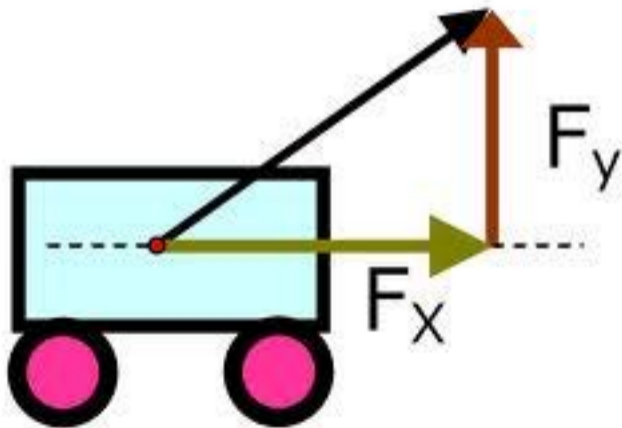


## Applied Force





# Lesson 1

## Physics 12 – Dynamics: Net Force ( $\Sigma F$ )

Recall from Physics 11:

- A force is a push or pull (measured in Newtons, N)
- Forces are vector quantities and direction must always be indicated when solving problems

### Newton's First Law of Motion (Law of Inertia)

Newton's first law of motion states that an object will remain at a constant velocity (including zero) unless acted upon by an **unbalanced force**.

**Restated this means:** An object will remain at rest or will continue to move with no change in speed or direction, if **AND ONLY IF**, Net Force = 0N ( $\Sigma F = 0N$ )

### Newton's Second Law of Motion

$$\Sigma F = ma$$

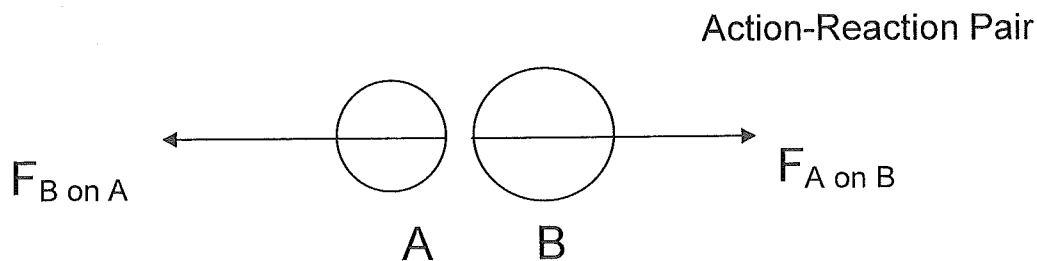
Once the net force ( $\Sigma F$ ) has been determined, we can find the acceleration that the net force will cause on the object. The acceleration that results from the net force is entirely dependent on the mass of the object.

### Newton's Third Law of Motion

EVERY force in the universe occurs as one part of an **action-reaction** pair of forces.

Both members of the action-reaction pair:

- Act on two **different** objects
- Are equal in magnitude but opposite in direction



## Static Friction:

The force that opposes the start of motion. This force is greater than the force required to keep the object in motion!

→ Coefficient of static friction ( $\mu_s$ )

i.e.  $F_f = \mu_s F_N$  → for an object being pushed but not moving!!!

## Kinetic or Sliding Friction:

The force between objects in relative motion.

→ Coefficient of sliding friction ( $\mu_k$ )

i.e.  $F_f = \mu_k F_N$  → for an object in motion

---

□ On a flat surface (Different cases from object at rest  
→ to accelerated motion)

Example 1: What is the coefficient of static friction between a 15-kg box and the floor if a 250 N force is required to start it moving?

Example 2: A 50-N sled is pulled across a cement sidewalk at constant speed. A horizontal force of 35 N is exerted. What is the coefficient of sliding friction between the sidewalk and the metal runners of the sled?

Example 3: A 10-kg box is pulled along a horizontal surface by a force of 40 N at a  $30^\circ$  angle. Calculate the acceleration if the coefficient of sliding friction between the surfaces is 0.30.

Example 1: A 20.0-kg box rests on a table. <sup>b)</sup> then a 10 kg box is added

- a) What is the weight and the normal force acting on it?
- b) Determine the normal force that the table exerts on the 20.0-kg box and the normal force that the 20.0-kg box exerts on the 10.0 kg box.

Example 2: A box weighing 55 N rests on a table. A rope tied to the box runs vertically upward over a pulley and a weight is hung from the other end. Determine the force that the table exerts on the box if the weight hanging on the other side of the pulley weighs

- a) 30 N    b) 70 N

Example 3: Kim shovels snow in the winter applying a force of 60.0 N and moving at a constant velocity. She angles her 2.0-kg shovel at  $37^\circ$  to the horizontal.

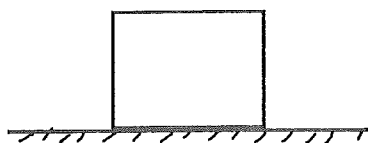
- a) What is the retarding force on the shovel?
  - b) What is the normal force on the shovel by the ground?
-

According to Newton's Second Law, **the acceleration (or change in motion) is dependent on two things:** the mass of the object and the net force.

Now we need to remember what net force is and how to determine it.

In Physics 11, the net force was found in a single plane only. In these cases, we take all of the forces and add them together (taking direction into account). The force that "wins" will result in the direction and magnitude of the net force.

Horizontally:



Vertically:



The net force directs the acceleration:

The net force directs the acceleration:

The acceleration achieved depends  
on the mass:

The acceleration achieved depends  
on the mass:

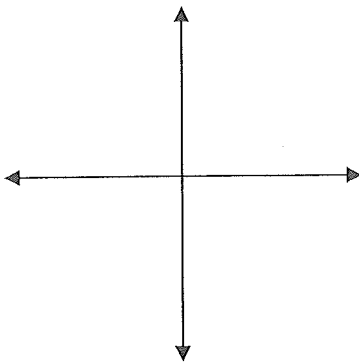
Net force can also be found as a resultant force when there are unbalanced forces acting on the object in two planes.



A force of 7.0 N east and a force of 5.0 N south act on an object. What is the net force acting on the object?



A force of 3.0 N west and a force of 4.0 N  $33^\circ$  S of E acts on an object. What is the net force on the object?



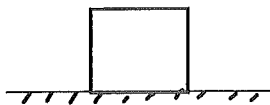
Using Net Force:

Once we have determined the net force acting on an object, we can determine the resulting acceleration by taking the mass of the object into account.

*A net force of 30.0 N south acts on a 10.0 kg object. What is the acceleration of the object?*



*A 22 kg object accelerates uniformly from rest to a velocity of 2.5 m/s west in 8.7 s. What is the net force acting on the object during this acceleration?*



*A force of 6.0 N East and a force of 8.0 N  $21^\circ$  N of E act on a 5.0 kg object. What is the acceleration that the object experiences?*



## Lesson 1

### Net Force Assignment: (Draw Free-Body Diagrams for ALL problems)

1. A force of 4.0 N East and a force of 3.0 N East act on an object. What is the net force on the object? (7.0 N [E])
2. A force of 6.0 N East and a force of 4.0 N West act on an object. What is the net force on the object? (2.0 N [E])
3. A force of 2.0 N East and a force of 5.0 N South act on an object. What is the net force acting on the object? (5.4 N  $68^\circ$  S of E)
4. A force of 2.7 N East and a force of 3.0 N  $25^\circ$  N of E act on an object. What is the net force acting on the object? (5.6 N  $77^\circ$  E of N)

5. Forces of  $4.5\text{ N } 38^\circ \text{ N of W}$  and  $3.0\text{ N } 61^\circ \text{ S of W}$  act on an object. What is the net force acting on the object? ( $4.9\text{ N } 2.3^\circ \text{ N of W}$ ; without rounding in calc:  $1.7^\circ \text{ N of W}$ )

6. A force of  $2.2\text{ N East}$ , a force of  $3.3\text{ N West}$  and a force of  $4.4\text{ N north}$  act on an object. What is the net force on the object? ( $4.5\text{ N } 14^\circ \text{ W of N}$ )

7. A  $15.0\text{ kg}$  object is accelerated by a net force of  $12.6\text{ N East}$ . What is the objects acceleration? ( $0.840\text{ m/s}^2 \text{ [E]}$ )

8. A  $925\text{ kg}$  car accelerates uniformly from  $5.0\text{ m/s}$  to  $12.0\text{ m/s south}$  in  $11.0\text{ s}$ . What is the net force causing this acceleration? ( $5.9 \times 10^2\text{ N [S]}$ )

9. A net force of 6.6 N East acts on a 9.0 kg object. The object accelerates uniformly from rest to a velocity of 3.0 m/s east. A) How far did the object travel while accelerating? B) What is the time of the acceleration? (6.1 m [E], 4.1 s)

10. A force of 5.0 N East and a force of 6.7 N East act on an 8.0 kg object. What is the acceleration of this object? (1.5 m/s<sup>2</sup> [E])

11. A force of 22.0 N East and a force of 25.0 N South act on a 4.2 kg cart. What is the acceleration caused by these forces? (7.9 m/s<sup>2</sup> 49° S of E)

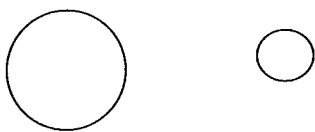
12. A force of 220 N North and a force of 110 N  $15^\circ$  N of E act on an object. If the mass of this object is 12.0 kg, what will its acceleration be? ( $23 \text{ m/s}^2$   $23^\circ$  E of N)

13. Forces of 39 N  $65^\circ$  N of E and 20 N  $31^\circ$  S of W act on an object. The object has a mass of 3.2 kg. What will its resulting acceleration be? ( $7.8 \text{ m/s}^2$   $1.4^\circ$  W of N)

**A. GRAVITY**GRAVITATIONAL FIELD STRENGTH

A gravitational field is a model that is used to describe the effects of gravity. The field strength can be determined using Newton's Law of Universal Gravitation.

A gravitational field forms around every particle or larger mass and is a vector field in which every vector points directly toward the particle or mass.



The magnitude of the field at every point is the same and represents the force per unit of mass on any object in that region of space.

On Earth, we know that the force of gravity acts as an unbalanced force and accelerates objects toward the earth at a rate of  $9.8 \text{ m/s}^2$ .

For every  $1.0\text{kg}$  of mass on the surface of the Earth, it will be pulled down by a force of gravity of  $9.8\text{N}$ .

## FORCE OF GRAVITY

From this, we are able to determine our weight on Earth which is equivalent to the force of the gravity.

Near the Earth's surface we use 9.80 as the value of gravity. However, it is not a constant value. This value depends on the mass of the Earth (or other planet) and the distance from the center of the Earth. This is seen in the formula:

Due to this, the force of gravity on an object on the surface of the Earth will vary depending on location. The Earth is not a perfect sphere and bulges at the equator. Therefore, the  $F_g$  on an object at the equator is less than at one of the poles (larger distance to center of the Earth).

Weight is often confused with mass. →

Mass only depends on the matter an object contains (the atoms + molecules).

Weight is the amount of force pulling an object down toward a surface.

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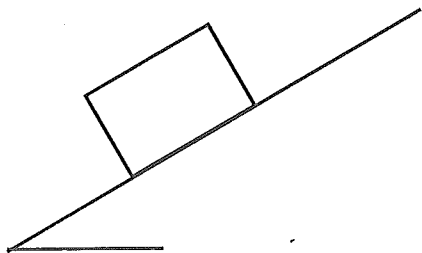
## B. NORMAL (SURFACE) FORCE

The normal force is the perpendicular force that a surface exerts with an object which it is in contact.

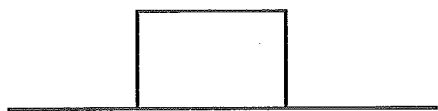
*On a horizontal surface*, the normal force is equal to the weight of the object.



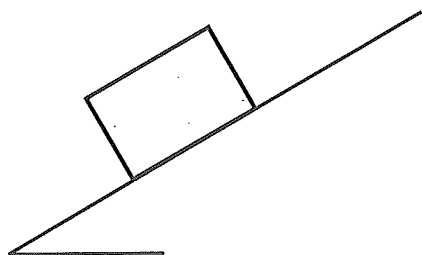
*On an incline, the normal force is equal to the component of weight that is perpendicular to the surface.*



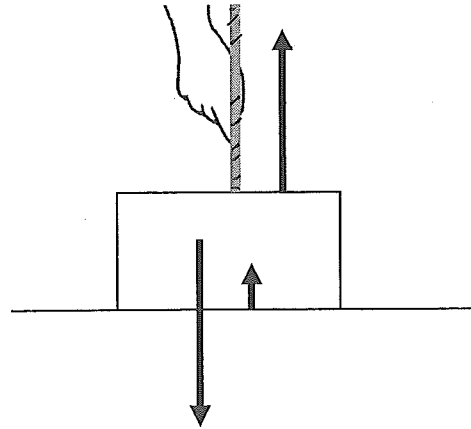
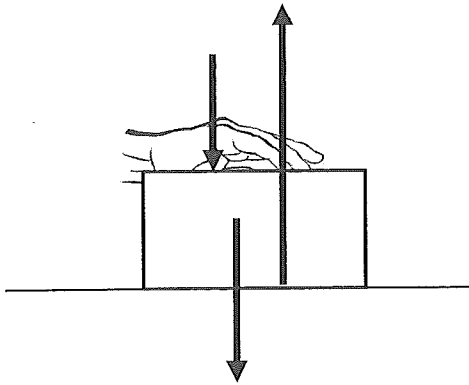
*A 6.7 kg object is resting on a horizontal surface. What is the normal force acting on the object?*



*A 7.6 kg object is at rest on an inclined plane. If the inclined plane makes an angle with the horizontal of  $33^\circ$ , what is the normal force acting on the object?*



There are many other situations when the normal force is **not equal** to the force of gravity, even when on a horizontal surface.



The normal force is also referred to as "apparent weight". This is due to acceleration of an object on a horizontal surface [up] or [down]. Due to inertia, an object will want to stay in its previous state of motion.

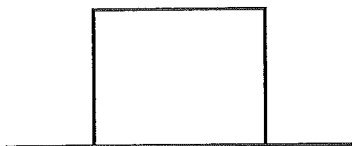
**When accelerating up**, the object will push into the surface below as its inertia maintains its previous state of motion until it is overcome by the unbalanced force of the accelerating surface below it.



**When accelerating down**, the object will pull away from the lower surface for the same reason as above.



When moving up or down at a constant velocity, the forces acting on the object are balanced and the object maintains its state of motion. This means that its weight is the same as if it were at rest. (In both cases, the forces are balanced = no change to state of motion)



### Vertical Motion:

Horizontal  $\sum F \rightarrow$   
 $\downarrow$  or  $\rightarrow$   
 $ma = F_{app} + F_g$

1. With  $F_a$  or  $T$ :  $\sum F = ma = F_{applied} + F_g$



A 1200 kg rocket is launched. The engines have a thrust of  $1.65 \times 10^5$  N. What is the rocket's acceleration?

2. With  $F_N$ :  $\sum F = ma = F_N + F_g$



A 100 kg man is standing in an elevator on a scale. What would the scale read when the elevator is accelerating upward at  $3.0 \text{ m/s}^2$ ?  
 a) down at  $2.0 \text{ m/s}^2$  b) down at  $2.0 \text{ m/s}$  c) down at  $2.0 \text{ m/s}$

## Lesson 2

**Forces Practice Assignment – Complete the following showing all work and with correct free-body diagrams.**

1. A 2.3 kg object is resting on a horizontal surface. What is the normal force acting on the object? (+23 N)
2. A 5.5 kg object is at rest on an inclined plane. If the inclined plane makes an angle with the horizontal of  $43^\circ$ , what is the normal force acting on the object? (+39N)
3. A 10.0 kg object is at rest on an inclined plane. If the inclined plane makes an angle with the horizontal of  $14.0^\circ$ , what is the normal force acting on the object? (+95.0N)
4. An 55Kg passenger is travelling in an 799Kg elevator. The apparent weight of the passenger is 700N. Is the elevator accelerating? If so in which direction? (accelerating up)
5. While on a ride at the amusement park a 33.0Kg boy feels as if he weighs only 203N. What must the acceleration of the ride be? ( $-3.65 \text{ m/s}^2$ )
6. A 61Kg passenger is travelling directly upwards in a helicopter upon take-off. If the helicopter accelerates at a rate of  $3.4 \text{ m/s}^2$ , what is the apparent weight of the passenger? ( $8.0 \times 10^2 \text{ N}$ )

7. While accelerating during take-off in a rocket, the 65 kg astronaut feels much heavier than normal. If he is sitting on a scale, he would calculate his apparent weight to be 1090 N. At what rate is the rocket accelerating? ( $+7.0 \text{ m/s}^2$ )

8. Two masses that each have a mass of 10.0 kg are sitting 10.0 m apart. What is the gravitational field strength produced by the masses? ( $6.67 \times 10^{-11} \text{ N}$ )

9. A 68 kg mass is sitting on the surface of the moon. If the moon's mass is  $7.35 \times 10^{22} \text{ kg}$  and its radius is  $1.74 \times 10^6 \text{ m}$ , what is the force of gravity acting on the 68 kg mass? (110 N)

10. What is the weight of a 15.0 kg object on the surface of the Earth? (-147 N)

11. What is the mass of an object if it has a weight of 110 N on the surface of the Earth? (11.2 kg)

12. An object of mass 10.0 kg is subjected to the following forces: 25.0 N due east, 50.0 N at an angle of  $53.0^\circ$  N of E, and 40.0 N due west. What is the acceleration of the object? ( $4.27 \text{ m/s}^2$   $69.3^\circ$  N of E)

13. You are standing on a scale in an elevator, the scale reads 450 N when the elevator is at rest. The elevator begins to accelerate upward at  $9.00 \text{ m/s}^2$ .

a) What is the new reading on the scale? (+863N)

b) Suppose the elevator was now accelerating downward at  $9.00 \text{ m/s}^2$ . What does the scale read as now? (+36.9N)

14. You push down on an 8.0 kg object with 44 N of force. What is the normal force exerted by the horizontal table surface that it is resting on. (+122N)

15. A rope tied to a 5.0 kg block is pulling up with a force of 38 N. What is the normal force acting on the object from the horizontal surface below? (+11N)

16. A 55.0 kg person is riding in a hot air balloon and a scale aboard shows the person's weight to be 549N. Determine the acceleration of the balloon (magnitude and direction). (+0.182m/s<sup>2</sup>)

17. A 12.0 kg object is thrown vertically into the air with an applied force of 145 N. What is the initial acceleration of the object? (+2.28 m/s<sup>2</sup>)

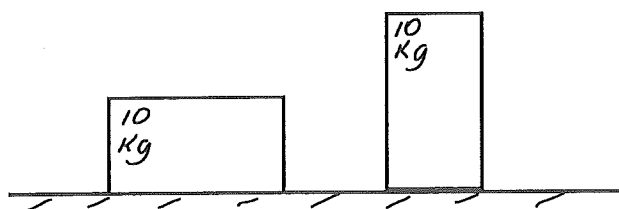
18. A 15.0 kg object is thrown vertically into the air. If the initial acceleration of the object is  $8.80 \text{ m/s}^2$ , what is the applied force? (+279N)

### C. Friction

Frictional Forces are forces that always oppose motion. Recall that the frictional force depends on only two things:

- The normal force acting on the object
- The nature of the two surfaces (indicated by the coefficient of friction  $\mu$ )

Frictional forces DO NOT depend on the area of contact between the two surfaces.



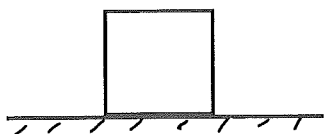
There are two types of friction: Sliding (Kinetic) Friction ( $\mu_k$ ) and Static Friction ( $\mu_s$ )

Static Friction refers to the friction force that must be overcome to start an object moving. This is always a larger frictional force than Sliding Friction which refers to the friction force that must be overcome to keep an object moving.

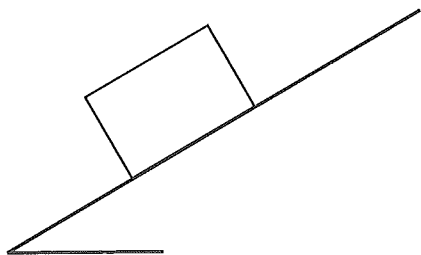
When an object is at rest, it wants to remain at rest (inertia). When an object is moving, it wants to remain moving (inertia). This is why it takes more force to get an object moving than to keep it moving. Hence, static friction is greater than kinetic friction.

$$\mu_s > \mu_k$$

A 7.6 kg object is being pulled along a horizontal surface. If the coefficient of sliding friction between the two surfaces is 0.20, what is the force of friction acting on the object?

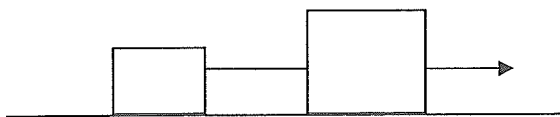


A 7.6 kg object is pulled up an inclined plane. If the inclined plane makes an angle with the horizontal of  $33^\circ$ , and the coefficient of friction is 0.20, what is the force of friction acting on the object?



#### D. Applied Force or Tension

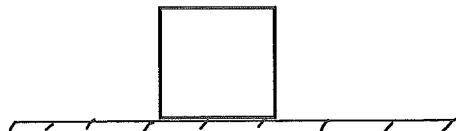
Recall that tension (T) is a force applied through a rope, cable or string. The tension in a string or rope pulls in both directions with the same force.



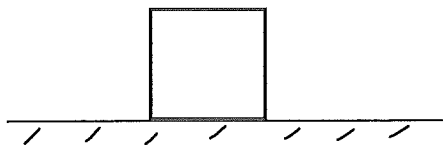
As always, we draw a **free-body diagram** to indicate all forces acting on an object.

An elevator with a mass of  $9.00 \times 10^2$  kg is accelerating downward at a rate of  $1.30 \text{ m/s}^2$ . What is the tension in the cable?

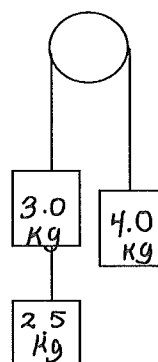
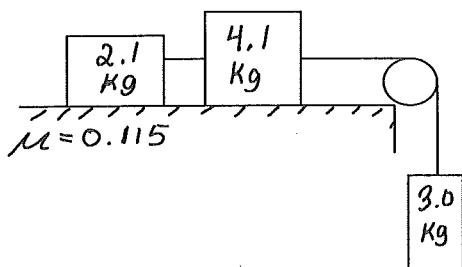
An object that has a mass of 25.0 kg is pulled along a horizontal surface with a force of 95.0N. If the coefficient of friction between the surfaces is 0.34, what is the acceleration of the object?



An object that has a mass of  $45.0\text{kg}$  is pulled along a horizontal surface by a rope that makes an angle of  $32.0^\circ$  with the horizontal. If the coefficient of kinetic friction between the surfaces is  $0.150$ , and the tension in the rope is  $95.0\text{N}$ , what is the acceleration of the object?



Two systems of masses are connected as shown below. Determine the acceleration of the system, and tensional forces along each segment of massless string. (see examples on next two pages)

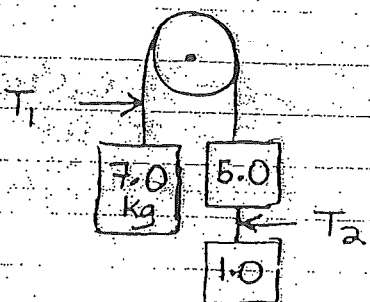


## Pulleys

- Pulleys change the direction of the force so we can combine horizontal ( $ma = F_{app} + F_f$ ) and vertical ( $ma = F_{app} + F_g$ ) problems in one question.
- Pulley and Tension systems are really just special types of elevator questions.
- Systems are questions where all the masses move as one unit which means the acceleration of each mass is the SAME,  $\therefore$  once we know  $F_{net}$  we can find "a" from  $F_{net} = m_{total} \times a$ .

