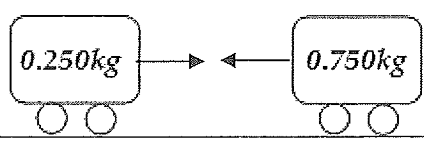
If the final velocity of the 0.75 kg cart is 0.15 m/s to the right, calculate the **final velocity** of the 0.250 kg cart.

Scenario #5 – A 0.750 kg cart is moving to the right at 0.20 m/s when it hits a stationary 0.250 kg cart. It is an **elastic collision.**

BEFORE	AFTER	
	Prediction	Actual
		

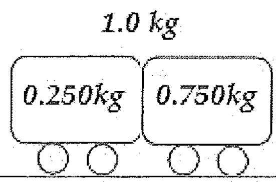
If the final velocity of the 0.25 kg cart is 0.45 m/s, calculate the **final velocity** of the 0.750 kg cart.

Scenario #6 – A 0.250 kg cart is moving to the right at 0.20 m/s when it hits a 0.750 kg cart moving at 0.20 m/s to the left. It is an **elastic collision.**

BEFORE	AFTER	
	Prediction	Actual
		

If the final velocity of the 0.75 kg cart is 0.05 m/s to the left, calculate the **final velocity** of the 0.250 kg cart.

Scenario #7 – A 1.0 kg mass cart system sits stationary on a track when an explosion happens. The system is split into a 0.75 kg cart moving to the right and a 0.25 kg cart moving to the left.

BEFORE	AFTER	
	Prediction	Actual
		

If the final velocity of the 0.75 kg cart is 0.10 m/s to the right, calculate the **final velocity** of the 0.250 kg cart.

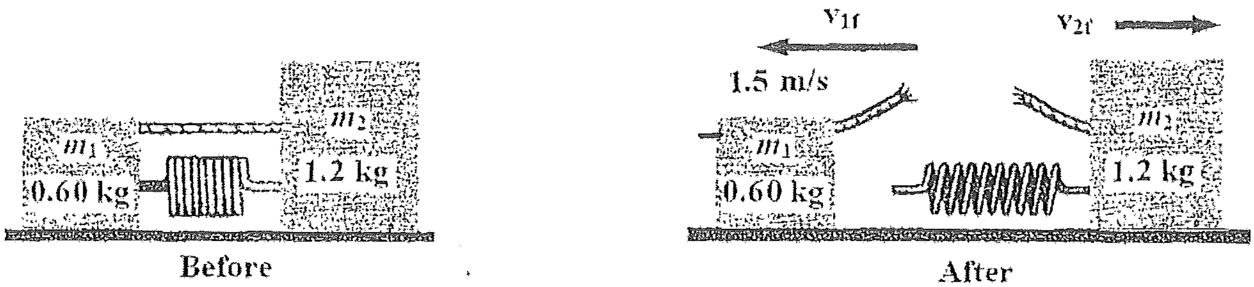
This worksheet will be handed in and marked so please review and make sure you have shown all work including formulas before you hand it in.

Physics 12 – **<u>2D Momentum – Explosions and Impulse</u>**

Conservation of Momentum and Work-Energy Theorem

<u>Recall:</u> $W = \Delta KE = KE_f - KE_0 = (\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2) - (\frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2)$

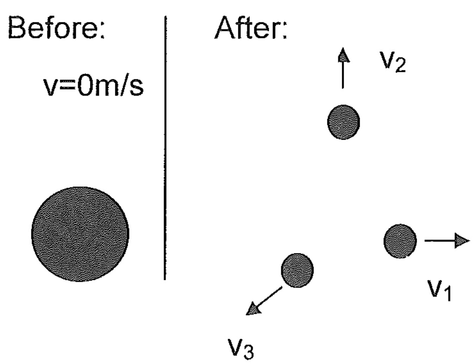
Once again we are able to relate momentum to work and energy.

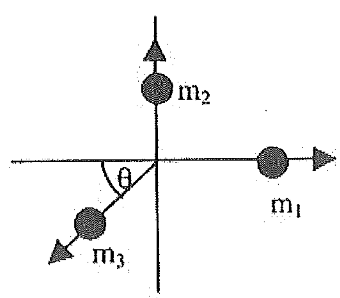


We can determine the work done on the block by the spring when the cord holding everything together breaks apart.

Explosions into <u>three</u> pieces:

Unlike past examples, all of the pieces will not explode into exactly opposite directions. Therefore we now need to work in **<u>more than one dimension</u>**.





Now let's assign some values to this explosion. A 10.0 kg object that is at rest breaks into three pieces as shown. $v_1 = 12$ m/s, $v_2 = 6.5$ m/s, $m_1 = 2.5$ kg, $m_2 = 5.5$ kg, $m_3 = 2.0$ kg

What is the velocity of the 2.0 kg mass?

Since we are in 2 dimensions now, we need to break the problem into its X and Y components:

X Component:

Y Component:

Now we find the resultant for v_{3f}:

FINAL ANSWER:

Impulse in Two Dimensions:

Remember that impulse is the change in momentum – the before and after. This is generally due to a change in an object's velocity.

A 2.5 kg model car moving at 6.0 m/s due east experiences a 16 N·s impulse southward. What is the magnitude and direction of the final momentum of the car?

This is in two dimensions – so we need X and Y:

Conservation of momentum (x-component):

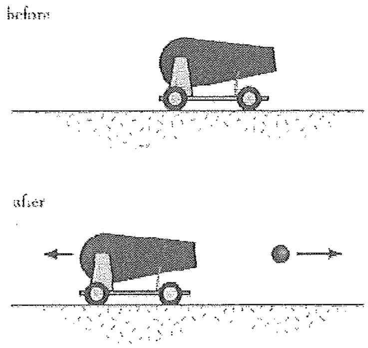
Conservation of momentum (y-component):

Find resultant to determine final momentum:

FINAL ANSWER:

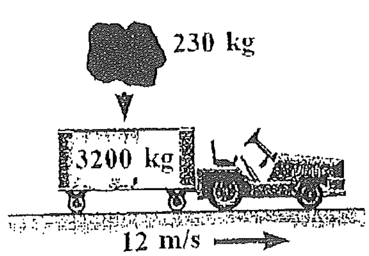
Explosion and Impulse Problems:

1. A 55 kg cannon and a 7.5 kg cannonball are sitting at rest. When the gunpowder has ignited, the cannonball is shot out of the cannon at a velocity of 45 m/s. What work was done on the cannon and cannonball by the gunpowder? (8.6×10^3 J)

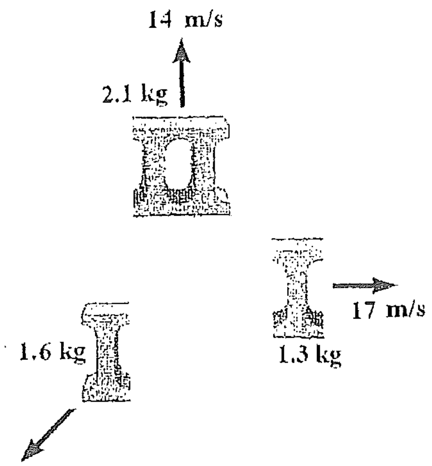


2. An 8.0 kg and 3.0 kg cart are held together and at rest. An internal spring is released between the two carts and they are pushed in opposite directions. If the 3.0 kg cart is traveling at 12 m/s after the spring is released, what is the work done on the carts by the spring? (297 J)

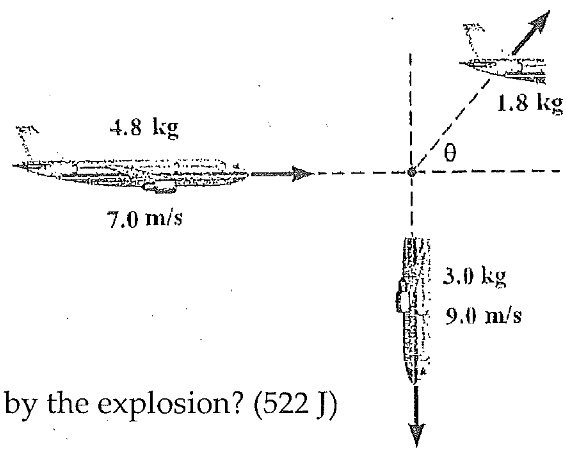
3. A 3200 kg dump car travels along the road at 12 m/s. Suddenly, a 230 kg chunk of coal is dumped into the car. Find the final velocity of the truck. (+11.2 m/s)



4. A 5.0 kg block at rest is dropped and breaks into three pieces as shown. What is the velocity (speed and direction) of the 1.6 kg piece? (The block is at rest when it hits the ground and breaks apart) (v_3' = 23.0 m/s @ 37° W of S)



5. A 4.8 kg model airplane flying at 7.0 m/s to the east explodes into two fragments as shown in the diagram. The larger 3.0 kg fragment moves at 9.0 m/s south.



a) What were the initial momentum and kinetic energy of the model airplane before the explosion? (+33.6 kg•m/s, 117.6J)

b) What are the velocity (speed and direction) and the kinetic energy of the smaller 1.8kg fragment? (24.0 m/s @ 39° N of E, 518 J)

c) What work was done on the model airplane by the explosion? (522 J)

6. A 0.40 kg ball moving at 20 m/s due south strikes a rock and moves 15 m/s at 30° west of south.

    a. Find the magnitude and direction of the impulse (change in momentum). (4.10 kg•m/s @ 43° N of W)
    b. If the ball is in contact with the rock for 0.06 s, what is the average force exerted on the ball by the rock? (68 N @ 43° N of W)

7. A 0.50 kg stone moving at 12 m/s due north makes contact with an electric pole for 0.04s, resulting in a final velocity of 8.0 m/s due west. What is the magnitude and direction of the net force exerted on the stone by the electric pole? (180N @ 34° W of S)

Physics 12 – <u>**2D Momentum - Collisions**</u>

Last lesson, we began dealing with non-linear momentum through impulse and explosions. Now, we are going to consider non-linear collisions.Collisions between objects are governed by laws of momentum and energy. When a collision occurs in an isolated system, the total momentum of the system of objects is conserved.

Elastic collisions are collisions in which **both momentum and kinetic energy are conserved**. The total system kinetic energy before the collision equals the total system kinetic energy after the collision.

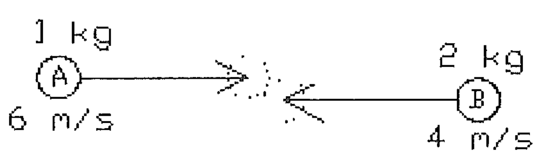
If total kinetic energy is not conserved, then the collision is referred to as an **inelastic collision.**

NON-LINEAR ELASTIC COLLISIONS:

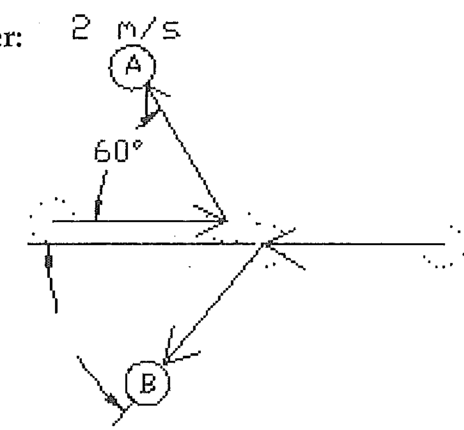
In this type of collision, no external forces act on the system. Kinetic Energy is conserved.

Two objects initially travelling east and west as shown have a collision. The object on the left is bounced 'up' at 60° with a velocity of 2 m/s. What is the final velocity of the object initially travelling west?

Before:



After:



Method Two:

A 4.0 kg object is moving east at an unknown velocity when it collides with a 6.1 kg stationary object. After the collision, the 4.0 kg object is travelling with a velocity of 2.8 m/s 32° N of E and the 6.1 kg object is travelling at a velocity of 1.5 m/s 41° S of E. What was the velocity of the 4.0 kg object before the collision?

Before collision: After collision:

Find p_1' and p_2':

Resolve both into their vector components:

Find $\Sigma p_x'$ and $\Sigma p_y'$:

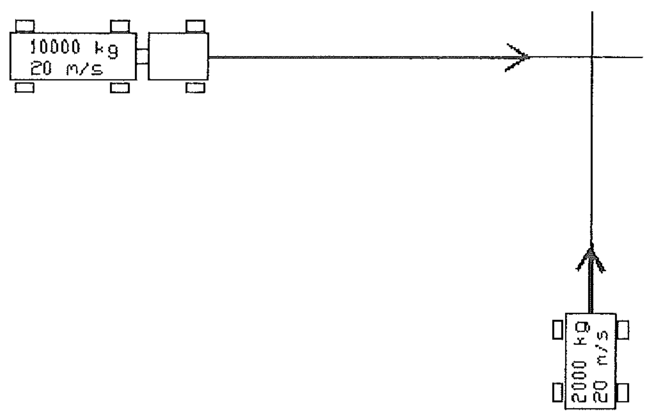
Find $\Sigma p'$:

NON-LINEAR INELASTIC COLLISIONS:

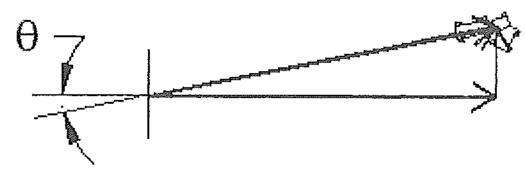
In this type of collision there are external forces acting on the system. Kinetic energy is converted into other forms such as sound, thermal etc.

A 2000 kg car is travelling at 20 m/s north and has a collision with a 10000 kg truck travelling 20 m/s east. After the collision the vehicles stick together. What is the speed and direction of the car and truck after the collision?

BEFORE



AFTER



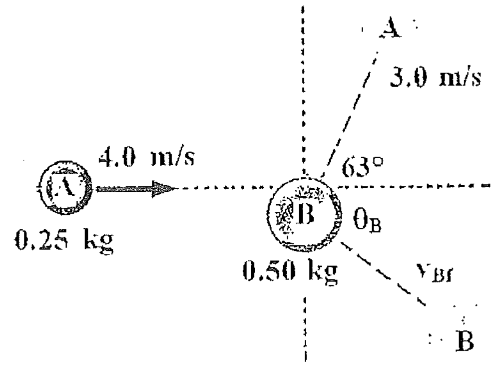
A 4.0 kg object is travelling south at a velocity of 2.8 m/s when it collides with a 6.0 kg object travelling east at a velocity of 3.0 m/s. If these two objects stick together upon collision, what is the speed and direction of the combined masses?

"Water Balloon" or Ball Impulse Lab (Lesson 6)

Lesson 6

2D Momentum – Collision Problems:

1.



A 0.25 kg puck (A) moving at 4.0 m/s to the right undergoes a collision with a 0.50 kg puck (B) at rest. As a result, puck A moves at 3.0 m/s at an angle of 63° north of east. What is the velocity (magnitude and direction) of puck B after the collision? (1.88 m/s @45° S of E)

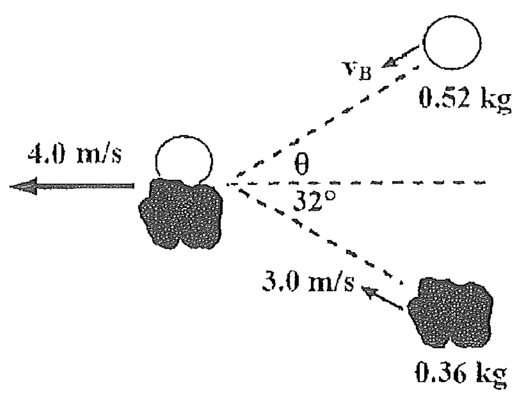
2. Two objects move on a level frictionless surface. Object A moves east with a momentum of 24 kg·m/s. Object B moves north with a momentum of 10 kg·m/s. They collide and stick together. What is the magnitude of the combined momentum after the collision? (26 kg·m/s)

3. A 1500 kg car traveling at 50 m/s 30° N of E collides with a 1000 kg car traveling at 40 m/s 45° S of E. The two cars collide and stick together. What is the speed and direction of the cars after the collision? (37.5 m/s @ 5.6° N of E)

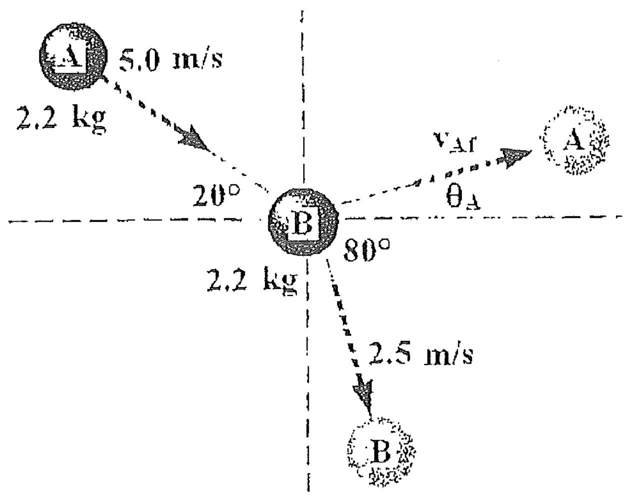
4. A 50.0 kg object is moving east at an unknown velocity when it collides with a stationary 60.0 kg object. After the collision, the 50.0 kg object is travelling at a velocity of 6.0 m/s 50.0° N of E, and the 60.0 kg object traveling at a velocity of 6.3 m/s 38.0° S of E. What was the velocity of the 50.0 kg object before the collision? (9.8 m/s [E])

5. A 15.0 kg object is moving east at a velocity of 7.0 m/s when it collides with a stationary 10.0 kg object. After the collision, the 15.0 kg object is moving at a velocity of 4.2 m/s 20.0° S of E. What is the velocity of the 10.0 kg object after the collision? (5.06 m/s @ 25° N of E)

6. A 0.36 kg lump of clay moving at 3.0 m/s collides with a 0.52 kg ball and they stick together as shown in the diagram. Find the speed and direction of the ball before the collision. (5.13 m/s @ 12° S of W)



7. A 2.2 kg ball (A) moving with a speed of 5.0 m/s strikes a second ball (B) of the same mass, 2.2 kg, initially at rest as shown in the diagram. As a result of the collision, ball B moves at 2.5 m/s at 80° S of E. What is the speed and direction of ball A after the collision? (4.34 m/s @ 10° N of E)



PHYSICS 12

ADDITIONAL PROBLEMS – MOMENTUM

1. What is the weight of an object that has a velocity of 10.0 m/s and momentum of 2.0×10^2 kg•m/s?

(2.0×10^2 N)

2. A 1.20×10^3 kg car accelerates uniformly from rest to 25.0 m/s in 10.3 s. What was the net force acting on the car?

(2.91×10^3 N)

3. 

A golfer hits a 5.0×10^{-2} kg ball from a ledge as shown in the diagram. If this ball leaves the face of the club with a horizontal velocity of 30.0 m/s, what is the impulse due to the club?

(1.5 N•s)

4. A 1.1×10^3 kg car travelling at a velocity of 10.0 m/s collides head on with a brick wall. If the car comes to a complete rest in 0.25 s, what was the average force exerted on the car during the collision?

(4.4×10^4 N)

5. If a 0.15 kg object has kinetic energy of 9.0 J, what is the magnitude of its momentum?

(1.6 kg•m/s)

6. If a 0.85 kg object is dropped from a height of 2.2 m above the floor, what is its momentum as it hits the floor?

(5.6 kg•m/s)

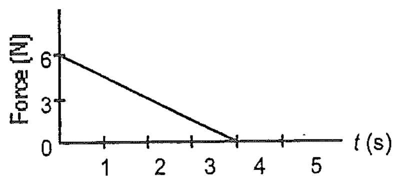
7. A 50.0 g bullet travelling at a velocity of 375 m/s becomes embedded 25.0 cm into a massive wood block. Calculate the average force exerted on the bullet by the wood.

(1.41×10^4 N)

8. If a 0.50 kg object is fired vertically and reaches a maximum height of 15 m, what was the maximum momentum of this object?

(8.6 kg•m/s)

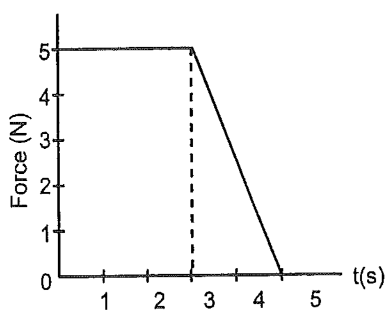
9.



Given the above force-time graph for a 0.75 kg object that was accelerated from rest, calculate the velocity at 4.0 s. (NOTE: Area under the graph represents the impulse.)

(16 m/s)

10.



Given the above force-time graph for a 0.50 kg object that was accelerated from rest, calculate the velocity at 5.0 s.

(4.0×10^1 m/s)

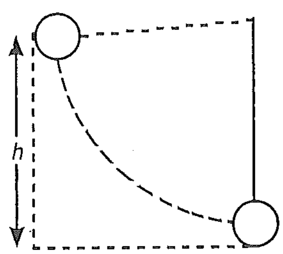
11. If a 2.0 kg object accelerates horizontally from rest at a uniform rate of 3.5 m/s^2, what is the momentum of this object after 2.5 s?

(18 kg•m/s)

12. If a 0.15 kg ball is thrown vertically up at a velocity of 12 m/s, what is the momentum of this ball when it is halfway to its maximum height?

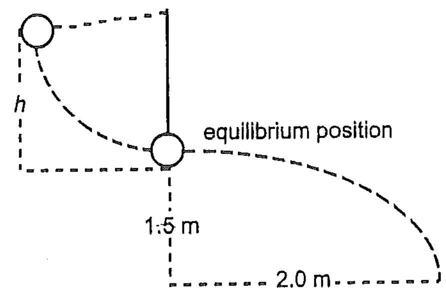
(1.3 kg•m/s)

13.



A 0.020 kg pendulum bob is dropped from a height, h. At its equilibrium position it has a momentum of 0.070 kg•m/s. What is the value of h?

(0.63 m)

14. 

A 0.010 kg pendulum bob is dropped from a height h above its equilibrium position as shown in the diagram. When the bob reaches its equilibrium position, the string breaks and the bob now acts as a projectile. After the string breaks the bob falls 1.5 m while moving 2.0 m horizontally. Calculate the height (h) from which the bob was released.

(0.67 m)

15. A 0.45 kg ball is moving east at a velocity of 11.0 m/s when it hits a wall. If the ball rebounds with a velocity of 10.0 m/s west, what was the impulse of the wall on the ball?

(9.5 N•s west)

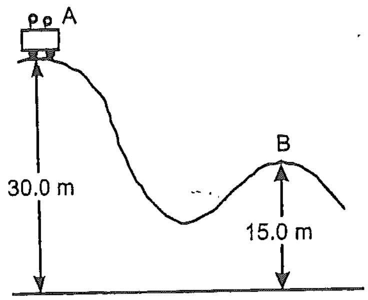
16. A 5.0 g bullet is moving at a velocity of 375 m/s when it hits a stationary block of wood 6.0 cm thick. If the bullet emerges from the wood at a velocity of 225 m/s, and the wood did not move, what was the average force exerted on the bullet by the wood?

(3.8×10^3 N)

17. A 0.15 kg ball is thrown north at a velocity of 25 m/s while a 2nd ball of identical mass is thrown west at a velocity of 22 m/s. Calculate the sum of the momenta of these two balls.

(5.0 kg•m/s 41° W of N or 5.0 kg•m/s 49° N of W or 5.0 kg•m/s 131°)

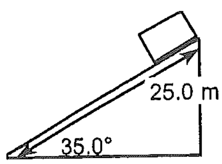
18.



A 5.0×10^2 kg roller coaster travels from point A to B along a frictionless track as shown in the diagram. If the momentum of the roller coaster is zero at point A, what is it at point B?

(8.6×10^3 kg•m/s)

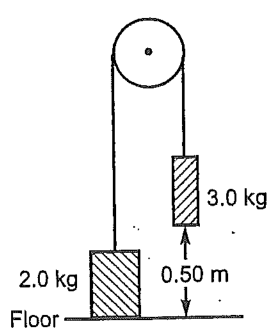
19.



A 98.0 N box slides 25.0 m along a 35.0° incline as shown in the diagram. If the force of friction along the incline is 32.0 N, and the box starts from rest at the top, what is the momentum of the box at the bottom?

(1.10×10^2 kg•m/s)

20.



A system containing a frictionless pulley is described in the diagram. If this system is released, what is the momentum of

a) the 2.0 kg box when the 3.0 kg box hits the floor?

(2.8 kg•m/s up)

b) the 3.0 kg box when it hits the floor?

(4.2 kg•m/s down)

21. A 45 kg student standing on a frictionless surface throws a 0.25 kg object at a velocity of 9.0 m/s east. Calculate the velocity of the student after she releases the object.

(5.0×10^{-2} m/s west or 5.0×10^{-2} m/s 180°)

22. A car moving east at a velocity of 10.0 m/s collides with a stationary truck with exactly twice the mass. If the two vehicles lock together, calculate the velocity of their combined mass immediately after collision.