$F_{net} \rightarrow$  what is dropping and what is stopping the dropping.

eg 1



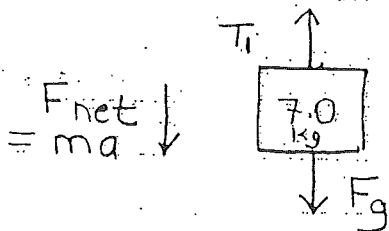
$F_f = 0$  or  $\mu = 0$   
Find <sup>a)</sup>  $T_1$  and <sup>b)</sup>  $T_2$

$$1. F_{net} = ma_g = 1.0 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} = 9.8 \text{ N}$$

(7.0 kg - 6.0 kg = 1.0 kg)  
down up

$$2. a = \frac{F_{net}}{m_{total}} = \frac{9.8 \text{ N}}{13 \text{ kg}} = 0.754 \frac{\text{m}}{\text{s}^2}$$

3. Find  $T_1$ : focus on ONE component of the diagram, draw a FBD

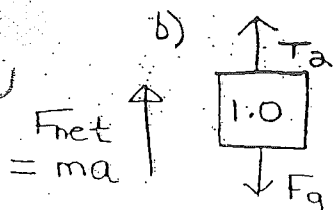


$$ma = F_{app} + F_g$$

$$(7.0 \text{ kg})(-0.754 \text{ m/s}^2) = T_1 + 7.0 \text{ kg}(-9.8 \text{ m/s}^2)$$

(You MUST use NEGATIVE if DOWN)

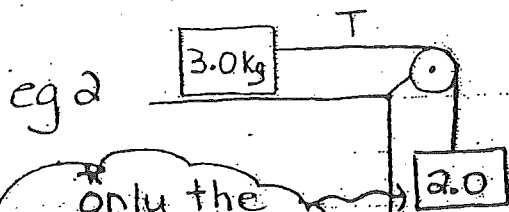
$$-5.28 \text{ N} = T_1 + -68.6 \text{ N} \therefore T_1 = 63.3 \text{ N} = 63 \text{ N}$$



$$ma = F_{app} + F_g$$

$$(1.0 \text{ kg})(+0.754 \text{ m/s}^2) = T_2 + 1.0 \text{ kg}(-9.8 \text{ m/s}^2)$$

$$0.754 \text{ N} = T_2 - 9.8 \text{ N} \therefore T_2 = 10.554 \text{ N} = 11 \text{ N}$$

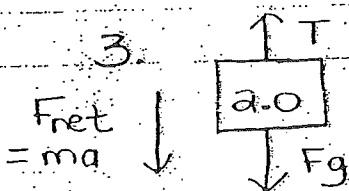


Given:  $\mu = 0$  Find T

only the 2.0 kg mass is causing motion of system

1.  $F_{net} = ma = 2.0 \text{ kg} \times 9.80 \text{ m/s}^2 = 19.6 \text{ N}$

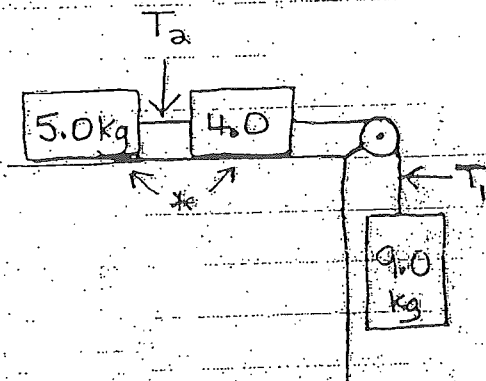
2.  $a = \frac{F_{net}}{m_{total}} = \frac{19.6 \text{ N}}{5.0 \text{ kg}} = 3.9 \text{ m/s}^2$



$ma = T + F_g$   
 $(2.0 \text{ kg})(-3.9 \text{ m/s}^2) = T + (2.0 \text{ kg})(-9.80 \text{ m/s}^2)$   
 DOWN DOWN

$-7.8 \text{ N} = T - 19.6 \text{ N}$   
 $T = 11.8 \text{ N} = 12 \text{ N}$

eg 3



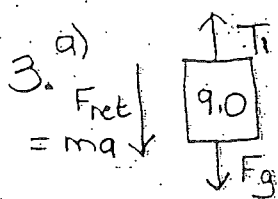
Given:  $\mu = 0.40$  Find a)  $T_1$  b)  $T_2$

1.  $F_{net} = ?$   
 $F_{dropping} = 9.0 \text{ kg}(9.80 \text{ m/s}^2) = 88.2 \text{ N}$

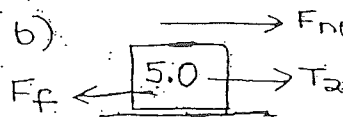
$F_f = \mu F_N = (0.40)(9.0 \text{ kg})(9.80 \text{ m/s}^2) = 35.28 \text{ N}$

$\therefore F_{net} = 88.2 \text{ N} - 35.28 \text{ N} = 52.92 \text{ N} = 53 \text{ N}$   
 opposes dropping

2.  $a = \frac{F_{net}}{m_{total}} = \frac{52.92 \text{ N}}{18.0 \text{ kg}} = 2.94 \text{ m/s}^2 = 2.9 \text{ m/s}^2$



$ma = F_{app} + F_g$   
 $(9.0 \text{ kg})(-2.94 \text{ m/s}^2) = T_1 + (9.0 \text{ kg})(-9.80 \text{ m/s}^2)$   
 $T_1 = 62 \text{ N}$



$ma = F_{app} + F_f$   
 $(5.0 \text{ kg})(2.94 \text{ m/s}^2) = T_2 - 19.6 \text{ N}$   
 $14.7 \text{ N} = T_2 - 19.6 \text{ N}$   
 $T_2 = 34.3 \text{ N} = 34 \text{ N}$

opposes direction of motion

$F_f = \mu F_N$   
 $F_f = (0.40)(5.0)(9.8) = 19.6 \text{ N}$

**Forces- Friction and Tension Assignment – Complete the following showing all work and with correct free-body diagrams.**

1. A block of mass 4.0 kg rests on a horizontal surface. What horizontal force is required to accelerate the block at  $5.0 \text{ m/s}^2$  if,

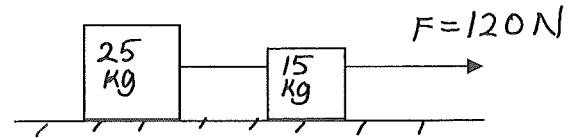
a) There is no friction? (+20N)

b) The coefficient of kinetic friction is 0.25? (+30N)

2. Two boxes are connected by a cord on a horizontal surface. A force  $F$  pulls on the blocks as shown in the diagram. Find the acceleration of the blocks and the tension in the connecting cord if,

a) The surface is frictionless? ( $\pm 3.0 \text{ m/s}^2$ , +75.0N)

b) The coefficient of kinetic friction is 0.20?  
( $\pm 1.04 \text{ m/s}^2$ , +75.0N)



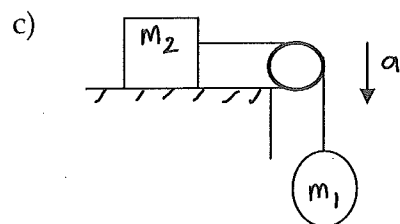
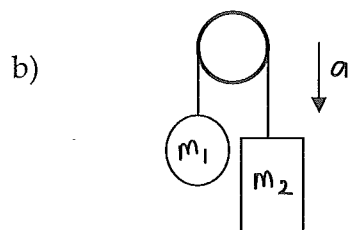
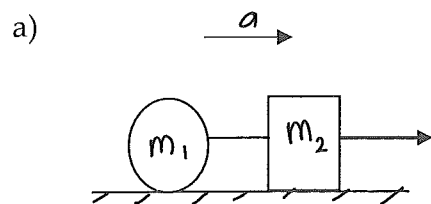
3. Traveling at a speed of 58.0 km/h, the driver of an 800 kg automobile suddenly slams on the brakes. The coefficient of kinetic friction between the tires and the road is 0.720. How far does the car skid before coming to a stop? (Ignore air friction) (18.4 m)

4. A 250 N block rests on a horizontal table. The coefficient of kinetic friction between the table and the block is 0.30. What force is required to pull the block at a constant speed if,

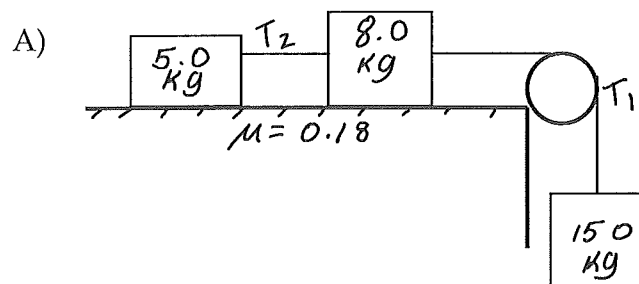
a) The rope is horizontal? (+75 N)

b) The rope makes an angle of  $25^\circ$  above the horizontal? (+83 N)

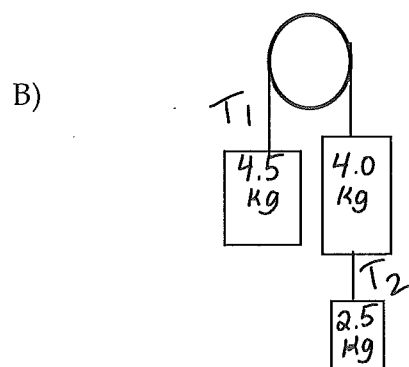
5. Draw a diagram showing the forces acting on each object, and label the forces with appropriate symbols. Ignore the mass and friction of the pulley.



6. Find the acceleration of the system and the tensional forces as labeled in each of the following: (ignore air friction and friction of the pulley)



( $\pm 4.4 \text{ m/s}^2$ ,  $+81 \text{ N}$ ,  $+31 \text{ N}$ )



( $\pm 1.78 \text{ m/s}^2$ ,  $+52 \text{ N}$ ,  $+20 \text{ N}$ )

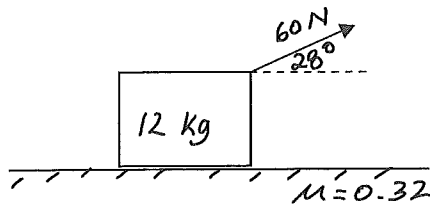
7. A 20.0 N object is placed on a horizontal surface. A force of 3.0 N is required to keep the object moving at a constant speed. What is the coefficient of friction between the two surfaces? (0.15)

8. A 17.0 N object is pulled up an inclined plane. If the inclined plane makes an angle with the horizontal of  $44.0^\circ$ , and the coefficient of friction is 0.300, what is the force of friction acting on the object? (-3.67 N)

9. A 23.0 kg object is pushed with a horizontal force of 25.0 N east across a horizontal surface. If the force of friction between the two surfaces is 12.0 N, what is the acceleration of the object? (+0.565 m/s<sup>2</sup>)

10. An object is pulled east along a horizontal frictionless surface with a steady horizontal force of 12.0 N. If the object accelerates from rest to a velocity of 4.0 m/s while moving 5.0 m, what is the mass of the object? (7.5 kg)

11. A 12 kg crate is pulled along a horizontal floor by a force of 60 N which is applied at an angle of  $28^\circ$  above the horizontal. The coefficient of friction between the crate and floor is 0.32.



- a) Find the magnitude of the normal force acting on the object. (89.4 N)
- b) Find the friction acting on the crate. (-28.6N)
- c) Find the acceleration of the crate. (+2.03 m/s<sup>2</sup>)

# Elevator Lab

+ Day 9 (Additional)  
Exercises 1-4

Purpose: to determine the acceleration of the school  
elevator going UP and DOWN

Down:

$$ma = F_{app} + F_g$$

$$m_{actual} = \text{_____ kg} \therefore F_g = \text{_____} \times -9.80 = \text{_____ N}$$

$$m_{apparent} = \text{_____ kg} \therefore F_{app} = \text{_____}$$

FBD

$$ma = F_{app} + F_g$$

$$a = \text{_____}$$

Up

$$ma = F_{app} + F_g$$

$$m_{actual} = \text{_____ kg} \therefore F_g = \text{_____ N}$$

$$m_{apparent} = \text{_____ kg} \therefore F_{app} = \text{_____ N}$$

FBD

$$ma = F_{app} + F_g$$

$$a = \text{_____}$$

## Physics 12 – Newton's Third Law of Motion

A force is a push or a pull upon an object which results from its interaction with another object. **Forces result from interactions.** According to Newton, whenever two objects interact with each other, they exert equal and opposite forces upon one another.

This is stated as:

For example, when you sit in a chair, your body exerts a downward force on the chair and the chair exerts an upward force on your body.

There are two forces resulting from this interaction — a force on the chair and a force on your body. These two forces are called action and reaction forces.

The statement means that in every interaction, there is a pair of forces acting on the two interacting objects. *The size of the force on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object.*

Recall that in order to be an action-reaction pair, there must be two objects involved.

For example, while the normal force is equal and opposite to the force of gravity on a horizontal surface, this **IS NOT** an action reaction pair as both forces are acting on the same object.

However, if we look at the interaction between the object and the surface below it (table), there is an action-reaction pair.

Newton's Third Law can be illustrated and solved using free-body diagrams and mathematics:

Formula:

Free-Body Diagram:

However, just because the magnitude of the forces acting on each of the objects is the same, does not mean that the resulting motion will be the same.

*Example* – A linebacker ( $m_A = 200\text{kg}$ ) runs into a gymnast ( $m_B = 40\text{kg}$ ) by accident.

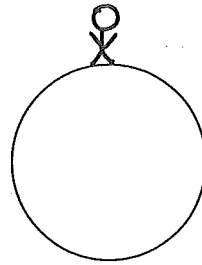
What happens as a result?

The Linebacker's acceleration  $\rightarrow$

The Gymnast's acceleration  $\rightarrow$

## Gravitation: You Attract The Earth!

$$F_{g \text{ you on Earth}} = - F_{g \text{ Earth on you}}$$



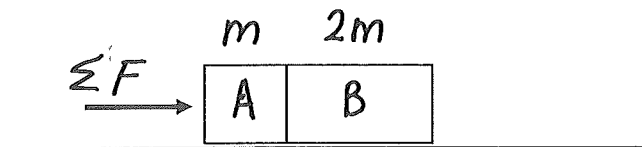
But the acceleration that you produce on the Earth is very, very small.

Two blocks of masses  $m$  and  $2m$  are pushed together along a horizontal, frictionless surface by a force  $F$ . The magnitude of the net force on block B is:

A.  $1/3 F$

B.  $2/3 F$

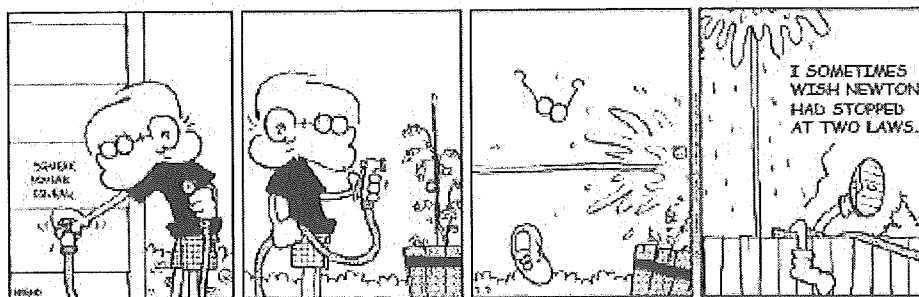
C.  $F$



The magnitude of the force on block A by block B is:

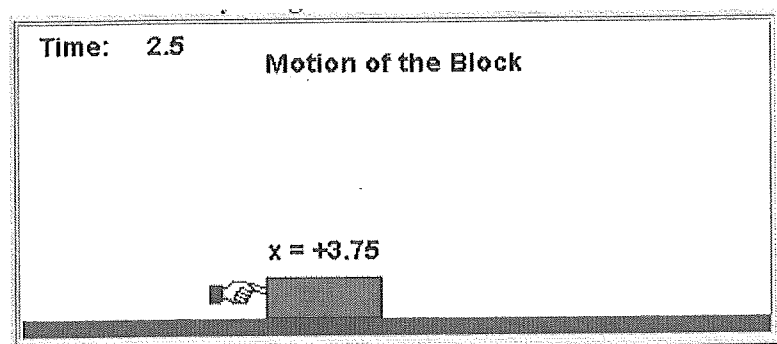
While a football is in flight, what forces act on it? What are the action and reaction pairs while the football is being kicked and while it is in flight?

A  $3.40 \times 10^3$  kg rocket is travelling east along a frictionless track at a velocity of 14.0 m/s. The rocket accelerates uniformly to a velocity of 20.0 m/s in a time of 1.10s by the expulsion of hot gases. What is the force in which the gases are expelled by the rocket?



## Assignment: Newton's 3rd Law and Free Body Diagrams

### Part I: Free Body Diagrams



An 8-kg block is pushed across the floor at a constant speed.

1. Sketch a possible free-body diagram for the block:

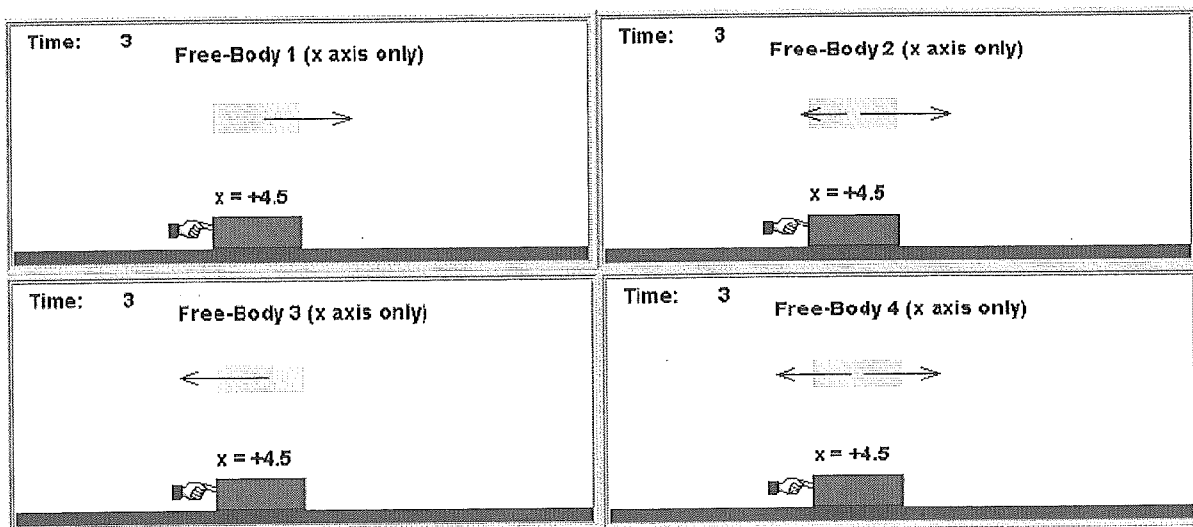
2. What is the acceleration of the block? \_\_\_\_\_

How do you know?

3. Therefore, the net force (sum of all the forces) in the x-direction = \_\_\_\_\_

4. Similarly, the net force (sum of all the forces) in the y-direction = \_\_\_\_\_

Consider the forces in the x-direction (horizontal forces). Consider each of the four possible free-body diagrams below (each diagram is above the block). The length of the vector represents the magnitude of the force.

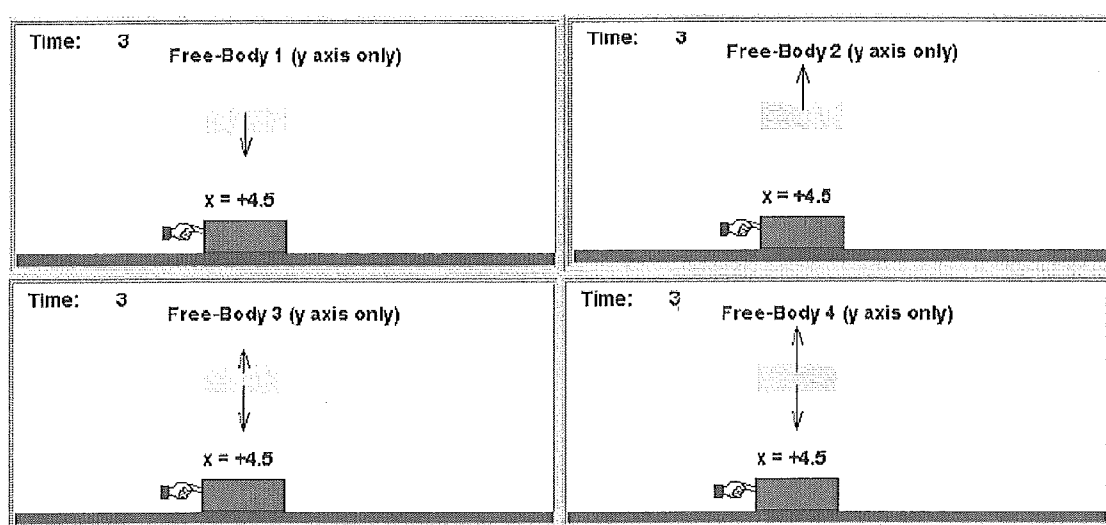


5. Only one is a possible free-body diagram. For each one, indicate why it is or is not a possible force diagram:

Horizontal components:

	Possible force diagram?	Reason:
Free-Body 1x		
Free-Body 2x		
Free-Body 3x		
Free-Body 4x		

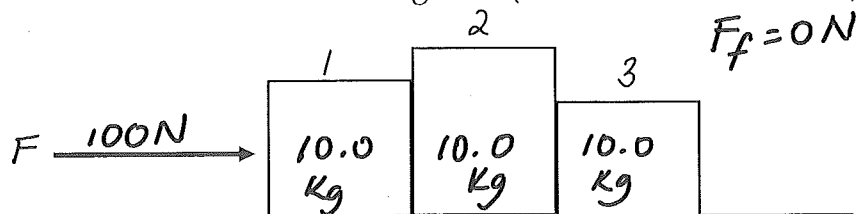
6. Vertical components:



	Possible force diagram?	Reason:
Free-Body 1y		
Free-Body 2y		
Free-Body 3y		
Free-Body 4y		

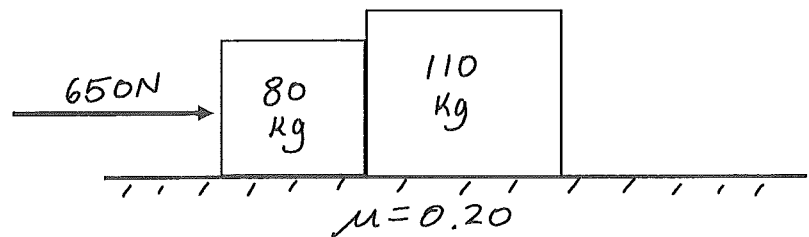
7. Based on this, sketch a complete free-body diagram below:

7. If your weight is force created by gravity on your body, and therefore is the Earth pulling you down, what is the reaction force?
8. There is nothing in outer space for rocket exhaust gasses to push against. How then can a rocket accelerate in outer space?
9. No matter how hard a horse pulls on a cart, the cart must pull back with exactly the same force according to Newton's Third Law. How can a cart pull a horse? (For this question assume the horse and cart are on level ground.)
10. A bug splatters on a fast-moving car's windshield. Is the force on the car from the bug the same as the force on the bug from the car? Explain.
11. Three blocks on a frictionless horizontal surface are in contact with each other. A force of 100N is applied to block 1. Each block has a mass of 10.0 kg.
- A) Determine the acceleration of the system. (+3.33 m/s<sup>2</sup>)
- B) Determine the force that each box exerts on its neighbor. ( $F_{1 \rightarrow 2 \& 3} = +67\text{N}$ ,  $F_{2 \rightarrow 3} = +33\text{N}$ )



12. Two crates, of mass 80 kg and 110 kg, are in contact and at rest on a horizontal surface. A 650 N force is exerted on the 80 kg crate. If the coefficient of friction is 0.20, calculate –

- A) The acceleration of the system ( $+1.46\text{m/s}^2$ )  
B) The force that each crate exerts on the other. (376 N)



13. While standing on a horizontal frictionless surface, two students push against each other. Angela has a mass of 38 kg and, during the push, is accelerating east at a rate of  $0.60\text{ m/s}^2$ . If Bob is accelerating west during the push at a rate of  $0.75\text{ m/s}^2$ , what is his mass? (30 kg)

14. While standing on a horizontal frictionless surface, a 65.0 kg student pushes against a wall with a force of 148 N west for 0.250 s. Calculate the velocity of this student at the end of the 0.250 s. ( $0.569\text{ m/s [E]}$ )

## Lesson 5

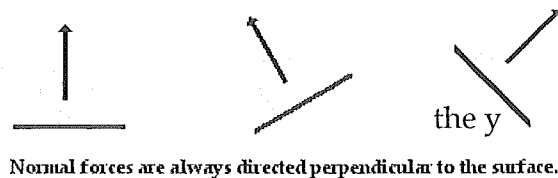
### Physics 12 – The Physics of Inclined Planes

In physics we call any tilted surface an **inclined plane**. As long as the force of friction is not greater than the component of gravity parallel to the surface, the object will slide down the surface.

The rate in which the object will accelerate down an inclined plane depends on how tilted the inclined plane is.

The object will accelerate according to Newton's Laws of motion = unbalanced forces acting on the object.

We already know that the normal force is always perpendicular to the surface and that we can use component of  $F_g \rightarrow F_{\perp}$  to determine the normal force acting on an object on an inclined plane.



$F_{\perp}$  is opposed to the normal force and therefore balances the normal force.

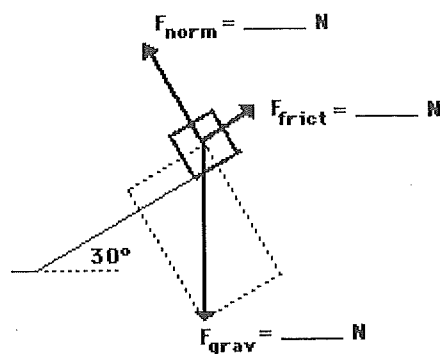
However, we know that an unbalanced force causes the object to accelerate down the inclined plane.

$F_{\parallel}$  is not balanced by any other force.

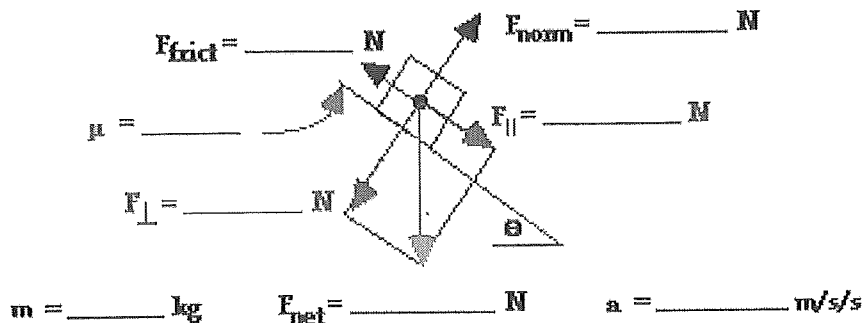
The equations for the parallel and perpendicular components are:

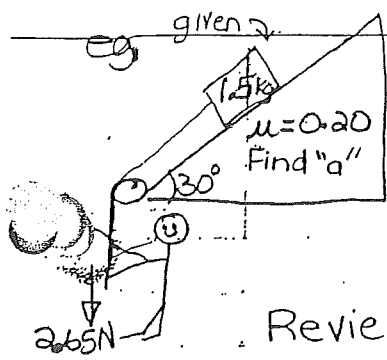
When there is friction or other forces (applied force, tensional forces, etc.) we need to take those into account when determining acceleration.

The diagram shows the forces acting upon a 25.0-kg crate that is sliding down an inclined plane. The plane is inclined at an angle of  $30.0^\circ$ . The coefficient of friction between the crate and the incline is 0.220. Determine the acceleration of the crate.



A 3.5 kg object is accelerating down an inclined plane inclined at  $40.0^\circ$  (with the horizontal) and having a coefficient of friction of 0.415. Fill in all of the blanks.





given:  $m = 1.5 \text{ kg}$ ,  $\mu = 0.20$ ,  $\theta = 30^\circ$ ,  $F_{\text{app}} = 2.65 \text{ N}$  (up),  $F_g$  (down)

①  $F_L = \cos 30^\circ \times 1.5 \times 9.80 = 12.7 \text{ N}$

②  $F_{\text{app}} = 2.65 \text{ N}$  (up)

③  $F_g = \sin 30^\circ \times 1.5 \times 9.80 = 7.35 \text{ N}$  (down)

④  $F_f = \mu F_N = 0.20(12.7 \text{ N}) = 2.55 \text{ N} = 2.6 \text{ N}$  (down)

⑤  $ma = F_{\text{app}} + F_f + F_{\text{Lg}}$

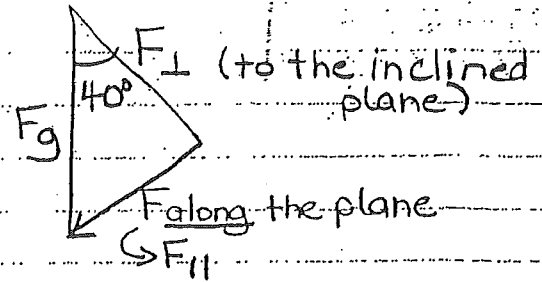
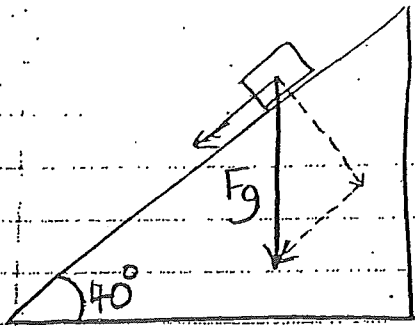
⑥  $1.5a = -2.65 \text{ N} + 2.55 \text{ N} + 7.35 \text{ N}$

⑦  $a = 5.0 \text{ m/s}^2$  (down)

Fabulous Inclined Planes

## Review of Inclined Plane Diagrams:

① Redraw vector diagram:



② Use  $F_g$  to find:  $F_{\perp} = \cos \theta F_g$

$$F_{\parallel} = \sin \theta F_g$$

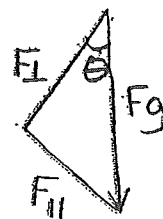
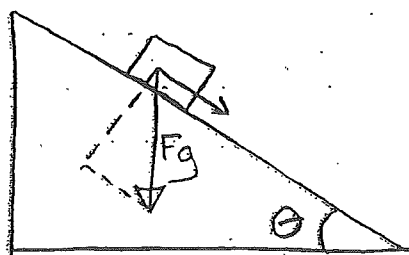
$$(\Sigma F = ma)$$

③ Use:  $ma = F_{\text{Lg}} + F_f + F_{\text{app}}$

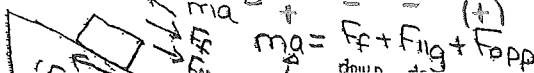
calculate from  $F_{\parallel}$   
 $F_{\parallel} = \sin \theta F_g$

given  $F_{\perp}$  calculate from  $F_N$   
 $F_f = \mu F_N$   
 $(m \rightarrow F_g \rightarrow F_{\perp} \rightarrow F_N \rightarrow F_f)$

Remember to reverse the diagram if the inclined plane is reversed:



$\Sigma F = ma$  a) moving up



b) moving down.



eg What if you have an inclined plane: tension system

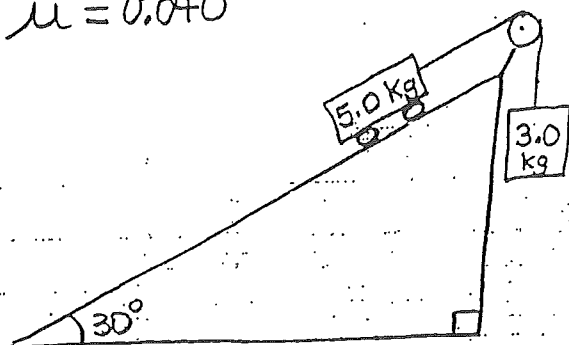
Given:

$$\mu = 0.040$$

Find:

$$a =$$

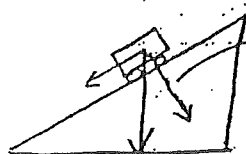
$$T =$$



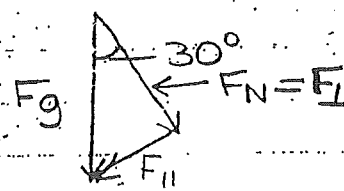
1.  $F_{net} = ?$  (or  $\Sigma F = ?$ )

$$F_{dropping} = m a_g = 3.0 \text{ Kg} \times 9.80 \text{ m/s}^2 = 29.4 \text{ N}$$

$$F_f =$$



re draw  
vector  
diagram



$$F_{\perp} = F_N$$

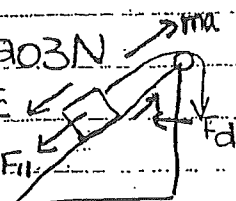
$F_{\text{stopping the dropping}} \rightarrow F_f = \mu F_N = (0.040)(\cos 30^\circ)(5.0 \text{ Kg})(9.80 \text{ m/s}^2) = 1.697 \text{ N}$

$F_{\parallel} = \sin 30^\circ \times (5.0 \times 9.80 \text{ m/s}^2) = 24.5 \text{ N}$

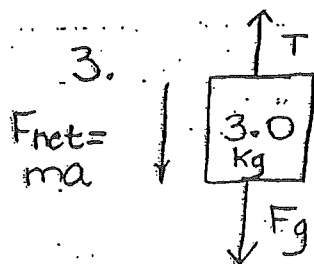
$\therefore F_{net} = 29.4 \text{ N} + 24.5 \text{ N} - 1.697 \text{ N} = 3.203 \text{ N}$

\* from  $(F_{net} = F_{\text{up}} + F_{\text{down}} + F_{\text{down}})$  \*

\* friction direction depends on the direction system moves



2.  $a = \frac{F_{net}}{m_{total}} = \frac{3.203 \text{ N}}{8.00 \text{ Kg}} = 0.400 \frac{\text{m}}{\text{s}^2}$



$$ma = T + F_g$$

$$(3.0 \text{ Kg})(-0.400 \frac{\text{m}}{\text{s}^2}) = T + 3.0 \text{ Kg}(-9.8 \text{ m/s}^2)$$

$$-1.20 \text{ N} = T + -29.4 \text{ N}$$

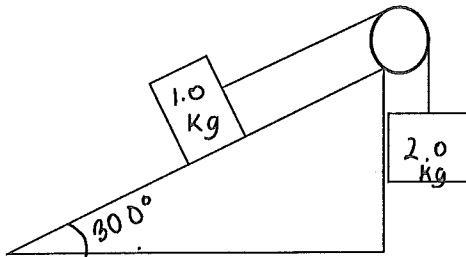
$$\therefore T = 28.2 \text{ N} = 28 \text{ N}$$

## Lesson 5

### Inclined Plane Assignment – Draw a free-body diagram for each question.

- ✓ 1. A 445 N box is sliding down a frictionless  $25.0^\circ$  inclined plane. Find the force (component of weight) that causes the box to slide. (188 N)
  
2. A 325 N box is sliding down a frictionless inclined plane. If the incline makes a  $30.0^\circ$  angle with the horizontal, what is the acceleration along the incline? ( $4.91 \text{ m/s}^2$  down incline)
  
- ✓ 3. A 275 N box is sliding down a  $35.0^\circ$  incline. If the force of friction acting on the box along the incline is 96.0 N, what is the acceleration of the box? ( $2.21 \text{ m/s}^2$ )
  
- ✓ 4. A 435 N box is sliding down a  $40.0^\circ$  incline. If the acceleration of the box is  $0.250 \text{ m/s}^2$ , what is the force of friction on the box? (-269 N)
  
- ✓ 5. A student pulls a 125 N object up a  $23.0^\circ$  inclined plane. If the coefficient of friction between the object and the incline is 0.180, what force must the student apply to pull the object up the incline at a constant velocity? (+69.5 N)

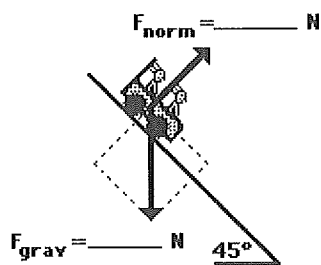
- ✓6. Two blocks are tied together with a string as shown in the diagram. If both the pulley and the incline are frictionless,
- A) What is the acceleration of the 1.0 kg block up the incline? ( $\pm 4.9 \text{ m/s}^2$ )
- B) What is the tension in the string joining the two blocks? ( $+9.8 \text{ N}$ )



7. If the pulley was frictionless but the coefficient of friction between the incline and the 1.0 kg block is 0.25, what would the acceleration of the 1.0 kg block in the question 6 be? ( $\pm 4.2 \text{ m/s}^2$ )

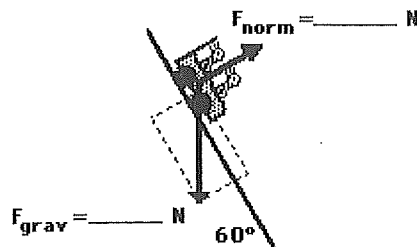
8. The two diagrams below depict the free-body diagram for a 1000-kg roller coaster on the first drop of two different roller coaster rides. Determine the **net force** and **acceleration** of the roller coaster cars. Assume no friction and no air resistance.

Diagram A



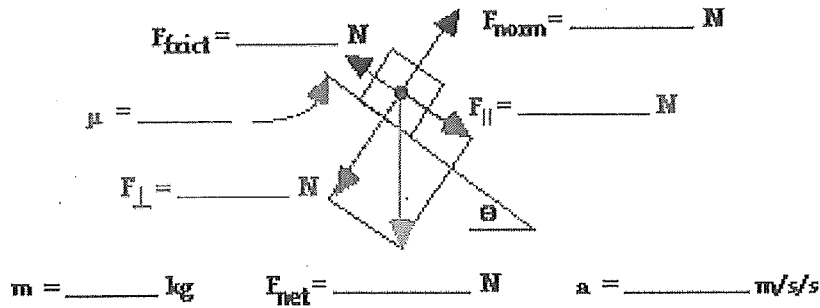
$m = 1000 \text{ kg}$   
 $a = \text{_____ m/s/s}$   
 $F_{\text{net}} = \text{_____ N}$

Diagram B



$m = 1000 \text{ kg}$   
 $a = \text{_____ m/s/s}$   
 $F_{\text{net}} = \text{_____ N}$

9. A 2.75-kg object is accelerating down an inclined plane inclined at  $26.0^\circ$  (with the horizontal) and having a coefficient of friction of 0.433.



10. A 22.5-kg box slides down an inclined plane (inclined at  $33.0$  degrees) at a constant speed of 2.1 m/s. What is the force of friction acting on the box? ( $-1.2 \times 10^2 \text{ N}$ )

11. A 1200 N truck slides down a frictionless plane inclined at an angle of  $25.0^\circ$  from the horizontal.

- Find the acceleration of the trunk. (frictionless) ( $+4.16 \text{ m/s}^2$ )
- Find the acceleration of the trunk if the coefficient of friction was 0.300. ( $+1.48 \text{ m/s}^2$ )

12. A 200 g pen placed on the cover of a book just begins to move when the cover makes an angle of  $41^\circ$  with the horizontal. What is the coefficient of static friction? (0.87)

✓ 13. A car weighing 1600 N is parked on a  $32^\circ$  slope. The brakes fail and the car starts to slide down the hill. Assume no friction.

A. What is the acceleration of the car? ( $+5.2\text{m/s}^2$ )

B. After it has moved 41.0 m, how fast is it moving? ( $+21\text{ m/s}$ )

✓ 14. A 10.0 kg box sits on an incline. Its coefficient of friction is 0.270, and the incline is at an angle of  $30.0^\circ$  to the horizontal. Find the acceleration of the block. ( $+2.61\text{m/s}^2$ )

✓ 15. A 16.0 kg block is released from the top of an incline that is 5.00 m long and makes an angle of  $38.0^\circ$  to the horizontal. A force of friction of 45.0 N acts on the box.

A. Find the acceleration of the box. ( $+3.22\text{m/s}^2$ )

B. How long will it take the box to reach the bottom of the incline? (1.76s)

C. What is the coefficient of friction? (0.363)

## Lesson 6

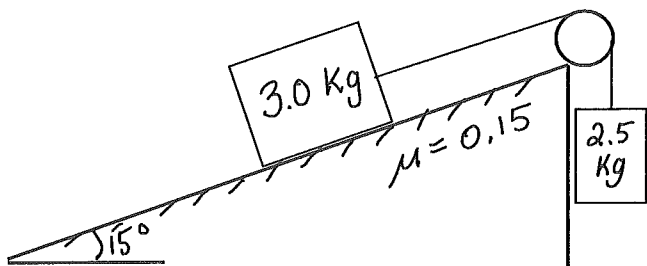
(+ Additional Exercises)  
10, 11, 12 & 14

### Physics 12 – Inclined Planes and Tension

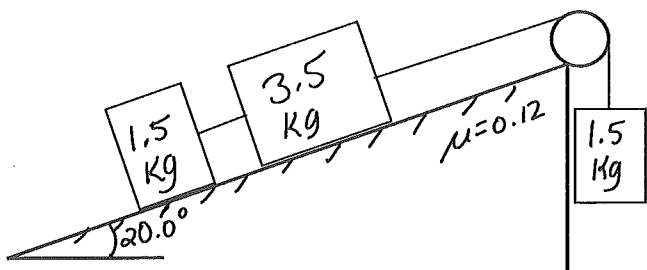
Name: \_\_\_\_\_

Now that you understand how to determine basic forces and acceleration on an inclined plane, we will now add systems into our observations and calculations.

Recall:



Example Two:

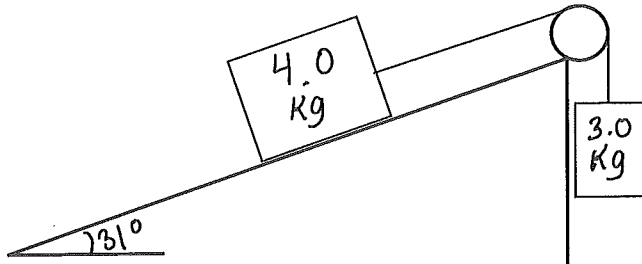


# Lesson 6

Day 4  
+ Additional Exercises

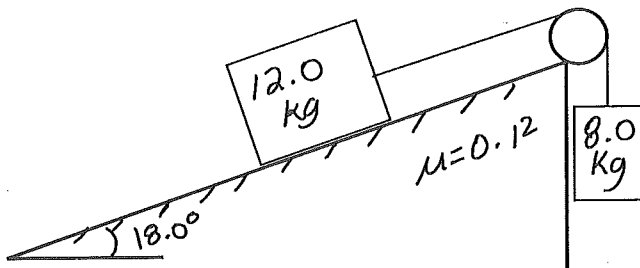
## Assignment Questions:

1. Find the acceleration and tension for the following frictionless system.



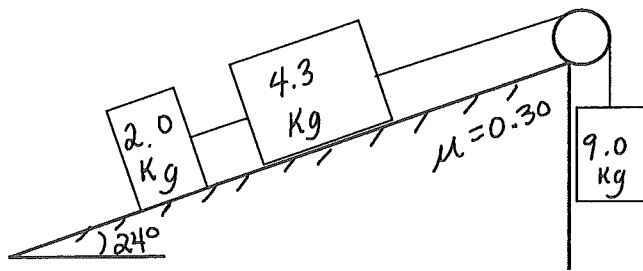
( $\pm 1.32 \text{ m/s}^2$ , +25 N)

2. Find the acceleration and tension for the following system.



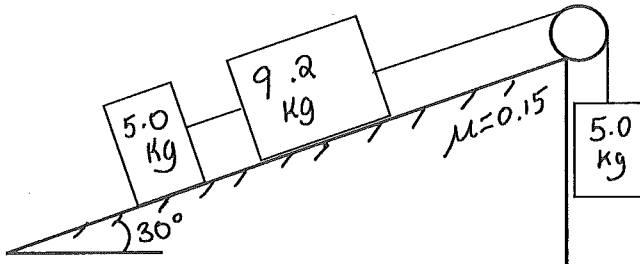
( $\pm 1.44 \text{ m/s}^2$ , +67 N)

3. Find the acceleration and the tension for both strings for the system.



( $\pm 3.02 \text{ m/s}^2$ ,  $+61 \text{ N}$ ,  $+19 \text{ N}$ )

4. Find the acceleration and the tension for both strings for the system.



( $\pm 0.130 \text{ m/s}^2$ ,  $+50 \text{ N}$ ,  $+18 \text{ N}$ )

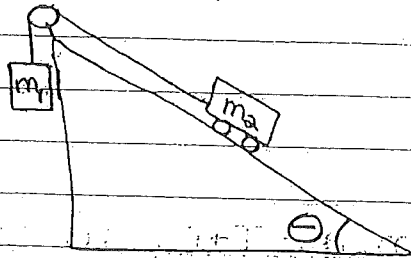
# Lesson 7

## Inclined Plane Lab - Physics 12

Purpose: to calculate " $\mu$ " (the coefficient of friction) for steel on steel.

A. Label the following incline planes showing the direction of  $F_f$ ,  $F_{app}$  (or  $T$ ),  $F_{lig}$  and  $\Sigma F \leftrightarrow ma$

1. going UP the incline plane:



$$m_1 = 273g$$

$$m_2 = 300g$$

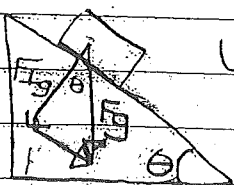
$$\theta = 40.0$$

2. Calculate " $a$ " from:  $d = 0.80m$ ,  $v_0 = 0.0m/s$ ,  $t = 2.0s$

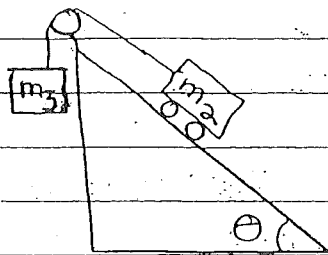
b. Use  $m_1 a = T + F_g$  to calculate  $T$  ( $F_{app}$ ):

c. Use  $m_2 a = F_{app} + F_{lig} + F_f$  to find  $F_f$  (or  $T$ ):

d. Use  $F_f = \mu F_N$  (where  $F_N = F_{\perp}$ ) to find  $\mu$ :



2. Going DOWN the incline plane:



$$m_2 = 300\text{g}$$

$$m_3 = 129\text{g}$$

$$\theta = 40.0^\circ$$

a Calculate "a" from:  $d = 0.80\text{m}$ ,  $v_0 = 0.0\text{m/s}$ ,  $t = 1.4\text{s}$

b. Use  $m_3 a = T + F_g$  to calculate  $T \leftrightarrow F_{app}$ :

c. Use  $m_2 a = F_{app} + F_f + F_{fg}$  to find  $F_f$ :

d. Use  $F_f = \mu F_N$  (where  $F_N = F_L$ ) to find  $\mu$ :

B. Calculate the % difference for the two " $\mu$ " values:

$$\% \text{ difference} = \frac{A - B}{\frac{A + B}{2}} \times 100$$

**Lab - Velocity and Acceleration on an Incline**

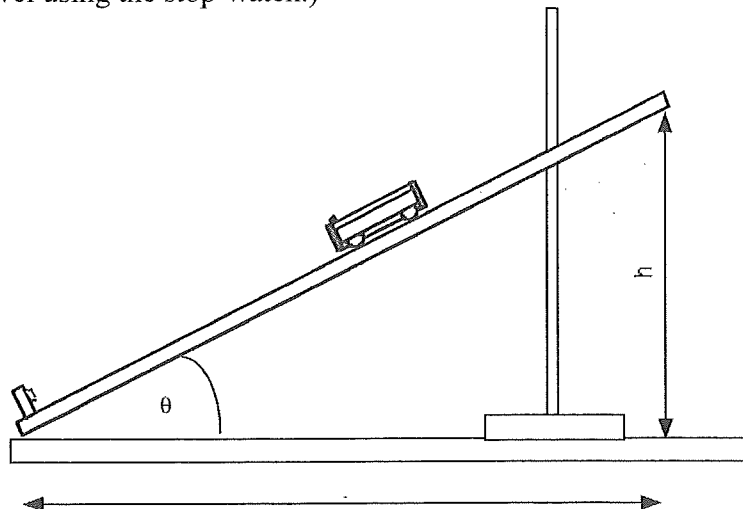
**Theory:** According to Newton's law an unbalanced force results in acceleration. Therefore, an object kept on a frictionless inclined plane will move down the plane. We experience this first hand while skiing downhill or bicycling down an incline. We know from experience that steeper the slope, the faster the acceleration. However, you may wonder if the acceleration is similar if you were twice or half as heavy?

**Purpose:** To determine if there is any dependence of the mass on the acceleration due to gravity by calculating the acceleration due to gravity on different masses rolling down an inclined plane.  
To determine the effect of changing the angle of incline on the acceleration due to gravity on a mass rolling down an inclined plane.

**Materials:** Cart, masses, meter stick, stopwatch, ring stand.

**Procedure:**

1. Configure the as shown in the diagram below. Set the inclination of the track to approximately  $2.0^\circ$ , but measure  $h$  and  $d$  to determine the angle  $\theta$  accurately. (Note that the inclination of the track is greatly exaggerated in the diagram. The experiment will work equally well at larger inclination angles but you may encounter more difficulties in timing the travel using the stop-watch.)



2. Place the cart on the track at least 80 cm from the end-stop. Try a few trial runs to see how the cart behaves as you allow it to roll on the incline. Do you observe that the cart starts from rest and the speed increases and appears to have its maximum velocity just as it collides with the end stop?

3. Next use the stop-watch to measure the time the cart takes to travel the distance (d) down the incline. Repeat your measurements five times. Note the individual times and the average time in your data table. Are all the recorded times the same? How confident are you that you started the stopwatch as the cart was released and stopped the timer when the cart collided with the end-stop?
4. Next, we investigate if the acceleration depends on the mass of the cart. Use the two supplied masses and chose the same distance d as you had in your first trial (step 3). Measure the time t for 5 trials. Calculate the acceleration directly using the formula  $d = v_0 t + \frac{1}{2} a t^2$  ( $v_0 = 0$ ) in the data analysis section. Is the acceleration corresponding to the three masses of the cart constant?
5. Next, increase the angle of inclination of the track to  $3.0^\circ$ . You may use only the cart itself without the additional masses. Measure the time taken to travel the same distance (d) and record the trials and average time in the data table. Calculate the acceleration in the data analysis section. Does the acceleration change? Repeat for an angle of  $4.0^\circ$ .
6. Draw a free-body diagram for each situation in your Data Analysis section and calculate the acceleration on the object using the parallel component of the weight. Use a ruler.
7. Compare the acceleration determined by the kinematics formula and that determined by the forces method.

#### Data and Observations:

#### PART ONE:

Data for a cart of different masses rolling down an incline at  $2.0^\circ$  for a distance of 0.80m

Mass (kg)	Time Trials					Average Time
	1	2	3	4	5	
0.500 kg						
0.750 kg						
1.00 kg						

## PART TWO

Data for a 0.500kg cart rolling down 0.80m with multiple incline angles

Angle $\angle (^{\circ})$	Time Trials					Average Time
	1	2	3	4	5	
2.0 <sup>o</sup>						
3.0 <sup>o</sup>						
4.0 <sup>o</sup>						

Data Analysis:

## PART ONE:

Calculate the acceleration for each mass using  $d = v_0 t + \frac{1}{2} a t^2$ .

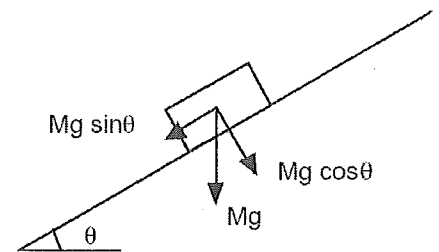
0.500 kg →

0.750 kg →

1.00 kg →

Draw a diagram and calculate the acceleration based on its weight.

0.500 kg  $\rightarrow$



0.750 kg  $\rightarrow$

1.00 kg  $\rightarrow$

## PART TWO:

Calculate the acceleration of the cart at the different angles of incline using  $d = v_0 t + \frac{1}{2} a t^2$ .

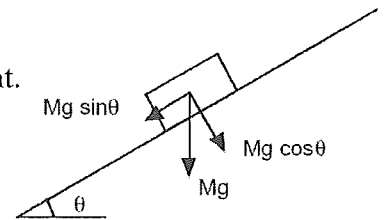
2.0°  $\rightarrow$

3.0° →

4.0° →

Draw a diagram and calculate the acceleration based on its angle and weight.

2.0° →



3.0° →

4.0° →

**Conclusion:**

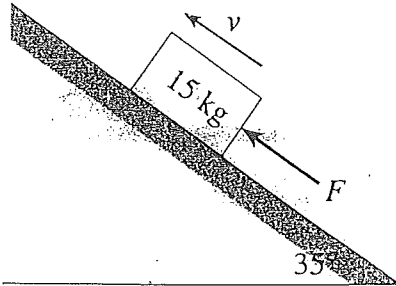
*In a paragraph, address the following. Did the change in mass affect the acceleration? Use your results in your explanation. Did the angle of incline affect the acceleration? Use your results in your explanation. Describe how could you use your results to determine the force of friction between the cart and incline.*

# Dynamics Provincial Exam Questions

## Lesson 8

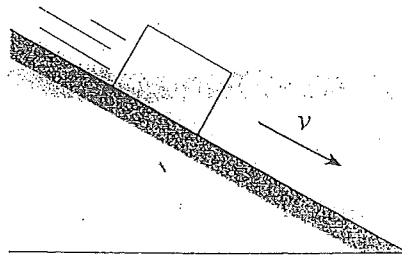
43

1. A 45 kg woman is standing in an elevator that is accelerating downwards at  $2.0 \text{ m/s}^2$ . What force (normal force) does the elevator floor exert on the woman's feet during this acceleration?
2. A 15 kg block is pushed up a  $35^\circ$  incline. A friction force of 110 N exists between the block and the incline.

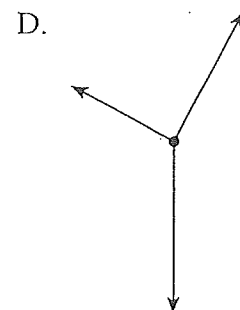
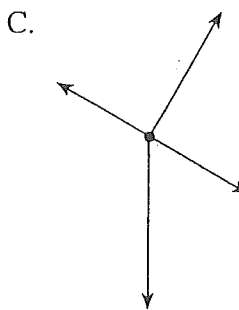
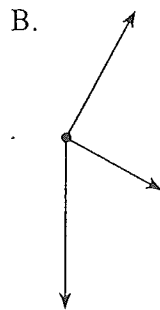
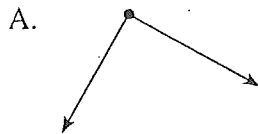


What minimum force  $F$ , would be necessary to move the block up the incline at a constant speed?

3. An object is sliding down an inclined plane at a constant speed.

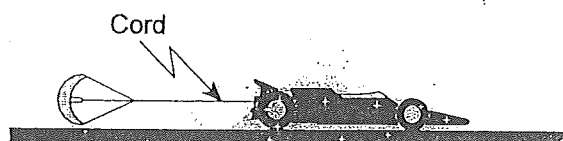


Which of the following represents the free-body diagram for the object?



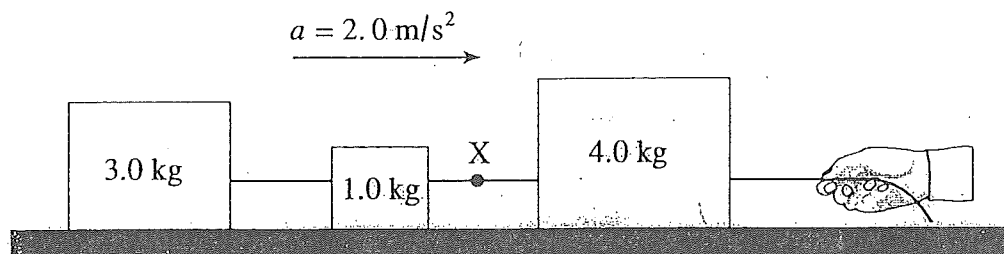
4. A book is at rest on a desk. Which of the following statements concerning the book is correct?
- The desk exerts no force on the book.
  - The book exerts no force on the desk.
  - There are no forces acting on the book.
  - The forces acting on the book are balanced.

5. An 810 kg dragster is being decelerated by a parachute at  $2.5 \text{ m/s}^2$  as shown in the diagram.



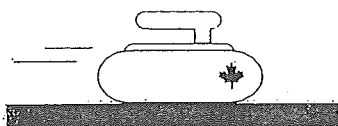
What is the tension in the cord at this moment?

6. The system of blocks on a frictionless surface in the diagram below is accelerating at  $2.0 \text{ m/s}^2$ .

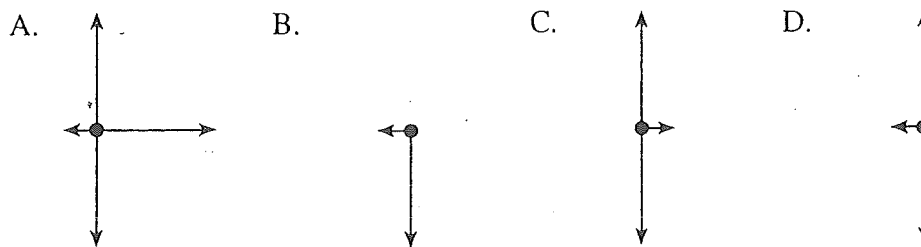


What is the tension in the cord at X?

7. A curling rock is travelling to the right across the ice as shown in the diagram.



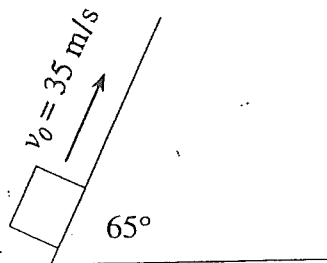
Which of the following best represents the forces acting on the curling rock?