(3.33 m/s east or 3.33 m/s 0°)

23. A 6.0 g ball moving north at a velocity of 3.0 m/s collides head on making a collision with an identical ball that is moving south at a velocity of 2.0 m/s. Immediately after collision the first ball is moving south at a velocity of 1.0 m/s. What is the magnitude of the velocity of the second ball?

(2.0 m/s)

24. A 5.0 g ball collides and sticks to a second ball which is at rest. If the combined mass moves at a velocity of one quarter the original velocity of the 5.0 g ball, what is the mass of the second ball?

(15 g)

25. A gun with a weight of 25 N fires a 6.0×10^{-2} kg bullet at a velocity of 325 m/s west. What is the recoil velocity of the gun?

(7.6 m/s east or 7.6 m/s 0°)

26. A 40.0 kg object is moving east at a velocity of 2.00 m/s when it collides with a 30.0 kg object moving north at a velocity of 2.00 m/s. If the objects stick together upon collision, what is the velocity of the combined mass immediately after collision?

(1.43 m/s 36.9° N of E or 1.43 m/s 53.1° E of N or 1.43 m/s 36.9°)

27. A 2.0 kg object is moving with a velocity of 5.0 m/s west when it collides with a stationary 3.0 kg object. After collision, the 2.0 kg object is moving west at a velocity of 1.5 m/s.

a. Calculate the velocity of the 3.0 kg object after the collision.

(2.3 m/s west)

b. Is this an elastic or inelastic collision? Provide mathematical evidence for your answer. What happened to the kinetic energy lost?

(E_k loss = 15 J, ∴ inelastic)

28. A 7.0 kg object is moving north at an unknown velocity when it collides with a 5.0 kg stationary object. After the collision the 7.0 kg object is moving at a velocity of 3.0 m/s 30.0° E of N, and the 5.0 kg object is moving at a velocity of 5.0 m/s 25.0° W of N.

a. Calculate the velocity of the 7.0 kg object before collision.

(5.8 m/s north)

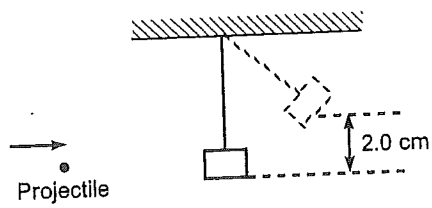
b. Is this an elastic or inelastic collision? Provide mathematical evidence for your answer. What happened to the kinetic energy lost?

(E_k loss = 25 J, ∴ inelastic)

29. Two cars collide at an intersection. The first car has a mass of 775 kg and was travelling west. The second car has a mass of 1125 kg and was travelling north. Immediately after impact, the first car had a velocity of 65.0 km/h 33.0° W of N, while the second car had a velocity of 42.0 km/h 46.0° W of N. What were the velocities of these two cars immediately before collision?

(car 1 - 79.3 km/h west) (car 2 - 66.7 km/h north)

30.



A 4.0 g projectile is fired at a 2.0 kg wooden pendulum as shown in the diagram. If the pendulum swings to a height of 2.0 cm after the projectile becomes embedded in it, how fast was the projectile travelling when it hit the pendulum? Explain, using conservation laws.

(3.1×10^2 m/s)

* * * * * * * * * *

Work, Energy + Momentum Provincial Exam Questions Lesson 8

1. Is power a scalar or vector quantity, and which are the correct units for measuring it?

	TYPE OF QUANTITY	UNITS
A.	Scalar	J/m
B.	Scalar	J/s
C.	Vector	J/m
D.	Vector	J/s

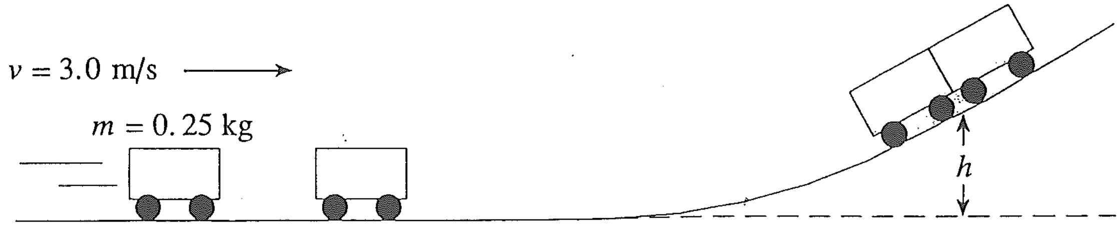
2. A climber's gravitational potential energy increases from 14 000 J to 21 000 J while climbing a cliff. She expends 18 000 J of energy during this activity. What is the efficiency of this process?

A. 3%
B. 39%
C. 61%
D. 97%

3. A 40 000 kg rail car travelling at 2.5 m/s collides with and locks to a stationary 30 000 kg car. Determine the speed of the locked cars and state whether the collision is elastic or inelastic.

	SPEED OF LOCKED CARS	TYPE OF COLLISION
A.	1.4 m/s	Elastic
B.	1.4 m/s	Inelastic
C.	1.9 m/s	Elastic
D.	1.9 m/s	Inelastic

4. A 0.25 kg cart travelling at 3.0 m/s collides with and sticks to an identical stationary cart on a level track. (Ignore friction.)



To what height h do the combined carts travel up the hill? **(7 marks)**

5. A cyclist must do 1 000 J of work to speed up from 0 m/s to 5.0 m/s. The same cyclist must do 3 000 J of work to speed up from 5.0 m/s to 10.0 m/s. (In both instances friction has been ignored.) Using principles of physics, explain why more work must be done to speed up from 5.0 m/s to 10.0 m/s than from 0 m/s to 5.0 m/s. (Remember, friction plays no role in this problem.) **(4 marks)**

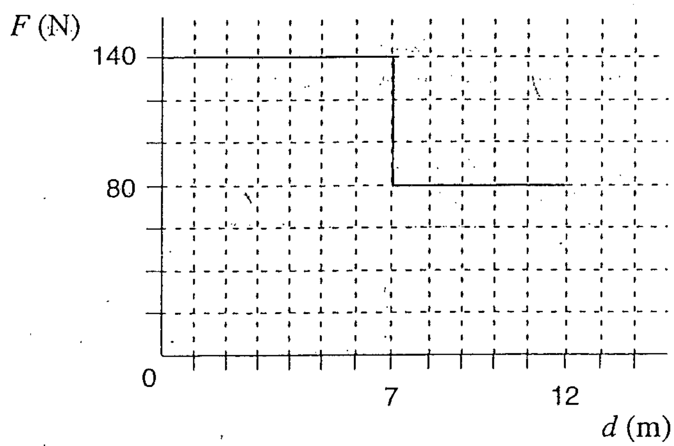
6. A cyclist increases his kinetic energy from 1 100 J to 5 200 J in 12 s. His power output during this time is

7. Which of the following best represents the momentum of a small car travelling at a city speed limit?

   A. 1 000 kg · m/s
   B. 10 000 kg · m/s
   C. 100 000 kg · m/s
   D. 1 000 000 kg · m/s

8. A 0.080 kg tennis ball travelling east at 15 m/s is struck by a tennis racquet, giving it a velocity of 25 m/s, west. What are the magnitude and direction of the impulse given to the ball?

9. Starting from rest, a farmer pushed a cart 12 m. The graph shows the force *F* which he applied, plotted against the distance *d*.

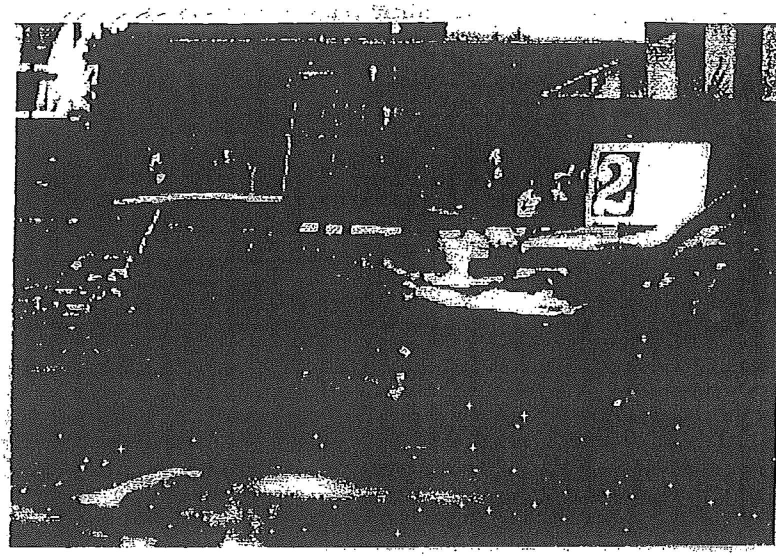


a) How much work did the farmer do moving the cart 12 m? **(3 marks)**

b) After the farmer had pushed the 240 kg cart 12 m, it was moving with a velocity of 2.2 m/s. What was the cart's kinetic energy? **(2 marks)**

c) What was the efficiency of this process? **(2 marks)**

10. Consider the collision between the vehicles in the photograph below.



Cars have crumpled fenders and are joined.

The collision is inelastic. Define inelastic. Give at least two pieces of evidence that show this to be an inelastic collision. **(4 marks)**

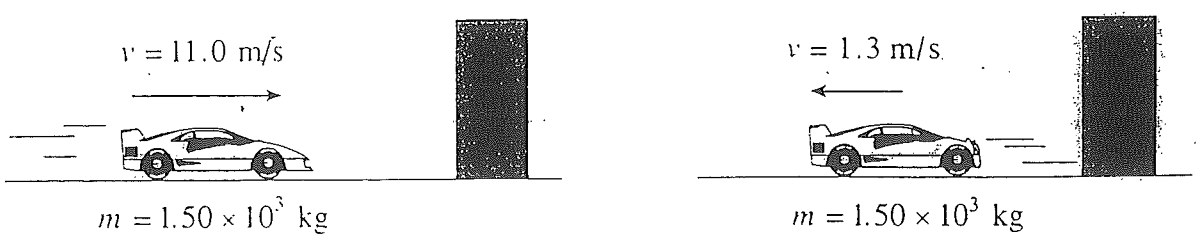
11. Which of the following correctly describes momentum and impulse?

	MOMENTUM	IMPULSE
A.	vector	vector
B.	vector	scalar
C.	scalar	vector
D.	scalar	scalar

12. A stationary object explodes into two fragments. A $4.0\ \text{kg}$ fragment moves westwards at $3.0\ \text{m/s}$. What are the speed and kinetic energy of the remaining $2.0\ \text{kg}$ fragment?

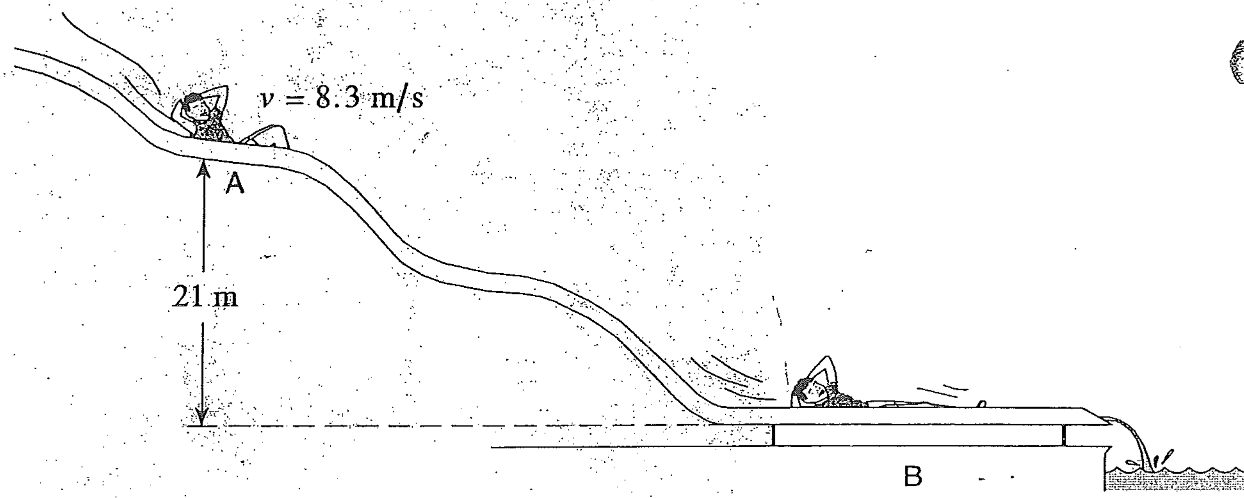
13. A $1\ 000\ \text{kg}$ vehicle travelling westward at $15\ \text{m/s}$ is subjected to a $1.0 \times 10^4\ \text{N} \cdot \text{s}$ impulse northward. What is the magnitude of the final momentum of the vehicle?

14. A $1.50 \times 10^3\ \text{kg}$ car travelling at $11.0\ \text{m/s}$ collides with a wall as shown.



The car rebounds off the wall with a speed of $1.3\ \text{m/s}$. If the collision lasts for $1.7\ \text{s}$, what force does the wall apply to the car during the collision?

15. A 45 kg child on a water slide passes point A at 8.3 m/s.



As the child descends from A to B, 3 600 J of heat energy is created because of friction. What is his speed at B? **(7 marks)**

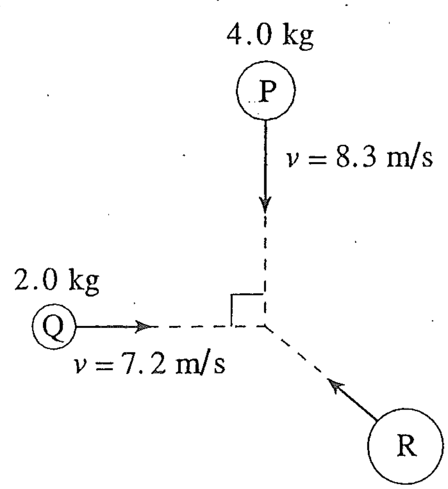
16 What is the minimum work done when a 65 kg student climbs an 8.0 m-high stairway in 12 s?

17. Which of the following is equal to impulse?

A. Energy
B. Momentum
C. Change in energy
D. Change in momentum

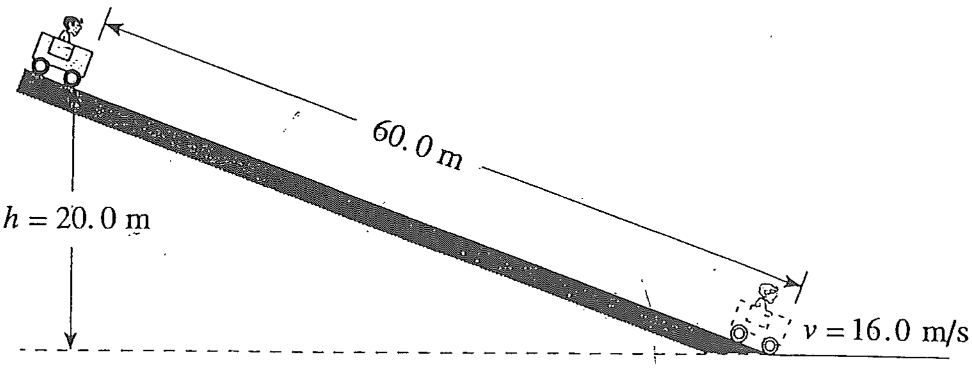
18. A 1 500 kg car travelling at 25 m/s collides with a 2 500 kg van stopped at a traffic light. As a result of the collision the two vehicles become entangled. With what initial speed will the entangled mass move off, and is the collision elastic or inelastic?

19. Three objects travel as shown.



What is the magnitude of the momentum of object R so that the combined masses remain stationary after they collide?

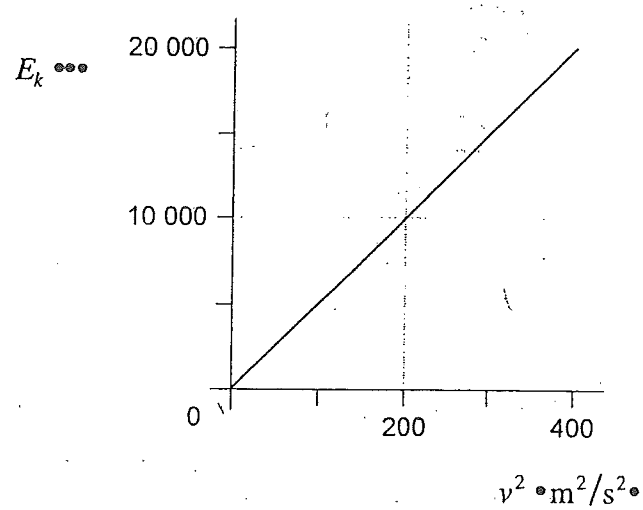
20. A 170 kg cart and rider start from rest on a 20.0 m high incline.



a) How much energy is transformed to heat? **(5 marks)**

b) What is the average force of friction acting on the cart? **(2 marks)**

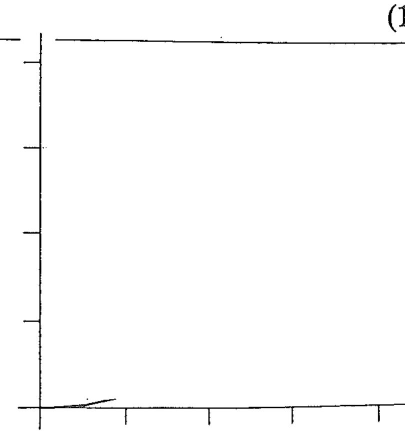
21. A student plots the graph below, showing the kinetic energy E_k of a motorbike versus the square of its velocity v^2.



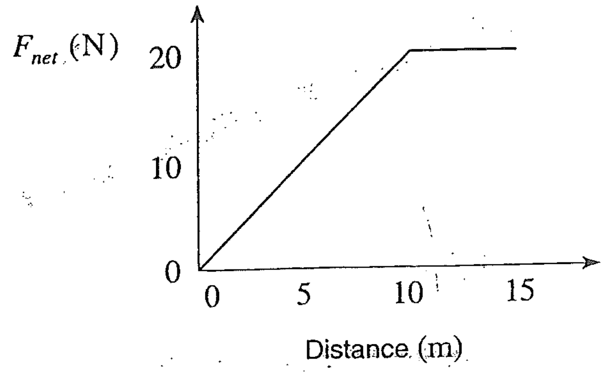
a) What is the slope of this graph? **(2 marks)**

b) What does the slope represent? **(2 marks)**

c) Using the axes below, sketch the graph of kinetic energy E_k versus velocity v for this motorbike. There is no need to plot any data points. **(1 mark)**



22. A force is applied to an 8.0 kg object initially at rest. The magnitude of the net force varies with distance as shown.



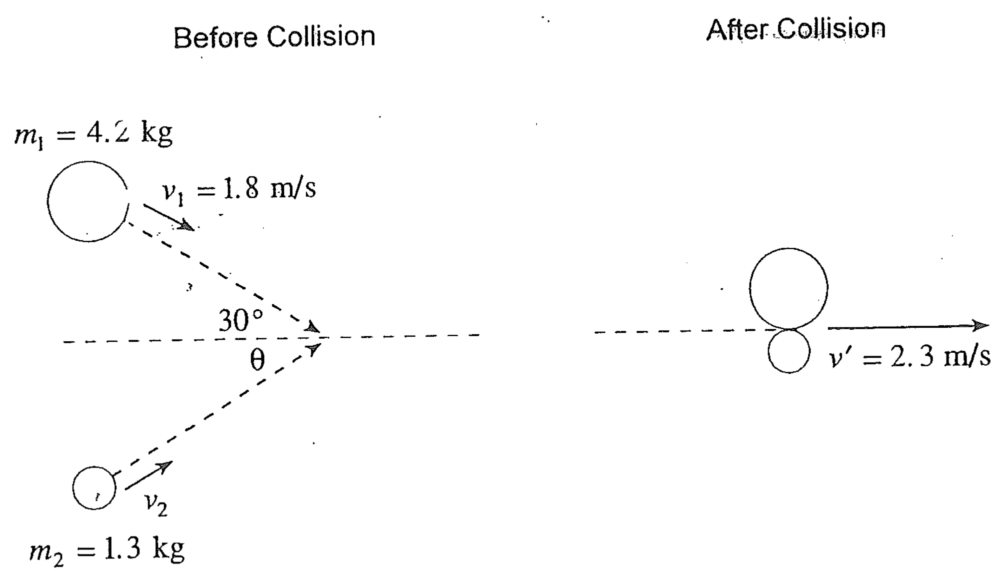
What is the speed of the object after moving 15 m?

23. A machine rated at 1 500 W lifts a 100 kg object 36 m vertically in 45 s. What is the efficiency of this machine?

24. Two cars collide head-on and come to a complete stop immediately after the collision. Which of the following is correct?

	TOTAL MOMENTUM	TOTAL ENERGY
A.	is conserved	is conserved
B.	is conserved	is not conserved
C.	is not conserved	is conserved
D.	is not conserved	is not conserved

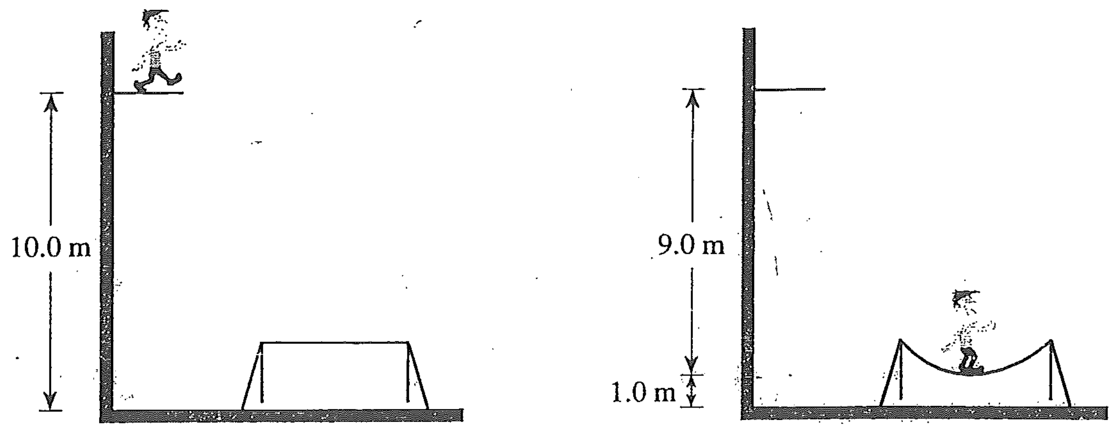
25. Two steel pucks are moving as shown in the diagram. They collide inelastically.



Determine the speed and direction (angle θ) of the 1.3 kg puck before the collision. **(7 marks)**

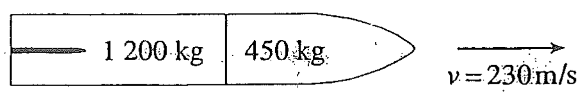
26. A 950 kg elevator ascends a vertical height of 410 m with an average speed of 9.1 m/s. What average power must the lifting motor supply?

27. A 55.0 kg athlete steps off a 10.0 m high platform and drops onto a trampoline. As the trampoline stretches, it brings him to a stop 1.00 m above the ground.

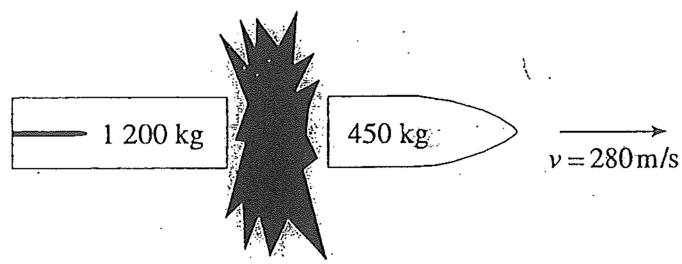


How much energy must have been momentarily stored in the trampoline when he came to rest?

28. A space vehicle made up of two parts is travelling at 230 m/s as shown.



An explosion causes the 450 kg part to separate and travel with a final velocity of 280 m/s as shown.



a) What was the momentum of the space vehicle before the explosion? **(2 marks)**

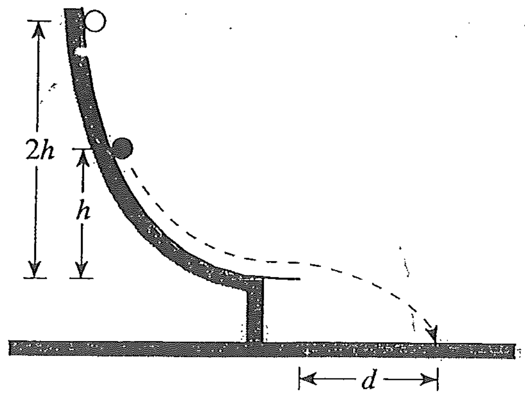
b) What was the magnitude of the impulse on the 1 200 kg part during the separation? **(3 marks)**

c) Using principles of physics, explain what changes occur, if any, to the

i) momentum of the system as a result of the explosion. **(2 marks)**

ii) kinetic energy of the system as a result of the explosion. **(2 marks)**

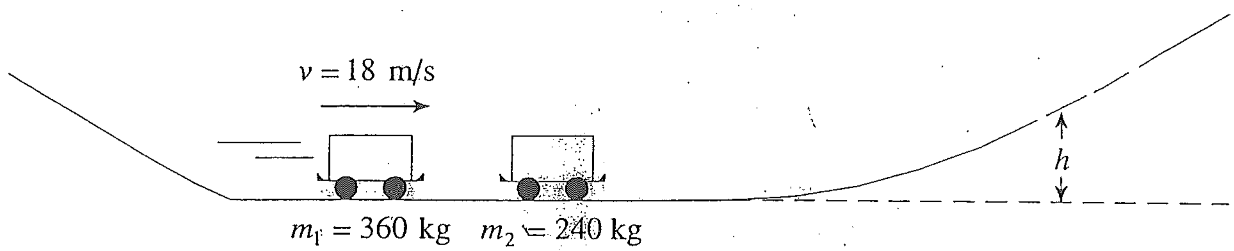
29. An object starts from rest and slides down a frictionless track as shown. It leaves the track horizontally, striking the ground at a distance d as shown.



The same object is now released from twice the height, $2h$. How far away will it land?

A. d
B. $\sqrt{2}\,d$
C. $2d$
D. $4d$

30. A 360 kg roller coaster car travelling at 18 m/s collides inelastically with a stationary 240 kg car on a section of horizontal track as shown in the diagram below.



To what maximum height, h, do the combined cars travel before rolling back down the hill? (Assume no friction.) **(7 marks)**

Work, Energy + Momentum Answers

1. B
2. B
3. B
4. 0.11 m
5. $W = \Delta E$

   $= \Delta E_k$ in this case ← **1 mark**

   $E_k = \frac{1}{2}mv^2$ **(1 mark)**, so velocity changing by a factor of two will cause kinetic energy to change by a factor of four **(1 mark)** and so the work done will become ever greater as the velocity increases by uniform amounts. **(1 mark)**

   OR

   $W = F \cdot d$ **(1 mark)**, but if the cyclist travels faster while exerting a constant force, for each uniform increment of velocity the distance travelled will become greater **(1 mark)** and greater. Hence $W = F \cdot d$ yields greater values for W as the distance becomes larger. **(2 marks)**
6. 3.4×10^2 ~~W~~ J
7. B
8. 3.2 N·s Westward
9. a) 1.4×10^3 J

   b) 5.8×10^2 J

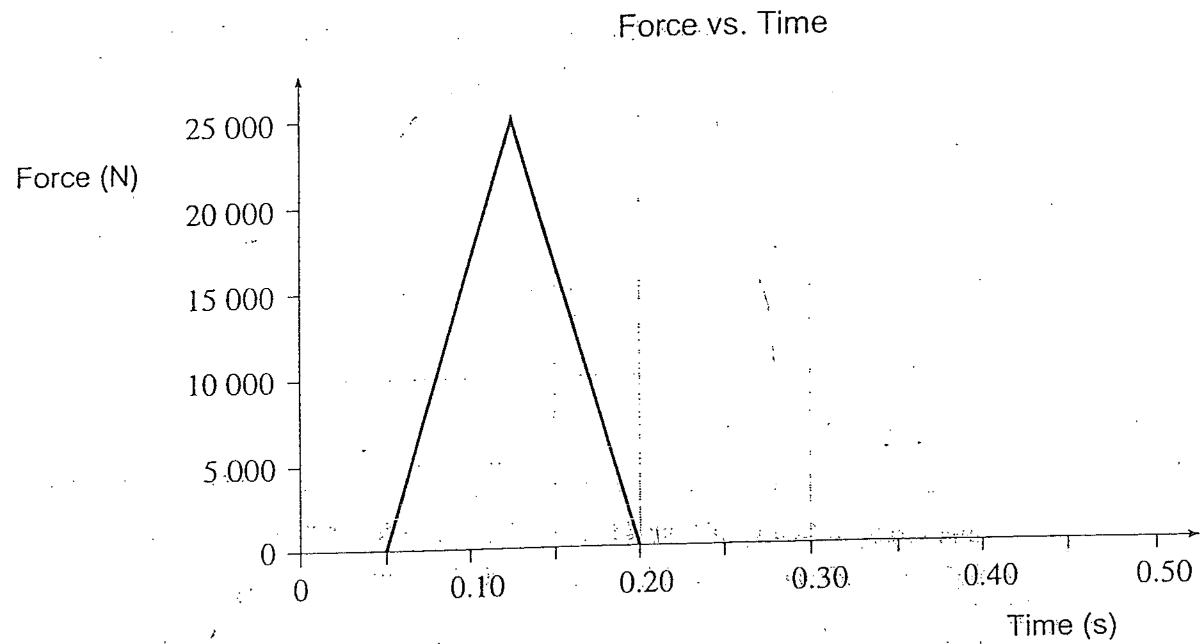
   c) 0.42 or 42%
10. **In inelastic collisions, kinetic energy is not conserved.**

    In collisions between cars there are skid marks, dents, pieces of twisted metal and loud sounds.

    Each of these requires energy. This energy comes from the original kinetic energy.

    Since an elastic collision requires conservation of kinetic energy, any collision producing one or more of the above observations must be inelastic.
11. A
12. 6.0 m/s, 36 J
13. 1.8×10^4 kg·m/s
14. 1.1×10^4 N
15. NOTE: Provincial Exam long answer questions require DETAILED solutions. Study the marking criteria for this question;

31. During a motor vehicle accident an unbelted passenger experienced a force which varied with time as shown on the graph.



a) Calculate the area of the shaded region in the graph. **(1 mark)**

b) What does this area represent? **(2 marks)**

c) If the passenger was wearing a seatbelt properly, the maximum force would have been of third the force experienced without the seatbelt. Sketch on the graph below how the force of the belted passenger might have varied with time. **(2 marks)**

15.(continued)

$$E = E'$$ ← 1 mark

$$E_k + E_p + E_H = E_k' + E_p' + E_H'$$ ← 2 marks

$$\tfrac{1}{2}mv^2 + mgh = \tfrac{1}{2}m(v')^2 + E_H'$$ ← 1 mark

$$\tfrac{1}{2}(45)(8.3)^2 + 45(9.8)(21) = \tfrac{1}{2}(45)(v')^2 + 3\,600$$ ← 1 mark

$$1\,550 + 9\,260 = 22.5(v')^2 + 3\,600$$ ← 1 mark

$$v' = 18 \text{ m/s}$$ ← 1 mark

OR

$$\left.\begin{aligned} &E = E' \\ &E_k + E_p + E_H = E_k' + E_p' + E_H' \end{aligned}\right\}$$ ← 2 marks

$E_k = \tfrac{1}{2}mv^2 = 1\,550$ J ← 1 mark

$E_p = mgh = 9\,260$ J ← 1 mark

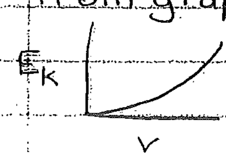
$E_k' = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(45)(v')^2$ ← 1 mark

$E_H' = 3\,600$ J ← 1 mark

$v' = 18$ m/s ← 1 mark

16. 5.1×10^3 J
17. D
18. 9.4 m/s, inelastic
19. 36 kg·m/s
20. 1.16×10^4 J, 193 N
21. 50 J/m²/s² or 50 kg

from graph: $E_k = kv^2$ and $E_k = 50v^2$, since $E_k = \tfrac{1}{2}mv^2$,



∴ slope = ½ the mass of the motorbike

22. 7.1 m/s
23. 0.52 or 52%

24. A

25. 5.5 m/s, 32° below horizontal

26. 8.5×10^4 W

27. 4.85×10^3 J

28. 3.80×10^5 kg m/s

2.25×10^4 N·s

c i) **In an explosion, momentum must be conserved.**

ii) **Since the explosion adds energy to the system, the system will gain kinetic energy.**

29. B

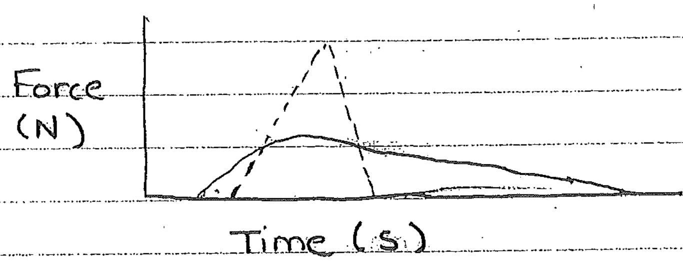
30. 6.0 m

31. a) 1.9×10^3 N·s

b) impulse or change in momentum

c) ⅓ (25000) = 8000N but over a longer time ← 1 mark

the area should be the same ← 1 mark



Work and Energy KEY

Physics 12 – **Work and Energy** Name:

WORK is defined as the **transfer** of energy from one body to another. $\boxed{W = \Delta E}$

We can calculate the work done on an object with: $\boxed{W = F \cdot d}$

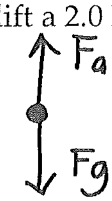
Units - $N \cdot m$ or J

Note that these are the same units as <u>torque</u> yet they are used to describe very difference quantities.

<u>Work against Gravity:</u>

How much work is required to lift a 2.0 kg textbook from the floor to a height of 1.5 m at a constant velocity?

$\Sigma F = 0N \rightarrow$



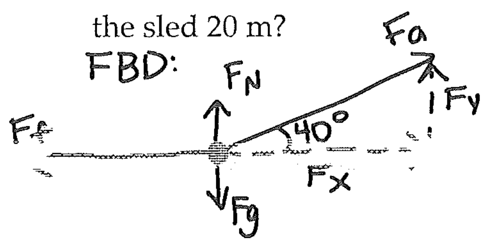
$F_a = F_g$ so... $W = F_g \cdot d$ or $W = mgd$

We know that W = F•d, so we need to determine the force needed to lift the book at a constant velocity. $W = (2.0)(9.8)(1.5) = \underline{29.4\,J}$

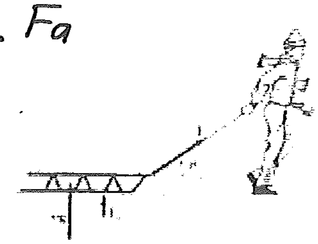
<u>Forces at an angle:</u>

A boy is pulling his sled at a constant velocity of 1.2 m/s. He pulls the 15 kg sled with a force of 35 N at an angle of 40° to the horizontal. How much work does he do in pulling the sled 20 m?

FBD:



*need to break F_a into F_x & F_y



Rule: When finding the work done on an object we only consider = <u>the component of the force in the direction of movement</u>

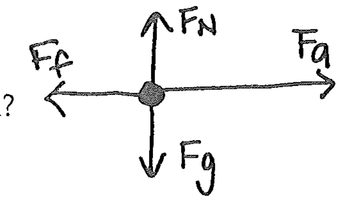
$W = F_x \cdot d$
$= \cos 40(35) \cdot 20$
$= \underline{536\,J}$

$W = (F \cdot \cos\theta)d$

<u>Most of the time the angle is 180° and cos180 = 1, so the formula becomes W = F• d</u>

Net Force vs. Applied Force – Which one do we use when calculating work?

A physics student is pushing a rope 15 m across a flat surface. The student pushes the rope with a force of 220 N, while the force of friction is 120 N. How much work is the student doing?



To find the amount of work done by the student should we use ΣF or F_A?

The student puts in F_A of 220 N so . . .

$$W = F_A \cdot d = (220)(15) = \underline{3300\ J}$$

So when would we use net force? to find the net work!!!

= includes all forces & the work they are doing on the object. ie. friction

Rule: When finding the **total work** done on an object, we always use the **applied force.**

Why can work be positive or negative when it is a scalar quantity?

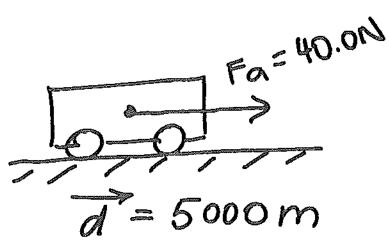
Work is the product of a scalar and a vector, but work is ***scalar***. However, work can be positive or negative...

The concept of work plays an important role in physics since it connects Newton's second law of motion to the important scalar quantities of kinetic energy and potential energy (through the work-energy theorem).

Example 1

A constant force of 40.0 N is needed to accelerate a car as it moves 5.0 km down the road. How much work is done? Does the energy of the car increase or decrease?

KE ↑



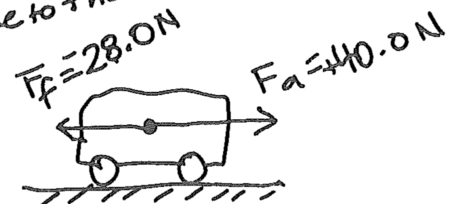
$$W = (F \cdot \cos 180^\circ) \cdot d$$
$$= (40.0)(5000) = +2.0 \times 10^5\ J$$

The work is POSITIVE

Example 2

In reality, we know there will also be a force of friction acting between the surface of the road and the tires. How much work does the force do? Is this work causing an increase or decrease in the energy of the car?

due to thermal E



$\vec{d} = 5000\ m$

$F_f = -28.0\ N$

$W = F_f \cdot d$

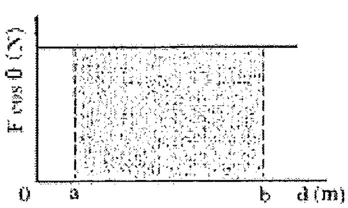
$= (-28.0)(5000) = -1.40 \times 10^5\ J$

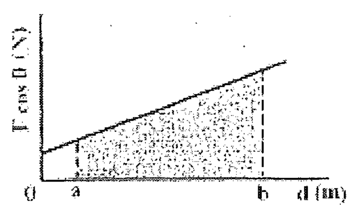
The work is <u>NEGATIVE</u>

<u>**Using a Force vs. Displacement Graph –**</u>

The graph below shows the component F of the net force that acts on a 5.0kg object as it moves along a flat horizontal surface. This information is graphed against the displacement of the object.

The work done by a force between two points equals the area under the curve of force vs. distance between two points.

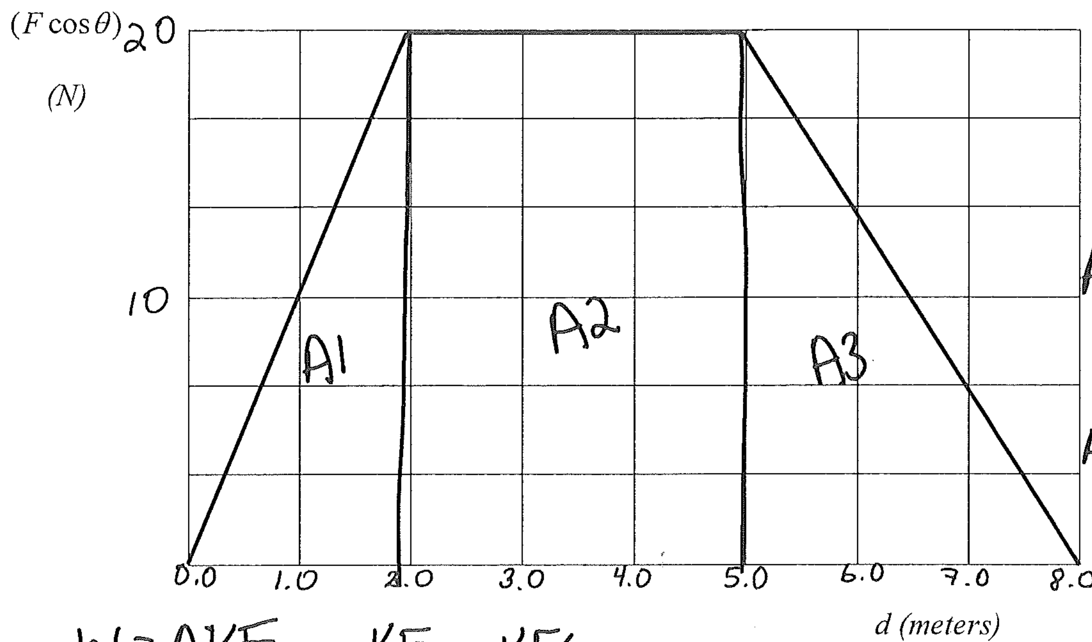




<u>**Work Done by a Variable Force**</u>

The work done by a variable force (a force that changes throughout the motion) is equal to the area under the F vs.d curve.

The graph shows a variable force acting on a 15.0 kg mass on a level surface which is initially at rest. Find the total work done on the block. Find the final speed of the mass assuming friction is negligible.   total $W = \boxed{110\,J}$



$A_1 = \frac{b \cdot h}{2}$

$= \frac{2.0 \cdot 20}{2}$

$= 20\,J$

$A_2 = 3.0 \cdot 20$

$= 60J$

$A_3 = \frac{3.0 \cdot 20}{2}$

$= 30J$

$W = \Delta KE = KE_f - \cancel{KE_o}$ (0)

$W = \frac{1}{2}mv_f^2$

$110 = \frac{1}{2}(15)(v_f^2)$ $v_f = \boxed{3.8\,m/s}$

WORK-ENERGY THEOREM –

$$\boxed{W_T = \Delta KE + \Delta PE = \Delta E_T}$$

$(E_{k_2} - E_{k_1}) + (E_{p_2} - E_{p_1})$

Work-Energy Relationships so far:

$\boxed{W_T = \Delta E_T = F_A \cdot d}$

$\boxed{W = \Delta KE = \Sigma F \cdot d}$

$\Sigma F \cdot d = \frac{1}{2}m\Delta v^2$

$\boxed{W = \Delta PE = \Sigma F \cdot d}$

$\Sigma F \cdot d = mg\Delta h$

heat/thermal

$\boxed{\Delta E_h = F_f \cdot d}$

With Kinetic Energy: $W = \Delta KE$ or $\Sigma F \cdot d = \frac{1}{2} m (v_F^2 - v_0^2)$

When we **accelerate** an object we **do work by changing the object's kinetic energy**.

We can use the work-energy theorem to find the speed of an object and this gives us another framework to solve problems involving motion.

Example 1

If an 85.0 kg has a Net Work of 600 J done on it in order to accelerate it across a level horizontal floor starting from rest, what is the final velocity.

work energy theorem tells us that $W = KE_f - KE_0$

rest

$$600 = \frac{1}{2}(85.0)(\vec{v}_F^{\,2} - 0^2)$$

$$\vec{v}_F = 3.76\ m/s$$

Another common use of the work-energy theorem is finding information about the forces acting on the object.

Example 2

0.090 kg

A baseball pitcher can throw a 90.0g baseball with speed measured by a radar gun of 130 km/h. Assuming that the force exerted by the pitcher on the ball acts over a distance of 0.90m, what is the applied force exerted by the pitcher on the ball (with no friction)?

36.1 m/s

$$W = \Delta KE = \frac{1}{2}(0.090)(36.1^2 - 0^2) = 58.7 J$$

$W = \Sigma F \cdot d$

$$58.7 = \Sigma F \cdot 0.90$$

$$\Sigma F = 65 N + 0(F_f)$$

$$F_a = \underline{\underline{65 N}}$$

With Gravitational Potential Energy -

The gravitational force is one of a class of "special" forces called conservative forces. What makes the gravitational force special is that the work done on an object by gravity ONLY depends on its initial and final position. The work does NOT depend the path taken between the initial and final position.

$$\boxed{W = \Delta PE} \qquad \boxed{\Sigma F \cdot d = mg\Delta h}$$

When an object is lifted, work is done by gravity on the object over a distance. The height is changed as an object is lifted. **A change in height produces a change in potential energy.**

Example 3

A 3.00 kg model rocket is launched vertically upward with sufficient initial speed to reach a height of 100m. However, air resistance (a non-conservative force), performed 8.00 x 10^2 of work on the rocket. If we were to ignore air resistance, how high would the rocket have gone?

N.C.

$W_{N.C.} = \Delta KE + \Delta PE$

$+800 = \Delta KE + (3)(9.8)(100)$

$\Delta KE = +3740 J$

(loss of velocity = $\downarrow$KE)

($\uparrow$h = $\uparrow$PE)

if the rocket were lauched with no air resistance then

$\Delta KE = -\Delta PE = -mg\Delta h$

$+3740 = -(3.00)(9.8)\Delta h$

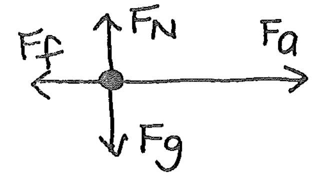
$\Delta h = \underline{127 m}$

Assignment – Work and Energy Problems

1. A 10.0 kg object is accelerated horizontally from rest to a velocity of 11.0 m/s in 5.00s by a horizontal force.

a) How much work is done on this object if the object is on a frictionless surface? (605J)

b) How much work is done in slowing the car if there is a coefficient of kinetic friction of 0.115 between the object and the horizontal surface? (-310J)



a) $\vec{a} = \frac{11-0}{5.5} = 2.2\, m/s^2$

$(10)(2.2) = F_a + 0 \quad F_a = \underline{22\,N}$

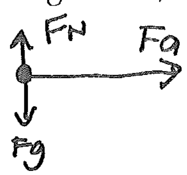
$\vec{d} = 0 + \frac{1}{2}(2.2)(5.0)^2 \quad \vec{d} = \underline{27.5\,m}$

$W = 22 \cdot 27.5$
$= \underline{605\,J}$

b) $F_f = (0.115)(10)(9.8) = -11.3$

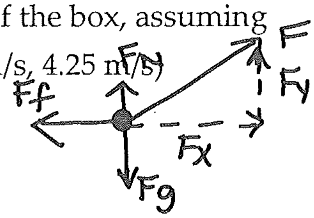
$W_{Ff} = F_f \cdot d$
$= (-11.3)(27.5) = \underline{-310\,J}$

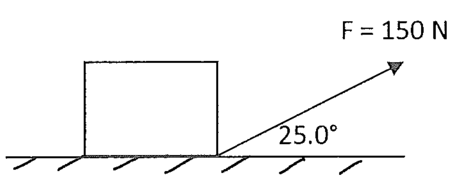
2. How much work is required to accelerate a 1.10 x 10³ kg car from rest to 5.00 km/h (1.39 m/s) along a level, frictionless surface? (1.06 x 10³ J)



$W = \Delta KE = \frac{1}{2}m(v_F^2 - v_0^2)$
$= \frac{1}{2}(1100)(1.39^2 - 0^2) = 1063\,J$
$(1.06 \times 10^3\,J)$

3. A 150 N force is pulling a 50.0 kg box along a horizontal surface. The force acts at an angle of 25.0º as shown in the diagram. If this force acts through a displacement of 12.0 m, and the coefficient of kinetic friction is 0.250, what is the speed of the box, assuming it started from rest? (a) What if its initial velocity was 2.00 m/s? (b) (3.75 m/s, 4.25 m/s)





(a) $W = \Delta KE$

$\Sigma F \cdot d = \frac{1}{2}m(v_F^2 - v_0^2)$

$(29.3)(12.0) = \frac{1}{2}(50)(v_F^2 - 0^2) \quad v_F = \underline{3.75\,m/s}$

$F_N = F_g - F_y$
$= (50 \cdot 9.8) - (\sin 25(150))$
$= 427\,N$

(b) $(29.3)(12.0) = \frac{1}{2}(50)(v_F^2 - 2^2)$

$v_F = \underline{4.25\,m/s}$

$F_f = (0.250)(427) = -107\,N$

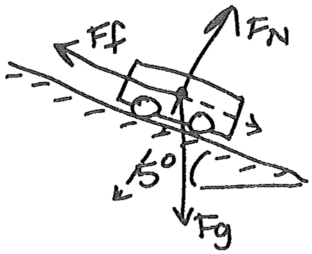
$\Sigma F = \cos 25(150) + (-107) = 29.3\,N$

4. A 2000 kg truck descending on a 5.0º hill is brought to a stop in 250 m. The driver applies the brakes so that the wheels lock.

a) If the coefficient of kinetic friction between the truck tires and the road is 0.60, how much work is done by friction in stopping the truck? (-2.93×10^6 J)

b) How much work is done on the truck by gravity? (4.27×10^5 J)

c) How fast was the truck travelling immediately before the brakes were applied? (50 m/s)



a) $F_f = \mu F_N = (0.60)(2000 \cdot 9.8 \cdot \cos 5)$
$= -11715$ N

$W_{F_f} = (-11715)(250) = -2.93 \times 10^6$ J

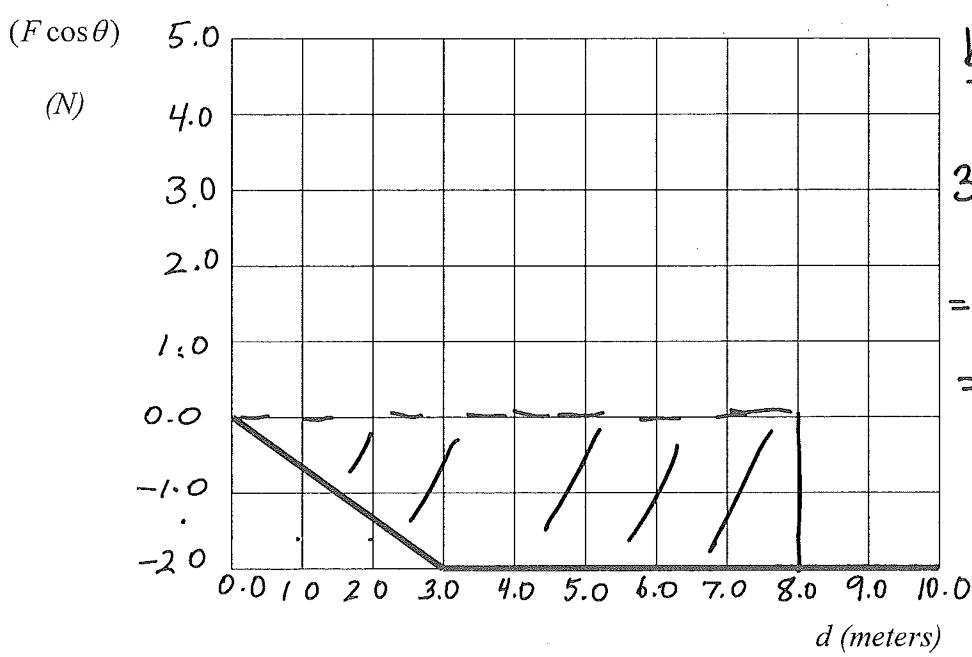
b) $F_{\parallel} = \sin 5(2000 \cdot 9.8) = 1708$ N

$W_{F_g} = (1708)(250) = 4.27 \times 10^5$ J

c) $-11715 + 1708 = 2000\vec{a}$
$\vec{a} = -5.00$ m/s^2

$0^2 = V_0^2 + 2(-5.00)(250)$ $\vec{V_0} = 50.0$ m/s

5. Given the following force-displacement graph of an object being pulled along a level surface, what is the work done in moving the object 8.0 m? Which force is responsible for the work on the object shown here? area → △ + ▭



$\frac{b \cdot h}{2} + l \cdot w$

$\frac{3.0 \cdot (-2.0)}{2} + (5.0)(-2.0)$

$= -3.0 + -10$

$= -13$ J

frictional forces are responsible.