8. A constant net force acting on an object results in the object having a constant

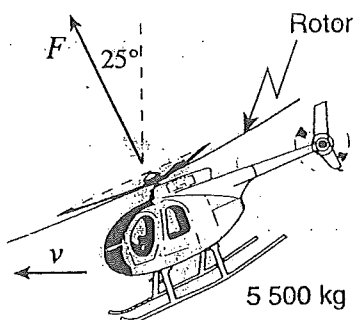
- velocity.
- momentum.
- acceleration.
- kinetic energy.

9. An object is fired up a frictionless ramp as shown in the diagram.



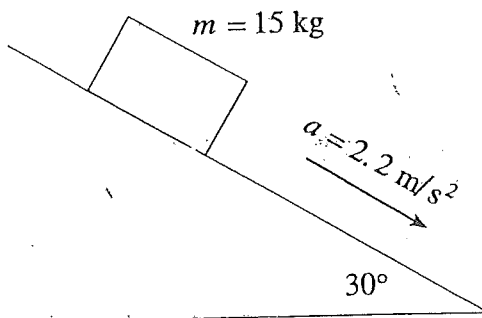
If the initial velocity is  $35 \text{ m/s}$ , how long does the object take to return to the starting point?

10.



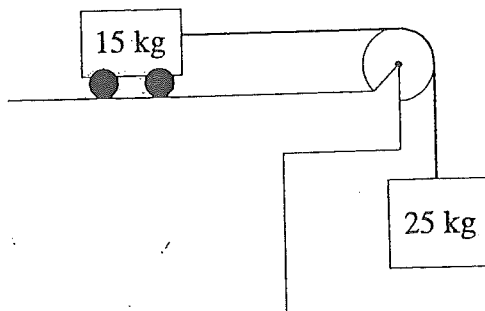
What is the force  $F$  provided by the rotor?

11. A  $15 \text{ kg}$  block has a constant acceleration of  $2.2 \text{ m/s}^2$  down a  $30^\circ$  incline.



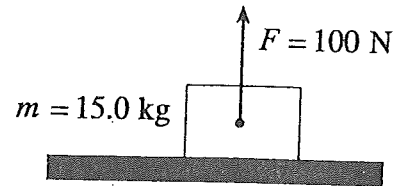
What is the magnitude of the friction force on the block?

12. A  $15 \text{ kg}$  cart is attached to a hanging  $25 \text{ kg}$  mass. Friction is negligible.



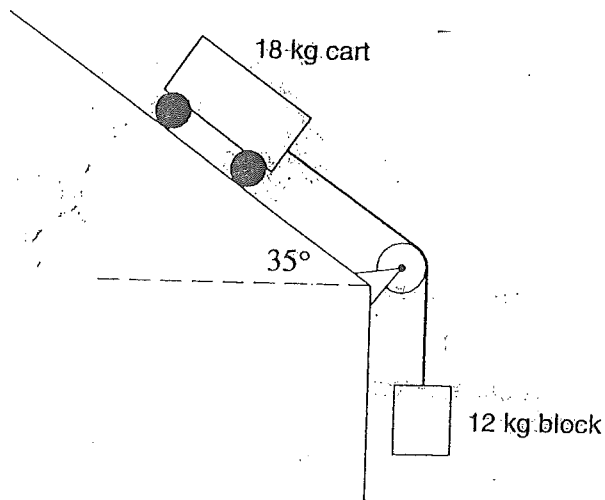
What is the acceleration of the  $15 \text{ kg}$  cart?

13. A 15 kg block on a horizontal surface has a 100 N force acting on it as shown.



What is the normal force?

14. An 18 kg cart is connected to a 12 kg hanging block as shown. (Ignore friction.)



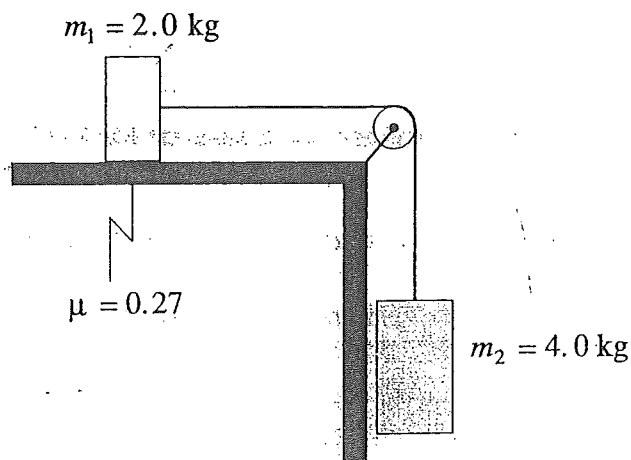
- a) Draw and label a free body diagram for the 18 kg cart.

(2 marks)

- b) What is the magnitude of the acceleration of the cart?

(5 marks)

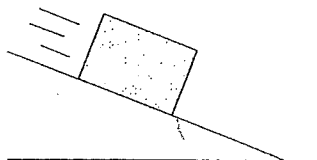
15. Two masses are connected by a light string over a frictionless massless pulley. There is a coefficient of friction of 0.27 between mass  $m_1$  and the horizontal surface.



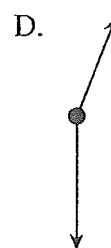
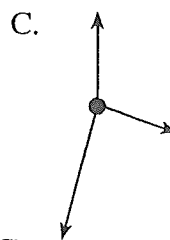
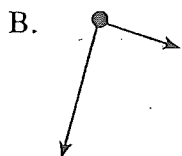
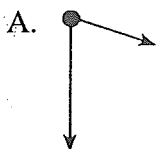
- a) Draw and label a free body diagram showing the forces acting on mass  $m_1$ . (2 marks)

- b) What is the acceleration of mass  $m_2$ ? (5 marks)

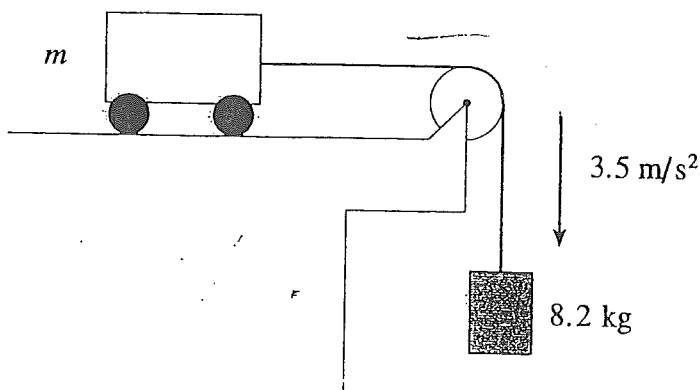
16. A block is on a frictionless incline.



Which of the following is a correct free body diagram for the block?

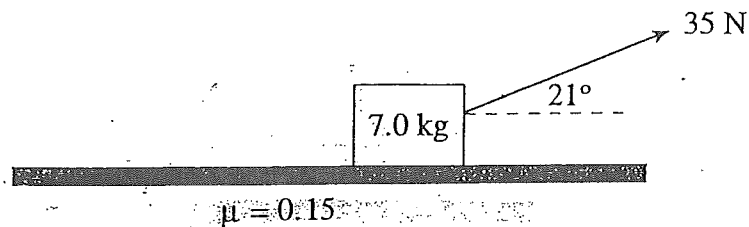


17. A cart on a frictionless surface is attached to a hanging mass of 8.2 kg.



If this system accelerates at  $3.5 \text{ m/s}^2$ , what is the mass  $m$  of the cart?

18. A 35 N force applied at  $21^\circ$  to the horizontal is used to pull a mass as shown.

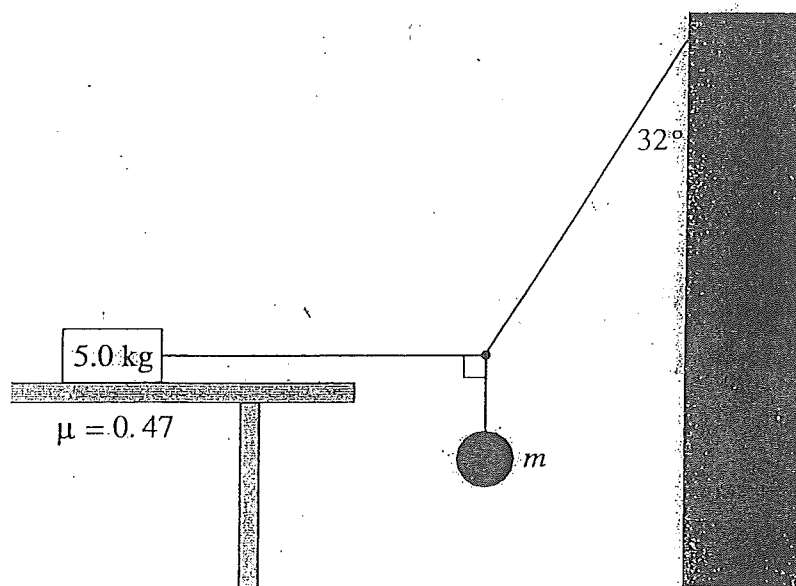


The coefficient of friction between the floor and the mass is 0.15.

- a) Draw and label a free body diagram showing the forces acting on the mass. (2 marks)

- b) What is the acceleration of the mass? (5 marks)

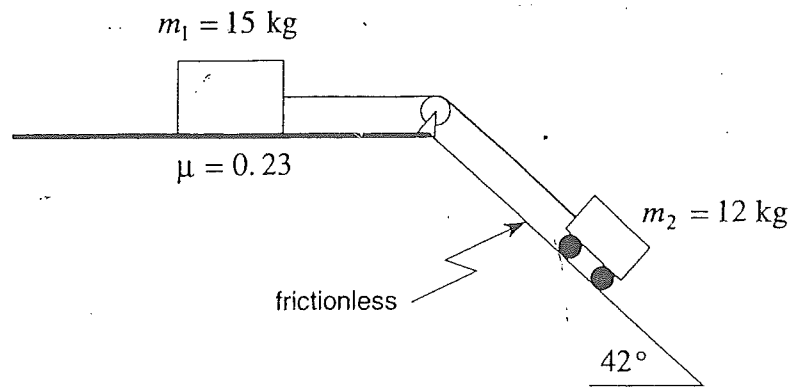
19. An object of mass,  $m$ , is suspended by two cords connected to a wall and to a 5.0 kg block resting on a table as shown.



A coefficient of friction of 0.47 exists between the 5.0 kg block and the table. What is the maximum mass,  $m$ , that can be hung from the cords before the 5.0 kg block begins to move?

(7 marks)

20. Two objects are connected as shown. The 12 kg cart is on a frictionless  $42^\circ$  incline while the 15 kg block is on a horizontal surface having a coefficient of friction  $\mu = 0.23$ .



Determine the acceleration of the system of masses.

(7 marks)

# Dynamics Provincial Exam Questions

1.  $3.5 \times 10^2 \text{ N}$

2.  $1.9 \times 10^2 \text{ N up}$

3. D

4. D

5.  $2.0 \times 10^3 \text{ N}$

6.  $8.0 \text{ N}$

7. D

8. C

9.  $7.9 \text{ s}$

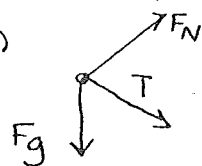
10.  $5.9 \times 10^4 \text{ N}$

11.  $41 \text{ N}$

12.  $6.1 \text{ m/s}^2$

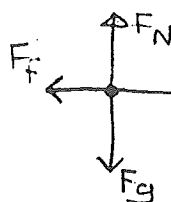
13.  $47 \text{ N}$

14. a)



b)  $7.3 \frac{\text{m}}{\text{s}^2}$

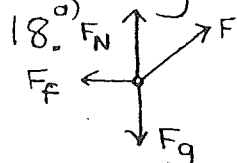
15.



b)  $5.7 \frac{\text{m}}{\text{s}^2}$

16. D

17.  $15 \text{ kg}$



b)  $3.5 \frac{\text{m}}{\text{s}^2}$

19.  $m = 3.8 \text{ kg}$

20.  $1.7 \text{ m/s}^2$

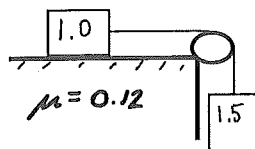
# Additional Exercises

## Physics 12 – Forces Assignment

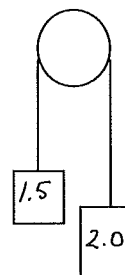
Name: \_\_\_\_\_

Draw a free-body diagram labelling all forces acting on the object and the direction of acceleration. **SHOW ALL OF YOUR WORK AND A FREE-BODY DIAGRAM FOR EACH QUESTION** (or you will not receive full credit for each question). \*This will be handed in for assessment.

1. A 1.0 kg box on a horizontal surface is accelerated by attaching a 1.5 kg mass as shown in the diagram. What is the acceleration of the box if the coefficient of friction between the surfaces is 0.12? ( $\pm 5.4 \text{ m/s}^2$ )

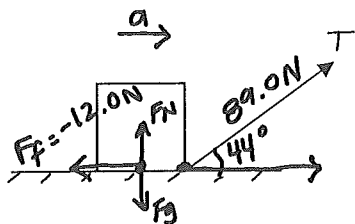


2. Two masses of 1.5 kg and 2.0 kg are hung on a frictionless pulley as shown in the diagram below. What is the acceleration of -  
A) the 1.5 kg mass? ( $+1.4 \text{ m/s}^2$ )



- B) the 2.0 kg mass? ( $-1.4 \text{ m/s}^2$ )

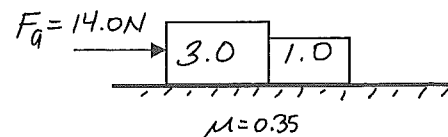
3. A 125 N box is pulled east along a horizontal surface with a force of 89.0 N acting at an angle of  $44.0^\circ$  as shown in the diagram. If the force of friction on the box is 12.0 N, what is the acceleration produced? ( $+4.06 \text{ m/s}^2$ )



4. A 725 N student stands on a bathroom scale while riding in an elevator. The student observes that the scale reads 775 N as the elevator begins to move. Find the acceleration and direction of acceleration of the elevator. ( $+0.676 \text{ m/s}^2$ )

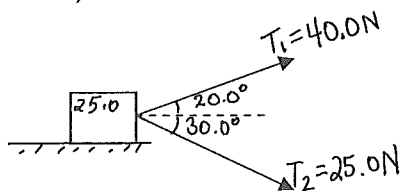
5. An 8.0 kg object is pulled vertically upward by a rope. If the tension in the rope is constant at 95 N, what is the velocity of the object after 1.1 s if it is starting at rest? ( $+2.3 \text{ m/s}$ )

6. A force of 14.0 N is applied to the block as shown in the diagram. If the coefficient of friction between the blocks and horizontal surface is 0.35,  
A) What is the acceleration of the two blocks? ( $+0.070 \text{ m/s}^2$ )



- B) What is the net force that the 3.0 kg block exerts on the 1.0 kg block? ( $+0.21 \text{ N}$ )

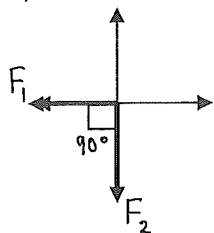
7. Two students are dragging a 25.0 kg object along the hall as shown in the diagram below. If the force of friction acting on the object is 5.0 N, what is the acceleration of the object? ( $+2.2 \text{ m/s}^2$ )



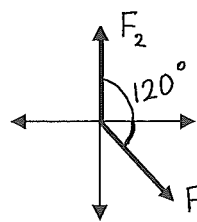
8. A 500 N force acts in the northwestern direction. A second 500 N force must be exerted in what direction so that the resultant of the two vectors points westward? (SW)

9. The two forces  $F_1$  and  $F_2$  shown in the diagram act on a 10 kg object. If  $F_1 = 12 \text{ N}$  and  $F_2 = 15 \text{ N}$ , find the acceleration produced. (Assume no friction.)

A)



B)



( $1.9 \text{ m/s}^2$  @  $39^\circ \text{ W of S}$ )

( $1.4 \text{ m/s}^2$  @  $41^\circ \text{ N of E}$ )

10. A man pushes a 16 kg lawn mower at a constant speed with a force of 90 N directed along the handle which is at an angle of  $45^\circ$  to the horizontal. Calculate

A) The horizontal frictional force acting on the mower. (-63.9 N)

B) The normal force exerted vertically upward on the mower. (+221 N)

11. A student pulls a 225 N object up a  $33.0^\circ$  inclined plane. If the coefficient of friction between the object and the incline is 0.105, what force must the student apply to pull the object up the incline at a constant velocity? (+143N)

12. At the instant a race begins, a 60 kg sprinter was found to exert a force of 850 N on the starting block at a  $20^\circ$  angle with respect to the horizontal. (assume no air friction)

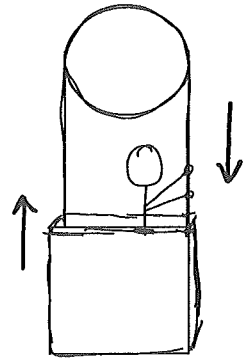
A) What was the horizontal acceleration of the sprinter? (+13 m/s<sup>2</sup>)

B) If the force was exerted for 0.38 s, with what speed did the sprinter leave the starting block? (+4.9 m/s)

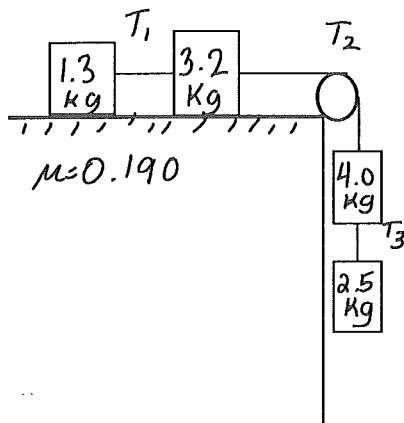
13. A window washer pulls herself upward using the bucket-pulley apparatus shown in the diagram. The mass of the bucket plus the person is 65 kg.

A) How hard must she pull downward to raise herself at a slow constant speed? (+637 N)

B) If she increases this force by 10%, what will her acceleration be? (+0.98 m/s<sup>2</sup>)



14. Find the acceleration for the system and the tension force for each segment of cable.



(±5.03 m/s<sup>2</sup>, +9.0 N, +31 N, +12 N)

# Dynamics

## Key

## Physics 12 – Dynamics: Net Force ( $\Sigma F$ )

Recall from Physics 11:

- A force is a push or pull (measured in Newtons, N)
- Forces are vector quantities and direction must always be indicated when solving problems

### Newton's First Law of Motion (Law of Inertia)

Newton's first law of motion states that an object will remain at a constant velocity (including zero) unless acted upon by an **unbalanced force**.

**Restated this means:** An object will remain at rest or will continue to move with no change in speed or direction, if **AND ONLY IF**, Net Force = 0N ( $\Sigma F = 0N$ )

### Newton's Second Law of Motion

$$\Sigma F = ma$$

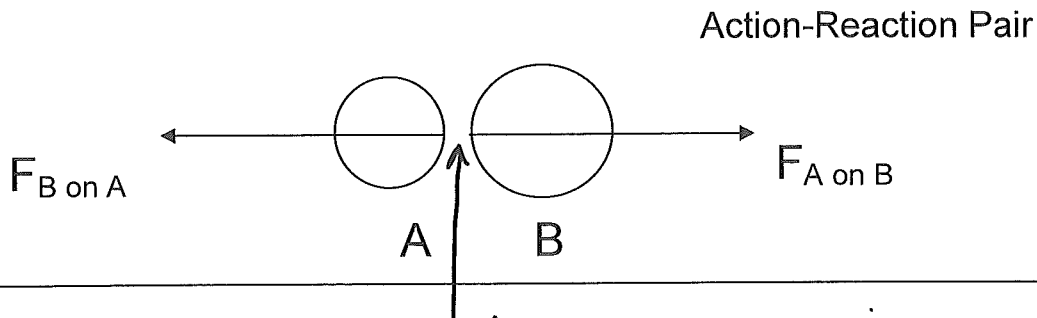
Once the net force ( $\Sigma F$ ) has been determined, we can find the acceleration that the net force will cause on the object. The acceleration that results from the net force is entirely dependent on the mass of the object.

### Newton's Third Law of Motion

EVERY force in the universe occurs as one part of an *action-reaction* pair of forces.

Both members of the action-reaction pair:

- Act on two **different** objects
- Are equal in magnitude but opposite in direction



interaction caused equal sized force  
on each object in opposite directions  
\*the actual change in motion (acceleration)  
is dependent on the mass

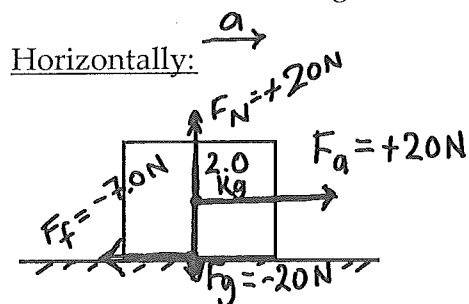
According to Newton's Second Law, the acceleration (or change in motion) is dependent on two things: the mass of the object and the net force.

$$\Sigma F = m\vec{a} \quad \Sigma F \propto \vec{a} \text{ (as } \Sigma F \uparrow, \vec{a} \uparrow \text{ and vice versa)}$$

$$\vec{a} \propto \frac{1}{m} \text{ (as } m \uparrow, a \downarrow \text{ and vice versa)}$$

Now we need to remember what net force is and how to determine it.

In Physics 11, the net force was found in a single plane only. In these cases, we take all of the forces and add them together (taking direction into account). The force that "wins" will result in the direction and magnitude of the net force.



$$\begin{aligned} \Sigma F &= F_a + F_f + F_N + F_g \\ &= 20 + (-7) + 20 + (-20) \\ \Sigma F &= +13 \text{ N} \end{aligned}$$

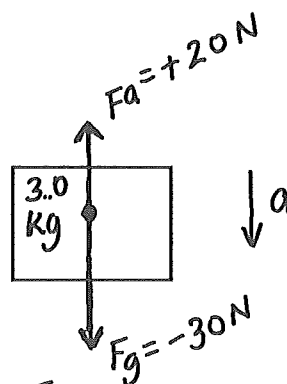
The net force directs the acceleration:

$$\Sigma F = +13 \text{ N} = +\vec{a}$$

The acceleration achieved depends on the mass:

$$\begin{aligned} \Sigma F &= m\vec{a} \\ 13 &= (2.0)\vec{a} \\ \vec{a} &= +6.5 \text{ m/s}^2 \end{aligned}$$

Vertically:



$$\begin{aligned} \Sigma F &= F_a + F_g \\ &= 20 + (-30) \\ \Sigma F &= -10 \text{ N} \end{aligned}$$

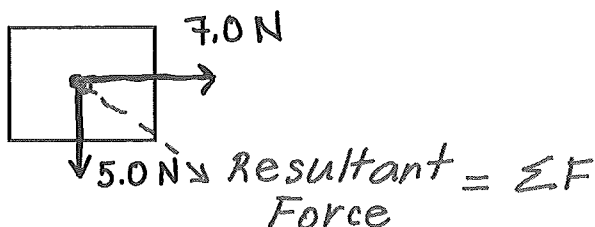
The net force directs the acceleration:

$$\Sigma F = -10 \text{ N} = -\vec{a}$$

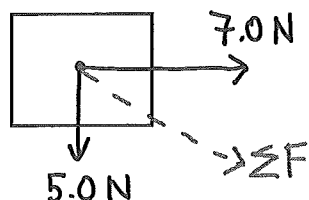
The acceleration achieved depends on the mass:

$$\begin{aligned} \Sigma F &= m\vec{a} \\ -10 &= (3.0)\vec{a} = -3.3 \text{ m/s}^2 \end{aligned}$$

Net force can also be found as a resultant force when there are unbalanced forces acting on the object in two planes.

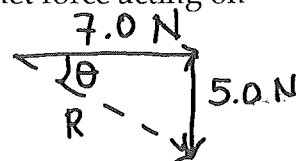


A force of 7.0N east and a force of 5.0N south act on an object. What is the net force acting on the object?



① Add the vectors:

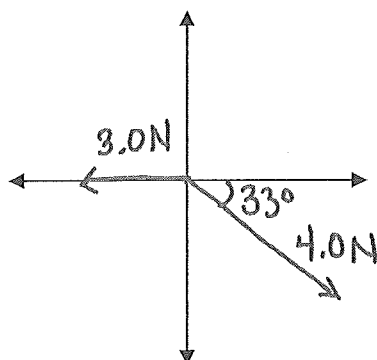
$$R = \sqrt{7.0^2 + 5.0^2} \\ = 8.6 \text{ N}$$



② determine direction  
 $\tan^{-1}\left(\frac{5.0}{7.0}\right) = 36^\circ \text{ S of E}$

$$\Sigma F = 8.6 \text{ N } 36^\circ \text{ S of E}$$

A force of 3.0N west and a force of 4.0 N  $33^\circ$  S of E acts on an object. What is the net force on the object?



Resolve the vectors:

$$\cos 33(4.0) = R_x = 3.35 \text{ N [E]}$$

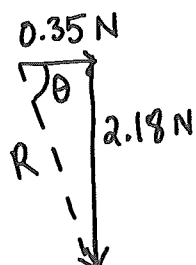
$$\sin 33(4.0) = R_y = 2.18 \text{ N [S]}$$

Add x & y components together:

$$X: +3.35 + (-3.0 \text{ N}) = 0.35 \text{ N [E]}$$

$$Y: 0 + (-2.18) = 2.18 \text{ N [S]}$$

Draw new triangle:



$$R = \sqrt{2.18^2 + 0.35^2} \\ = 2.2 \text{ N}$$

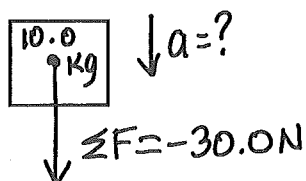
$$\tan^{-1}\left(\frac{2.18}{0.35}\right) = 81^\circ \text{ S of E}$$

$$\Sigma F = \underline{2.2 \text{ N } 81^\circ \text{ S of E}}$$

### Using Net Force:

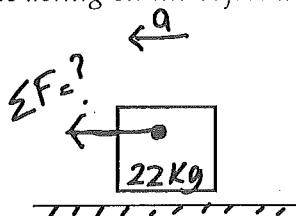
Once we have determined the net force acting on an object, we can determine the resulting acceleration by taking the mass of the object into account.

A net force of 30.0 N south acts on a 10.0 kg object. What is the acceleration of the object?