6. A student wants to lift a 20 kg crate off a floor.

a) If the crate is lifted a height of 0.80 m at a constant speed, what is the work done by the student? (157 J)

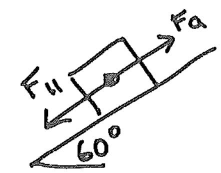
b) If the crate is lifted to a height of 0.80 m at a constant speed using a frictionless ramp with an angle of 60° above the horizontal, what is the required force and the work done by the student? (170 N, 157 J)

c) If the crate is lifted to a height of 0.80 m at a constant speed using a frictionless ramp with an angle of 30° above the horizontal, what is the required force and the work done by the student? (98 N, 157 J)

d) If the crate is lifted to a height of 0.80 m at a constant speed using a ramp with an angle of 30° above the horizontal, and the coefficient of kinetic friction between the crate and the ramp is 0.30, what is the required force and the work done by the student? (149N, 238 J)

a) $W = mgh = (20)(9.8)(0.80) = \boxed{157\,J}$

b)

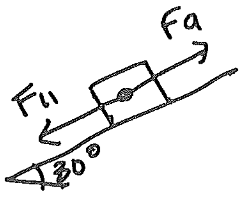


$F_{\parallel} = (\sin 60 \cdot 20 \cdot 9.8) = \underline{170\,N} = F_a \quad (\Sigma F = 0N)$

$W = (170)(0.924) = \boxed{157\,J}$

0.80m $\rightarrow \sin 60 = \frac{0.80}{d} \quad d = 0.924m$

c)

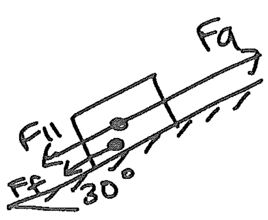


$F_{\parallel} = (\sin 30 \cdot 20 \cdot 9.8) = \underline{98\,N} = F_a \quad (\Sigma F = 0N)$

$W = (98)(1.6) = \boxed{157\,J}$

0.80m $\rightarrow \sin 30 = \frac{0.80}{d} \quad d = 1.6m$

d)



$F_{\parallel} = -98N$

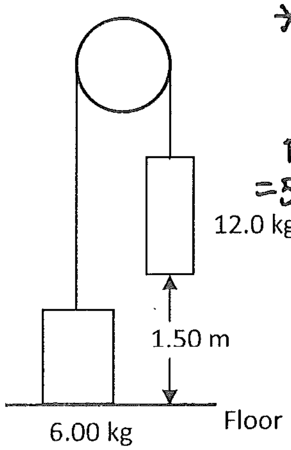
$F_f = (0.30)(20 \cdot 9.8 \cdot \cos 30) = -51\,N$

$0 = F_a + F_{\parallel} + F_f$

$= F_a + (-98) + (-51) \quad F_a = +\underline{149\,N}$

$W = (149)(1.6) = \boxed{238\,J}$

7. A system containing a frictionless pulley is illustrated below. If this system is released, at what speed does the 12.0 kg object hit the floor? (3.14 m/s)



*system : Find ΣF

$F_g = 58.8\,N$

$F_g = 118\,N$

$\Sigma F = 118 + (-58.8)$
$= \underline{59.2\,N}$

* system will accel at $\underline{\text{same rate}}$.

$W = \Delta KE$

$\Sigma F \cdot d = \frac{1}{2} m (v_F^2 - v_0^2)$

$(59.2)(1.50) = \frac{1}{2}(18.0)(v_F^2 - 0^2)$

$\underline{v_F = 3.14\,m/s}$

8. A 45.0 kg box is pulled across a horizontal surface by a constant horizontal force of 192 N. If the box starts from rest, and the coefficient of kinetic friction is 0.35, what is the final speed of the box when it has travelled 8.0 m? (3.68 m/s)

$F_f = -154\,N$ $\quad F_a = 192\,N$

$\Sigma F = 192 + (-154) = 38\,N$

$[F_f = (0.35)(45.0 \cdot 9.8) = 154\,N]$

$W = \Delta KE$

$\Sigma F \cdot d = \frac{1}{2} m (v_F^2 - v_0^2)$

$38 \cdot 8.0 = \frac{1}{2}(45)(v_F^2 - 0^2)$

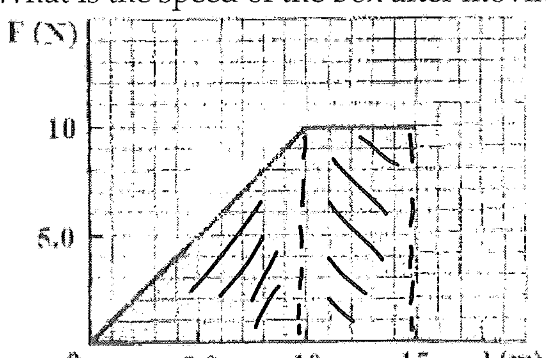
$v_F = \underline{3.68\,m/s}$

9. How much energy is needed to accelerate a 1.1×10^3 kg object along a horizontal frictionless surface from 15 km/h (4.17 m/s) to 25 km/h (6.94 m/s) in 5.0 s? (1.7×10^4 J)

$W = \Delta KE = \frac{1}{2} m \Delta v^2 = \frac{1}{2}(1100)(6.94^2 - 4.17^2)$

$= 16926\,J \rightarrow \underline{1.7 \times 10^4\,J}$

10. A 4.0 kg box moves on a floor by a force that varies with distance as shown in graph. What is the speed of the box after moving 15 m starting from rest? (100 J)



$W = \triangle + \square$

$= \frac{b \cdot h}{2} + l \cdot w$

$= \frac{10 \cdot 10}{2} + (5 \cdot 10)$

$= 50 + 50 \quad = \underline{100\,J}$

11. A 1200 kg car is travelling at 10 m/s.

a) If the car is accelerated from 10 m/s to 15 m/s, how much work is required? (7.5×10^4 J)

b) If the car is accelerated again from 15 m/s to 20 m/s, how much work is required? (1.1×10^5 J)

c) The car is traveling at 10 m/s comes to a stop in 12 m when its brakes are applied. If the speed of the car is doubled to 20 m/s and all else is the same, how far will the car move before coming to a stop? (48 m)

a) $W = \Delta KE = \frac{1}{2}(1200)(15^2 - 10^2) = 75000 J \rightarrow \underline{7.5 \times 10^4 J}$

b) $W = \Delta KE = \frac{1}{2}(1200)(20^2 - 15^2) = 105000 J \rightarrow \underline{1.1 \times 10^5 J}$

c) $\Sigma F \cdot d = \frac{1}{2} m \Delta v$

$\Sigma F \cdot 12 = \frac{1}{2}(1200)(0^2 - 10^2)$

$\Sigma F = -5000 N$

$-5000 \cdot d = \frac{1}{2}(1200)(0 - 20^2)$

$d = \underline{48 m}$

12. A 0.08 kg arrow is drawn back from a bow whose string exerts an average net force of 120 N on the arrow over a distance of 0.90 m.

a) How much energy is stored in the bow? (108 J)

b) What is the speed of the arrow when it leaves the bow? (52.0 m/s)

c) When the arrow hits a wooden target, an average force of 4500 N brings the arrow to rest. What distance does the arrow penetrate the wooden target? Ignore air resistance. (0.024m)

a) $W = \Delta PE = \Sigma F \cdot d = 120 \cdot 0.90 = \underline{108 J}$

b) $\Delta PE = \Delta KE$

$108 = \frac{1}{2}(0.08)(v_F^2 - 0^2)$ $v_F = \underline{52.0 m/s}$

c) $W = \Delta KE$

$\Sigma F \cdot d = \frac{1}{2} m (v_F^2 - v_0^2)$

$-4500 \cdot d = \frac{1}{2}(0.08)(0^2 - 52^2)$ $d = \underline{0.024 m}$

Physics 12 - **Conservation of Energy**

Energy is neither created nor destroyed. It may only be changed from one type to another. This means that if we are looking at a **closed system** (a situation that has no outside sources of energy), the **total change in energy** is always zero.

There are many forms of energy: mechanical (potential and kinetic), thermal, electrical, nuclear, chemical etc. **One form of energy can be converted into another form by** "doing work".

We will be mainly focusing on potential and kinetic (and a little bit of thermal) energy.

Potential Energy (PE):

stored energy

$$\boxed{PE = mgh}$$

Remember:
Potential Energy is always... relative to a reference point.

Kinetic Energy (KE):

energy of motion

$$\boxed{KE = \frac{1}{2}mv^2}$$

Total Initial Energy = Total Final Energy

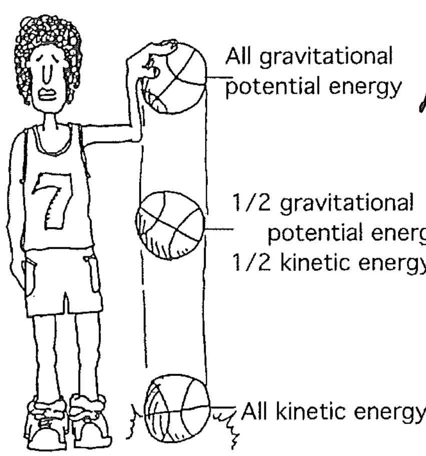
$$KE_0 + PE_0 = KE_f + PE_f$$

$$\boxed{\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_f^2 + mgh_f}$$

Total Change in Energy = 0

$$\Delta KE + \Delta PE = 0$$

$$\boxed{\Delta KE = -\Delta PE}$$



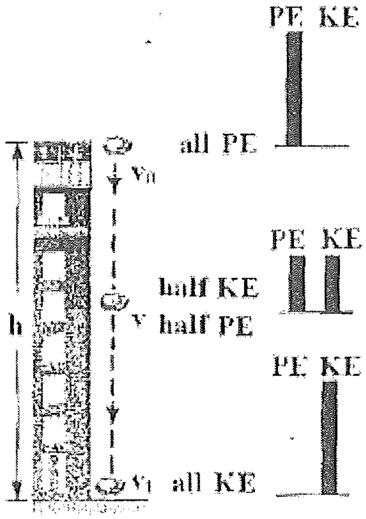
$PE = 100J$ $KE = 0J$ $E_T = 100J$

$PE = 50J$ $KE = 50J$ $E_T = 100J$

$PE = 0J$ $KE = 100J$ $E_T = 100J$

But at all times the **SUM** of the gravitational potential energy and the kinetic energy is constant. E_T

The principle of the conservation of energy states that the energy can be transformed into another form, but the total energy remains the same. This is true even when there is friction.



transformed from mech. E to thermal E by friction

$$E_o = E_f$$

$$KE_o + PE_o = KE_f + PE_f + (F_f d)$$

$$\frac{1}{2}mv_o^2 + mgh_o = \frac{1}{2}mv_f^2 + mgh_f + (F_f d)$$

Some terms to know:

<u>**Conservative Force**</u> – A force which does work on an object that is independent of the path taken by the object between its starting point and ending point.

Both gravitational and elastic forces are considered conservative forces. This means that work done against the force (energy) can be recovered = it is stored.

∴ remains mechanical E throughout motion

<u>**Non-Conservative Force**</u> – A force whose work on an object <u>IS DEPENDENT</u> on the path taken by the object from starting point to ending point.

An example of a **non-conservative force is friction**. When work is done against friction, the energy cannot be recovered as it is converted to another form = mainly thermal energy. ∴ no longer mechanical E

When work is done by a non-conservative force, it produces a change in the total mechanical energy of the object.

$$\boxed{W_{nc} = E_f - E_o}$$

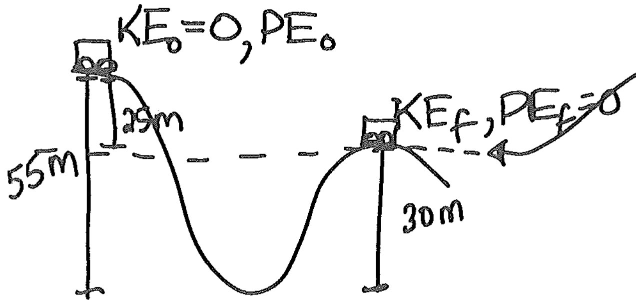
Last class we learned through the work-energy theorem that:

The change in *Potential Energy* of an object is equal to the work done on the object.

The change in *Kinetic Energy* of an object is equal to the work done on the object.

But what if the Potential Energy <u>*and*</u> the Kinetic Energy change?????

Example: The first peak of a roller coaster is 55 m above the ground. The 1200 kg car starts from rest and goes down the hill and up a second hill which is 30 m high. How fast is the car traveling at the top of the second hill? (assume no friction)



$$\cancel{KE_0} + PE_0 = KE_f + \cancel{PE_f}$$

if we base our measurements on this ref. line being $h = 0$ m then $KE_0 = 0$ AND $PE_F = 0$

so . . . $PE_0 = KE_f$

$$\cancel{m} g h_0 = \frac{1}{2} \cancel{m} v_F^2$$

$$(9.8)(25) = \frac{1}{2} v_F^2$$

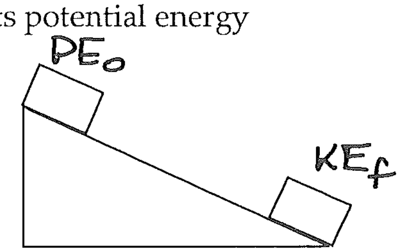
note

$$\boxed{v_F = 22.1 \text{ m/s}}$$

When we are dealing with **non-conservative forces** (such as *friction*) acting on an object, not all energy will be transferred between KE and PE.

The "work" done by friction on an object will change some of the mechanical energy into HEAT (thermal energy). This energy is quickly conducted or radiated in all directions and is effectively dispersed.

If we observe a block of wood sliding down a ramp with a small amount of friction, how would the blocks kinetic energy at the bottom compare with its potential energy at the top? $PE_0 > KE_f$ because some $PE \rightarrow E_H$

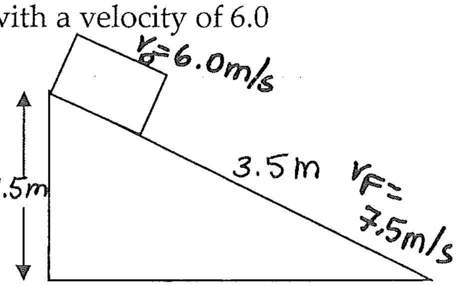


This does **not** change the fact that the total energy in the system is **CONSTANT.** Now…

$$\boxed{KE_0 + PE_0 = KE_f + PE_f + E_H}$$

$$\boxed{\Delta KE + \Delta PE + \Delta E_h = 0}$$

Example: A 5.0 kg block of wood is now pushed down a ramp with a velocity of 6.0 m/s. At the bottom of the ramp it is traveling at 7.5 m/s.



a. How much thermal energy is generated due to friction?
b. Determine the force of friction.

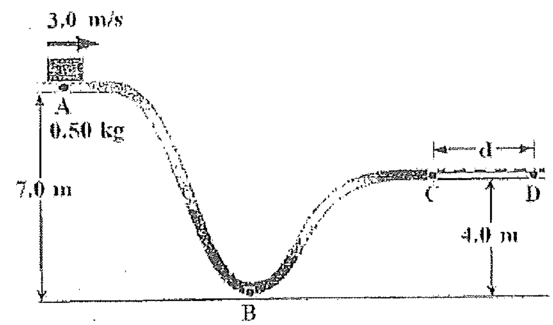
a) $KE_o + PE_o = KE_f + PE_f + E_H$

$\frac{1}{2}(5.0)(6.0)^2 + (5.0)(9.8)(1.5) = \frac{1}{2}(5.0)(7.5)^2 + 0 + E_H$

$E_H = \underline{22.9\,J}$

b) $\Delta E_H = F_f \cdot d \qquad 22.9 = F_f \cdot 3.5 \qquad F_f = \underline{6.5\,N}$

Example: A 0.50 kg block moving at 3.0 m/s slides from A to C along a frictionless surface, and then passes through the horizontal surface CD, where a frictional force acts on it. As a result, the block slows down and comes to a stop at point D. The coefficient of kinetic friction between the block and the surface in the region CD is 0.40. Ignore air resistance.



a) What is the total energy of the block at point A?
b) What is the speed of the block at point B?
c) What are the speed and the kinetic energy of the block when it reaches C?
d) How far will the block move before coming to a stop at point D?

a) $E_T = KE + PE$

$= \frac{1}{2}(0.50)(3.0)^2 + (0.50)(9.8)(7.0) = \underline{36.6\,J}$

b) $E_T = KE + PE \qquad V_B = \underline{12.1\,m/s}$

$36.6 = \frac{1}{2}(0.50)v^2 + 0$

c) $E_T = KE + PE$

$36.6 = \frac{1}{2}(0.50)v^2 + (0.50)(9.8)(4.0) \qquad V_c = \underline{8.25\,m/s}$

d) $W_{Ff} = \Delta KE = \frac{1}{2}m(v_F^2 - v_0) = \frac{1}{2}(0.50)(0^2 - 8.25^2)$

$W_{Ff} = -17.0\,J = -1.96 \cdot d \qquad F_f = (0.40)(0.50)(9.8)$

$d = \underline{8.67\,m} \qquad = -1.96\,N$

Conservation of Energy Assignment

1. A physics student lifts his lifts his pet rock 2.8m straight up. He then lets it drop to the ground. Use the Law of Conservation of Energy to calculate how fast the rock will be moving (a) half way down and (b) just before it hits the ground (ignore air resistance). (5.2 m/s, 7.4 m/s)

*no mass given → but it remains the same so ratios remain constant to determine speed.

PE_0, KE_F

$\frac{1}{2}PE, \frac{1}{2}KE$

PE_F, KE_F

$PE_0 = (9.8)(2.8)$ $KE_0 = 0\,J$

$= 27.4\,J$

$\frac{1}{2}27.4 = 13.7\,J = \frac{1}{2}v^2$ $v = \underline{5.2\,m/s}$

$PE_F = 0$ $KE_F = 27.4 = \frac{1}{2}v^2$ $v = \underline{7.4\,m/s}$

2. A 65 kg girl is running with a speed of 2.5 m/s. How much kinetic energy does she have? She grabs on to a rope which is hanging from the ceiling, and swings from the end of the rope. How high off the ground will she swing? (ignore air resistance) (203 J, 0.32m)

$KE = \frac{1}{2}(65)2.5^2 = \underline{203\,J}$

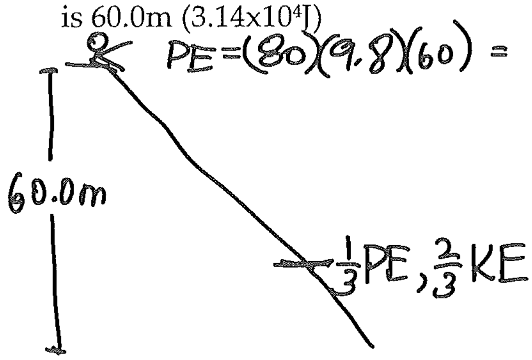
$\Delta KE + \Delta PE = 0$

$-203 + (65)(9.8)h = 0$ $h = \underline{0.32\,m}$

(lost KE by top)

3. How much kinetic energy will an 80.0 kg skier sliding down a frictionless slope have when he is two-thirds of the way down the slope? The vertical height of the slope is 60.0m (3.14×10^4J)

$PE = (80)(9.8)(60) = 47040\,J$



$\frac{2}{3}(47040) = \underline{3.14 \times 10^4\,J}$

4. A golfer wishes to hit his drives further by increasing the kinetic energy of the golf club when it strikes the ball. Which would have the greater effect on the energy transferred to the ball by the driver – doubling the mass of the club head or doubling the speed of club head? (double speed)

$KE = \frac{1}{2}mv^2$

double mass $= \frac{1}{2}(2)(1)^2 = 1\,J$

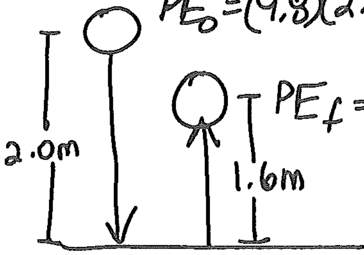
double speed $= \frac{1}{2}(1)(2)^2 = 2\,J$

*double speed has greater effect.

5. A rubber ball falls from a height of 2.0m, bounces off the floor and goes back up to a height of 1.6m. What percentage of its initial gravitational potential energy has been lost? Where does this energy go? Has the Law of Conservation of Energy been broken? (20% lost)

$PE_0 = (9.8)(2.0) = 19.6\,J$

$\frac{15.6}{19.6} \times 100\% = 79.6$



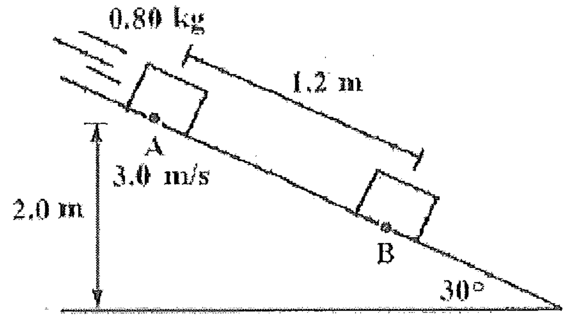
$PE_f = (9.8)(1.6) = 15.7\,J$

$100 - 79.6 = 20.4\%$ lost

→ thermal E

→ No, total E is constant

6. A 0.80 kg block slides along a frictionless surface of a 30.0° incline. When the block passes point A, the velocity of the block is 3.0 m/s. After the block moves 1.2 m from point A, it reaches point B as shown in the diagram.



a) Find the kinetic energy, potential energy, and total energy at point A.
b) Find the kinetic energy, potential energy, and total energy at point B.
c) Find the changes in kinetic energy, potential energy, and total energy between point A and B.

a) $KE = \frac{1}{2}(0.80)(3)^2 = \underline{3.6\,J}$ $\qquad E_T = 3.6 + 15.68 = \underline{19.3\,J}$

$PE = (0.80)(9.8)(2.0) = \underline{15.68\,J}$

b) $F_{\parallel} = \sin 30(0.80)(9.8) = 3.92\,N \quad \Sigma F = m\vec{a} \quad 3.92 = 0.80\,\vec{a} \quad \vec{a} = 4.9\,m/s^2$

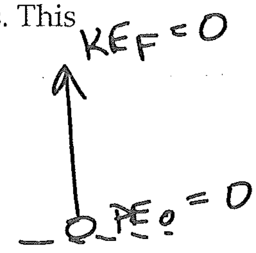
$v_F^2 = 3.0^2 + 2(4.9)(1.2) \quad v_F = 4.56\,m/s$

$KE = \frac{1}{2}(0.80)(4.56)^2 = \underline{8.3\,J} \qquad PE = 19.3 - 8.3 = \underline{11\,J} \quad E_T = \underline{19.3\,J}$

c) $\Delta KE = 8.3 - 3.6 = \underline{4.6\,J} \qquad \Delta PE = 11 - 15.68 = \underline{-4.6\,J}$

$\Delta E_T = 4.6 + (-4.6) = \underline{0\,J}$

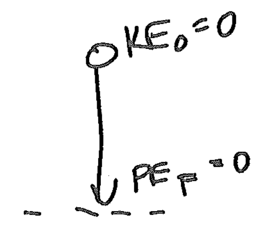
7. A 3.0 kg stone is projected directly upward with an initial speed of 12 m/s. This stone experiences an average air resistance force of 20 N.
a) What maximum height does the stone reach? (4.37 m)
b) What is the speed of the stone when it lands on the ground? (5.2 m/s)



a) $KE_0 + PE_0 = KE_f + PE_f + W_{Ff}$

$\frac{1}{2}(3.0)(12)^2 + 0 = 0 + (3.0)(9.8)h + 20 \cdot h$

$216 = 29.4h + 20h \qquad 216 = 49.4h \qquad h = \underline{4.37\ m}$



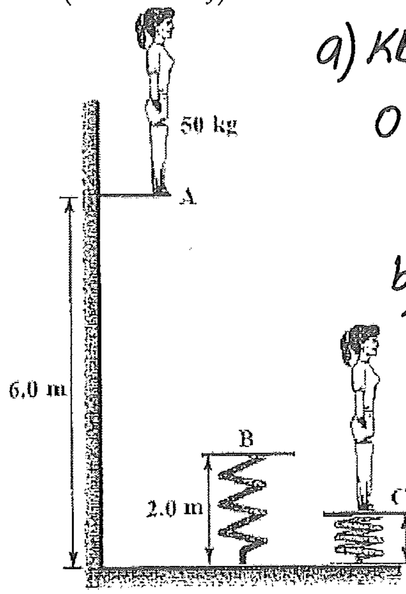
b) $KE_0 + PE_0 = KE_f + PE_f + W_{Ff}$

$0 + (3.0)(9.8)(4.37) = \frac{1}{2}(3.0)V_F^2 + 0 + 20(4.37)$

$128.5 = 1.5V_F^2 + 87.4$

$41.1 = 1.5V_F^2 \qquad V_F = \underline{5.2\ m/s}$

8. A 50 kg student steps off a 6.0 m high platform and drops onto a 2.0 m tall spring-loaded board. As the spring-loaded board is compressed, it brings her to a stop 1.0 m above the ground. Ignore air resistance.
a) What is the speed of the student when she hits the board? (8.85 m/s)
b) How much energy is momentarily stored in the spring when she comes to rest? (2.45 x 10^3 J)



a) $KE_0 + PE_0 = KE_f + PE_f$

$0 + (50)(9.8)(6.0) = \frac{1}{2}(50)v_F^2 + (50)(9.8)(2.0)$

$2940 = 25v_F^2 + 980 \qquad V_F = \underline{8.85\ m/s}$

b) $KE_0 + PE_0 = KE_f + PE_f + E_{stored}$

$0 + (50)(9.8)(6.0) = 0 + (50)(9.8)(1.0) + E_{stored}$

$E_{stored} = 2450\ J$

$= \underline{2.45 \times 10^3\ J}$

9. A 7.0 kg block is fired up a 35° incline with an initial speed of 9.0 m/s as shown by the diagram. If a frictional force of 55 N acts on the block as it moves up the incline, what maximum vertical height will the block reach? (1.7 m)

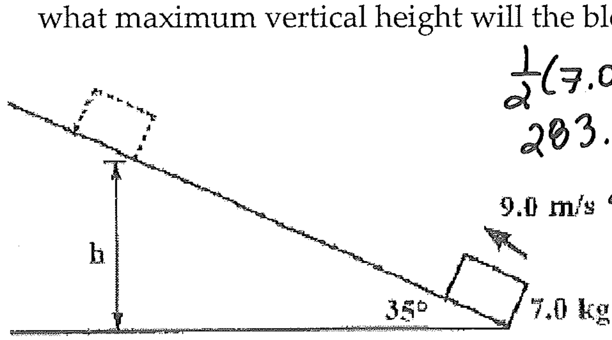
$F_f \cdot d$

$\frac{1}{2}(7.0)(9.0)^2 + 0 = 0 + (7.0)(9.8)h_f + (55)\left(\frac{h_f}{\sin 35}\right)$

$283.5 = 68.6h_f + 95.9h_f$

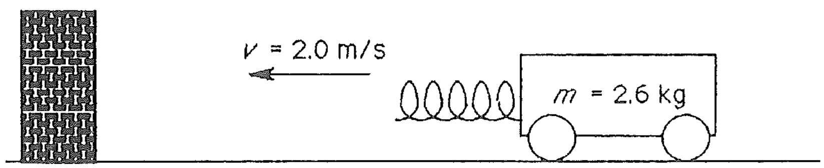
$283.5 = 164.8h_f$

$h_f = \underline{1.7\ m}$



10. A 2.6kg laboratory cart is given a push and moves with a speed of 2.0 m/s toward a solid barrier, where it is momentarily brought to a rest by its spring bumper. How much elastic potential energy will be stored in the spring at the moment when the spring is fully compressed? (5.2J)

$KE = \frac{1}{2}mv^2 = PE$



$= \frac{1}{2}(2.6)(2.0)^2$

$= \underline{5.2\ J}$

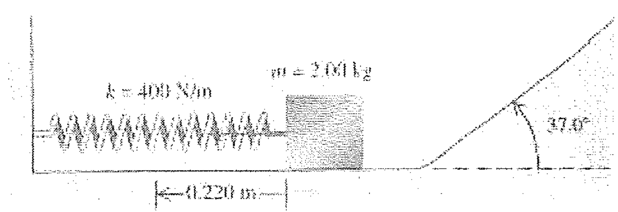
11. A 2.00-kg block is pushed against a spring with negligible mass and force constant $k = 400$ Nm, compressing it 0.220m. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope 37.0⁰ **Spring forces are conservative.** **(PE of an ideal spring that is completely compressed can be found using $PE_{compressed\ spring} = 1/2kx^2$)**

A) What is the speed of the block as it slides along the horizontal surface after having left the spring? (3.11 m/s)

B) How far does the block travel up the incline before starting to slide back down? (0.82m)

a) since spring force is conservative

$\Delta KE = -\Delta PE$

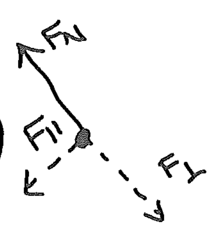


$PE = \frac{1}{2}kx^2 = \frac{1}{2}(400)(0.220)^2$

$= 9.68J$

$KE = \frac{1}{2}mv^2 \quad 9.68 = \frac{1}{2}(2.00)v^2$

$v = \underline{3.11 m/s}$

b) 

$F_{//} = \sin 37(2.00 \cdot 9.8)$

$= 11.8\ N$

$(2.00)(\vec{a}) = 11.8 \quad \vec{a} = -5.90 m/s^2$

$0^2 = 3.11^2 + 2(-5.90)\vec{d} \quad \vec{d} = \underline{0.82 m}$

Physics 12 – **Power and Efficiency**

POWER is the *rate* of doing work or the *rate* of using energy. In other words, power is concerned with the amount of time it takes to do a certain amount of work.

Mathematically we define power as:

$$\boxed{P = \frac{W}{t} = \frac{\Delta E}{t}}$$

The unit of power is J/s or Watts (W)

<u>Example</u>: A wrestler is setting up a body slam and lifts his 80 kg opponent clear over his head to a height of 2.2 m in 0.675 s. How much power did the wrestler generate?

$$P = \frac{\Delta PE}{t} = \frac{mgh}{t} = \frac{(80)(9.8)(2.2)}{0.675} = \boxed{2529\ W}$$
$$(2.5 \times 10^3\ W)$$

<u>Example</u>: While cruising along level ground in a sleigh at 4.0 m/s, the driver cracks the whip and speeds up to 12.0 m/s in 4.5 s. If the sleigh and driver have a combined mass of 850 kg, how much power did it generate? Ignore friction.

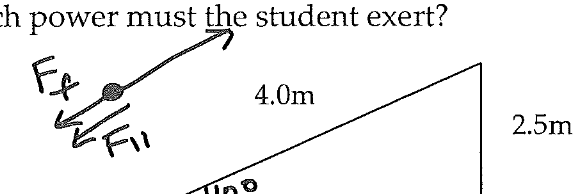
$$P = \frac{\Delta KE}{t} = \frac{\frac{1}{2}m(v_F^2 - v_0^2)}{t} = \frac{\frac{1}{2}(850)(12.0^2 - 4.0^2)}{4.5}$$
$$= \boxed{12{,}089\ W}\ (1.2 \times 10^4\ W)$$

Another useful equation for power can be derived:

$$P = \frac{W}{t} \qquad W = Fd$$
$$= \frac{Fd}{t} \qquad v = \frac{d}{t}$$
$$\boxed{P = Fv} \quad \text{* constant velocity}$$

<u>Example:</u> A student pushes 14 kg of their physics homework up a 40° ramp at a constant velocity of 3.2 m/s. The friction force is 26 N. How much power must the student exert?

$P = Fv \qquad F_{\parallel} + F_f + F_a = 0$



$-\sin 40(14 \cdot 9.8) + (-26) + F_a = 0$

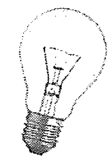
$F_a = 114\ N \qquad P = (114)(3.2) = \boxed{365\ W}$

Whenever we use a machine to do work, some of the energy we put into the machine is always lost, mainly due to **friction.**

For example: An electric heater is approximately 95% efficient.

A car is approximately 30% efficient.

A light bulb is approximately 3% efficient.



We can define **efficiency** in one of two ways:

$$\frac{W_{out}}{W_{in}} \times 100\% \quad or \quad \frac{P_{out}}{P_{in}} \times 100\%$$

The most common confusion when calculating efficiency is in understanding which values apply to work/power IN and which applies to work/power OUT.

Work/Power In: total energy/power used by the process.

Work/Power Out: what was actually accomplished by the process.

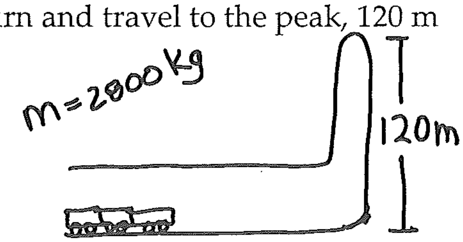
Remember that energy is always LOST somewhere in using the machine, so…

Work IN > Work OUT

Power IN > Power OUT

and Efficiency ≤ 100%

Example: The Top Thrill Dragster is one of the tallest roller coasters in the world. The car is accelerated along a level track until they take a 90° vertical turn and travel to the peak, 120 m high. A typical fully loaded car has a mass of 2800 kg.



a) Calculate the minimum amount of work done on the car in order for it to reach the peak?

$$W = \Delta PE = mg\Delta h = (2800)(9.8)(120)$$
$$= 3{,}292{,}800 \text{ J}$$
$$= 3.29 \times 10^6 \text{ J}$$

$\rightarrow 53.6\ m/s$

b) In reality, the roller coaster is accelerated from 0 to 193 km/h in 3.8 s. Find the actual power input of the ride.

$$P = \frac{W}{t} = \frac{4.02\times10^6}{3.8} = \boxed{1.06\times10^6\ W}$$

$$W = \Delta KE = \frac{1}{2}m(v_f^2 - v_o^2) = \frac{1}{2}(2800)(53.6^2 - 0^2) = 4.02\times10^6 J$$

c) Determine the efficiency of the ride from start to peak.

$$Eff = \frac{W_{out}}{W_{in}} \times 100\% \quad \frac{3.29\times10^6}{4.02\times10^6}\times 100\% = \boxed{82\%}$$

Example: On the Incredible Hulk roller coaster the car is initially launched up a hill 34 m high, travelling from 0 to 64 km/h in 2.0 s. A full car has a mass of 4500 kg.

$\searrow 17.8\ m/s$

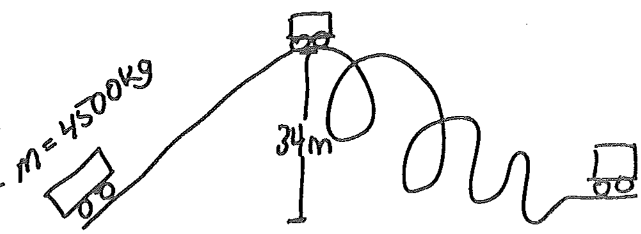
a) Find the power output of the ride.

$$P_{out} = \frac{W_{out}}{t} = \frac{1.499\times10^6 + 7.13\times10^5}{2.0} = \boxed{1.11\times10^6\ W}$$

$$W_{out} = \Delta PE + \Delta KE = 2.21\times10^6 J$$

$$\Delta PE = (4500)(9.8)(34) = 1.499\times10^6 J$$

$$\Delta KE = \frac{1}{2}(4500)(17.8^2 - 0^2) = 7.13\times10^5 J$$



b) The power consumption during the initial launch is actually 1.45 MW. Determine the efficiency of the ride during the initial launch.

$\nwarrow 1\,000\,000$

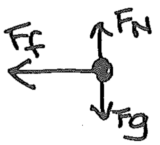
$$Eff = \frac{1.11\times10^6}{1.45\times10^6} \times 100\% = \boxed{77\%}$$

c) If the car pulls into the station at 8.0 m/s. How much heat has been generated since the first peak? $E_{T_0} = KE_f + E_H$

$$2.21\times10^6 = \frac{1}{2}(4500)(8.0)^2 + E_H$$

$$E_H = \boxed{2.07\times10^6 J}$$

d) The car is finally brought to rest over a distance of 2.0 m. How much force is required?



$$W = \Delta KE = \Sigma F \cdot d$$

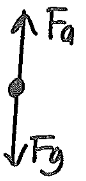
$$\left(\frac{1}{2}m(0)^2 - \frac{1}{2}(4500)(8.0)^2\right) = \Sigma F \cdot 2.0$$

$$\boxed{\Sigma F = -72{,}000\ N}$$

opposite to motion

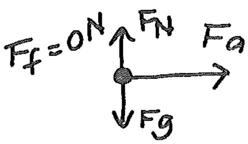
Power and Efficiency Problems:

1. A 20.0 kg object is lifted vertically at a constant velocity 2.50 m in 2.00 s by a student. Calculate the power output of the student. (245 W)



$$P = \frac{W}{t} = \frac{(20.0)(9.8)(2.50)}{2.00} = \boxed{245\ W}$$

2. A 2.00 kg object is accelerated uniformly from rest to 3.00 m/s while moving 1.5 m across a level frictionless surface. Calculate the power output. (9.0 W)



$$P = Fv = (6.0)(1.5) = \boxed{9.0\ W}$$

$$3.0^2 = 0^2 + 2\vec{a}\,1.5$$

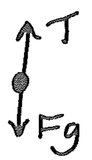
$$\vec{a} = 3.0\ m/s^2$$

$$(2.0)(3.0) = F_a \text{ to}$$

$$F_a = 6.0\ N$$

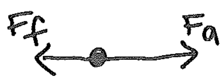
$$V_{av} = \frac{0 + 3.0}{2} = 1.5\ m/s$$

3. An 8.5 x 10^2 kg elevator is pulled up at a constant velocity of 1.00 m/s by a 10.0 kW electric motor. Calculate the efficiency of the motor. (83%)



$$\frac{P_{out}}{P_{in}} \times 100\% \quad \frac{(850)(9.8)(1.00)}{10\,000} \times 100 = \boxed{83\%}$$

4. A 5.0 kg object is accelerated uniformly from rest to 6.0 m/s while moving 2.0 m across a level surface. If the force of friction is 4.0 N, what is the power output? (135 W)



$$P_{out} = \Sigma F \cdot v = (45)(3.0) = \boxed{135\ W}$$

$$6.0^2 = 0^2 + 2\vec{a}\,2.0$$

$$\vec{a} = 9.0\ m/s^2$$

$$(5.0)(9.0) = \Sigma F = 45\ N$$

$$V_{av} = \frac{0 + 6}{2} = 3.0\ m/s$$

5. If a 100 kW motor has an efficiency of 82%, how long will it take to lift a 50.0 kg object to a height of 8.00m? (0.048 s)

$$0.82 = \frac{P_{out}}{100\,000}$$

$$P_{out} = 82\,000\ W = \frac{(50)(9.8)(8.00)}{t}$$

$$\boxed{t = 0.048\ s}$$

6. A 2.10×10^4 N airplane requires a power of 7.46×10^4 W at the propeller to climb at an angle of 20.0° to the horizontal at a constant speed. How much altitude could it gain in 10.0 minutes if air resistance is ignored? (2133 m)

$$P = \frac{W}{t} \qquad 7.46\times10^4 = \frac{W}{(10)(60)} \qquad W = 4.48\times10^7\ \text{J}$$

$$W = \Delta PE \qquad 4.48\times10^7 = (2143)(9.8)h \qquad \boxed{h = 2133\,m}$$

7. A 1500 kg car accelerates from rest to 75 km/h (20.8 m/s) in 45 s. How much power is supplied by the engine to accelerate the car? (7.2×10^3 W)

$$P = \frac{W}{t} = \frac{\Delta KE}{t} = \frac{\frac{1}{2}(1500)(20.8)^2}{45} = \boxed{7211\ W}$$

8. A locomotive engine can supply 1.49×10^6 W to accelerate a train car from rest to 20.0 m/s in 9.0 min. Find the mass of the train. (Ignore friction) (4.02×10^6 kg)

$$P = \frac{\Delta KE}{t} \qquad 1.49\times10^6 = \frac{\frac{1}{2}m(20.0^2)}{(9.0)(60)} \qquad \boxed{m = 4.02\times10^6\,Kg}$$

9. A skateboarder increases his kinetic energy from 800 J to 1600 J in 20 s. He expends 1500 J of energy during this activity.

a) What is his power output? (40 W)

$$a)\ P_{out} = \frac{\Delta KE}{t} = \frac{1600-800}{20} = \boxed{40\,W}$$

b) What is the efficiency of this process? (53%)

$$b)\ P_{in} = \frac{1500}{20} = \boxed{75\ W} \qquad \frac{40}{75}\times100 = 53\%$$

10. A 1000 kg automobile starts from rest and accelerates along a road to 30 m/s in 15s. Assume that the air resistance and frictional force remain constant at 500 N during this time?

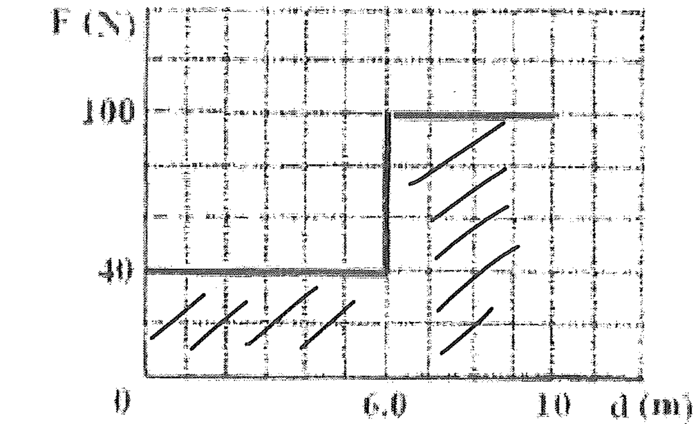
What is the power developed by the engine?

$$P = F_a v = (2500)(15) = \boxed{3.75\times10^4 W} \qquad \vec{a} = \frac{30-0}{15} = 2.0\ m/s^2$$

$$(1000)(2.0) = F_a + (-500)$$

$$F_a = 2500\ N$$

11. A student pushes a lawn mower 10 m from rest. The graph shows the applied force versus distance.



a) $W = \text{area} = (6\cdot 40)(4\cdot 100)$

$\boxed{= 640 J}$

b) $KE = \frac{1}{2}(30)(5.0)^2 = \boxed{375 J}$

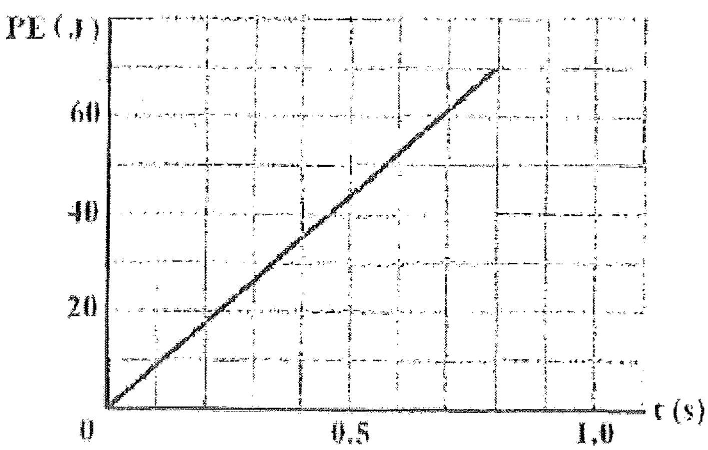
c) $\frac{375}{640} \times 100 = \boxed{59\%}$

a) How much work does she do moving the lawn mower 10 m? (640 J)

b) After she pushes the 30 kg lawn mower 10 m, it moves at 5.0 m/s. What is the kinetic energy of the lawn mower? (375 J)

c) What is the efficiency of this process? (59%)

12. The graph shows the potential energy of a model rocket versus time.



a) $W = \frac{\Delta PE}{t}$

$= \frac{70}{0.80} = \boxed{87.5 W}$

b) $0.60 = \frac{87.5}{P_{in}}$

$P_{in} = \boxed{146 W}$

a) Find the power output of the model rocket. (87.5 W)

b) If the model rocket is 60% efficient, find the power delivered to the rocket by the engine? (146 W)

13. A force acting upon an object to cause a displacement is known as d _____.
a. energy b. potential c. kinetic d. work

14. Two acceptable units for work are ________. Choose two.
a. Joule b. newton c. watt d. N•m

15. Power is defined as the _______ is done.
a. amount of work which b. direction at which work c. angle at which work d. the rate at which work

16. Two machines (e.g., elevators) might do identical jobs (e.g., lift 10 passengers three floors) and yet the machines might have different power outputs. Explain how this can be so.
depends on the time they complete the work in "rate"

17. There are a variety of units for power. Which of the following would be *fitting* units of power (though perhaps not standard)? Include all that apply.
a. Watt b. Joule c. Joule / second d. horsepower

18. Two physics students, Will and Ben, are in the weightlifting room. Will lifts the 100-pound barbell over his head 10 times in one minute; Ben lifts the 100-pound barbell over his head 10 times in 10 seconds.
Which student does the most work? both do the same

Which student delivers the most power? Ben Explain your answers. smaller amount of time

19. Jack and Jill ran up the hill. Jack is twice as massive as Jill; yet Jill ascended the same distance in half the time. Who did the most work? Jack (higher mass)

Who delivered the most power? same Explain your answers.
He has 2x mass, she has 2x velocity
=cancel out

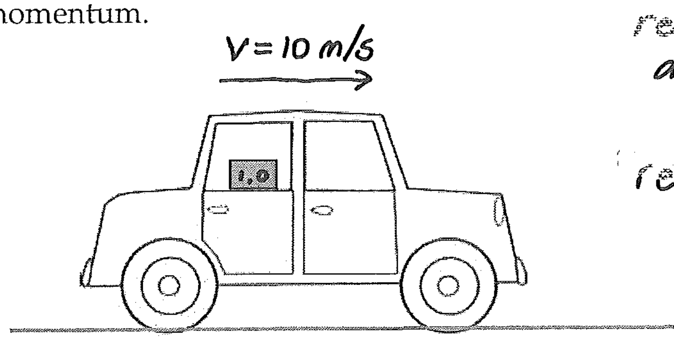
Physics 12 - **Impulse – Momentum Theorem**

What is **Momentum** and how is it different from **Kinetic Energy?** In physics 11, you learned that momentum is equal to the *mass* of an object multiplied by the *velocity* of an object. The larger the mass a moving object has, the larger the momentum, and the faster an object moves, the larger the momentum. ***But how is this different from kinetic energy???***

Momentum = vector! Kinetic Energy = scalar!

MOMENTUM HAS DIRECTION WHILE ENERGY IS ONLY A QUANTITY!!!

Both energy and momentum are ***relative*** quantities. If you are driving in your car down the highway holding a brick in your hand, relative to the ground the brick has both energy and momentum, yet relative to the car the brick has zero energy and zero momentum.



relative to ground = brick moves at 10m/s v

$\vec{p} = mv$
$= (1.0)(10)$
$\vec{p} = +10$ Kg·m/s

relative to car = brick does not move $v=0$

$\vec{p} = mv$
$= (1.0)(0) = 0$ Kg·m/s

Momentum can also be thought of in terms of force. A force applied for a certain *time* on an object will change the velocity of the object (therefore changing its momentum).