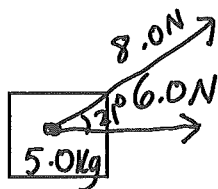
$$\begin{aligned}\Sigma F &= m\vec{a} \\ -30.0 &= (10.0)\vec{a} \\ \vec{a} &= -3.00 \text{ m/s}^2 \\ &\text{or} \\ &3.00 \text{ m/s}^2 \text{ [S]}\end{aligned}$$

A 22 kg object accelerates uniformly from rest to a velocity of 2.5 m/s west in 8.7 s. What is the net force acting on the object during this acceleration?



$$\begin{aligned}a &= \frac{2.5 - 0}{8.7} & \vec{a} &= -0.29 \text{ m/s}^2 \\ \Sigma F &= ma \\ &= (22)(-0.29) \\ \Sigma F &= -6.3 \text{ N or } 6.3 \text{ N [W]}\end{aligned}$$

A force of 6.0 N East and a force of 8.0 N 21° N of E act on a 5.0 kg object. What is the acceleration that the object experiences?



$$\begin{aligned}\textcircled{4} \Sigma F &= m\vec{a} \\ 14 &= (5.0)\vec{a} \\ \vec{a} &= 2.8 \text{ m/s}^2 \\ &78^\circ \text{ E of N}\end{aligned}$$

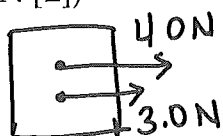
$$\begin{aligned}\textcircled{1} \cos 21(8.0) &= R_x = 7.47 \text{ N [E]} \\ \sin 21(8.0) &= R_y = 2.87 \text{ N [N]}\end{aligned}$$

$$\begin{aligned}\textcircled{2} X: 7.47 + 6.0 &= 13.5 \text{ N [E]} \\ Y: 2.87 + 0 &= 2.87 \text{ N [N]}\end{aligned}$$

$$\begin{aligned}\textcircled{3} \begin{array}{c} 13.5 \text{ N} \\ \nearrow \\ 2.87 \text{ N} \end{array} \quad \begin{array}{c} R \\ \nearrow \\ \end{array} \quad \begin{array}{c} 78^\circ \\ \nearrow \\ \end{array} \\ R &= \sqrt{13.5^2 + 2.87^2} \\ &= 14 \text{ N} \\ \tan^{-1}\left(\frac{13.5}{2.87}\right) &= 78^\circ \text{ E of N} \\ \Sigma F &= 14 \text{ N } 78^\circ \text{ E of N}\end{aligned}$$

# Net Force Assignment: (Draw Free-Body Diagrams for ALL problems)

1. A force of 4.0 N East and a force of 3.0 N East act on an object. What is the net force on the object? (7.0 N [E])



$$\Sigma F = 4.0 + 3.0 = 7.0 \text{ N [E]}$$

2. A force of 6.0 N East and a force of 4.0 N West act on an object. What is the net force on the object? (2.0 N [E])

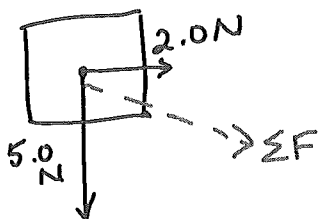


$$\Sigma F = 6.0 + (-4.0) = +2.0 \text{ N}$$

or

$$2.0 \text{ N [E]}$$

3. A force of 2.0 N East and a force of 5.0 N South act on an object. What is the net force acting on the object? (5.4 N 68° S of E)

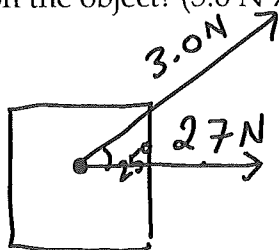


$$R = \sqrt{5.0^2 + 2.0^2} = 5.4 \text{ N}$$

$$\tan^{-1}\left(\frac{5.0}{2.0}\right) = 68^\circ \text{ S of E}$$

$$\Sigma F = 5.4 \text{ N } 68^\circ \text{ S of E}$$

4. A force of 2.7 N East and a force of 3.0 N 25° N of E act on an object. What is the net force acting on the object? (5.6 N 77° E of N)



$$\cos 25(3.0) = R_x = 2.72 \text{ N [E]}$$

$$\sin 25(3.0) = R_y = 1.27 \text{ N [N]}$$

$$X: 2.7 + 2.72 = 5.42 \text{ N [E]}$$

$$Y: 1.27 + 0 = 1.27 \text{ N [N]}$$

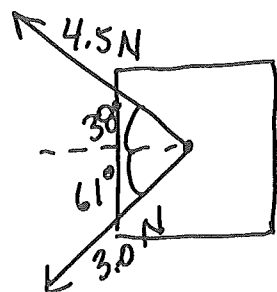
$$R = \sqrt{5.42^2 + 1.27^2}$$

$$= 5.6 \text{ N}$$

$$\Sigma F = 5.6 \text{ N } 77^\circ \text{ E of N}$$

$$\tan^{-1}\left(\frac{5.42}{1.27}\right) = 77^\circ \text{ E of N}$$

5. Forces of 4.5 N 38° N of W and 3.0 N 61° S of W act on an object. What is the net force acting on the object? (4.9 N 2.3° N of W; without rounding in calc: 1.7° N of W)



$$\cos 38(4.5) = R_x = 3.55 \text{ N [W]} \quad \cos 61(3.0) = R_x = 1.45 \text{ N [W]}$$

$$\sin 38(4.5) = R_y = 2.77 \text{ N [N]} \quad \sin 61(3.0) = R_y = 2.62 \text{ N [S]}$$

$$X: -3.55 + (-1.45) = -5.0 \text{ N} = 5.0 \text{ N [W]}$$

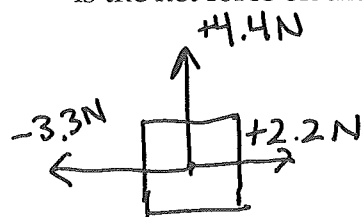
$$Y: 2.77 + (-2.62) = 0.15 \text{ N [N]}$$

$$R = \sqrt{0.15^2 + 5.0^2} = 5.0 \text{ N}$$

$$\tan^{-1}\left(\frac{0.15}{5.0}\right) = 1.7^\circ \text{ N of W}$$

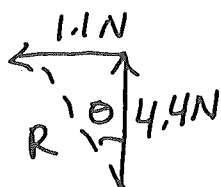
$$\Sigma F = 5.0 \text{ N } 1.7^\circ \text{ N of W}$$

6. A force of 2.2 N East, a force of 3.3 N West and a force of 4.4 N north act on an object. What is the net force on the object? (4.5 N 14° W of N)



$$X: +2.2 + (-3.3) = -1.1 \text{ N} = 1.1 \text{ N [W]}$$

$$Y: 4.4 + 0 = 4.4 \text{ N [N]}$$

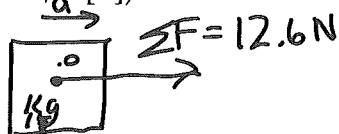


$$R = \sqrt{1.1^2 + 4.4^2} = 4.5 \text{ N}$$

$$\tan^{-1}\left(\frac{1.1}{4.4}\right) = 14^\circ \text{ W of N}$$

$$\Sigma F = 4.5 \text{ N } 14^\circ \text{ W of N}$$

7. A 15.0 kg object is accelerated by a net force of 12.6 N East. What is the object's acceleration? (0.840 m/s² [E])

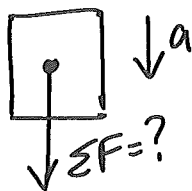


$$\Sigma F = ma \quad 0) \vec{a}$$

$$12.6 = (15.0) \vec{a}$$

$$\vec{a} = 0.840 \text{ m/s}^2 \text{ [E]}$$

8. A 925 kg car accelerates uniformly from 5.0 m/s to 12.0 m/s south in 11.0 s. What is the net force causing this acceleration? (5.9 × 10² N [S])

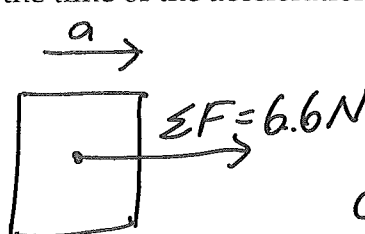


$$\vec{a} = \frac{12.0 - (-5.0)}{11.0} \quad \vec{a} = -0.636 \text{ m/s}^2$$

$$\Sigma F = (925)(-0.636) = -589 \text{ N}$$

$$= 5.9 \times 10^2 \text{ N [S]}$$

9. A net force of 6.6 N East acts on a 9.0 kg object. The object accelerates uniformly from rest to a velocity of 3.0 m/s east. A) How far did the object travel while accelerating? B) What is the time of the acceleration? (6.1 m [E], 4.1 s)



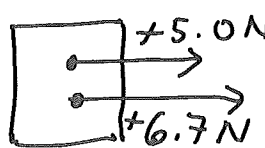
$$6.6 = (9.0)\vec{a}$$

$$\vec{a} = 0.733 \text{ m/s}^2 [\text{E}]$$

a)  $3.0^2 = 0^2 + 2(0.733)d$        $\vec{d} = \underline{6.1 \text{ m} [\text{E}]}$

b)  $0.733 = \frac{3.0 - 0}{t}$        $t = \underline{4.1 \text{ s}}$

10. A force of 5.0 N East and a force of 6.7 N East act on an 8.0 kg object. What is the acceleration of this object? (1.5 m/s<sup>2</sup> [E])



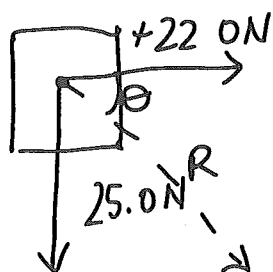
$$\Sigma F = 5.0 + 6.7 = 11.7 \text{ N} [\text{E}]$$

$$\Sigma F = m\vec{a}$$

$$11.7 = (8.0)\vec{a}$$

$$\vec{a} = 1.5 \text{ m/s}^2 [\text{E}]$$

11. A force of 22.0 N East and a force of 25.0 N South act on a 4.2 kg cart. What is the acceleration caused by these forces? (7.9 m/s<sup>2</sup> 49° S of E)



$$R = \sqrt{22.0^2 + 25.0^2} = 33 \text{ N}$$

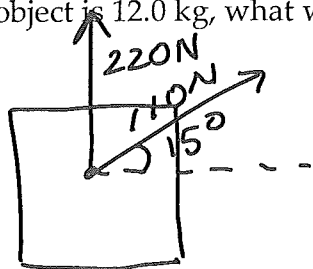
$$\tan^{-1}\left(\frac{25.0}{22.0}\right) = 49^\circ \text{ S of E}$$

$$\Sigma F = 33 \text{ N } 49^\circ \text{ S of E}$$

$$33 = (4.2)\vec{a}$$

$$\vec{a} = 7.9 \text{ m/s}^2 \text{ } 49^\circ \text{ S of E}$$

12. A force of 220 N North and a force of 110 N 15° N of E act on an object. If the mass of this object is 12.0 kg, what will its acceleration be? (23 m/s<sup>2</sup> 23° N of E)

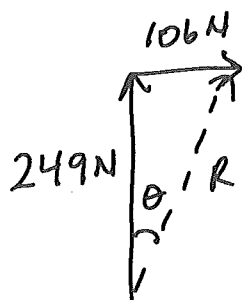


$$\cos 15(110) = R_x = 106 \text{ N [E]}$$

$$\sin 15(110) = R_y = 28.5 \text{ N [N]}$$

$$X: 106 + 0 = 106 \text{ N [E]}$$

$$Y: 28.5 + 220 = 248.5 \text{ [N]}$$



$$R = \sqrt{249^2 + 106^2} = 271 \text{ N}$$

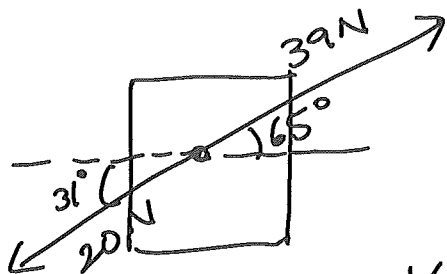
$$\Sigma F = 271 \text{ N } 23.1^\circ \text{ N of E}$$

$$\tan^{-1}\left(\frac{106}{249}\right) = 23.1^\circ \text{ E of N}$$

$$271 = (12.0) \vec{a}$$

$$\vec{a} = 23 \text{ m/s}^2 \quad 23^\circ \text{ N of E}$$

13. Forces of 39 N 65° N of E and 20 N 31° S of W act on an object. The object has a mass of 3.2 kg. What will its resulting acceleration be? (7.8 m/s<sup>2</sup> 1.4° W of N)



$$\cos 65(39) = R_x = 16.5 \text{ N [E]}$$

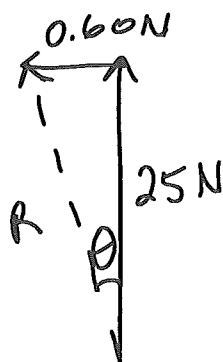
$$\sin 65(39) = R_y = 35.3 \text{ N [N]}$$

$$\cos 31(20) = R_x = 17.1 \text{ N [W]}$$

$$\sin 31(20) = R_y = 10.3 \text{ N [S]}$$

$$X: 16.5 + (-17.1) = -0.60 \text{ N} = 0.60 \text{ N [W]}$$

$$Y: 35.3 + (-10.3) = 25 \text{ N [N]}$$



$$R = \sqrt{25^2 + 0.60^2} = 25 \text{ N}$$

$$\tan^{-1}\left(\frac{0.60}{25}\right) = 1.4^\circ \text{ W of N}$$

$$\Sigma F = 25 \text{ N } 1.4^\circ \text{ W of N}$$

$$25 = (3.2) \vec{a}$$

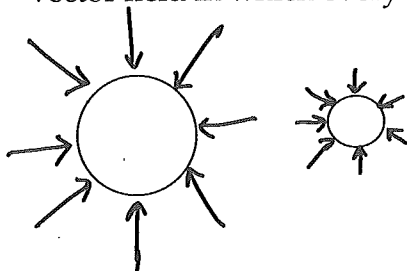
$$\vec{a} = 7.8 \text{ m/s}^2 \quad 1.4^\circ \text{ W of N}$$

**A. GRAVITY**GRAVITATIONAL FIELD STRENGTH

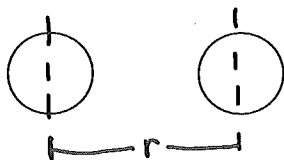
A gravitational field is a model that is used to describe the effects of gravity. The field strength can be determined using Newton's Law of Universal Gravitation.

$$F_g = \frac{Gm_1m_2}{r^2}$$

A gravitational field forms around every particle or larger mass and is a vector field in which every vector points directly toward the particle or mass.



larger mass = larger field strength  
(↑ m) (↑  $F_g$ )



larger distance between masses = smaller field strength  
(↑ r) (↓  $F_g$ )

The magnitude of the field at every point is the same and represents the force per unit of mass on any object in that region of space.

On Earth, we know that the force of gravity acts as an unbalanced force and accelerates objects toward the earth at a rate of  $9.8 \text{ m/s}^2$ .

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11})(1.0)(5.98 \times 10^{24})}{(6.38 \times 10^6)^2}$$

← Earth's mass (kg)  
 ← Earth's radius (m)

Field strength on Earth =  $9.8 \text{ N}$

$\vec{a}$  due to gravity =  $9.8 \text{ m/s}^2$

For every  $1.0 \text{ kg}$  of mass on the surface of the Earth, it will be pulled down by a force of gravity of  $9.8 \text{ N}$ .

$$* g = 9.8 \text{ N/kg}$$

## FORCE OF GRAVITY

From this, we are able to determine our weight on Earth which is equivalent to the force of the gravity.

$$F_g = mg$$

(weight)

mass - kg  
 $g = 9.8 \text{ N/kg}$  on Earth

Near the Earth's surface we use 9.80 as the value of gravity. However, it is not a constant value. This value depends on the mass of the Earth (or other planet) and the distance from the center of the Earth. This is seen in the formula:

$$F_g = G \frac{m_1 m_2}{r^2}$$

as  $m \uparrow = F_g \uparrow$   
as  $r \uparrow = F_g \downarrow$

Due to this, the force of gravity on an object on the surface of the Earth will vary depending on location. The Earth is not a perfect sphere and bulges at the equator. Therefore, the  $F_g$  on an object at the equator is less than at one of the poles (larger distance to center of the Earth).

Weight is often confused with mass. →

Mass only depends on the matter an object contains (the atoms + molecules).

Weight is the amount of force pulling an object down toward a surface.

\* Key concept  
↓  
it does not vary with location

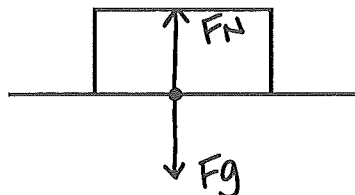
\* Key concept  
↓  
this varies depending on location (the interacting masses)  
Ex. Earth's  $g = 9.8 \text{ N/kg}$  moon's  $g = 1.6 \text{ N/kg}$

## B. NORMAL FORCE

The normal force is the perpendicular force that a surface exerts with an object which it is in contact.

On a horizontal surface, the normal force is equal to the weight of the object.

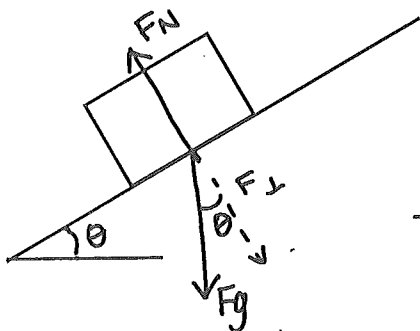
↑  
the pressure between the two surfaces



$$F_N = F_g$$

vertical accel =  $0 \text{ m/s}^2$

On an incline, the normal force is equal to the component of weight that is perpendicular to the surface.



$F_N$  is always perpendicular to the surface. (forms  $90^\circ \angle$ )

$F_g$  is always directly down to the surface of the Earth

\* to find  $F_N$ , we determine the  $\perp$  (perpendicular) component of  $F_g$

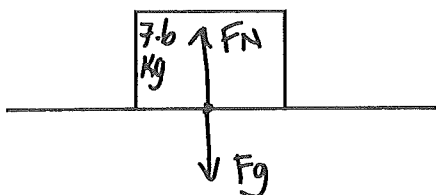
$$F_N = F_{\perp} = F_g \cdot \cos \theta$$

$\downarrow$   
 $mg \cos \theta$

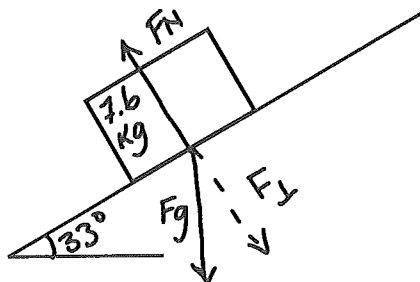
A 6.7 kg object is resting on a horizontal surface. What is the normal force acting on the object?

$$F_N = F_g = mg = (6.7)(9.8)$$

$$= \underline{+74 \text{ N}}$$



A 7.6 kg object is at rest on an inclined plane. If the inclined plane makes an angle with the horizontal of  $33^\circ$ , what is the normal force acting on the object?

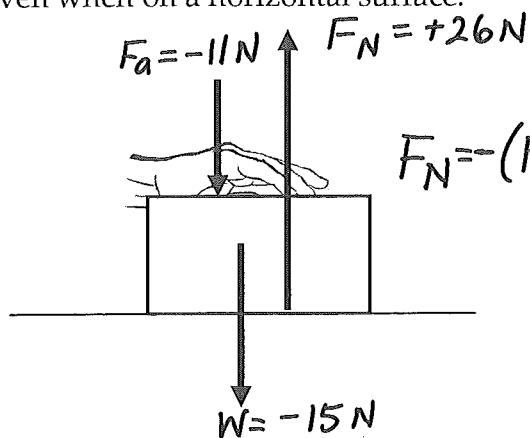


$$F_N = F_{\perp} = mg \cos \theta$$

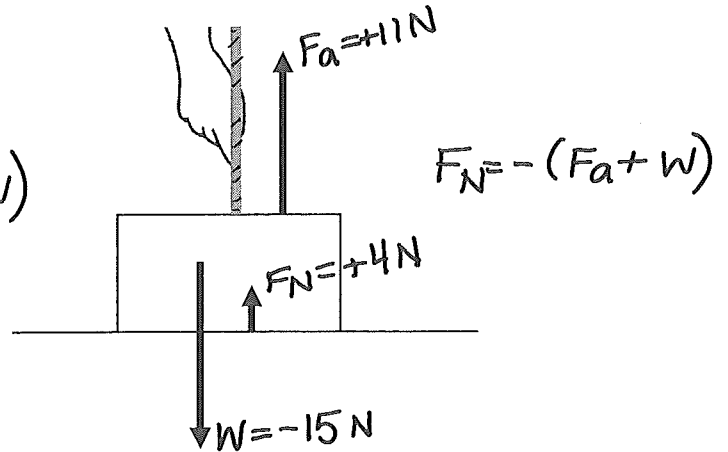
$$= (7.6)(9.8)(\cos 33)$$

$$= +62 \text{ N}$$

There are many other situations when the normal force is not equal to the force of gravity, even when on a horizontal surface.



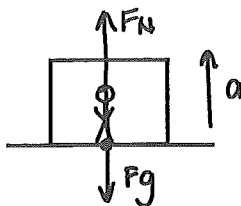
\*  $F_N$  is greater than weight ( $F_g$ ) as it supports the  $F_g + F_a$



\*  $F_N$  is less than weight because the rope supplies an upward force.

The normal force is also referred to as "apparent weight". This is due to acceleration of an object on a horizontal surface [up] or [down]. Due to inertia, an object will want to stay in its previous state of motion.

When accelerating up, the object will push into the surface below as its inertia maintains its previous state of motion until it is overcome by the unbalanced force of the accelerating surface below it.



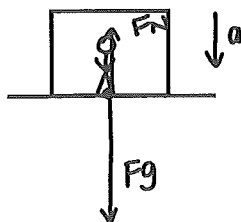
The surfaces are "pushed" together (more than normal) <sub>pressure</sub>

$$F_N > F_g \rightarrow m \cdot g \text{ remain the same}$$

to calculate:  $ma = F_N + F_g$

When accelerating down, the object will pull away from the lower surface for the same reason as above.

(Ex. drop ride)

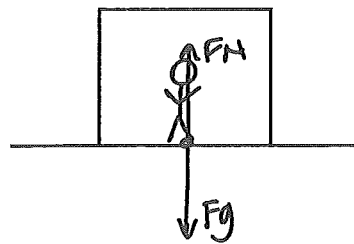


surfaces are "pulled" apart  
= less pressure than normal

$$F_N < F_g \rightarrow m \cdot g \text{ remain the same}$$

$$ma = F_N + F_g$$

When moving up or down at a constant velocity, the forces acting on the object are balanced and the object maintains its state of motion. This means that its weight is the same as if it were at rest. (In both cases, the forces are balanced = no change to state of motion)



vertical  $\vec{a} = 0 \text{ m/s}^2$

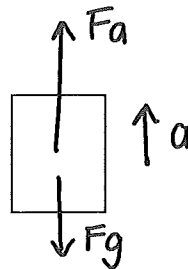
$$F_N = -F_g$$

- vertical  $\Sigma F$  must =  $0 \text{ N}$
- so forces must be balanced

### Vertical Motion:

1. With  $F_a$  or  $T$ :  $\Sigma F = ma = F_{\text{applied}} + F_g$

$F_g$  must be negative as it is in the opposite direction to  $F_a$

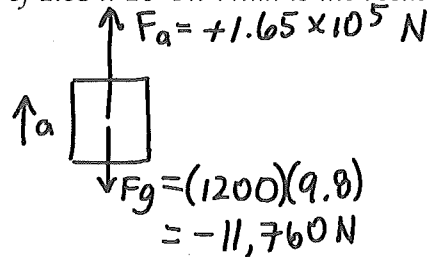


A 1200 kg rocket is launched. The engines have a thrust of  $1.65 \times 10^5 \text{ N}$ . What is the rocket's acceleration?

$$ma = F_a + F_g$$

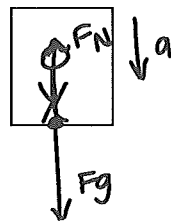
$$(1200)(a) = 1.65 \times 10^5 + (-11,760)$$

$$a = +128 \text{ m/s}^2$$



2. With  $F_N$ :  $\Sigma F = ma = F_N + F_g$

(+) (-)  
opposite directions

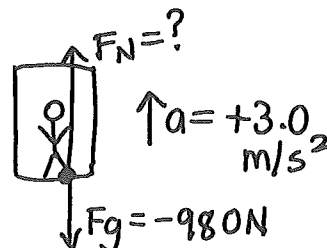


A 100 kg man is standing in an elevator on a scale. What would the scale read when the elevator is accelerating upward at  $3.0 \text{ m/s}^2$ ?

$$ma = F_N + F_g$$

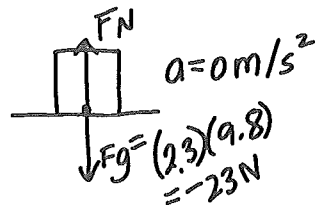
$$(100)(3) = F_N + (-980)$$

$$F_N = +1.28 \times 10^3 \text{ N}$$



**Forces Practice Assignment – Complete the following showing all work and with correct free-body diagrams.**

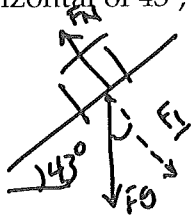
1. A 2.3 kg object is resting on a horizontal surface. What is the normal force acting on the object? (+23 N)



$$F_N = -F_g$$

$$\underline{F_N = +23 \text{ N}}$$

2. A 5.5 kg object is at rest on an inclined plane. If the inclined plane makes an angle with the horizontal of  $43^\circ$ , what is the normal force acting on the object? (+39 N)

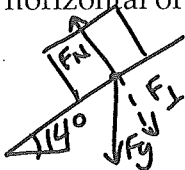


$$F_N = F_{\perp} = mg \cos \theta$$

$$= (5.5)(9.8) \cos(43)$$

$$= \underline{+39 \text{ N}}$$

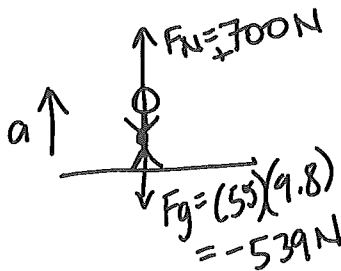
3. A 10.0 kg object is at rest on an inclined plane. If the inclined plane makes an angle with the horizontal of  $14.0^\circ$ , what is the normal force acting on the object? (+95.0 N)



$$F_N = mg \cos \theta = (10.0)(9.8)(\cos 14)$$

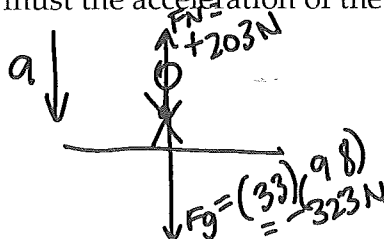
$$= \underline{+95.0 \text{ N}}$$

4. An 55Kg passenger is travelling in an 799Kg elevator. The apparent weight of the passenger is 700N. Is the elevator accelerating? If so in which direction? (accelerating up)



$$F_N > F_g \quad \text{accelerating up}$$

5. While on a ride at the amusement park a 33.0Kg boy feels as if he weighs only 203N. What must the acceleration of the ride be? ( $-3.65 \text{ m/s}^2$ )

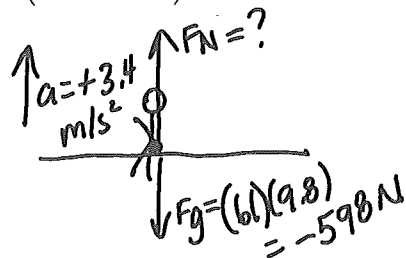


$$ma = F_N + F_g$$

$$(33.0)a = 203 + (-323)$$

$$\vec{a} = -3.65 \text{ m/s}^2$$

6. A 61 kg passenger is travelling directly upwards in a helicopter upon take-off. If the helicopter accelerates at a rate of  $3.4 \text{ m/s}^2$ , what is the apparent weight of the passenger? ( $+8.0 \times 10^2 \text{ N}$ )