$\Sigma F = ma$ net force applied will result in accel. $= \Delta v$

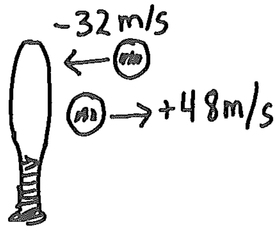
We have a special name for this *amount* of force given to the object. We call this *amount* of force **IMPULSE.**

Momentum is a quantity of motion that depends on both mass and velocity of the object in question. *vector quantity*

$$\boxed{\vec{p} = m\vec{v}}$$

The units of momentum are → kg•m/s or N•s

Example: A baseball pitcher hurls a ball (mass = 0.100 kg) at 32 m/s. The batter crushes it and the ball leaves the bat at 48 m/s. What was the ball's change in momentum?



$\Delta\vec{p} = m\Delta\vec{v}$
$= (0.100)(48 - (-32))$
$= +8.0$ Kg·m/s

Impulse:

Since momentum is the product of <u>mass</u> and <u>velocity</u>. Since we will not be dealing with changing masses, we can define an object's change in momentum as:

$$\boxed{\Delta\vec{p} = m\Delta\vec{v} = \sum F \cdot t}$$

Whenever a net force acts on an object, an acceleration results and so its momentum must change.

How do **forces** relate to **changes in momentum?**

A student jumps off a desk. When they land they bend their knees on impact. Why does this help prevent damage to their knees?

$$\underbrace{m\Delta\vec{v}}_{\text{constant}} = \sum F\downarrow \cdot\, t\uparrow$$

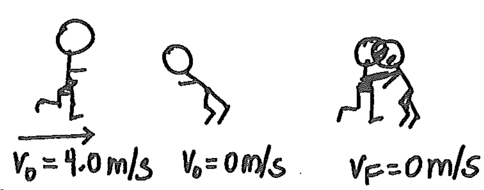
Coaches for many sports such as baseball, tennis and golf can often be heard telling athletes to "follow through" with their swing. Why is this so important?

$$\underbrace{m}_{\text{constant}}\Delta\vec{v}\uparrow = \underbrace{\sum F}_{\text{constant}} \cdot\, t\uparrow$$

Conventional wisdom suggests that cars should be made tough and rigid to prevent injury during a collision. However, newer vehicles are all built with large crumple zones. Why?

$$\underbrace{m\Delta\vec{v}}_{\text{constant}} = \sum F\downarrow \cdot\, t\uparrow$$

<u>Example</u>: A 115 kg fullback running at 4.0 m/s east is stopped in 0.75 s by a head-on tackle. Calculate:



a) The impulse felt by the fullback
b) The impulse felt by the tackler
c) The average net force exerted on the tackler

a) $\Delta\vec{p} = m\Delta\vec{v} = (115)(0 - 4.0) = \boxed{-460 \text{ Kg}\cdot\text{m/s}}$

b) $\Delta\vec{p} = \sum F \cdot t = \boxed{+460 \text{ Kg}\cdot\text{m/s}}$

same

equal & opposite (3rd Law)

c) $460 = \sum F \cdot 0.75$

$\sum F = \boxed{613 \text{ N}}$

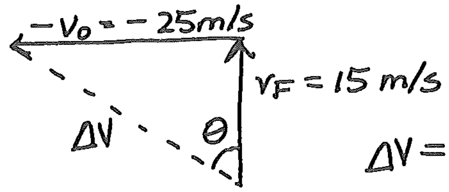
Example: A 1250 kg car traveling east at 25 m/s turns due north and continues on at 15 m/s. What was the impulse of the car exerted while turning the corner?

$\vec{\Delta p} = m\vec{\Delta v}$ $\qquad \vec{\Delta v} = \vec{v_F} - \vec{v_0}$ $\qquad v_0 = 25\ m/s \longrightarrow$ $\qquad \uparrow v_F = 15\ m/s$

$\vec{\Delta p} = (1250)(29.2)$

$= 36000\ kg \cdot m/s$

@ $59°$ W of N



$\Delta v = \sqrt{25^2 + 15^2} = 29.2\ m/s$

$\tan^{-1}\left(\frac{25}{15}\right) = 59°$ W of N

The Law of Conservation of Momentum

Momentum is a useful quantity because in a closed system it is always conserved. This means that in any collision, the total momentum before the collision must equal the total momentum after the collision.

① $\vec{P_0} = \vec{P_f}$

$m_1 v_{10} + m_2 v_{02} = m_1 v_{1F} + m_2 v_{2F}$

② $\vec{\Delta P}_{total} = 0$

Collisions can be grouped into two categories:

Elastics Collisions:	**Inelastic Collisions:**
KE is conserved $KE_0 = KE_f$	KE is not conserved

- $\vec{p}$ is always conserved
- total energy is always conserved

In reality, collisions are generally somewhere in between perfectly elastic and perfectly inelastic. It is actually impossible for a *macroscopic* collision to ever be perfectly elastic. Perfectly elastic collisions can only occur at the *atomic* or *subatomic level.*

Why?
- a change in shape → HEAT
- sound
- other vibrations

$\boxed{\text{* work is done}}$

INELASTIC:

When two or more objects collide and *stick* together.

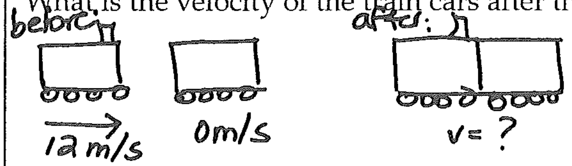
Is momentum conserved? YES

Is energy conserved? YES

Is kinetic energy conserved? NO

$KE_0 > KE_f$

A 9500 kg caboose is at rest on some tracks. An 11000 kg engine moving east at 12.0 m/s collides with it and they stick together. What is the velocity of the train cars after the collision?



$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$

$(11000)(12) + (9500)(0) = (11000 + 9500)v'$

$v' = \boxed{6.44\ m/s}$

ELASTIC:

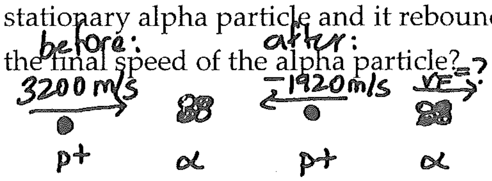
When two or more objects collide and *bounce* off each other (do not stick).

Is momentum conserved? yes

Is energy conserved? yes

Is kinetic energy conserved? yes $\rightarrow KE_0 = KE_f$

An alpha particle has a mass approximately 4 times larger than a proton. A proton travelling to the right at 3200 m/s strikes a stationary alpha particle and it rebounds at 1920 m/s. What is the final speed of the alpha particle?



$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

$m(3200) + \cancel{4m(0)} = m(-1920) + 4mv_2'$

$5120\cancel{m} = 4\cancel{m}v_2' \qquad v_2' = 1280\ m/s$

EXPLOSION:

Initial momentum is zero and therefore the sum of momentums after the explosion must also equal zero.

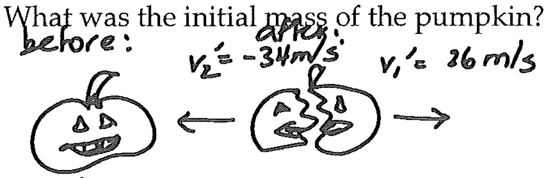
Is momentum conserved? yes

Is energy conserved? yes

Is kinetic energy conserved? NO

$KE_0 < KE_f$

A firecracker is placed in a pumpkin which explodes in into exactly two pieces. The first piece has a mass of 2.2 kg and flies due east at 26 m/s. The second chunk heads due west at 34 m/s. What was the initial mass of the pumpkin?



$\vec{P_0} = 0$

$0 = (2.2)(26) + m_2(-34) \qquad m_2 = 1.68\ kg$

$m_T = 1.68 + 2.2 = \boxed{3.9\ kg}$

Conservation of Momentum Problems – Inelastic Collisions and Explosions

1. A 54 kg women dives straight down into the water. Just before she strikes the water, her speed is 4.7 m/s. At a time of 2.1 s after she enters the water, her speed is 0.80 m/s.

a) What is the impulse given to the woman by the water? (+211 kg•m/s)

b) What is the net force exerted on her by the water? (+100 N)

c) What is her acceleration while she is entering the water? (+1.86 m/s²)

a) $\vec{\Delta p} = m\vec{\Delta v}$
$= (54)(-0.8-(-4.7))$
$= +211$ kg·m/s

b) $\vec{\Delta p} = \Sigma F \cdot t$
$211 = \Sigma F \cdot 2.1$
$\Sigma F = +100$ N

c) $\Sigma F = m\vec{a}$
$100 = 54\vec{a}$
$\vec{a} = +1.86$ m/s^2

2. A 0.06 kg tennis ball travels east at 15 m/s.

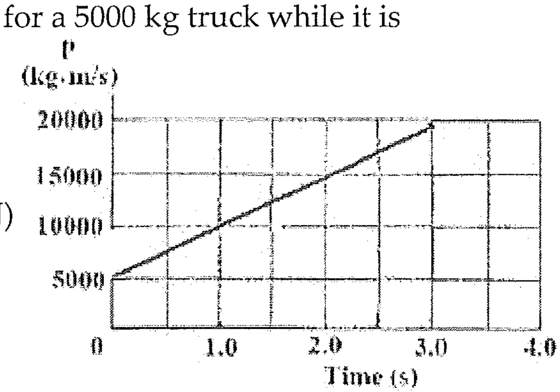
a) If a net force of 12 N is exerted on the ball for 0.030 s in the same direction, what is the final velocity of the ball? (+21 m/s)

b) If a net force of 18 N is now exerted on the ball (travelling at the final velocity from part a) in the opposite direction, how long should the impact last to stop the ball? (0.070s)

a) $m\vec{\Delta v} = \Sigma F \cdot t$
$(0.06)\Delta v = 12 \cdot 0.030$
$\vec{\Delta v} = 6.0$ m/s
$6.0 = \vec{v_F} - 15$ $\vec{v_F} = 21$ m/s [E]

b) $(0.06)(0-21) = -18 \cdot t$ $t = 0.070$ s

3. The graph below shows momentum versus time for a 5000 kg truck while it is accelerating uniformly.



a) What is the initial speed of the truck? (1.0 m/s)

b) What is the net force acting on the truck? (5000N)

c) What is the acceleration of the truck? (1.0 m/s²)

a) $\vec{p} = m\vec{v}$
$5000 = (5000)\vec{v}$
$\vec{v} = 1.0$ m/s

b) $\vec{\Delta p} = \Sigma F \cdot t$
$15000 = \Sigma F \cdot 3.0$
$\Sigma F = 5000$ N

c) $\Sigma F = m\vec{a}$
$5000 = 5000\vec{a}$
$\vec{a} = 1.0$ m/s^2

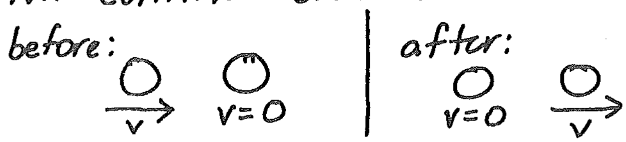
4. Two objects collide, one of which was initially moving and the other initially at rest.

a) Is it possible for *both* particles to be at rest after the collision? Give an example in which this happens, or explain why it cannot happen.

Not possible → one particle had momentum before the collision so there must be an equal amount after the collision.

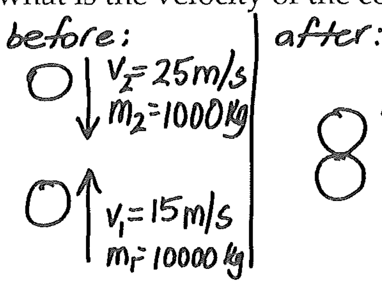
b) Is it possible for *one* particle to be at rest **after** the collision? Give an example in which this happens, or explain why it cannot happen.

Yes → if both have the same mass, one will stop and the other will continue on at the same velocity as the initial particle



* they do not "stick" together

5. A 1.0 x 10[5] N truck moving at a velocity of 15 m/s north collides head on with a 1.0x10[4] N car moving at a velocity of 25 m/s south. If they stick together upon impact, what is the velocity of the combined masses immediately after the collision? (+11.4 m/s)

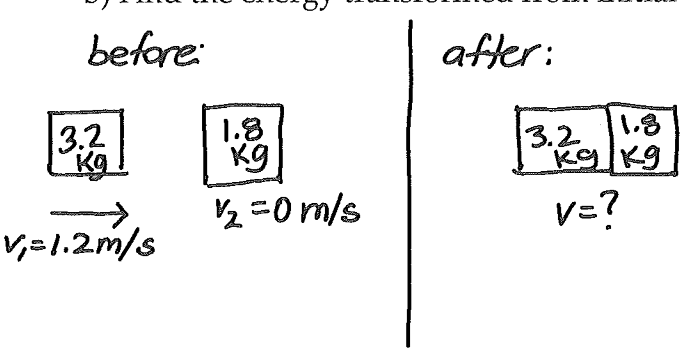


$m_1 V_1 + m_2 V_2 = (m_1 + m_2) v'$

$(10000)(15) + (1000)(-25) = (11000) v'$

$v' = +11.4 m/s$

6. A 3.2 kg cart moving at 1.2 m/s collides with a 1.8 kg wooden box at rest. After the collision, the cart and the wooden box stick together.
a) Find the final speed immediately after the collision. (0.77 m/s)
b) Find the energy transformed from initial kinetic energy to other forms. (0.82 J)



a) $(3.2)(1.2) + 0 = (5.0) v'$

$v' = \underline{0.77 m/s}$

transformed energy

b) $KE_o = KE_f + T.E.$

$\frac{1}{2}(3.2)(1.2)^2 = \frac{1}{2}(5.0)(0.77)^2 + T.E.$

$2.3 = 1.48 + T.E.$

$T.E. = \underline{0.82 J}$

7. A 5.0 kg block moving at 3.0 m/s to the right collides with a 10 kg block moving at 2.0 m/s to the left. After the collision, the two blocks stick together.
a) Find the speed of the combined blocks after the collision. (-0.33 m/s)
b) Find how much energy is transformed from the initial kinetic energy to other forms. (41.7 J)

before:

5.0 kg　　10 kg

$v_1 = 3.0$ m/s　　$v_2 = -2.0$ m/s

after:

5.0 kg 10 kg

$v = ?$

a) $(5.0)(3.0) + (10)(-2.0) = (15)v'$

$v' = \underline{-0.33 \text{ m/s}}$

b) $\frac{1}{2}(5.0)(3.0)^2 + \frac{1}{2}(10)(-2.0)^2 = \frac{1}{2}(15)(-0.33)^2 +$ T.E.

$42.5 = 0.82 + TE$

$TE = \underline{41.7 \text{ J}}$

8. A 4.0 kg stone moving at 6.4 m/s overtakes and becomes embedded in a 2.8 kg lump of clay moving in the same direction. After the collision, the combined object moves at 4.2 m/s.
a) Find the initial speed of the lump of clay. (1.06 m/s)
b) Find the kinetic energy lost in the collision. (23.5 J)

before:

4.0 kg　　2.8 kg

$v_1 = 6.4$ m/s　　$v_2 = ?$

after:

4.0 kg 2.8 kg

$v' = 4.2$ m/s

a) $(4.0)(6.4) + (2.8)v_2 = (6.8)(4.2)$

$v_2 = \underline{1.06 \text{ m/s}}$

b) $\frac{1}{2}(4.0)(6.4)^2 + \frac{1}{2}(2.8)(1.06)^2 - \left(\frac{1}{2}(6.8)(4.2)^2\right)$

$= \underline{23.5 \text{ J}}$

9. A 20 kg boy and his 40 kilogram sister are at rest on ice skates in the middle of a frozen lake. The boy pushes the girl and the boy moves to the left at 2 m/s. What is the velocity of the girl after the *explosion?* (+1.0 m/s)

before:

20 kg 40 kg

$v = 0$ m/s

after:

20 kg　　40 kg

$v_1' = -2.0$ m/s　　$v_2' = ?$

$0 = (20)(-2.0) + (40)v_2'$

$v_2' = +1.0$ m/s

10. A 12.0 kg object splits into two parts. If part A has a mass of 5.0 kg and a velocity of 7.0 m/s right, what is the velocity of part B? (-5.0 m/s)

before:

12.0 kg

$v = 0$ m/s

after:

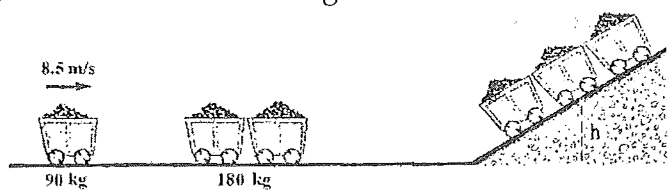
7.0 kg 5.0 kg

$v_B = ?$ $v_A = 7.0$ m/s

$0 = (7.0)v_B + (5.0)(7.0)$

$v_B = -5.0$ m/s

11. A 90 kg ore cart moving at 8.5 m/s collides with two carts of total mass 180 kg at rest on a frictionless horizontal track as shown in the figure. The three carts stick together and rise up the hill. If the hill has no friction, find the maximum height to which the combined ore cars rise up the hill. (0.41 m)



before:

90 kg 180 kg

$v_1 = 8.5$ m/s $V_2 = 0$

after:

90 kg 180 kg

$v' = ?$

$KE_0 = PE_f$

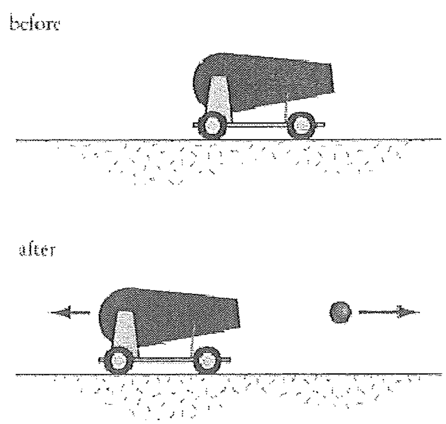
$\frac{1}{2}(270)(2.83)^2 = (270)(9.8)h$

$h = 0.41$ m

$90(8.5) + 0 = 270v'$

$v' = 2.83$ m/s

12. A 5.50 kg cannonball is fired from a 100 kg cannon. If the velocity of the cannonball is 300 m/s, what is the recoil velocity of the cannon? (-16.5 m/s)



before:

105.5 kg

$V = 0$ m/s

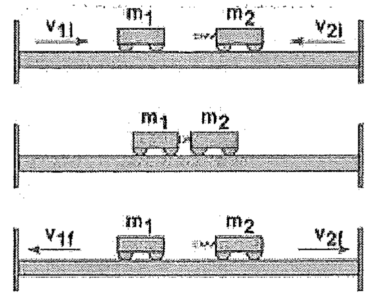
after:

100 kg 5.5 kg

$v_1' = ?$ $v_2' = 300$ m/s

$0 = 100v_1' + (5.5)(300)$ $v_1' = -16.5$ m/s

Physics 12- **Conservation of Momentum - Elastic Collisions**



Last class we dealt with problems involving **explosions and inelastic collisions**. We saw that in an inelastic collision, some of the initial kinetic energy of the system is transformed to other forms of energy resulted in less final kinetic energy for the system. We also saw that momentum is conserved in explosions and inelastic collisions.

Now, we are going to learn about **elastic collisions**. In an elastic collision, the objects 'bounce' off each other. They **do not 'stick together'** as they do during inelastic collisions.

In an elastic collision, **both momentum AND kinetic energy are conserved.**

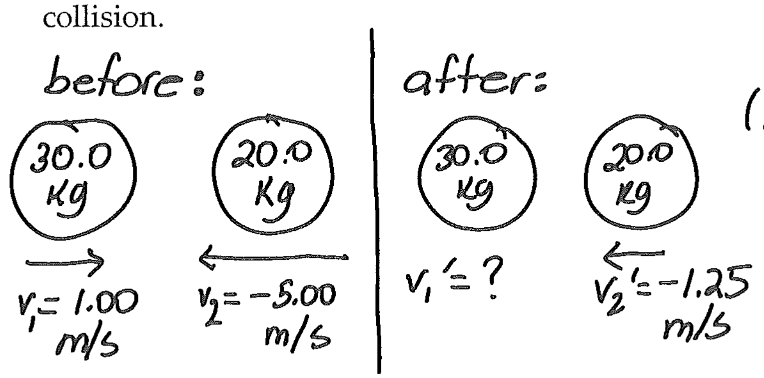
There is a "spectrum" of collisions and explosions with the extremes being perfectly inelastic and elastic collisions.

INELASTIC--------------------ELASTIC------------------EXPLOSION

$P_o = P_f$	$P_o = P_f$	$P_o = P_f$
$KE_o > KE_f$	$KE_o = KE_f$	$KE_o < KE_f$

Elastic Collisions:

Example 1: A 30.0 kg object moving to the right at a speed of 1.00 m/s collides with a 20.0 kg object moving to the left at a velocity of 5.00 m/s. If the 20.0 kg object continues to move left at a velocity of 1.25 m/s, what is the velocity of the 30.0 kg object? Assume a perfectly elastic collision.



$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$(30)(1.0) + (20)(-5.0) = (30)v_1' + (20)(-1.25)$$

$$\underline{v_1' = -1.25 \text{ m/s}}$$

Example 2: When Kinetic Energy is conserved we can find the velocities of BOTH of the objects after the collision: Two identical pool balls make a perfectly elastic head-on collision on a frictionless table. The speeds of the balls for the collision are 2.0 m/s and 3.0 m/s. What are the speeds and direction of motion for the balls after the collision?

before:

m_1 $\quad$ m_2

$\overrightarrow{v_1 =}$ $+3.0$ m/s $\quad$ $\overleftarrow{v_2 =}$ -2.0 m/s

after:

m_1 $\quad$ m_2

$v_1' = ?$ $\quad$ $v_2' = ?$

*same mass:

$$\cancel{m_1}v_1 + \cancel{m_2}v_2 = \cancel{m_1}v_1' + \cancel{m_2}v_2'$$

$$v_1 + v_2 = v_1' + v_2'$$

$$3.0 + (-2.0) = v_1' + v_2'$$

$$1.0 \text{ m/s} = v_1' + v_2'$$

Because in an elastic collision, $KE_0 = KE_f$ and since we are only dealing with velocity, not the KE, **AND the masses are the same,** we can simplify the formula to just the 'velocity' portion of the equation.

$$\boxed{\text{from above: } v_1' = 1.0 - v_2'}$$

$$KE_0 = KE_f$$

$$\cancel{\tfrac{1}{2}m}v_1^2 + \cancel{\tfrac{1}{2}m}v_2^2 = \cancel{\tfrac{1}{2}m}v_1'^2 + \cancel{\tfrac{1}{2}m}v_2'^2$$

$$(3.0)^2 + (-2.0)^2 = v_1'^2 + v_2'^2$$

$$13 = v_1'^2 + v_2'^2 \longrightarrow 13 = (1.0 - v_2')^2 + v_2'^2$$

$$13 = (1.0 - v_2')(1.0 - v_2') + v_2'^2$$

$$13 = 1.0 - 2v_2' + v_2'^2 + v_2'^2$$

$$13 = 1.0 - 2v_2' + 2v_2'^2$$

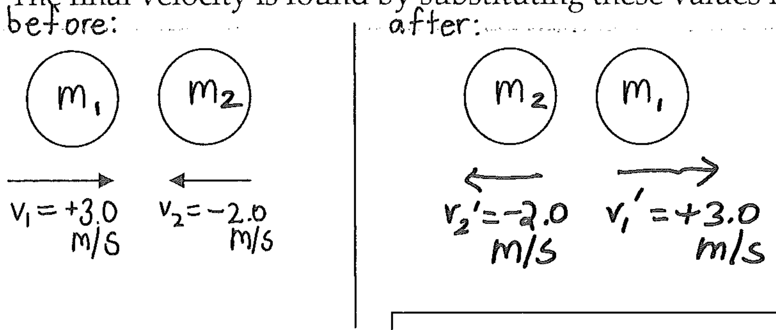
rearrange for quadratic formula:

$$ax^2 + bx + c = 0 \longrightarrow 2v_2'^2 - 2v_2' - 12 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longrightarrow v_2' = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-12)}}{2(2)}$$

$$v_2' = \frac{2.0 \pm 10.0}{4.0} \qquad v_2' = -2.0 \text{ m/s} \text{ or } +3.0 \text{ m/s}$$

The final velocity is found by substituting these values into → $\boxed{V_1 + V_2 = V_1' + V_2'}$

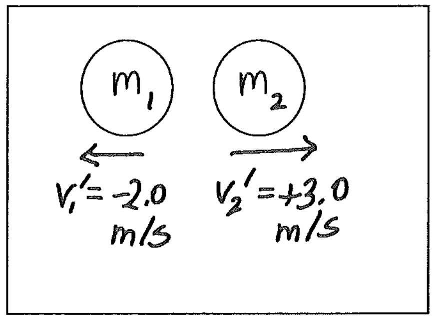


$$V_1' = 1.0 - V_2'$$
$$= 1.0 - (-2.0)$$
$$= +3.0 \text{ m/s}$$

or

$$V_1' = 1.0 - 3.0$$
$$= -2.0 \text{ m/s}$$

OR

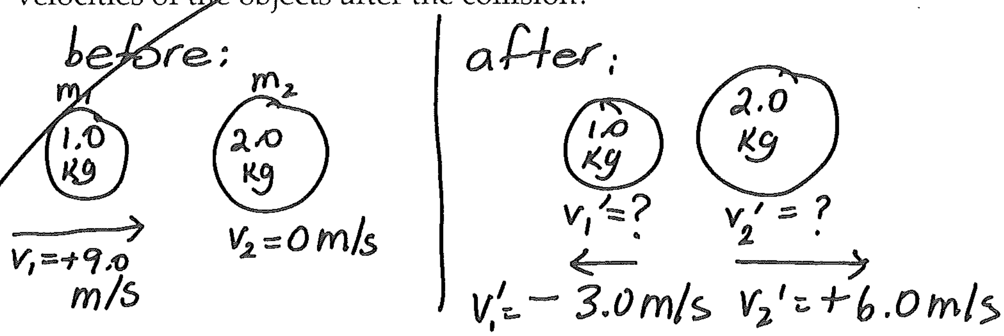


The only resulting velocities that make sense is for $v_1' = -2.0$ m/s and $v_2' = +3.0$ m/s. The other option shows velocities in which the objects pass right through each other which is not realistic.

The objects transferred their initial velocities to the other object. This is the case when the masses are the same.

<u>Example 3: Different masses with an initial stationary object:</u>

A 1 kg object travelling at 9 m/s to the right strikes a 2 kg stationary object. What are the velocities of the objects after the collision?



<u>Derived from the conservation of momentum and conservation of kinetic energy:</u>

$$\boxed{v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1}$$

*use when one object is stationary to beginwith

$$v_1' = \left(\frac{1.0 - 2.0}{1.0 + 2.0}\right) 9.0 = \underline{-3.0 \text{ m/s}}$$

$$v_2' = \left(\frac{2(1.0)}{1.0 + 2.0}\right) 9.0 = \underline{+6.0 \text{ m/s}}$$

Physics 12 – Conservation of Momentum Assignment – Elastic Collisions

1. A 2.0 kg object travelling at 12 m/s to the right strikes a 2.0 kg object travelling to the left at 3 m/s. What are the velocities of the objects after the collision? (v_1'= -3.0m/s, v_2'= +12m/s)

before: m_1 2.0 kg, m_2 2.0 kg

$v_1 = 12\,m/s$ $\quad v_2 = -3\,m/s$

after: m_1 $\quad m_2$

$v_1' = ?$ $\quad v_2' = ?$

same mass:

$$v_1 + v_2 = v_1' + v_2'$$
$$(12)+(-3) = v_1' + v_2'$$
$$9.0 = v_1' + v_2'$$
$$\boxed{v_1' = 9.0 - v_2'}$$

KE: $$v_1^2 + v_2^2 = v_1'^2 + v_2'^2$$
$$12^2+(-3)^2 = v_1'^2 + v_2'^2$$
$$153 = v_1'^2 + v_2'^2$$

substitute

$$153 = (9.0 - v_2')^2 + v_2'^2$$
$$153 = 81 - 18v_2' + v_2'^2 + v_2'^2$$
$$2v_2'^2 - 18v_2' - 72 = 0$$

quadratic: $$v_2' = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(2)(-72)}}{2(2)}$$

$$v_2' = \frac{18 \pm 30}{4.0} \qquad v_2' = -3.0\,m/s \text{ or } +12\,m/s$$

what makes sense?

m_1 $\quad m_2$

$-3.0\,m/s$ $\quad +12\,m/s$

$$\boxed{\begin{aligned} v_1' &= -3.0\,m/s \\ v_2' &= +12\,m/s \end{aligned}}$$

2. A 2.0 kg object travelling at 12 m/s to the right strikes a stationary 4.0 kg object. What are the velocities of the objects after the collision? (v_1'= -4.0m/s, v_2'= +8.0 m/s)

before: 2.0 kg $\quad$ 4.0 kg

$v_1 = +12\,m/s$ $\quad v_2 = 0\,m/s$

after: 2.0 kg $\quad$ 4.0 kg

$v_1' = ?$ $\quad v_2' = ?$

$-4.0\,m/s$ $\quad +8.0\,m/s$

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1$$
$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

$$v_1' = \left(\frac{2.0 - 4.0}{2.0 + 4.0}\right)12 \qquad v_2' = \left(\frac{2(2.0)}{2.0 + 4.0}\right)12$$

$$v_1' = -4.0\,m/s \qquad v_2' = +8.0\,m/s$$

3. A 2.0 kg object travelling at 12 m/s to the right strikes a 2.0 kg object travelling to the left at 12 m/s. What are the velocities of the objects after the collision? (v_1'= -12 m/s, v_2'= +12 m/s)

before:

2.0 kg — 2.0 kg

$v_1 = +12$ m/s → ← $v_2 = -12$ m/s

after:

2.0 kg — 2.0 kg

← $v_1' = -12$ m/s $v_2' = +12$ m/s →

momentum:

$$v_1 + v_2 = v_1' + v_2'$$

$$12 + (-12) = v_1' + v_2'$$

$$0 = \underbrace{v_1' + v_2'}$$

*same mass, only result that makes sense is for them to rebound at same speeds in opposite directions

4. A pool ball moving with a speed of 2.5 m/s makes an elastic head-on collision with an identical ball travelling in the opposite direction with a speed of 5.9 m/s. Find the velocities of the balls after the collision. (v_1'= -2.5 m/s, v_2'= +5.9 m/s)

before:

m_1 — m_2

$v_1 = 5.9$ m/s → ← $v_2 = -2.5$ m/s

after:

m_1 — m_2

$v_1' = ?$ ← -2.5 m/s $v_2' = ?$ → $+5.9$ m/s

$$v_1 + v_2 = v_1' + v_2'$$

$$5.9 + (-2.5) = v_1' + v_2'$$

$$3.4 = v_1' + v_2'$$

$$\boxed{v_1' = 3.4 - v_2'}$$

$$v_1^2 + v_2^2 = v_1'^2 + v_2'^2$$

$$5.9^2 + (-2.5)^2 = v_1'^2 + v_2'^2$$

$$\boxed{41.1 = v_1'^2 + v_2'^2}$$

$$41.1 = (3.4 - v_2')^2 + v_2'^2$$

$$41.1 = 11.6 - 6.8v_2' + v_2'^2 + v_2'^2$$

$$2v_2'^2 - 6.8v_2 - 29.5 = 0$$

$$v_2' = \frac{-(-6.8) \pm \sqrt{(-6.8)^2 - 4(2)(-29.5)}}{2(2)}$$

$$v_2' = \frac{6.8 \pm 16.8}{4.0}$$

$$\boxed{v_2' = +5.9 \text{ m/s} \quad \text{or} \quad v_2' = -2.5 \text{ m/s}}$$

5. A 0.25 kg puck moving at 5.0 m/s due east collides head-on with a 0.45 kg puck at rest. If the collision is elastic, find the velocities of the pucks after the collision. (v_1'= -1.43m/s, v_2'=+3.57 m/s)

before:

$v_1 = 5.0$ m/s $\rightarrow$ $\quad\quad$ $v_2 = 0$ m/s

0.25 kg $\quad\quad$ 0.45 kg

after:

$v_1' = -1.43$ m/s $\leftarrow$ $\quad$ $v_2' = +3.57$ m/s $\rightarrow$

A $\quad$ B

0.25 kg $\quad$ 0.45 kg

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_1' = \left(\frac{0.25 - 0.45}{0.25 + 0.45}\right) 5.0 \qquad v_1' = -1.43 \text{ m/s}$$

$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2(0.25)}{0.25 + 0.45}\right) 5.0 \qquad v_2' = +3.57 \text{ m/s}$$

6. A 225 g ball moves with a velocity of 30.0 cm/s to the right. This ball collides with a 125 g ball moving in the same direction at a velocity of 10.0 cm/s. After the collision, the velocity of the 125 g ball is 24.0 cm/s to the right.

a) What is the velocity of the 225 g ball after the collision? (22.0 cm/s [right])

before:

.225 kg $\quad$ .125 kg

$v_1 = 30.0$ cm/s $\rightarrow$ $\quad$ $v_2 = 10.0$ cm/s $\rightarrow$

after:

.225 kg $\quad$ .125 kg

$v_1' = ?$ $\quad$ $v_2' = 24.0$ cm/s $\rightarrow$

$$(.225)(30) + (.125)(10) = .225 v_1' + (.125)(24.0)$$

$$v_1' = 22.0 \text{ cm/s [right]}$$

b) Is this an elastic or inelastic collision? Provide mathematical evidence for your answer. (NOTE: Calculate the kinetic energy of the objects before and after collision. Re-read your notes on inelastic and elastic collisions and determine the type of collision based on whether the energies are the same or not.)

$$KE_f - KE_o$$

$$= \left(\tfrac{1}{2}(.225)(.22)^2 + \tfrac{1}{2}(.125)(.24)^2\right) - \left(\tfrac{1}{2}(.225)(.30)^2 + \tfrac{1}{2}(.125)(.10)^2\right)$$

$$= 0.009045 - 0.01075 \qquad = -1.71 \times 10^{-3} \text{ J} \qquad KE_o \neq KE_f$$

$$\boxed{= \text{inelastic}}$$

c) If kinetic energy is lost, what happened to it?

transformed to other forms $\rightarrow$ thermal, sound etc.

7. A 30.0 kg object moving to the right at a velocity of 1.00 m/s collides with a 20.0 kg object moving to the left at a velocity of 5.00 m/s. If the 20.0 kg object continues to move left at a velocity of 1.25 m/s, what is the velocity of the 30.0 kg object? (-1.50 m/s)

before:

30.0 kg 20.0 kg

$\rightarrow$ $v_1 = 1.00$ m/s $\leftarrow$ $v_2 = -5.00$ m/s

after:

30.0 kg 20.0 kg

$\leftarrow$ $v_1' = ?$ $\leftarrow$ $v_2' = -1.25$ m/s

$(30.0)(1.00) + (20.0)(-5.0) = (30)(v_1') + (20)(-1.$

$v_1' = \underline{-1.50 \text{ m/s}}$

8. A 10.0 g object is moving with a velocity of 20.0 cm/s to the right when it collides with a stationary 30.0 g object. After collision, the 10.0 g object is moving left at a velocity of 6.00 cm/s.

a) What is the velocity of the 30.0 g ball after the collision?

before:

.010 kg .030 kg

$\rightarrow$ $v_1 = 20.0$ cm/s $v_2 = 0$ m/s

after:

.010 kg .030 kg

$\leftarrow$ $v_1' = -6.00$ cm/s $v_2' = ?$

$(.01)(20) + 0 = (.01)(-6.0) + (.03)v_2'$

$v_2' = +8.67$ cm/s

b) Is this an elastic or inelastic collision? Provide mathematical evidence for your answer.

$= \left(\frac{1}{2} \cdot 0.01 \cdot (-6)^2 + \frac{1}{2} \cdot 0.03 \cdot (8.67)^2\right) \neq \left(\frac{1}{2} \cdot 0.01 \cdot (20)^2 + 0\right)$

$KE_f \neq KE_o$

inelastic

c) If kinetic energy is lost, what happened to it?

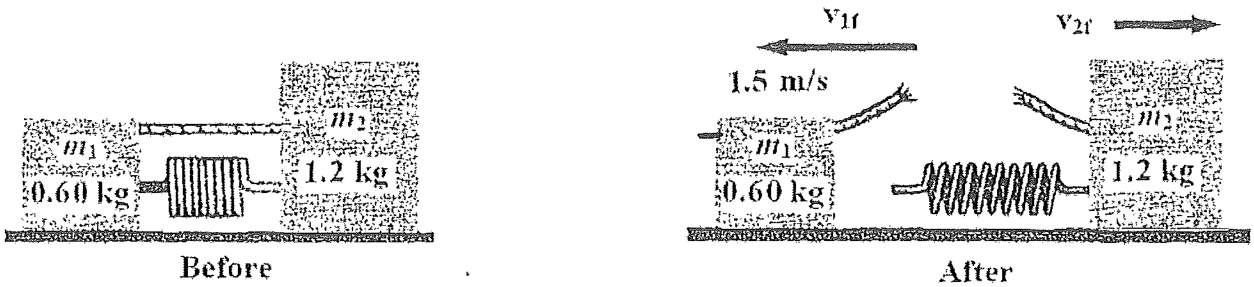
transformed to other forms

Physics 12 – **2D Momentum – Explosions and Impulse**

Conservation of Momentum and Work-Energy Theorem

Recall: $W = \Delta KE = KE_f - KE_0 = (\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2) - (\frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2)$

Once again we are able to relate momentum to work and energy.



We can determine the work done on the block by the spring when the cord holding everything together breaks apart.

(A) $0 = (0.60)(-1.5) + (1.2)(v_2')$; $v_2' = +0.75$ m/s

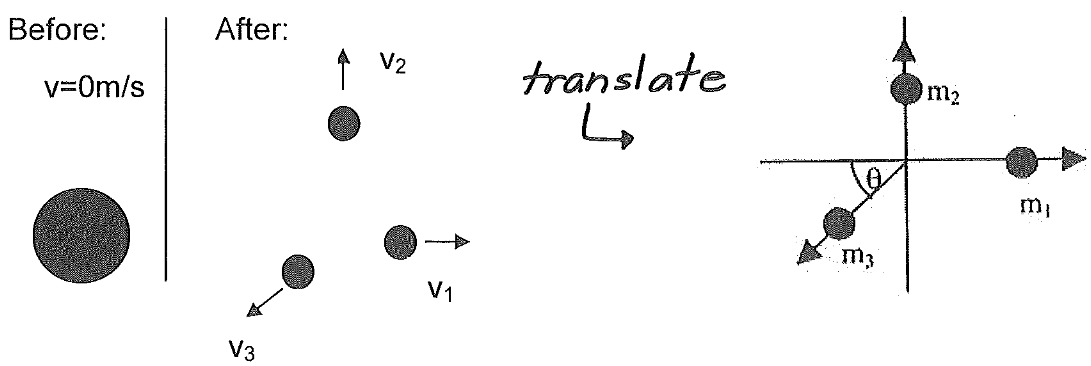
(B) Find ΔKE to find work done

$W = \Delta KE = KE_f - KE_0 = \left(\frac{1}{2}(0.60)(1.5)^2 + \frac{1}{2}(1.2)(0.75)^2\right) - 0 = \underline{1.01\ J}$

Work done $= \underline{1.01\ J}$

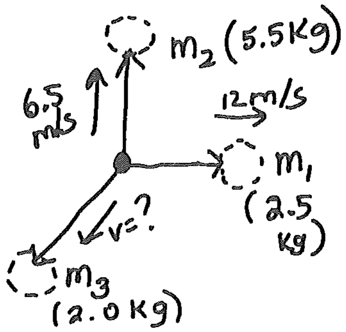
Explosions into <u>three</u> pieces:

Unlike past examples, all of the pieces will not explode into exactly opposite directions. Therefore we now need to work in **<u>more than one dimension</u>**.



Now let's assign some values to this explosion. A 10.0 kg object that is at rest breaks into three pieces as shown. $v_1 = 12$ m/s, $v_2 = 6.5$ m/s, $m_1 = 2.5$ kg, $m_2 = 5.5$ kg, $m_3 = 2.0$ kg

What is the velocity of the 2.0 kg mass?



$$\Sigma p_o = \Sigma p'$$

$$\Sigma p_o = m v_o$$
$$= (10)(0)$$
$$= 0 \text{ kg}\cdot\text{m/s} \qquad \therefore \Sigma p' = 0 \text{ kg}\cdot\text{m/s}$$

Since we are in 2 dimensions now, we need to break the problem into its X and Y components:

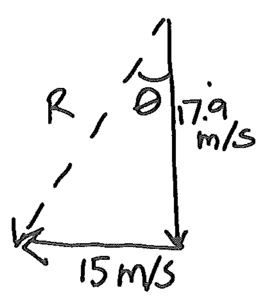
X Component: $\Sigma p_x' = 0$

$$0 = m_1 v_{1x}' + m_2 v_{2x}' + m_3 v_{3x}'$$
$$0 = (2.5)(+12) + (5.5)(0) + 2.0\, v_{3x}'$$
$$v_{3x}' = -15 \text{ m/s}$$

Y Component:

$$\Sigma p_y' = 0$$
$$0 = (2.5)(0) + (5.5)(6.5) + (2.0) v_{3y}'$$
$$v_{3y}' = -17.9 \text{ m/s}$$

Now we find the resultant for v_{3f}:



$$R = \sqrt{17.9^2 + 15^2} = 23.4 \text{ m/s}$$

$$\tan^{-1}\left(\frac{15}{17.9}\right) = 40^\circ \text{ W of S}$$

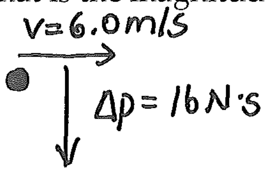
FINAL ANSWER:

23.4 m/s @ 40° W of S

Impulse in Two Dimensions:

Remember that impulse is the change in momentum – the before and after. This is generally due to a change in an object's velocity.

A 2.5 kg model car moving at 6.0 m/s due east experiences a 16 N·s impulse southward. What is the magnitude and direction of the final momentum of the car? $\rightarrow$ $p' = ?$



This is in two dimensions – so we need X and Y:

components:

$V_{ox} = +6.0 m/s$ $\quad$ $\Delta p_x = 0 \, N \cdot s$

$V_{oy} = 0 \, m/s$ $\quad$ $\Delta p_y = -16 \, N \cdot s$

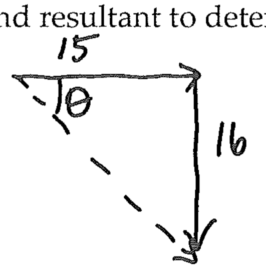
Conservation of momentum (x-component):

$$p_x' = mV_{ox} + \Delta p_x = (2.5)(6.0) + 0 = 15 \, kg \cdot m/s$$

Conservation of momentum (y-component):

$$p_y' = mV_{oy} + \Delta p_y = (2.5)(0) + (-16) = -16 \, kg \cdot m/s$$

Find resultant to determine final momentum:



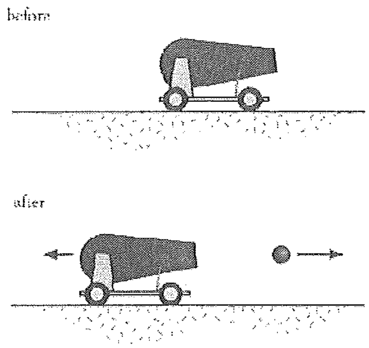
$$R = \sqrt{16^2 + 15^2} = 21.9 \, kg \cdot m/s$$

$$\tan^{-1}\left(\frac{16}{15}\right) = 47^\circ \text{ S of E}$$

FINAL ANSWER: $p' = 21.9 \, kg \cdot m/s$ @ 47° S of E

Explosion and Impulse Problems:

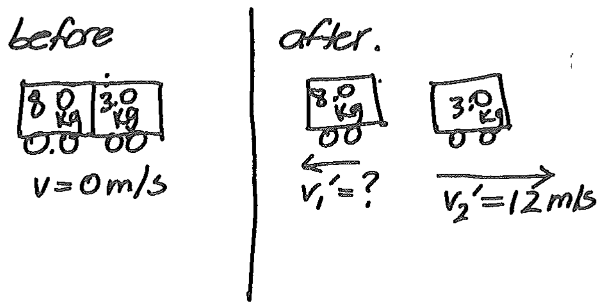
1. A 55 kg cannon and a 7.5 kg cannonball are sitting at rest. When the gunpowder has ignited, the cannonball is shot out of the cannon at a velocity of 45 m/s. What work was done on the cannon and cannonball by the gunpowder? (8.6×10^3 J)



$$0 = (55)v_1' + (7.5)(45)$$
$$v_1' = -6.14 \text{ m/s}$$

$$\Delta KE = KE_f - KE_o$$
$$= \left(\tfrac{1}{2}\cdot 55\cdot(-6.14)^2 + \tfrac{1}{2}\cdot 7.5\cdot(45)^2\right) - 0$$
$$= 8.6\times 10^3 \text{ J} = \text{Work done}$$

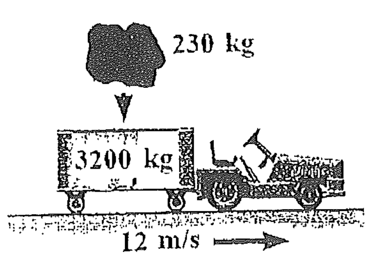
2. An 8.0 kg and 3.0 kg cart are held together and at rest. An internal spring is released between the two carts and they are pushed in opposite directions. If the 3.0 kg cart is traveling at 12 m/s after the spring is released, what is the work done on the carts by the spring? (297 J)



$$0 = (8.0)v_1' + (3.0)(12)$$
$$v_1' = -4.5 \text{ m/s}$$

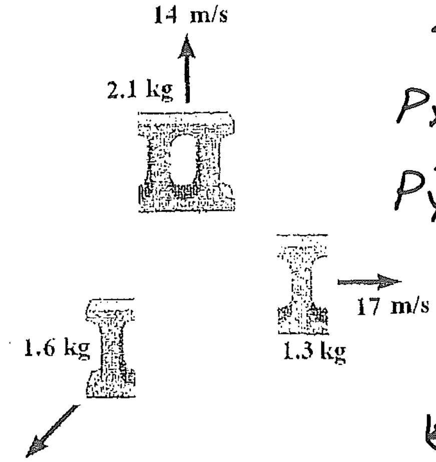
$$W = \Delta KE = KE_f - KE_o$$
$$= \left(\tfrac{1}{2}\cdot 8.0\cdot(4.5)^2 + \tfrac{1}{2}\cdot 3.0\cdot(12)^2\right) - 0$$
$$= 297 \text{ J of work done}$$

3. A 3200 kg dump car travels along the road at 12 m/s. Suddenly, a 230 kg chunk of coal is dumped into the car. Find the final velocity of the truck. (+11.2 m/s)



$$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$$
$$(3200)(12) + 0 = (3200 + 230)v'$$
$$38400 = 3430v'$$
$$v' = +11.2 \text{ m/s}$$

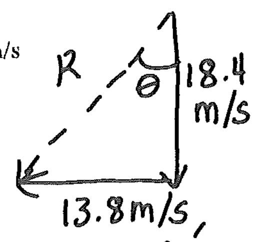
4. A 5.0 kg block at rest is dropped and breaks into three pieces as shown. What is the velocity (speed and direction) of the 1.6 kg piece? (The block is at rest when it hits the ground and breaks apart) (v_3' = 23.0 m/s @ 37° W of S)



$\Sigma p' = 0$ kg·ms

$p_x' = (2.1)(0) + (1.3)(17) + (1.6)v_{3x}'$ $\quad v_{3x}' = -13.8$ m/s

$p_y' = (2.1)(14) + (1.3)(0) + (1.6)v_{3y}'$ $\quad v_{3y}' = -18.4$ m/s



$R = \sqrt{13.8^2 + 18.4^2} = 23.0$ m/s

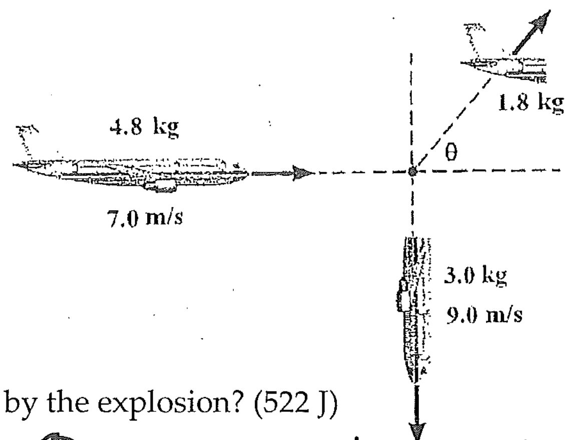
$\tan^{-1}\left(\frac{13.8}{18.4}\right) = 37°$ W of S

$v_3' = \underline{23.0 \text{ m/s} @ 37° \text{ W of S}}$

5. A 4.8 kg model airplane flying at 7.0 m/s to the east explodes into two fragments as shown in the diagram. The larger 3.0 kg fragment moves at 9.0 m/s south.

a) What were the initial momentum and kinetic energy of the model airplane before the explosion? (+33.6 kg•m/s, 117.6J)

b) What are the velocity (speed and direction) and the kinetic energy of the smaller 1.8kg fragment? (24.0 m/s @ 39° N of E, 518 J)



c) What work was done on the model airplane by the explosion? (522 J)

(a) $\Sigma p_0 = (4.8)(7.0) = +33.6$ kg·m/s

$KE_0 = \frac{1}{2}(4.8)(7.0)^2 = 117.6$ J

(b) $33.6 = 1.8v_x' + (3.0)(0)$

$v_x' = +18.7$ m/s

no initial y-direction momentum $\rightarrow 0 = 1.8v_y' + (3.0)(-9.0)$

$v_y' = +15$ m/s

$R = \sqrt{18.7^2 + 15^2} = 24.0$ m/s @ $\underline{39° \text{ N of E}}$

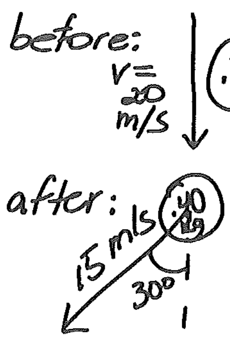
$KE = \frac{1}{2}(1.8)(24.0)^2 = \underline{518 \text{ J}}$

(c) $KE_f - KE_0$

$(518 + \frac{1}{2}\cdot 3 \cdot 9^2) - 117.6 = \underline{522 \text{ J}}$ of work done

6. A 0.40 kg ball moving at 20 m/s due south strikes a rock and moves 15 m/s at 30° west of south.

   a. Find the magnitude and direction of the impulse (change in momentum). (4.10 kg•m/s @ 43° N of W)
   b. If the ball is in contact with the rock for 0.06 s, what is the average force exerted on the ball by the rock? (68 N @ 43° N of W)

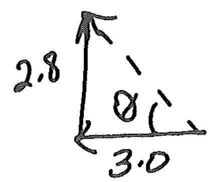


a $V_{ox} = 0\ m/s \quad V_{oy} = -20\ m/s$

$V'_x = -\sin 30(15) = -7.5\ m/s \quad V'_y = -\cos 30(15) = -13\ m/s$

$\Delta P_x = m(v'_x - V_{ox}) = (0.40)(-7.5 - 0) = -3.0\ kg \cdot m/s$

$\Delta P_y = m(v'_y - V_{oy}) = (0.40)(-13 - (-20)) = +2.8\ kg \cdot m/s$



$R = \sqrt{2.8^2 + 3.0^2} = 4.10\ kg \cdot m/s$ @ 43° N of W

b $\vec{\Delta P} = \Sigma F \cdot t$

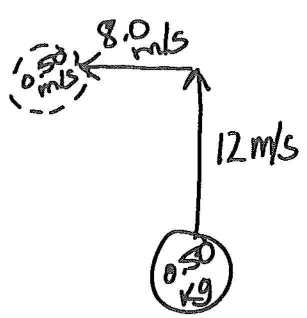
$4.1 = \Sigma F \cdot 0.06$

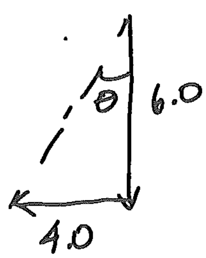
$\Sigma F = 68\ N$ @ 43° N of W

7. A 0.50 kg stone moving at 12 m/s due north makes contact with an electric pole for 0.04s, resulting in a final velocity of 8.0 m/s due west. What is the magnitude and direction of the net force exerted on the stone by the electric pole? (180N @ 34° W of S)

$\Delta P_x = 0.50(-8.0 - 0) = -4.0\ kg \cdot m/s$

$\Delta P_y = 0.50(0 - 12) = -6.0\ kg \cdot m/s$





$R = \sqrt{6.0^2 + 4.0^2} = 7.2\ kg \cdot m/s$ @ 34° W of S

$\Delta P_R = \Sigma F \cdot t$

$7.2 = \Sigma F \cdot 0.04$

$\Sigma F = 180\ N$ @ 34° W of S

Physics 12 – **2D Momentum - Collisions**

Last lesson, we began dealing with non-linear momentum through impulse and explosions. Now, we are going to consider non-linear collisions.Collisions between objects are governed by laws of momentum and energy. When a collision occurs in an isolated system, the total momentum of the system of objects is conserved.