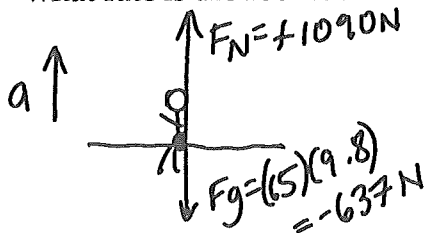


$$ma = F_N + F_g$$

$$(61)(+3.4) = F_N + (-598)$$

$$F_N = +8.0 \times 10^2 \text{ N}$$

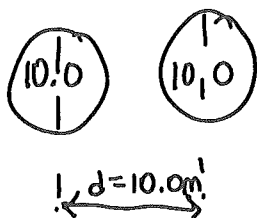
7. While accelerating during take-off in a rocket, the 65 kg astronaut feels much heavier than normal. If he is sitting on a scale, he would calculate his apparent weight to be 1090 N. At what rate is the rocket accelerating? ( $+7.0 \text{ m/s}^2$ )



$$(65)a = 1090 + (-637)$$

$$\vec{a} = +7.0 \text{ m/s}^2$$

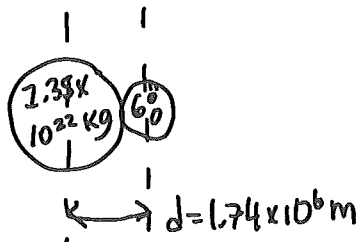
8. Two masses that each have a mass of 10.0 kg are sitting 10.0 m apart. What is the gravitational field strength produced by the masses? ( $6.67 \times 10^{-11} \text{ N}$ )



$$F_g = G \frac{m_1 m_2}{d^2} = \frac{(6.67 \times 10^{-11})(10)(10)}{10^2}$$

$$F_g = 6.67 \times 10^{-11} \text{ N}$$

9. A 68 kg mass is sitting on the surface of the moon. If the moon's mass is  $7.35 \times 10^{22} \text{ kg}$  and its radius is  $1.74 \times 10^6 \text{ m}$ , what is the force of gravity acting on the 68 kg mass? (110 N)



$$F_g = G \frac{m_1 m_2}{d^2} = \frac{(6.67 \times 10^{-11})(7.35 \times 10^{22})(68)}{(1.74 \times 10^6)^2}$$

$$= 1.1 \times 10^2 \text{ N}$$

10. What is the weight of a 15.0 kg object on the surface of the Earth? (-147 N)

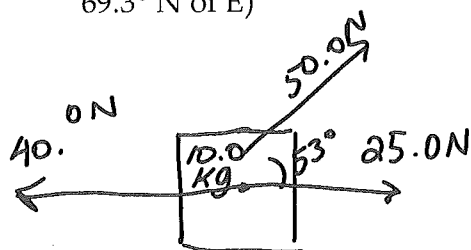
$$F_g = mg = (15.0)(9.8) = -147 \text{ N}$$

11. What is the mass of an object if it has a weight of 110N on the surface of the Earth?

(11.2kg)

$$F_g = mg \quad 110 = m(9.8) \quad m = 11.2 \text{ Kg}$$

12. An object of mass 10.0 kg is subjected to the following forces: 25.0 N due east, 50.0 N at an angle of 53.0° N of E, and 40.0 N due west. What is the acceleration of the object? (4.27 m/s<sup>2</sup> 69.3° N of E)



$$\cos 53(50) = R_x = 30.1 \text{ N east}$$

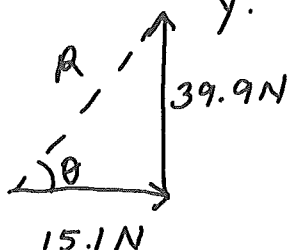
$$\sin 53(50) = R_y = 39.9 \text{ N north}$$

$$x: 30.1 + 25.0 + (-40) = 15.1 \text{ N east}$$

$$y: 0 + 0 + 39.9 = 39.9 \text{ N north}$$

$$R = \sqrt{39.9^2 + 15.1^2} = 42.7 \text{ N} = \Sigma F$$

$$\tan^{-1}\left(\frac{39.9}{15.1}\right) = 69.3^\circ \text{ N of E}$$



$$\Sigma F = ma$$

$$42.7 = 10.0 a$$

$$a = 4.27 \text{ m/s}^2 @ 69.3^\circ \text{ N of E}$$

13. You are standing on a scale in an elevator, the scale reads 450 N when the elevator is at rest. The elevator begins to accelerate upward at 9.00 m/s<sup>2</sup>.

mass @ rest

a) What is the new reading on the scale? (+863N)

$$450 \div 9.8 = 45.9 \text{ Kg}$$

b) Suppose the elevator was now accelerating downward at 9.00 m/s<sup>2</sup>. What does the scale read as now? (+36.9N)

$$a) ma = F_N + F_g$$

$$(45.9)(+9) = F_N + (-450)$$

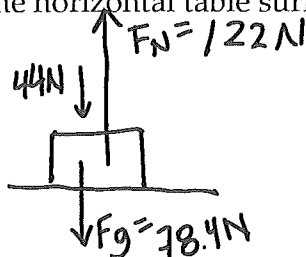
$$F_N = +863 \text{ N}$$

$$b) ma = F_N + F_g$$

$$(45.9)(-9) = F_N + (-450)$$

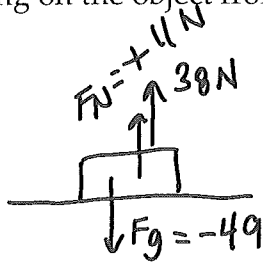
$$F_N = +36.9 \text{ N}$$

14. You push down on an 8.0 kg object with 44 N of force. What is the normal force exerted by the horizontal table surface that it is resting on. (+122N)



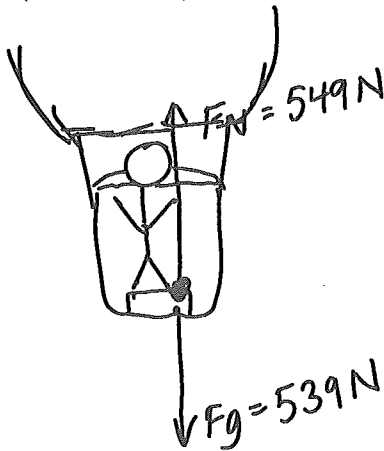
$$F_N = 122 \text{ N}$$

15. A rope tied to a 5.0 kg block is pulling up with a force of 38 N. What is the normal force acting on the object from the horizontal surface below? (+11N)



$$F_N = +11 \text{ N}$$

16. A 55.0 kg person is riding in a hot air balloon and a scale aboard shows the person's weight to be 549N. Determine the acceleration of the balloon (magnitude and direction). (+0.182 m/s<sup>2</sup>)

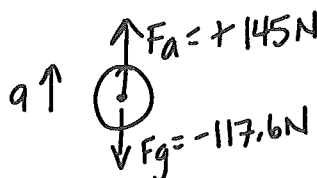


$$ma = F_N + F_g$$

$$(55.0)a = 549 + (-539)$$

$$\vec{a} = +0.182 \text{ m/s}^2$$

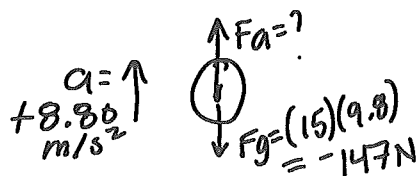
17. A 12.0 kg object is thrown vertically into the air with an applied force of 145 N. What is the initial acceleration of the object? (+2.28 m/s<sup>2</sup>)



$$ma = F_a + F_g$$

$$(12)(a) = 145 + (-117.6) \quad \vec{a} = +2.28 \text{ m/s}^2$$

18. A 15.0 kg object is thrown vertically into the air. If the initial acceleration of the object is 8.80 m/s<sup>2</sup>, what is the applied force? (+279N)



$$(15.0)(+8.80) = F_a + (-147)$$

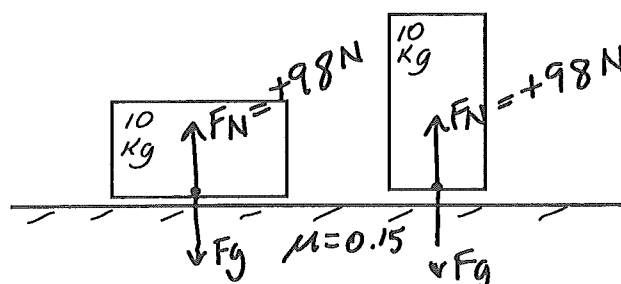
$$F_a = +279 \text{ N}$$

### C. Friction

Frictional Forces are forces that always oppose motion. Recall that the frictional force depends on only two things:

- The normal force acting on the object (*pressure betw. surfaces*)
- The nature of the two surfaces (indicated by the coefficient of friction  $\mu$ )

Frictional forces DO NOT depend on the area of contact between the two surfaces.



*same  $F_N$  &  $\mu$*

$$F_f = \mu F_N$$

There are two types of friction: Sliding (Kinetic) Friction ( $\mu_k$ ) and Static Friction ( $\mu_s$ )

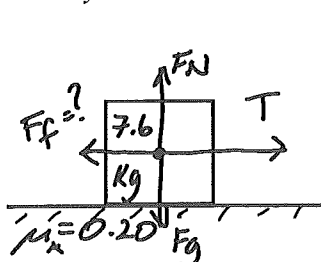
Static Friction refers to the friction force that must be overcome to start an object moving. This is always a larger frictional force than Sliding Friction which refers to the friction force that must be overcome to keep an object moving.

When an object is at rest, it wants to remain at rest (inertia). When an object is moving, it wants to remain moving (inertia). This is why it takes more force to get an object moving than to keep it moving. Hence, static friction is greater than kinetic friction.

$$\mu_s > \mu_k$$

*coefficient of static friction*      *coefficient of kinetic friction*

A 7.6 kg object is being pulled along a horizontal surface. If the coefficient of sliding friction between the two surfaces is 0.20, what is the force of friction acting on the object?



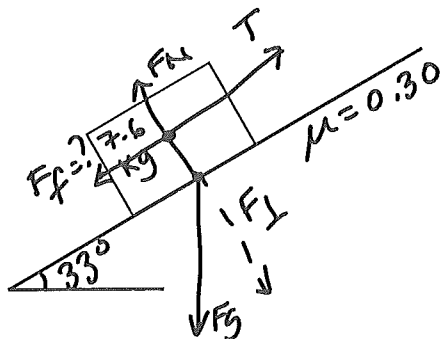
$$F_f = \mu F_N$$

$$= (0.20)(7.6)(9.8)$$

$$= -15\text{ N}$$

*friction force opposes motion*

A 7.6 kg object is pulled up an inclined plane. If the inclined plane makes an angle with the horizontal of  $33^\circ$ , and the coefficient of friction is 0.20, what is the force of friction acting on the object?



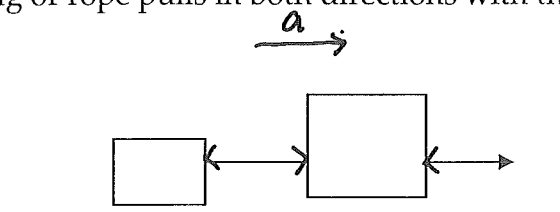
$$F_f = \mu F_N$$

\*calculate  $F_N = mg \cos 33 = 62 \text{ N}$

$$F_f = (0.30)(62) = -12 \text{ N}$$

## D. Applied Force or Tension

Recall that tension (T) is a force applied through a rope, cable or string. The tension in a string or rope pulls in both directions with the same force.

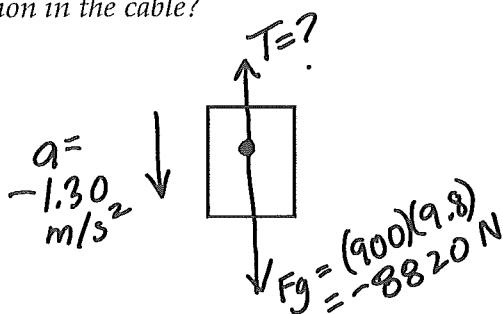


$\vec{a}$  will be the same  
for all masses

$$\vec{a} = \frac{\sum F}{\text{total mass}}$$

As always, we draw a **free-body diagram** to indicate all forces acting on an object.

An elevator with a mass of  $9.00 \times 10^2 \text{ kg}$  is accelerating downward at a rate of  $1.30 \text{ m/s}^2$ . What is the tension in the cable?

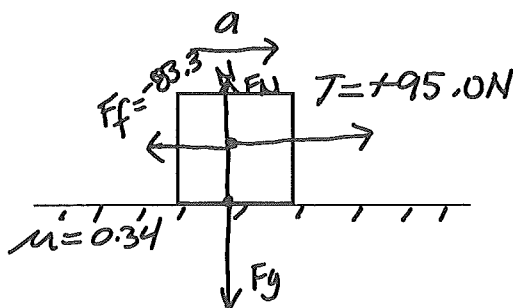


$$ma = T + F_g$$

$$(900)(-1.30) = T + (-8820)$$

$$T = +7.65 \times 10^3 \text{ N}$$

An object that has a mass of  $25.0 \text{ kg}$  is pulled along a horizontal surface with a force of  $95.0 \text{ N}$ . If the coefficient of friction between the surfaces is 0.34, what is the acceleration of the object?



$$F_f = \mu F_N = (0.34)(25.0)(9.8)$$

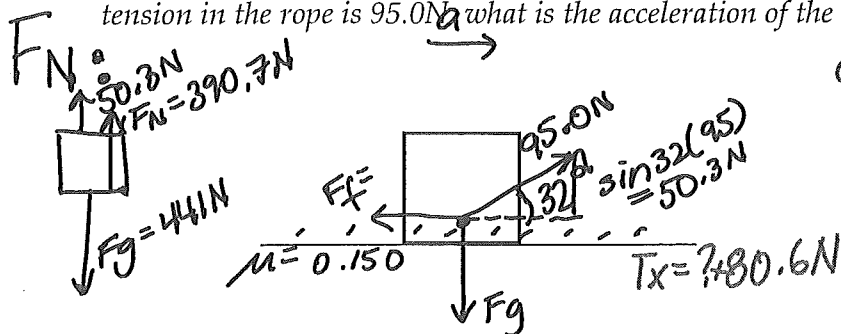
$$= -83.3 \text{ N}$$

$$ma = T + F_f$$

$$(25.0)a = 95.0 + (-83.3)$$

$$\vec{a} = +0.47 \text{ m/s}^2$$

An object that has a mass of 45.0 kg is pulled along a horizontal surface by a rope that makes an angle of  $32.0^\circ$  with the horizontal. If the coefficient of kinetic friction between the surfaces is 0.150, and the tension in the rope is 95.0 N, what is the acceleration of the object?

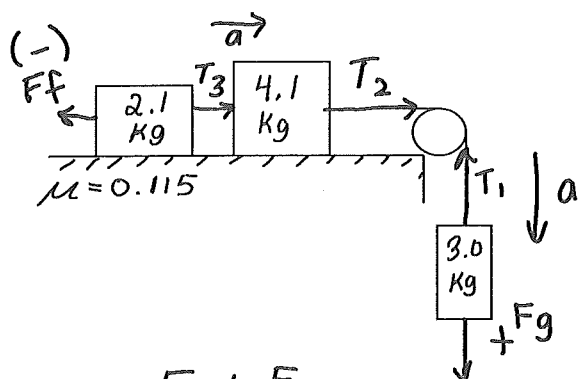


$$\cos 32(95.0) = T_x = 80.6 \text{ N}$$

$$F_f = \mu(F_N) = (0.150)(390.7) = 58.7 \text{ N}$$

$$\left[ \begin{aligned} m\vec{a} &= T_x + F_f \\ (45.0)\vec{a} &= 80.6 + (-58.7) \\ \vec{a} &= +0.487 \text{ m/s}^2 \end{aligned} \right]$$

Two systems of masses are connected as shown below. Determine the acceleration of the system, and tensional forces along each segment of massless string.



$$\sum F = F_g + F_f$$

$$= (3.0)(9.8) + (-0.115)(6.2)(9.8)$$

$$= 22.4 \text{ N}$$

$$\vec{a} = \frac{22.4}{9.2} = \pm 2.44 \text{ m/s}^2$$

$$T_1: ma = T_1 + F_g$$

$$(3.0)(-2.44) = T_1 + (-29.4)$$

$$T_1 = +22 \text{ N}$$

$$T_2: ma = T_2 + F_f$$

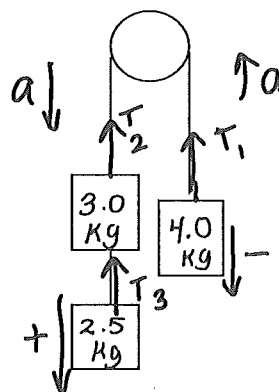
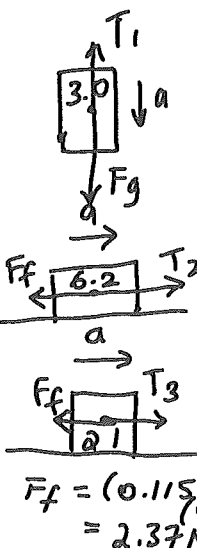
$$(6.2)(2.44) = T_2 + (-6.99)$$

$$T_2 = +22 \text{ N}$$

$$T_3: ma = T_3 + F_f$$

$$(2.1)(2.44) = T_3 + (-2.37)$$

$$T_3 = +7.5 \text{ N}$$



$$\sum F = F_g + F_g$$

$$= (5.5)(9.8) + (-4.0)(9.8)$$

$$= 53.9 + (-39.2) = 14.7 \text{ N}$$

$$\vec{a} = \frac{14.7}{9.5} = \pm 1.55 \text{ m/s}^2$$

$$T_1: ma = T_1 + F_g$$

$$(4.0)(1.55) = T_1 + (-39.2)$$

$$T_1 = +45 \text{ N}$$

$$T_2: ma = T_2 + F_g$$

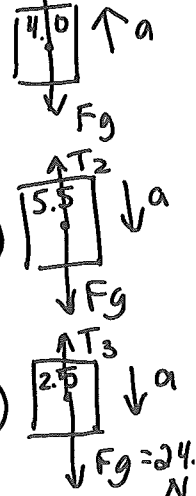
$$(5.5)(-1.55) = T_2 + (-53.9)$$

$$T_2 = +45 \text{ N}$$

$$T_3: ma = T_3 + F_g$$

$$(2.5)(-1.55) = T_3 + (-24.5)$$

$$T_3 = +21 \text{ N}$$



**Forces- Friction and Tension Assignment – Complete the following showing all work and with correct free-body diagrams.**

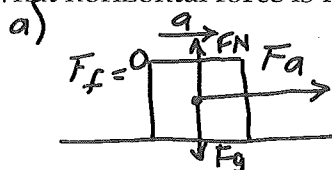
1. A block of mass 4.0 kg rests on a horizontal surface. What horizontal force is required to accelerate the block at 5.0 m/s<sup>2</sup> if,

a) There is no friction? (+20N)

b) The coefficient of kinetic friction is 0.25? (+30N)

a)  $ma = F_a + F_f \quad (4.0)(5) = F_a + 0$

b)  $ma = F_a + F_f \quad (4.0)(5) = F_a + (-9.8)$   
 $F_a = +30 \text{ N}$



b)

$$F_f = \mu F_N = (0.25)(4)(9.8) = -9.8 \text{ N}$$

2. Two boxes are connected by a cord on a horizontal surface. A force F pulls on the blocks as shown in the diagram. Find the acceleration of the blocks and the tension in the connecting cord if,

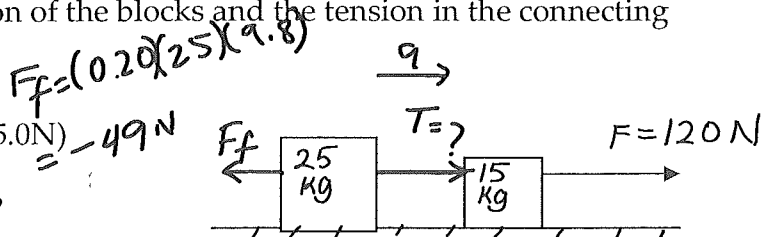
a) The surface is frictionless? ( $\pm 3.0 \text{ m/s}^2$ , +75.0N)

b) The coefficient of kinetic friction is 0.20?

( $\pm 1.04 \text{ m/s}^2$ , +75.0N)

a)  $\Sigma F = 120 + 0 = 120$   
 $\vec{a} = \frac{120}{40} = 3.0 \text{ m/s}^2$

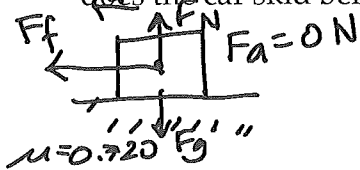
$ma = T + F_f$   
 $(25)(3.0) = T + 0 \quad T = 75.0 \text{ N}$



b)  $\Sigma F = 120 + (-0.20)(40)(9.8)$   
 $= 41.6 \text{ N}$   
 $\vec{a} = \frac{41.6}{40} = 1.04 \text{ m/s}^2$

$ma = T + F_f$   
 $(25)(1.04) = T + (-49) \quad T = 75 \text{ N}$

3. Traveling at a speed of 58.0 km/h, the driver of an 800 kg automobile suddenly slams on the brakes. The coefficient of kinetic friction between the tires and the road is 0.720. How far does the car skid before coming to a stop? (Ignore air friction) (18.4m)



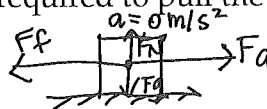
$F_f = (0.720)(800)(9.8)$   
 $= -5645 \text{ N}$

$ma = F_a + F_f$   
 $(800)a = 0 + (-5645)$   
 $\vec{a} = -7.06 \text{ m/s}^2$

$0^2 = 16.1^2 + 2(-7.06)d$   
 $\vec{d} = 18.4 \text{ m}$

4. A 250 N block rests on a horizontal table. The coefficient of kinetic friction between the table and the block is 0.30. What force is required to pull the block at a constant speed if,

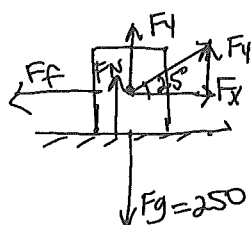
a) The rope is horizontal? (+75 N)



$$F_f = (0.30)(250) = -75 \text{ N}$$

$$F_a = +75 \text{ N (balanced)}$$

b) The rope makes an angle of  $25^\circ$  above the horizontal? (+72.6 N)

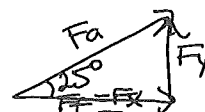


$$F_f = \mu F_N$$

$$F_x = F_f$$

$$*F_N = 250 - \sin 25(F_a)$$

$$F_f = (0.30)(250 - \sin 25(F_a))$$

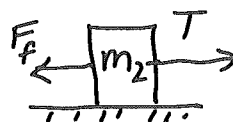
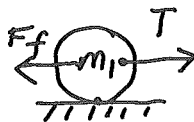
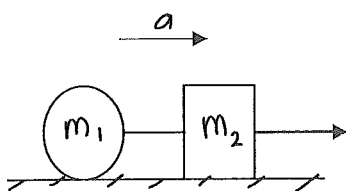


$$\cos 25 = \frac{(0.30)(250 - \sin 25(F_a))}{F_a}$$

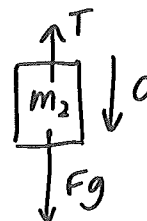
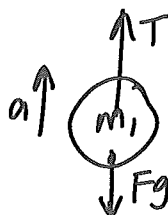
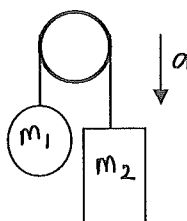
$$F_a = +72.6 \text{ N}$$

5. Draw a diagram showing the forces acting on each object, and label the forces with appropriate symbols. Ignore the mass and friction of the pulley.

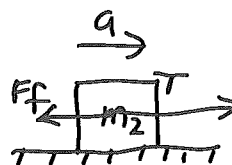
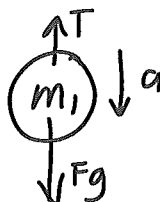
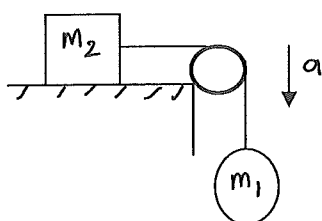
a)



b)



c)



6. Find the acceleration of the system and the tensional forces as labeled in each of the following: (ignore air friction and friction of the pulley)

A)

$\Sigma F = F_g + F_f$   
 $= (15)(9.8) + (-0.18)(13)(9.8)$   
 $= 124 \text{ N}$   
 $\vec{a} = \frac{124}{28} = \pm 4.43 \text{ m/s}^2$

$ma = T_1 + F_g$   
 $(15)(-4.43) = T_1 + (-147)$   
 $T_1 = +81 \text{ N}$   
 $(\pm 4.43 \text{ m/s}^2, +81 \text{ N}, +31 \text{ N})$

$T_2 \rightarrow F_f \leftarrow \boxed{5.0} \rightarrow T_2$   
 $ma = T_2 + F_f$   
 $(5.0)(4.43) = T_2 + (-8.82)$   
 $F_f = (0.18)(5)(9.8) = -8.82 \text{ N}$   
 $T_2 = +31 \text{ N}$

B)

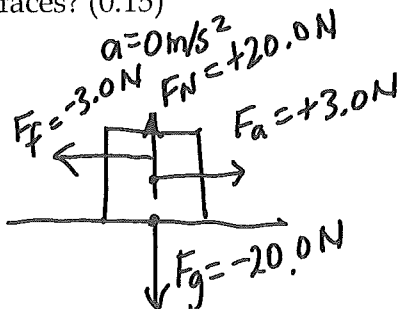
$\Sigma F = F_g + F_g = (6.5)(9.8) + (-4.5)(9.8) = 19.6$   
 $\vec{a} = \frac{19.6}{11} = \pm 1.78 \text{ m/s}^2$

$ma = T_1 + F_g$   
 $(4.5)(1.78) = T_1 + (-44.1)$   
 $T_1 = +52 \text{ N}$

$T_2 \rightarrow \boxed{2.5} \downarrow a$   
 $ma = T_2 + F_g$   
 $(2.5)(-1.78) = T_2 + (-24.5)$   
 $T_2 = +20 \text{ N}$

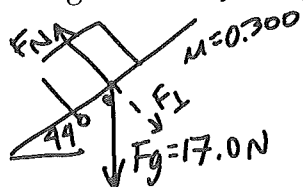
$(\pm 1.78 \text{ m/s}^2, +52 \text{ N}, +20 \text{ N})$

7. A 20.0 N object is placed on a horizontal surface. A force of 3.0 N is required to keep the object moving at a constant speed. What is the coefficient of friction between the two surfaces? (0.15)



$\Sigma F = 0 \text{ N}$   
 $F_f = \mu F_N$   
 $3.0 = \mu(20)$   
 $\mu = 0.15$

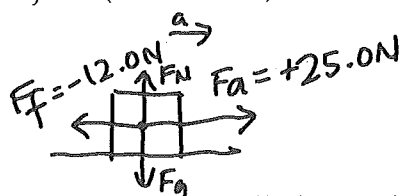
8. A 17.0 N object is pulled up an inclined plane. If the inclined plane makes an angle with the horizontal of  $44.0^\circ$ , and the coefficient of friction is 0.300, what is the force of friction acting on the object? (-3.67 N)



$$F_N = (17.0) \cos 44 = +12.2 \text{ N}$$

$$F_f = \mu F_N = (0.300)(12.2) = -3.67 \text{ N}$$

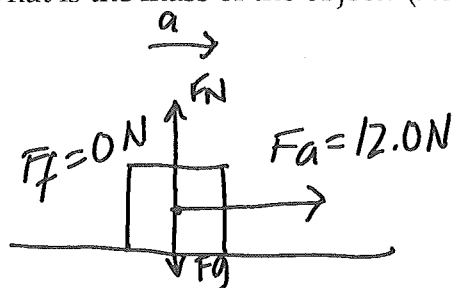
9. A 23.0 kg object is pushed with a horizontal force of 25.0 N east across a horizontal surface. If the force of friction between the two surfaces is 12.0 N, what is the acceleration of the object? (+0.565 m/s<sup>2</sup>)



$$ma = F_a + F_f$$

$$(23.0)a = 25.0 + (-12.0) \quad \vec{a} = +0.565 \text{ m/s}^2$$

10. An object is pulled east along a horizontal frictionless surface with a steady horizontal force of 12.0 N. If the object accelerates from rest to a velocity of 4.0 m/s while moving 5.0 m, what is the mass of the object? (7.5 kg)



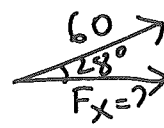
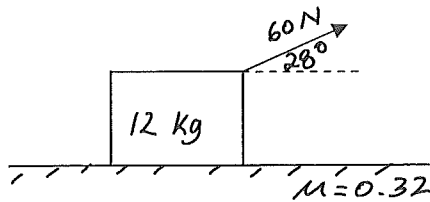
$$4.0^2 = 0^2 + 2a(5.0)$$

$$\vec{a} = +1.6 \text{ m/s}^2$$

$$m(1.6) = 12.0 + 0$$

$$m = 7.5 \text{ kg}$$

11. A 12 kg crate is pulled along a horizontal floor by a force of 60 N which is applied at an angle of  $28^\circ$  above the horizontal. The coefficient of friction between the crate and floor is 0.32.

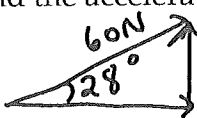


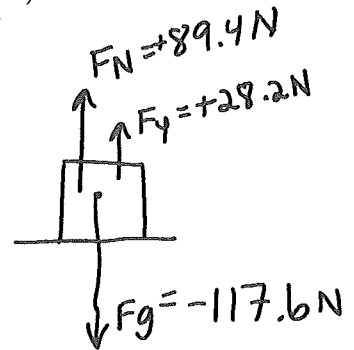
$$\cos 28(60) = 53.0 \text{ N}$$

a) Find the magnitude of the normal force acting on the object. (89.4 N)

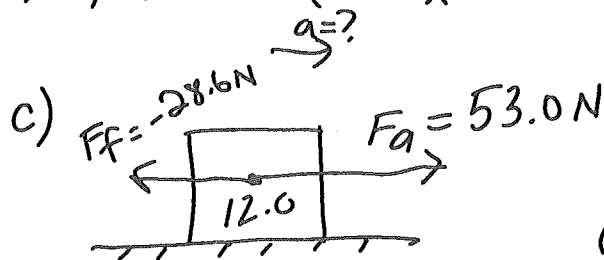
b) Find the friction acting on the crate. (-28.6 N)

c) Find the acceleration of the crate. (+2.03 m/s<sup>2</sup>)

a)   $F_y = ?$   $F_y = \sin 28(60) = 28.2 \text{ N}$   
 $F_N = 117.6 - 28.2 = \underline{\underline{89.4 \text{ N}}}$



b)  $F_f = \mu F_N = (0.32)(89.4) = \underline{\underline{-28.6 \text{ N}}}$



$$ma = F_a + F_f$$

$$(12.0)a = 53 + (-28.6)$$

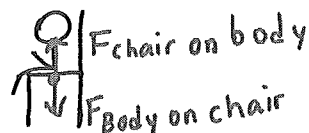
$$\vec{a} = 2.03 \text{ m/s}^2$$

## Physics 12 – Newton's Third Law of Motion

A force is a push or a pull upon an object which results from its interaction with another object. **Forces result from interactions.** According to Newton, whenever two objects interact with each other, they exert equal and opposite forces upon one another.

This is stated as: *For every action (force), there is an equal and opposite reaction (force)*

For example, when you sit in a chair, your body exerts a downward force on the chair and the chair exerts an upward force on your body.

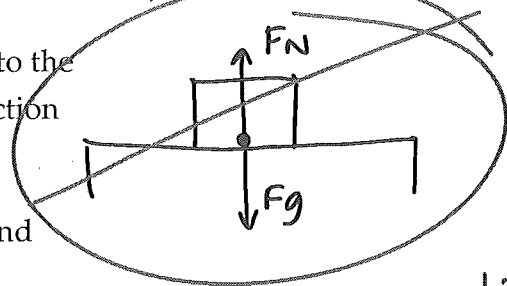


There are two forces resulting from this interaction — a force on the chair and a force on your body. These two forces are called action and reaction forces.

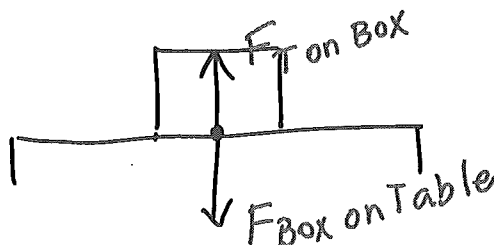
The statement means that in every interaction, there is a pair of forces acting on the two interacting objects. *The size of the force on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object.*

Recall that in order to be an action-reaction pair, there must be two objects involved.

For example, while the normal force is equal and opposite to the force of gravity on a horizontal surface, this **IS NOT** an action reaction pair as both forces are acting on the same object.



However, if we look at the interaction between the object and the surface below it (table), there is an action-reaction pair.



*= Action-Reaction*

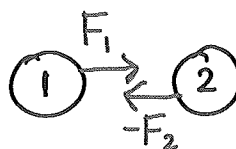
Newton's Third Law can be illustrated and solved using free-body diagrams and mathematics:

Formula:

$$\vec{F}_{A \rightarrow B} = -\vec{F}_{B \rightarrow A}$$

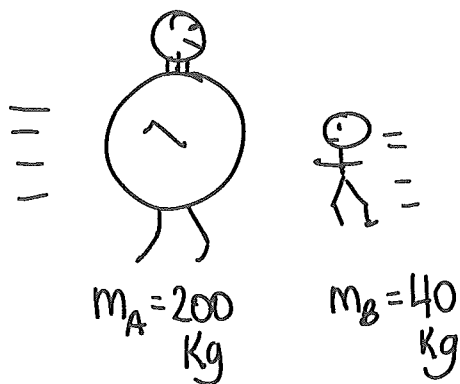
$$m_1 a_1 = -m_2 a_2$$

Free-Body Diagram:



However, just because the magnitude of the forces acting on each of the objects is the same, does not mean that the resulting motion will be the same.

Example – A linebacker ( $m_A = 200\text{kg}$ ) runs into a gymnast ( $m_B = 40\text{kg}$ ) by accident.



As they collide,  
each exerts a  
force on the other:

The linebacker exerts  $F_L = \underline{800 \text{ N}}$  to the right  
on the gymnast

The gymnast exerts  $F_G = \underline{800 \text{ N}}$  to the left  
on the linebacker

What happens as a result?

Both will experience an  
acceleration

The Linebacker's acceleration  $\rightarrow F_G = m_L a \quad -800 = (200)a$   
 $a = -4 \text{ m/s}^2$

The Gymnast's acceleration  $\rightarrow F_L = m_G a \quad +800 = (40)a$   
 $a = +20 \text{ m/s}^2$

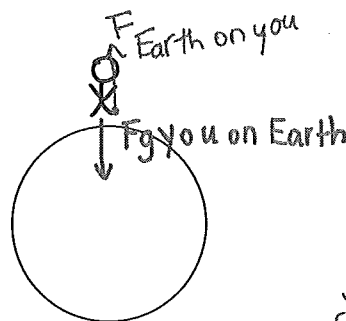
Same  $\vec{F}$  but different  $\vec{a}$   
= depends on mass (Newton's  
2nd Law)

## Gravitation: You Attract The Earth!

$$F_{g \text{ you on Earth}} = - F_{g \text{ Earth on you}}$$

But the acceleration that you produce on the Earth is very, very small.

$$a = \frac{\text{your weight}}{\text{Earth's mass}} \approx 10^{-22} \text{ m/s}^2$$



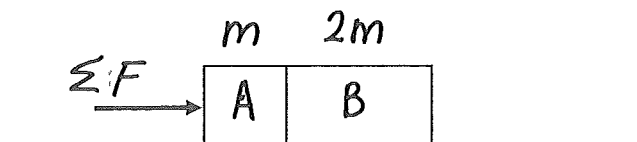
*\*Video clip  
(all jumped  
at the  
same time)*

Two blocks of masses  $m$  and  $2m$  are pushed together along a horizontal, frictionless surface by a force  $F$ . The magnitude of the net force on block B is:

A.  $1/3 F$

B.  $2/3 F$

C.  $F$



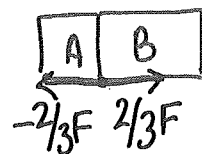
system:  $\Sigma F = 3ma$  so...  $ma = \frac{\Sigma F}{3}$  or  $ma = \frac{1}{3} F$   
( $m + 2m = 3m$ )

Block B:  $\Sigma F_B = 2ma = \frac{2\Sigma F}{3}$  or  $2/3 F$

Block A:  $\Sigma F_A = ma = \frac{\Sigma F}{3}$  or  $1/3 F$

The magnitude of the force on block A by block B is:

*the same*



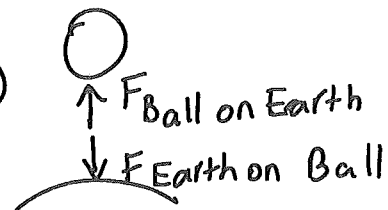
While a football is in flight, what forces act on it? What are the action and reaction pairs while the football is being kicked and while it is in flight?

Forces = gravity & air friction

Action-Reaction pairs =

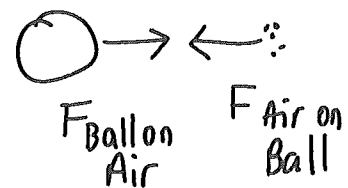
① Earth pulls down on ball,  
Ball pulls up on Earth

$$F_B = -F_E$$



② Ball pushes air forward  
Air pushes ball backward

$$F_B = -F_A$$



A  $3.40 \times 10^3$  kg rocket is travelling east along a frictionless track at a velocity of 14.0 m/s. The rocket accelerates uniformly to a velocity of 20.0 m/s in a time of 1.10s by the expulsion of hot gases. What is the force in which the gases are expelled by the rocket?

$$F_1 = -F_2$$

$$m_1 a_1 = -m_2 a_2$$



$$v_0 = 14.0 \text{ m/s}$$

$$v_f = 20.0 \text{ m/s}$$

$$a = ?$$

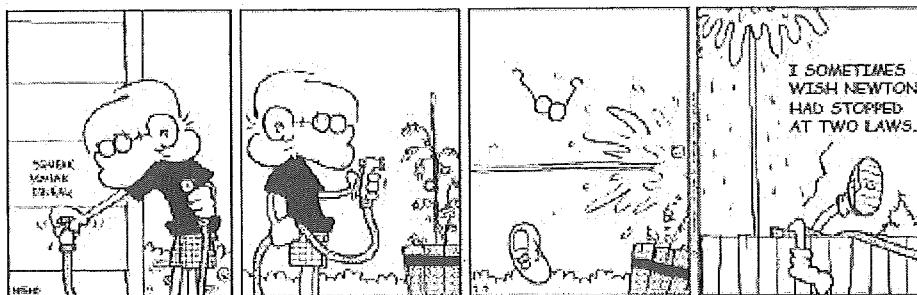
$$d = x$$

$$t = 1.10 \text{ s}$$

$$a = \frac{20 - 14}{1.10} \quad a = 5.45 \text{ m/s}^2$$

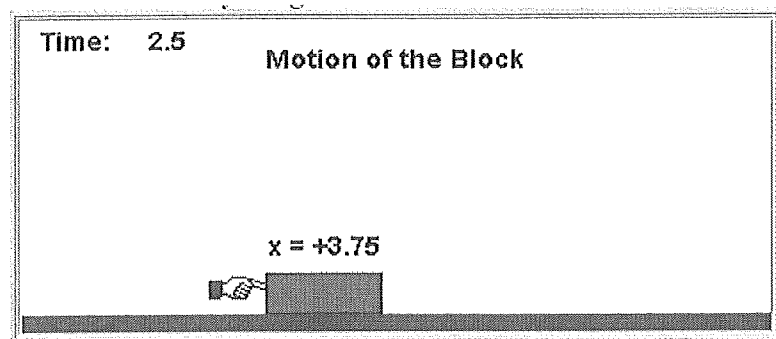
$$\begin{aligned} F_1 &= m_1 a_1 \\ &= (3400)(5.45) \\ &= 18,530 \text{ N} \end{aligned}$$

$$F_1 = -F_2 = -1.85 \times 10^4 \text{ N}$$



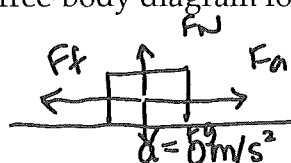
# Assignment: Newton's 3rd Law and Free Body Diagrams

## Part I: Free Body Diagrams



An 8-kg block is pushed across the floor at a constant speed.

1. Sketch a possible free-body diagram for the block:



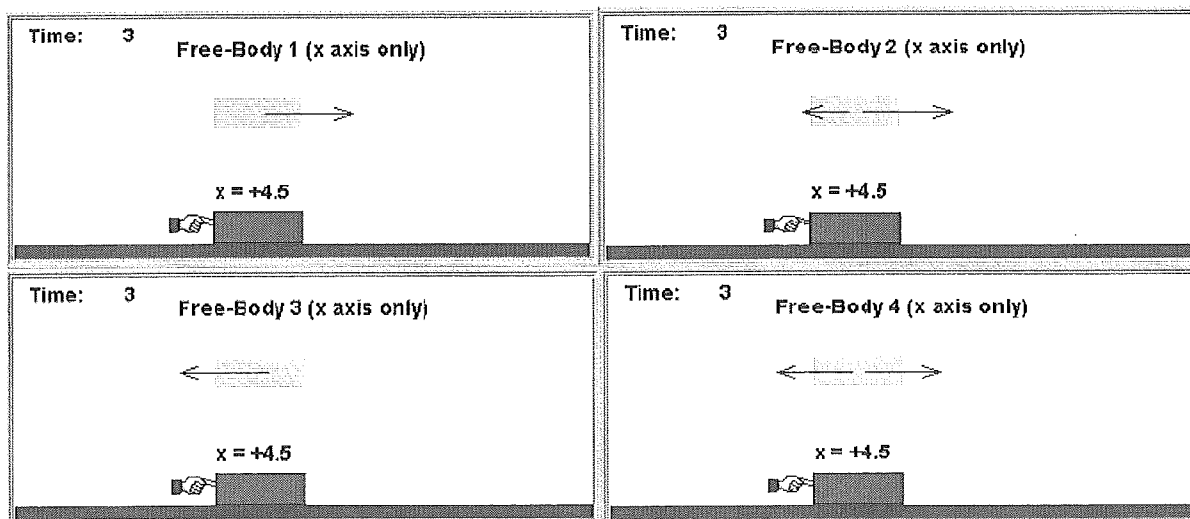
2. What is the acceleration of the block? 0 m/s<sup>2</sup>

How do you know? constant speed / balanced forces

3. Therefore, the net force (sum of all the forces) in the x-direction = 0 N

4. Similarly, the net force (sum of all the forces) in the y-direction = 0 N

Consider the forces in the x-direction (horizontal forces). Consider each of the four possible free-body diagrams below (each diagram is above the block). The length of the vector represents the magnitude of the force.

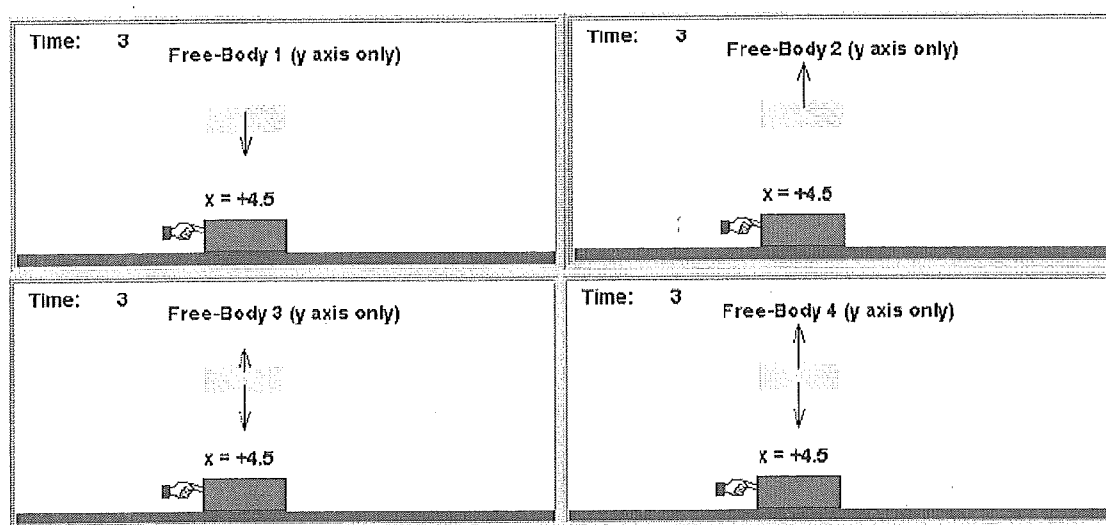


5. Only one is a possible free-body diagram. For each one, indicate why it is or is not a possible force diagram:

Horizontal components:

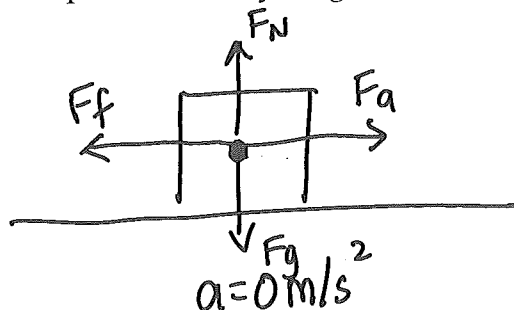
	Possible force diagram?	Reason:
Free-Body 1x	No	would be accel. to right
Free-Body 2x	No	"
Free-Body 3x	No	would accel to left
Free-Body 4x	Yes	balanced forces = $0 \text{ m/s}^2$

6. Vertical components:



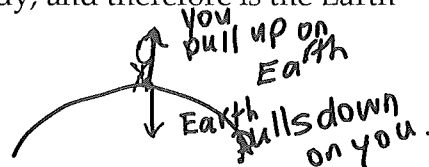
	Possible force diagram?	Reason:
Free-Body 1y	No	accel down
Free-Body 2y	No	accel up
Free-Body 3y	No	accel down
Free-Body 4y	Yes	balanced = $0 \text{ m/s}^2$

7. Based on this, sketch a complete free-body diagram below:



7. If your weight is force created by gravity on your body, and therefore is the Earth pulling you down, what is the reaction force?

*you pulling up on the Earth*



8. There is nothing in outer space for rocket exhaust gasses to push against. How then can a rocket accelerate in outer space?

*the rocket pushes out the gasses and the gasses push back on the rocket*



9. No matter how hard a horse pulls on a cart, the cart must pull back with exactly the same force according to Newton's Third Law. How can a cart pull a horse? (For this question assume the horse and cart are on level ground.)



*Exerts an equal & opposite reaction force but wheels decrease the force needed to do work (accel. the wagon)*

10. A bug splatters on a fast-moving car's windshield. Is the force on the car from the bug the same as the force on the bug from the car? Explain.



*mass is so tiny in comparison to car that acceleration is extremely high (very high impulse causes damage to bug!)*

$$\Delta p = m \Delta v$$

11. Three blocks on a frictionless horizontal surface are in contact with each other. A force of 100N is applied to block 1. Each block has a mass of 10.0 kg.

A) Determine the acceleration of the system. (+3.33 m/s<sup>2</sup>)

B) Determine the force that each box exerts on its neighbor. (F<sub>1→2&3</sub> = +67N, F<sub>2→3</sub> = +33N)

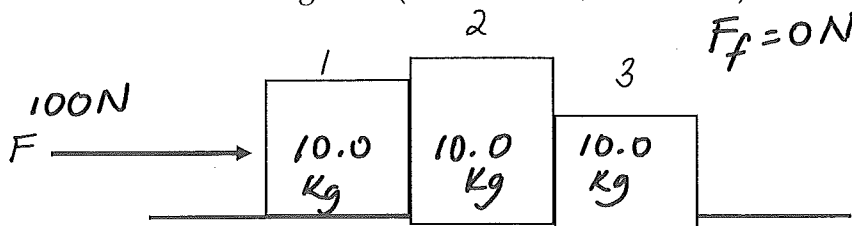
A)  $\Sigma F = ma$

$$100 + 0 = 30a$$

$$a = +3.33 \text{ m/s}^2$$

B)  $F_{1 \rightarrow 2 \& 3} \quad \Sigma F = (20)(3.3)$   
 $= +67 \text{ N}$

$$F_{2 \rightarrow 3} \quad \Sigma F = (10)(3.3)$$
  
 $= +33 \text{ N}$



12. Two crates, of mass 80 kg and 110 kg, are in contact and at rest on a horizontal surface. A 650 N force is exerted on the 80 kg crate. If the coefficient of friction is 0.20, calculate -

A) The acceleration of the system (+1.46 m/s<sup>2</sup>)

B) The force that each crate exerts on the other. (376 N)

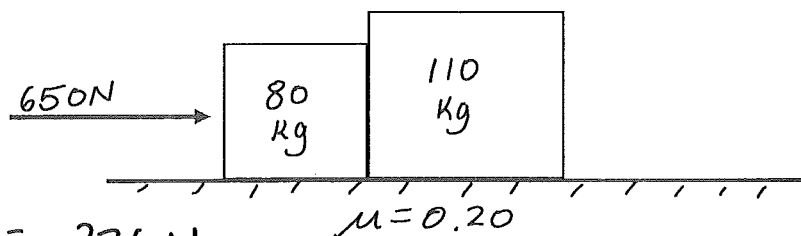
$$\begin{aligned} \text{A) } m\vec{a} &= F_a + F_f \\ 190\vec{a} &= 650 + (-0.20)(190)(9.8) \\ \vec{a} &= +1.46 \text{ m/s}^2 \end{aligned}$$

$$\text{B) } F_{80 \rightarrow 110}$$

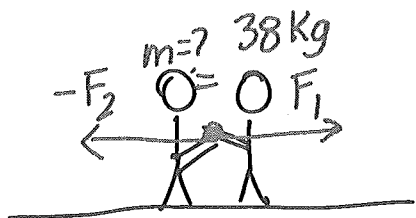
$$(110)(1.46) = F + (-215.6)$$

$$F_{80 \rightarrow 110} = +376 \text{ N}$$

$$F_{110 \rightarrow 80} = -376 \text{ N}$$

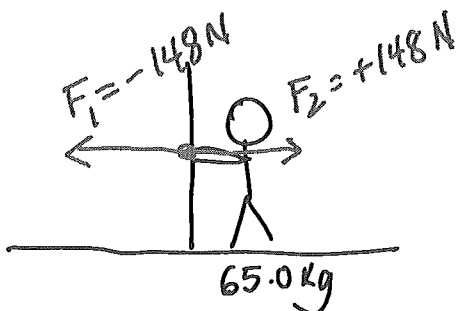


13. While standing on a horizontal frictionless surface, two students push against each other. Angela has a mass of 38 kg and, during the push, is accelerating east at a rate of 0.60 m/s<sup>2</sup>. If Bob is accelerating west during the push at a rate of 0.75 m/s<sup>2</sup>, what is his mass? (30 kg)



$$\begin{aligned} \sum F_1 &= ma = (38)(0.6) = +22.8 \text{ N} \\ F_1 &= -F_2 = -22.8 \text{ N} = F_2 \\ \sum F_2 &= ma \\ -22.8 &= m(-0.75) \quad m = 30 \text{ kg} \end{aligned}$$

14. While standing on a horizontal frictionless surface, a 65.0 kg student pushes against a wall with a force of 148 N west for 0.250 s. Calculate the velocity of this student at the end of the 0.250 s. (0.569 m/s [E])



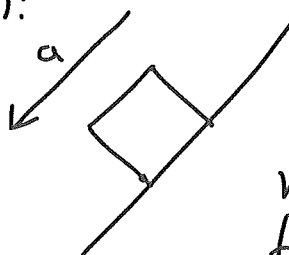
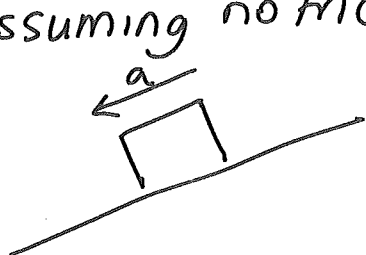
$$\begin{aligned} \sum F &= ma \\ 148 &= 65.0 a \\ a &= \frac{v_F - v_0}{t} \\ 2.28 &= \frac{v_F - 0}{0.250} \\ v_F &= 0.569 \text{ m/s [E]} \end{aligned}$$

## Physics 12 – The Physics of Inclined Planes

In physics we call any tilted surface an **inclined plane**. As long as the force of friction is not greater than the component of gravity parallel to the surface, the object will slide down the surface.

The rate in which the object will accelerate down an inclined plane depends on how tilted the inclined plane is.

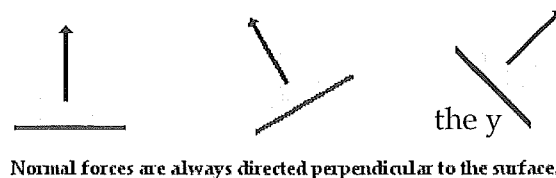
assuming no friction:



acceleration  
will be greater  
for the steeper  
slope

The object will be accelerated according to Newton's Laws of motion = unbalanced forces acting on the object.

We already know that the normal force is always perpendicular to the surface and that we can use component of  $F_g \rightarrow F_{\perp}$  to determine the normal force acting on an object on an inclined plane.



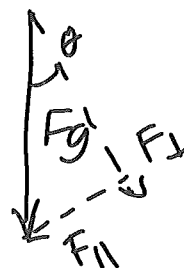
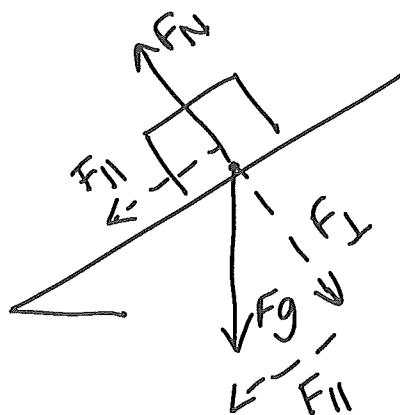
Normal forces are always directed perpendicular to the surface.

$F_{\perp}$  is opposed to the normal force and therefore balances the normal force.

However, we know that an unbalanced force causes the object to accelerate down the inclined plane.

$$= F_{\parallel}$$

(x-component  
of  $F_g$   
= parallel to  
surface)



$F_{\parallel}$  is not balanced by any other force.