Elastic collisions are collisions in which **both momentum and kinetic energy are conserved.** The total system kinetic energy before the collision equals the total system kinetic energy after the collision.

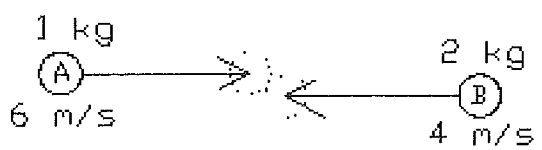
If total kinetic energy is not conserved, then the collision is referred to as an **inelastic collision.**

NON-LINEAR ELASTIC COLLISIONS:

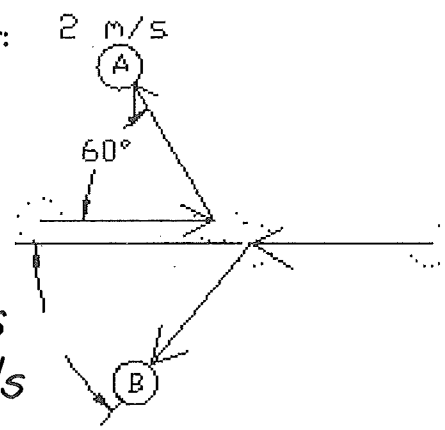
In this type of collision, no external forces act on the system. Kinetic Energy is conserved.

Two objects initially travelling east and west as shown have a collision. The object on the left is bounced 'up' at 60° with a velocity of 2 m/s. What is the final velocity of the object initially travelling west?

Before:



After:



after: $V_{Ax}' = -\cos 60(2.0) = -1.0\ m/s$

$V_{Ay}' = \sin 60(2.0) = +1.73\ m/s$

$\Sigma p_x = \Sigma p_x' \quad (1.0)(6.0) + (2.0)(-4.0) = (1.0)(-1.0) + (2.0)V_{Bx}'$

$V_{Bx}' = -0.50\ m/s$

$\Sigma p_y = \Sigma p_y' \quad 0 + 0 = (1.0)(1.73) + (2.0)V_{By}'$

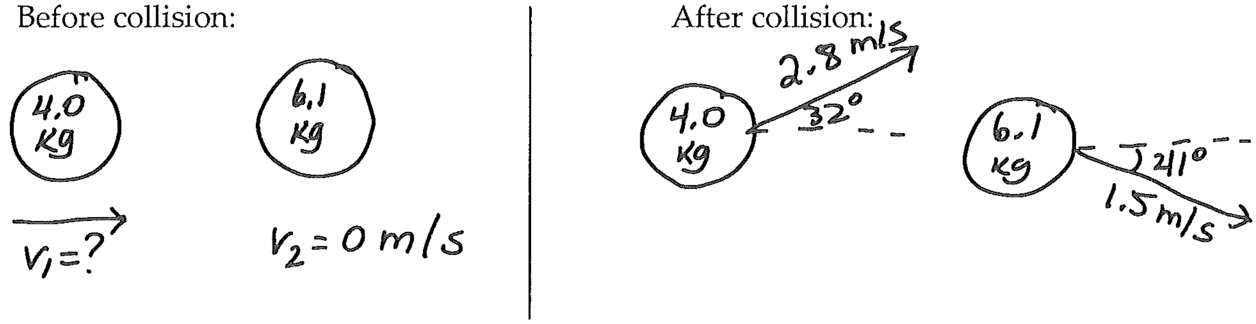
$V_{By}' = -0.865\ m/s$

$R = \sqrt{(-0.865)^2 + (-0.50)^2} = \underline{1.0\ m/s\ @\ 60°\ S\ of\ W}$

Method Two: (another way to solve):

A 4.0 kg object is moving east at an unknown velocity when it collides with a 6.1 kg stationary object. After the collision, the 4.0 kg object is travelling with a velocity of 2.8 m/s 32° N of E and the 6.1 kg object is travelling at a velocity of 1.5 m/s 41° S of E. What was the velocity of the 4.0 kg object before the collision?

Before collision: After collision:



Find p1′ and p2′:

$$P_1' = mv_1' = (4.0)(2.8) = 11.2 \text{ kg}\cdot\text{m/s} \ @ \ 32° \text{ N of E}$$
$$P_2' = mv_2' = (6.1)(1.5) = 9.15 \text{ kg}\cdot\text{m/s} \ @ \ 41° \text{ S of E}$$

Resolve both into their vector components:

$$P_{1x}' = \cos 32(11.2) = +9.50 \text{ kg}\cdot\text{m/s}$$
$$P_{1y}' = \sin 32(11.2) = +5.94 \text{ kg}\cdot\text{m/s}$$
$$P_{2x}' = \cos 41(9.15) = +6.91 \text{ kg}\cdot\text{m/s}$$
$$P_{2y}' = -\sin 41(9.15) = -6.00 \text{ kg}\cdot\text{m/s}$$

Find $\Sigma p_x'$ and $\Sigma p_y'$:

$$\Sigma p_x' = 9.50 + 6.91 = 16.4 \text{ kg}\cdot\text{m/s}$$
$$\Sigma p_y' = 5.94 + (-6.00) = -0.060 \text{ kg}\cdot\text{m/s}$$

Find $\Sigma p'$: $R = \sqrt{16.4^2 + (-0.06)^2} = 16.4 \text{ kg}\cdot\text{m/s}$

$$\Sigma p = \Sigma p' \rightarrow m_1 v_1 + m_2 v_2 = 16.4$$
$$(4.0)v_1 + 0 = 16.4$$

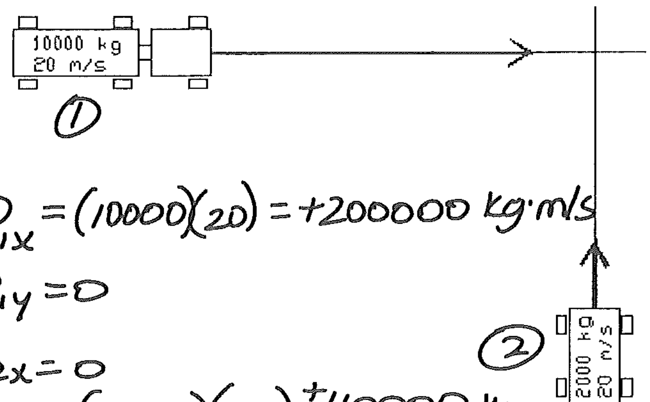
$v_1 = \underline{+4.1 \text{ m/s}}$

NON-LINEAR INELASTIC COLLISIONS:

In this type of collision there are external forces acting on the system. Kinetic energy is converted into other forms such as sound, thermal etc.

A 2000 kg car is travelling at 20 m/s north and has a collision with a 10000 kg truck travelling 20 m/s east. After the collision the vehicles stick together. What is the speed and direction of the car and truck after the collision?

BEFORE



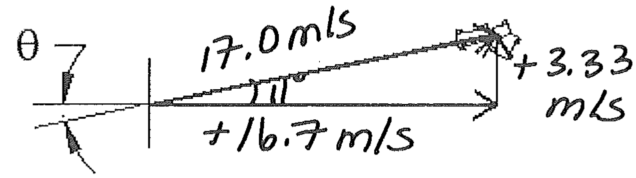
$P_{1x} = (10000)(20) = +200000$ kg·m/s

$P_{1y} = 0$

$P_{2x} = 0$

$P_{2y} = (2000)(20) = +40000$ kg·m/s

AFTER



$R = \sqrt{16.7^2 + 3.33^2} = 17.0$ m/s

@ 11° N of E

$\Sigma p_x = +200000$ kg·m/s

$\Sigma p_y = +40000$ kg·m/s

$X: \Sigma p_x = \Sigma p_x'$

$200000 = (10000 + 2000) v_x'$

$v_x' = +16.7$ m/s

$Y: \Sigma p_y = \Sigma p_y'$

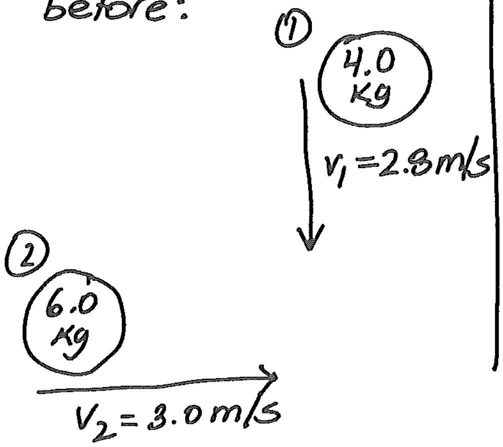
$40000 = (12000) v_y'$

$v_y' = +3.33$ m/s

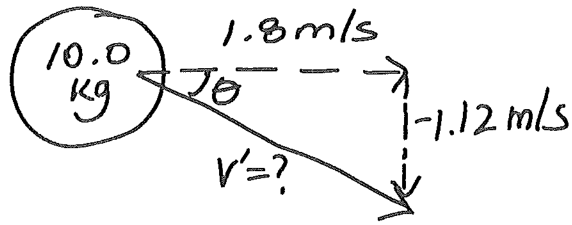
$v' =$ <u>17.0 m/s @ 11° N of E</u>

A 4.0 kg object is travelling south at a velocity of 2.8 m/s when it collides with a 6.0 kg object travelling east at a velocity of 3.0 m/s. If these two objects stick together upon collision, what is the speed and direction of the combined masses?

before:



after:



$v' = 2.12\ m/s$ @ $32°$ S of E

$P_{1x} = 0$

$P_{1y} = (4.0)(-2.8) = -11.2\ kg \cdot m/s$

$P_{2x} = (6.0)(3.0) = +18.0\ kg \cdot m/s$

$P_{2y} = 0$

$\Sigma P_x = +18.0\ kg \cdot m/s$

$\Sigma P_y = -11.2\ kg \cdot m/s$

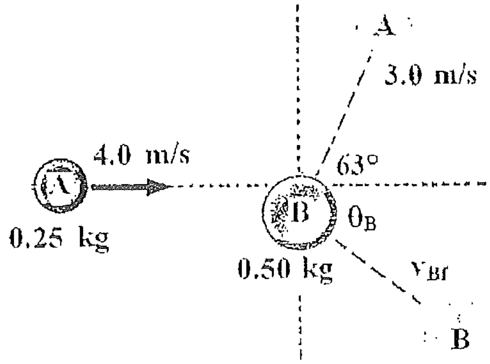
x: $18.0 = (10.0)v_x'$ $\quad v_x' = +1.80\ m/s$

y: $-11.2 = (10.0)v_y'$ $\quad v_y' = -1.12\ m/s$

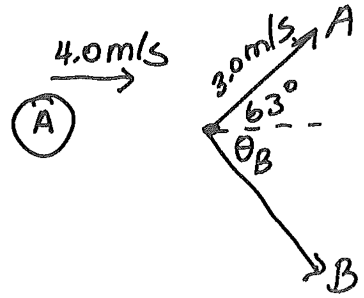
$R = \sqrt{1.8^2 + 1.12^2} = 2.12\ m/s$ @ $32°$ S of E

2D Momentum – Collision Problems:

1.



A 0.25 kg puck (A) moving at 4.0 m/s to the right undergoes a collision with a 0.50 kg puck (B) at rest. As a result, puck A moves at 3.0 m/s at an angle of 63° north of east. What is the velocity (magnitude and direction) of puck B after the collision? (1.88 m/s @45° S of E)



$V_{Ax}' = \cos 63(3.0) = +1.36\ m/s$

$V_{Ay}' = \sin 63(3.0) = +2.67\ m/s$

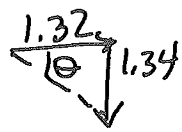
$X: (0.25)(4.0) + 0 = (0.25)(1.36) + (0.50)V_{Bx}'$

$V_{Bx}' = +1.32\ m/s$

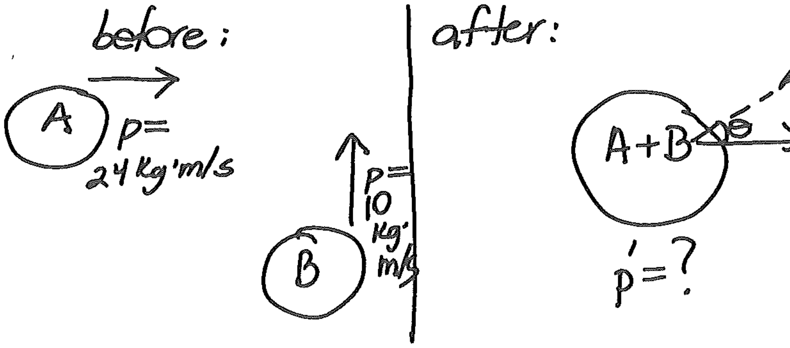
$y: 0 + 0 = (0.25)(2.67) + (0.50)V_{By}'$

$V_{By}' = -1.34\ m/s$

$\vec{R} = \sqrt{1.32^2 + (-1.34)^2}$ $\boxed{= 1.88\ m/s\ @\ 45°\ S\ of\ E}$



2. Two objects move on a level frictionless surface. Object A moves east with a momentum of 24 kg·m/s. Object B moves north with a momentum of 10 kg·m/s. They collide and stick together. What is the magnitude of the combined momentum after the collision? (26 kg·m/s)



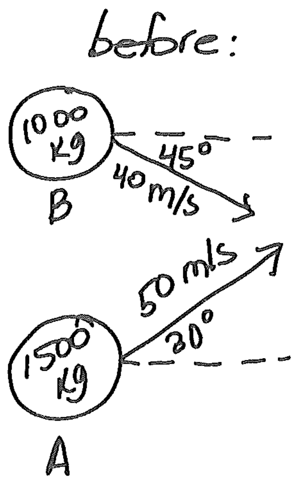
$\Sigma p_x = 24 + 0 = 24\ kg \cdot m/s$

$\Sigma p_y = 0 + 10 = +10\ kg \cdot m/s$

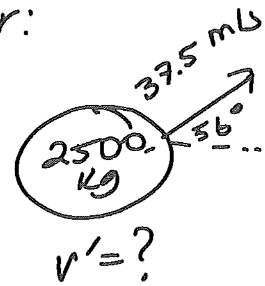
$R = \sqrt{24^2 + 10^2} = \underline{26\ kg \cdot m/s}$

3. A 1500 kg car traveling at 50 m/s 30° N of E collides with a 1000 kg car traveling at 40 m/s 45° S of E. The two cars collide and stick together. What is the speed and direction of the cars after the collision? (37.5 m/s @ 5.6° N of E)

before:



after:



$V_{Bx} = \cos 45(40) = +28.3\ m/s$

$V_{By} = -\sin 45(40) = -28.3\ m/s$

$V_{Ax} = \cos 30(50) = +43.3\ m/s$

$V_{Ay} = \sin 30(50) = +25.0\ m/s$

$X: (1000)(28.3) + 1500(43.3) = 2500 V_x'$

$V_x' = +37.3\ m/s$

$Y: (1000)(-28.3) + 1500(25.0) = 2500 V_y'$

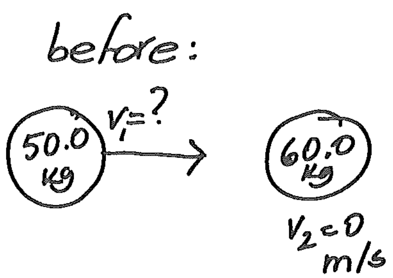
$V_y' = +3.68\ m/s$

$R = \sqrt{37.3^2 + 3.68^2}$

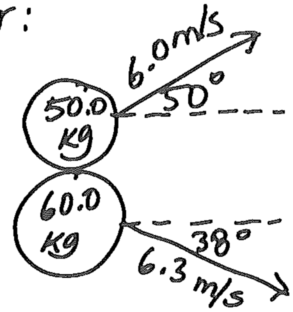
$V' = 37.5\ m/s$ @ $5.6°$ N of E

4. A 50.0 kg object is moving east at an unknown velocity when it collides with a stationary 60.0 kg object. After the collision, the 50.0 kg object is travelling at a velocity of 6.0 m/s 50.0° N of E, and the 60.0 kg object traveling at a velocity of 6.3 m/s 38.0° S of E. What was the velocity of the 50.0 kg object before the collision? (9.8 m/s [E])

before:



after:



$V_{1x}' = \cos 50(6.0) = +3.86\ m/s$

$V_{1y}' = \sin 50(6.0) = +4.60\ m/s$

$V_{2x}' = \cos 38(6.3) = +4.96\ m/s$

$V_{2y}' = -\sin 38(6.3) = -3.88\ m/s$

$\Sigma p_x' = (50)(3.86) + (60)(4.96) = 491\ kg \cdot m/s$

$\Sigma p_y' = (50)(4.60) + (60)(-3.88) = -2.8\ kg \cdot m/s$

$\Sigma p_R' = \sqrt{491^2 + (-2.8)^2} = 491\ kg \cdot m/s$

$\Sigma p = \Sigma p' \quad (50)V_1 + 0 = 491 \quad V_1 = 9.8\ m/s\ [E]$

5. A 15.0 kg object is moving east at a velocity of 7.0 m/s when it collides with a stationary 10.0 kg object. After the collision, the 15.0 kg object is moving at a velocity of 4.2 m/s 20.0° S of E. What is the velocity of the 10.0 kg object after the collision? (5.06 m/s @ 25° N of E)

before: 15.0 kg, 10.0 kg, $v_1 = 7.0\,m/s$, $v_2 = 0\,m/s$

after: 10.0 kg $v_2' = ?$, 15.0 kg, 20°, $v_1' = 4.2\,m/s$

$v_{1x}' = \cos 20(4.2) = +3.95\,m/s$

$v_{1y}' = -\sin 20(4.2) = -1.44\,m/s$

$\Sigma p_x = (15)(7.0) + 0 = 105\,kg\cdot m/s$

$\Sigma p_y = 0\,kg\cdot m/s$

$105 = (15)(3.95) + (10)v_{2x}'$ $\quad v_{2x}' = +4.58\,m/s$

$0 = (15)(-1.44) + (10)v_{2y}'$ $\quad v_{2y}' = +2.16\,m/s$

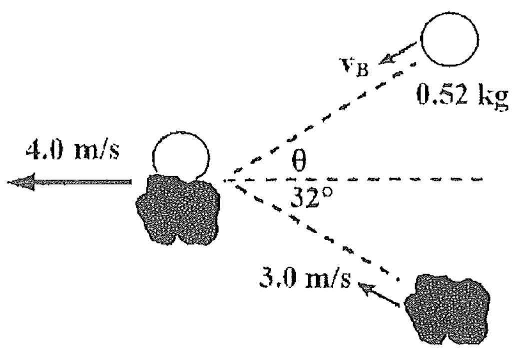
2.16, θ, 4.58

$v_2' = \sqrt{2.16^2 + 4.58^2} = 5.06\,m/s$ @ 25° N of E

6. A 0.36 kg lump of clay moving at 3.0 m/s collides with a 0.52 kg ball and they stick together as shown in the diagram. Find the speed and direction of the ball before the collision. (5.13 m/s @ 12° S of W)

$v_{Ax} = -\cos 32(3.0) = -2.54\,m/s$

$v_{Ay} = \sin 32(3.0) = +1.59\,m/s$



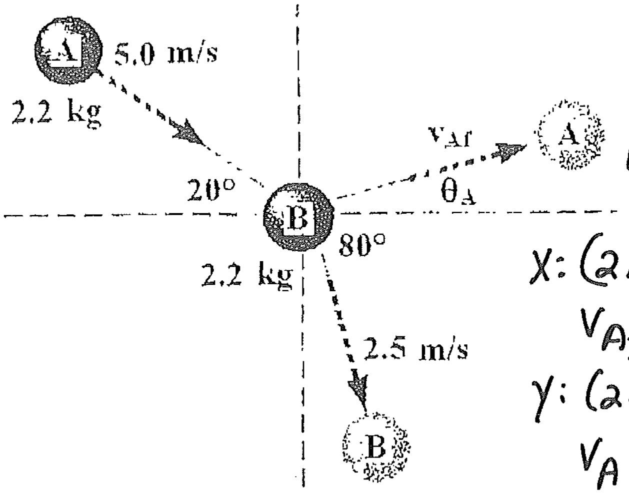
X: $(0.36)(-2.54) + (0.52)v_{Bx} = (0.36+0.52)(-4.0)$ $\quad v_{Bx} = -5.01\,m/s$

y: $(0.36)(+1.59) + (0.52)v_{By} = (0.36+0.52)(0)$ $\quad v_{By} = -1.10\,m/s$

5.01, θ, 1.10

$v_{By} = \sqrt{5.01^2 + 1.10^2} = 5.13\,m/s$ @ 12° S of W

7. A 2.2 kg ball (A) moving with a speed of 5.0 m/s strikes a second ball (B) of the same mass, 2.2 kg, initially at rest as shown in the diagram. As a result of the collision, ball B moves at 2.5 m/s at 80° S of E. What is the speed and direction of ball A after the collision? (4.34 m/s @ 10° N of E)



$V_{Ax} = \cos 20(5.0) = +4.7$ m/s

$V_{Ay} = -\sin 20(5.0) = -1.71$ m/s

$V_{Bx}' = \sin 10(2.5) = +0.434$ m/s

$V_{By}' = -\cos 10(2.5) = -2.46$ m/s

$x: (2.2)(4.7) + 0 = (2.2)V_{Ax}' + (2.2)(0.434)$

$V_{Ax}' = +4.27$ m/s

$y: (2.2)(-1.71) + 0 = (2.2)V_{Ay}' + (2.2)(-2.46)$

$V_{Ay}' = +0.75$ m/s

$V_A' = \sqrt{4.27^2 + 0.75^2} = 4.34$ m/s @ 10° N of E