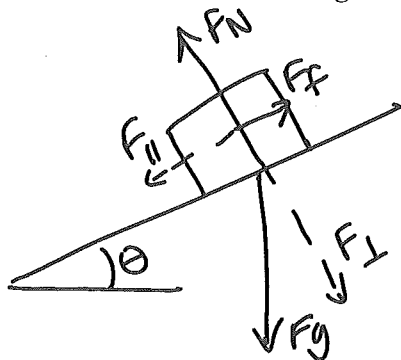
= therefore results in  
acceleration in that  
direction

The equations for the parallel and perpendicular components are:

$$F_{\perp} = mg \cos \theta$$

$$F_{\parallel} = mg \sin \theta$$

When there is friction or other forces (applied force, tensional forces, etc.) we need to take those into account when determining acceleration.



we still use

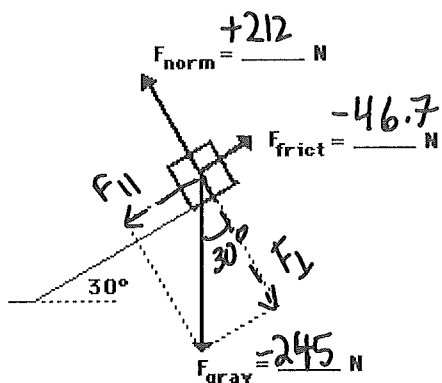
$$\Sigma F = ma$$

$\Sigma F$  = sum of forces

ex:

$$ma = F_{\parallel} + F_f$$

The diagram shows the forces acting upon a 25.0-kg crate that is sliding down an inclined plane. The plane is inclined at an angle of 30.0 degrees. The coefficient of friction between the crate and the incline is 0.220. Determine the acceleration of the crate.



$$F_{\perp} = mg \cos 30$$

$$= 212 \text{ N} = F_N$$

$$F_f = (0.22)(212) = -46.7 \text{ N}$$

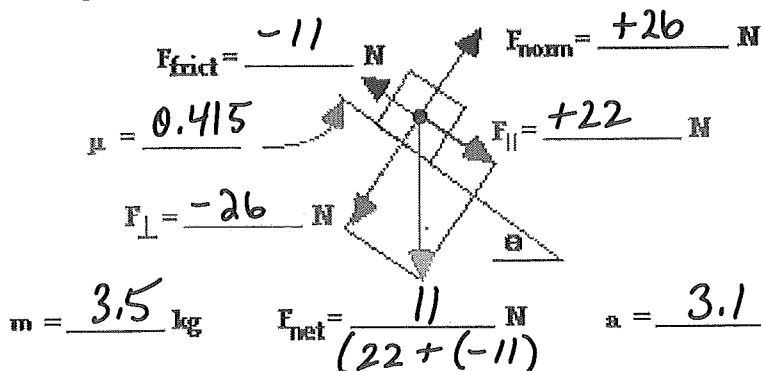
$$F_{\parallel} = mg \sin 30$$

$$= 123 \text{ N}$$

$$ma = F_{\parallel} + F_f$$

$$(25) a = 123 + (-46.7) \quad a = 3.05 \text{ m/s}^2$$

A 3.5 kg object is accelerating down an inclined plane inclined at 40.0° (with the horizontal) and having a coefficient of friction of 0.415. Fill in all of the blanks.



$$F_{\perp} = (3.5)(9.8) \cos 40$$

$$= 26 \text{ N}$$

$$F_{\parallel} = (3.5)(9.8) \sin 40$$

$$= 22 \text{ N}$$

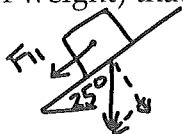
$$F_f = (0.415)(26) = -11 \text{ N}$$

$$3.5a = 22 + (-11)$$

$$a = 3.1 \text{ m/s}^2$$

**Inclined Plane Assignment – Draw a free-body diagram for each question.**

1. A 445 N box is sliding down a frictionless 25.0° inclined plane. Find the force (component of weight) that causes the box to slide. (188 N)



$$F_{||} = \sin 25.0 (445) = 188 \text{ N}$$

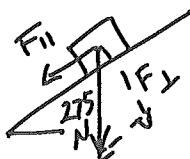
2. A 325 N box is sliding down a frictionless inclined plane. If the incline makes a 30.0° angle with the horizontal, what is the acceleration along the incline? (4.91 m/s² down incline)



$$\Sigma F = F_{||} = \sin 30 (325) = 163 \text{ N}$$

$$163 = 32.2 \vec{a} \quad \vec{a} = 4.91 \text{ m/s}^2$$

3. A 275 N box is sliding down a 35.0° incline. If the force of friction acting on the box along the incline is 96.0 N, what is the acceleration of the box? (2.21 m/s²)



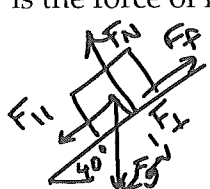
$$F_{||} = \sin 35 (275) = 158 \text{ N}$$

$$ma = F_{||} + F_f$$

$$(28.1) \vec{a} = 158 + (-96.0)$$

$$\vec{a} = 2.21 \text{ m/s}^2$$

4. A 435 N box is sliding down a 40.0° incline. If the acceleration of the box is 0.250 m/s², what is the force of friction on the box? (-269 N)

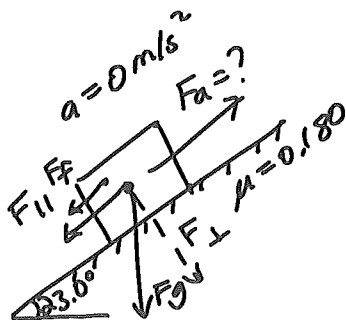


$$F_{||} = \sin 40 (435) = 280 \text{ N}$$

$$ma = F_{||} + F_f \quad (44.4)(0.250) = 280 + F_f$$

$$F_f = -269 \text{ N}$$

5. A student pulls a 125 N object up a 23.0° inclined plane. If the coefficient of friction between the object and the incline is 0.180, what force must the student apply to pull the object up the incline at a constant velocity? (+69.5 N)



$$\cos 23 (125) = F_{\perp} = F_N = 115 \text{ N}$$

$$\sin 23 (125) = F_{||} = -48.8 \text{ N}$$

$$F_f = (0.180)(115) = -20.7 \text{ N}$$

$$ma = F_a + F_f + F_{||}$$

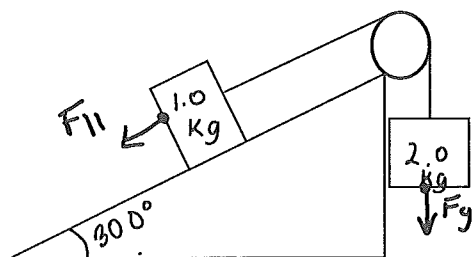
$$0 = F_a + (-20.7) + (-48.8)$$

$$F_a = +69.5 \text{ N}$$

6. Two blocks are tied together with a string as shown in the diagram. If both the pulley and the incline are frictionless,

A) What is the acceleration of the 1.0 kg block up the incline? ( $\pm 4.9 \text{ m/s}^2$ )

B) What is the tension in the string joining the two blocks? ( $+9.8 \text{ N}$ )



$$a) F_{||} = \sin 30(9.8) = -4.9 \text{ N} \quad (-)$$

$$F_g = 19.6 \text{ N} \quad (+)$$

$$\Sigma F = 19.6 + (-4.9) = \frac{14.7}{3.0} = +4.9 \text{ m/s}^2$$

$$b) (2.0)(-4.9) = T + (-19.6)$$

$$T = +9.8 \text{ N}$$

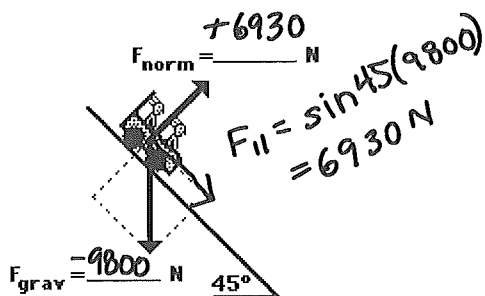
7. If the pulley was frictionless but the coefficient of friction between the incline and the 1.0 kg block is 0.25, what would the acceleration of the 1.0 kg block in the question 6 be? ( $\pm 4.2 \text{ m/s}^2$ )

$$F_N = \cos 30(9.8) = 8.49 \text{ N} \quad F_f = (0.25)(8.49) = -2.12 \text{ N}$$

$$\Sigma F = 19.6 + (-4.9) + (-2.12) = \frac{12.58}{3.0} = +4.2 \text{ m/s}^2$$

8. The two diagrams below depict the free-body diagram for a 1000-kg roller coaster on the first drop of two different roller coaster rides. Determine the **net force** and **acceleration** of the roller coaster cars. Assume no friction and no air resistance.

Diagram A



$$m = 1000 \text{ kg}$$

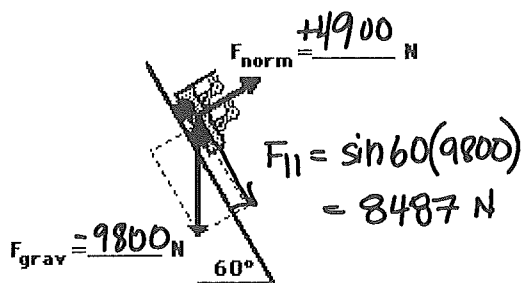
$$a = 6.930 \text{ m/s/s}$$

$$F_{\text{net}} = 6930 \text{ N}$$

$$F_N = \cos 45(1000 \cdot 9.8) = +6930 \text{ N}$$

$$6930 = 1000 \vec{a}$$

Diagram B



$$m = 1000 \text{ kg}$$

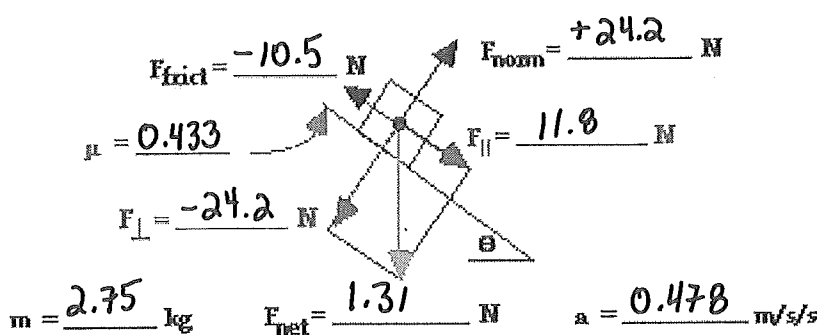
$$a = 8.487 \text{ m/s/s}$$

$$F_{\text{net}} = 8487 \text{ N}$$

$$F_N = \cos 60(1000 \cdot 9.8) = 4900 \text{ N}$$

$$8487 = 1000 \vec{a}$$

9. A 2.75-kg object is accelerating down an inclined plane inclined at  $26.0^\circ$  (with the horizontal) and having a coefficient of friction of 0.433.



$F_{\text{frict}} = -10.5 \text{ N}$   
 $F_{\text{norm}} = +24.2 \text{ N}$   
 $\mu = 0.433$   
 $F_{\perp} = -24.2 \text{ N}$   
 $F_{\parallel} = 11.8 \text{ N}$   
 $m = 2.75 \text{ kg}$   
 $F_{\text{net}} = 1.31 \text{ N}$   
 $a = 0.478 \text{ m/s}^2$

$$F_{\perp} = \cos 26.0 (2.75 \cdot 9.8) = 24.2 \text{ N}$$

$$F_f = (0.433)(24.2) = -10.5 \text{ N}$$

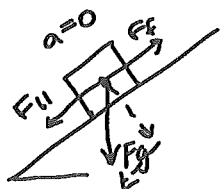
$$F_{\parallel} = \sin 26.0 (2.75 \cdot 9.8) = 11.8 \text{ N}$$

$$\Sigma F = 11.8 + (-10.5) = 1.31 \text{ N}$$

$$1.31 = 2.75 \vec{a}$$

$$\vec{a} = 0.478 \text{ m/s}^2$$

10. A 22.5-kg box slides down an inclined plane (inclined at  $33.0$  degrees) at a constant speed of 2.1 m/s. What is the force of friction acting on the box? ( $-1.2 \times 10^2 \text{ N}$ )



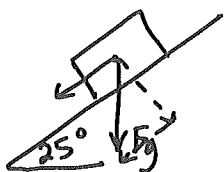
$$F_{\parallel} = \sin 33 (22.5 \cdot 9.8) = +120 \text{ N}$$

$$F_f = -120 \text{ N}$$

11. A 1200 N truck slides down a frictionless plane inclined at an angle of  $25.0^\circ$  from the horizontal.

A. Find the acceleration of the trunk. (frictionless) ( $+4.16 \text{ m/s}^2$ )

B. Find the acceleration of the trunk if the coefficient of friction was 0.300. ( $+1.48 \text{ m/s}^2$ )



$$A. F_{\parallel} = \sin 25 (1200) = 507 \text{ N}$$

$$\Sigma F = ma \quad 507 = 122 \vec{a}$$

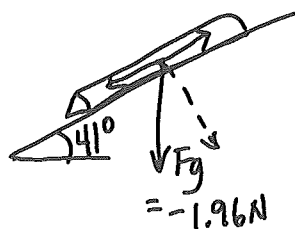
$$\vec{a} = +4.16 \text{ m/s}^2$$

$$B. F_{\perp} = \cos 25 (1200) = F_N = 1088 \text{ N} \quad F_f = (0.300)(1088) = -326 \text{ N}$$

$$(122) \vec{a} = 507 + (-326)$$

$$\vec{a} = +1.48 \text{ m/s}^2$$

12. A 200 g pen placed on the cover of a book just begins to move when the cover makes an angle of  $41^\circ$  with the horizontal. What is the coefficient of static friction? (0.87)



$$\sin 41 (1.96) = 1.29 \text{ N}$$

$$\cos 41 (1.96) = 1.48 \text{ N} = F_N$$

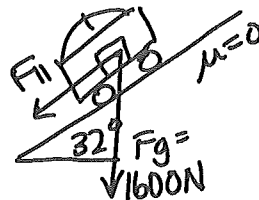
$$1.29 = \mu 1.48$$

$$\mu = 0.87$$

$$F_{\parallel} = F_f = 1.29 \text{ N}$$

$$a = 0 \text{ m/s}$$

13. A car weighing 1600 N is parked on a 32° slope. The brakes fail and the car starts to slide down the hill. Assume no friction.



A. What is the acceleration of the car? (+5.2 m/s<sup>2</sup>)

B. After it has moved 41.0 m, how fast is it moving? (+21 m/s)

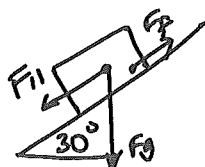
$$A) F_{||} = \sin 32(1600) = 849 \text{ N} \quad \Sigma F = ma$$

$$849 = 1600a \quad a = +5.2 \text{ m/s}^2$$

$$B) v_f^2 = 0^2 + 2(5.2)(41.0)$$

$$v_f = 21 \text{ m/s}$$

14. A 10.0 kg box sits on an incline. Its coefficient of friction is 0.270, and the incline is at an angle of 30.0° to the horizontal. Find the acceleration of the block. (+2.61 m/s<sup>2</sup>)



$$F_N = \cos 30(98) = 84.9 \text{ N} \quad F_f = (0.270)(84.9)$$

$$F_{||} = \sin 30(98) = 49 \text{ N} \quad = -22.9 \text{ N}$$

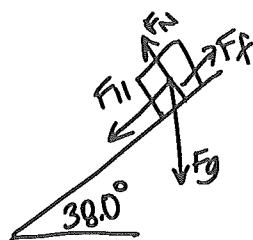
$$(10.0)a = 49 + (-22.9) \quad a = +2.61 \text{ m/s}^2$$

15. A 16.0 kg block is released from the top of an incline that is 5.00 m long and makes an angle of 38.0° to the horizontal. A force of friction of 45.0 N acts on the box.

A. Find the acceleration of the box. (+3.22 m/s<sup>2</sup>)

B. How long will it take the box to reach the bottom of the incline? (1.76 s)

C. What is the coefficient of friction? (0.363)



$$A) 16.0a = 96.5 + (-45.0) \quad F_{||} = \sin 38(16 \cdot 9.8)$$

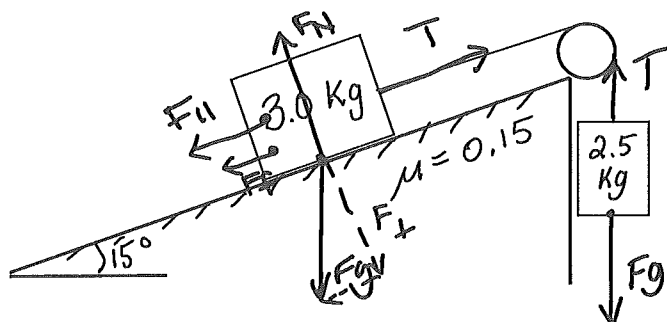
$$a = 3.22 \text{ m/s}^2 \quad = 96.5 \text{ N}$$

$$B) 5.00 = 0 + \frac{1}{2}(3.22)t^2 \quad t = 1.76 \text{ s}$$

$$C) F_N = \cos 38(16 \cdot 9.8) = 124 \text{ N}$$

$$45.0 = \mu 124 \quad \mu = 0.363$$

Now that you understand how to determine basic forces and acceleration on an inclined plane, we will now add systems into our observations and calculations.

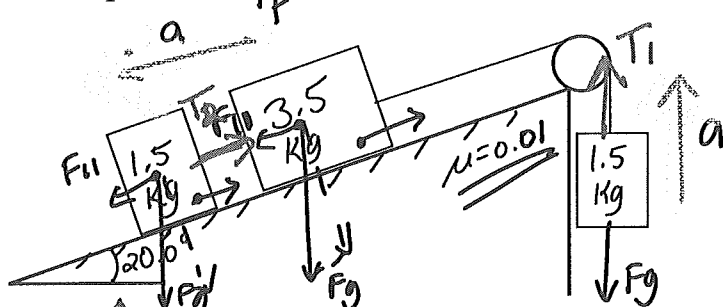


$$\Sigma F = 24.5 + (-7.6) + (-4.3) = 12.6 \text{ N}$$

$$\vec{a} = \frac{12.6}{5.5} = \pm 2.3 \text{ m/s}^2$$

T:  $(2.5)(-2.3) = T + (-24.5)$   
 $T = \underline{+19 \text{ N}}$

Example Two:  $\vec{F}_f = ?$



T<sub>1</sub>:  $(1.5)(0.25) = T_1 + (-14.7)$   
 $T_1 = \underline{+15 \text{ N}}$

$$F_g = (1.5)(9.8) = 14.7 \text{ N}$$

$$F_{||} = (5.0 \cdot 9.8) \sin 20 = 16.8 \text{ N}$$

$$F_f = (0.01)(46) = 0.46 \text{ N}$$

$$F_{\perp} = (5.0 \cdot 9.8) \cos 20 = 46 \text{ N}$$

$$\Sigma F = F_g + F_{||} + F_f$$

$$(-) (+) (-)$$

$$(1.5)(-0.25) = T_2 + 0.14 + (-5.0) = -14.7 + 16.8 + -0.46 = 1.64 \text{ N}$$

$$T_2 = \underline{+4.5 \text{ N}}$$

$$\vec{a} = \frac{1.64}{6.5} = \pm 0.25 \text{ m/s}^2$$

$F_f = (1.5 \cdot 9.8 \cdot \cos 20)(0.01)$   
 $= 0.4 \text{ N}$

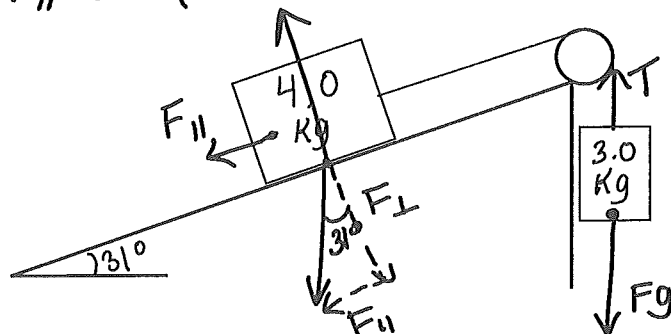
$F_{||} = 1.5 \cdot 9.8 \cdot \sin 20$   
 $= 5.0 \text{ N}$

# Assignment Questions:

1. Find the acceleration and tension for the following frictionless system.

$$F_g = (3.0)(9.8) = 29.4 \text{ N}$$

$$F_{11} = \sin 31(4.0 \cdot 9.8) = 20.2 \text{ N}$$



$$\Sigma F = F_g + F_{11}$$

$$(+)(-)$$

$$= 29.4 + (-20.2)$$

$$= 9.2 \text{ N}$$

$$\vec{a} = \frac{9.2}{7.0} = \underline{\underline{\pm 1.32 \text{ m/s}^2}}$$

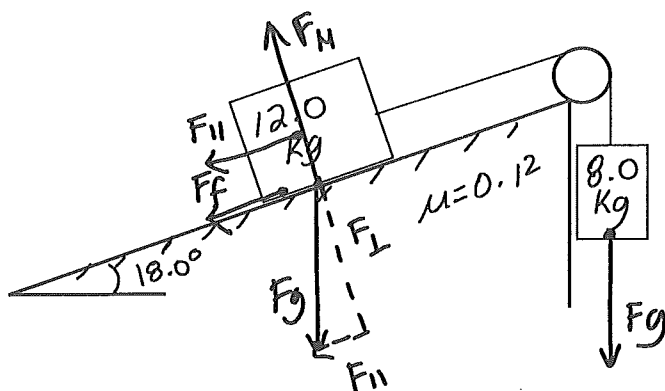
$$T = \underline{\underline{+25 \text{ N}}}$$

$$T: ma = T + F_g$$

$$(3.0)(-1.32) = T + (-29.4)$$

$$(\pm 1.32 \text{ m/s}^2, +25 \text{ N})$$

2. Find the acceleration and tension for the following system.



$$F_g = (8.0)(9.8) = 78.4 \text{ N}$$

$$F_N = F_{\perp} = \cos 18(12.0 \cdot 9.8)$$

$$= 112 \text{ N}$$

$$F_f = (0.12)(112) = 13.4 \text{ N}$$

$$F_{11} = \sin 18(12.0 \cdot 9.8)$$

$$= 36.3 \text{ N}$$

$$\Sigma F = F_g + F_{11} + F_f = 78.4 + (-13.4) + (-36.3)$$

$$(+)(-)(-)$$

$$\Sigma F = 28.7 \text{ N}$$

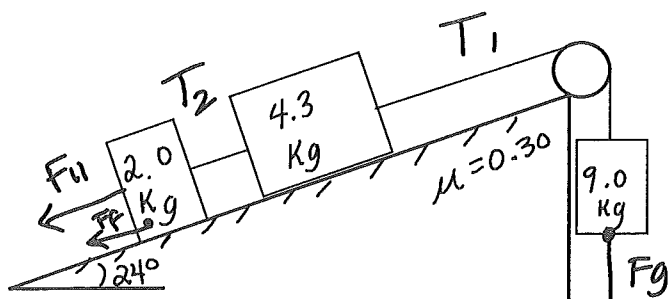
$$(\pm 1.44 \text{ m/s}^2, +67 \text{ N})$$

$$\vec{a} = \frac{28.7}{20} = \underline{\underline{\pm 1.44 \text{ m/s}^2}}$$

$$T: (8.0)(-1.44) = T + (-78.4)$$

$$T = \underline{\underline{+67 \text{ N}}}$$

3. Find the acceleration and the tension for both strings for the system.



$$\Sigma F = F_g + F_{||} + F_f$$

(+    -    -)

$$F_g = (9.0)(9.8) = 88.2 \text{ N}$$

$$F_{||} = \sin 24 (6.3 \cdot 9.8) = 25.1 \text{ N}$$

$$F_N = \cos 24 (6.3 \cdot 9.8) = 56.4 \text{ N}$$

$$F_f = (0.30)(56.4) = 16.9 \text{ N}$$

$$\Sigma F = 88.2 + (-25.1) + (-16.9)$$

$$\Sigma F = 46.2$$

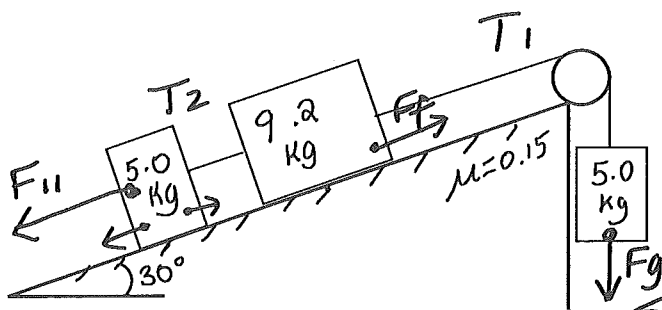
$$T_1: (9.0)(-3.02) = T_1 + (-88.2) \quad T_1 = +61 \text{ N}$$

$$T_2: (2.0)(3.02) = T_2 + (-7.97) + (-5.37)$$

$$\vec{a} = \frac{46.2}{15.3} = \pm 3.02 \text{ m/s}^2$$

$$T_2 = +19 \text{ N} (\pm 3.02 \text{ m/s}^2, +61 \text{ N}, +19 \text{ N})$$

4. Find the acceleration and the tension for both strings for the system.



$$\vec{a} = \frac{2.5}{19.2} = \pm 0.130 \text{ m/s}^2$$

$$F_g = (5.0)(9.8) = 49 \text{ N}$$

$$F_{||} = \sin 30 (14.2 \cdot 9.2) = 69.6 \text{ N}$$

$$F_N = \cos 30 (14.2 \cdot 9.2) = 121 \text{ N}$$

$$F_f = (0.15)(121) = 18.1 \text{ N}$$

$$\Sigma F = F_{||} + F_f + F_g$$

(+    -    -)

$$= 69.6 + (-18.1) + (-49) = 2.5 \text{ N}$$

$$T_1: (5.0)(0.130) = T_1 + (-49) \quad T_1 = +50 \text{ N}$$

$$T_2: (5.0)(-0.130) = T_2 + (-24.5) + 6.37$$

$$T_2 = +18 \text{ N}$$

$$F_{||} = 24.5 \text{ N}$$

$$F_{\perp} = 42.4 \text{ N}$$

$$F_f = (0.15)(42.4) = 6.37 \text{ N}$$

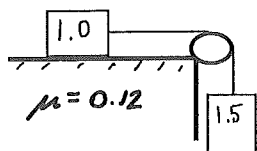
$$(\pm 0.130 \text{ m/s}^2, +50 \text{ N}, +18 \text{ N})$$

Physics 12 – Forces Assignment

Name:

Draw a free-body diagram labelling all forces acting on the object and the direction of acceleration. SHOW ALL OF YOUR WORK AND A FREE-BODY DIAGRAM FOR EACH QUESTION (or you will not receive full credit for each question). \*This will be handed in for assessment.

1. A 1.0 kg box on a horizontal surface is accelerated by attaching a 1.5 kg mass as shown in the diagram. What is the acceleration of the box if the coefficient of friction between the surfaces is 0.12? ( $\pm 5.4 \text{ m/s}^2$ )



$$\Sigma F = F_g + F_f = (1.5)(9.8) + (-0.12)(1.0)(9.8) = +13.5 \text{ N}$$

$$\vec{a} = \frac{13.5}{2.5} = \pm 5.4 \text{ m/s}^2$$

2. Two masses of 1.5 kg and 2.0 kg are hung on a frictionless pulley as shown in the diagram below. What is the acceleration of -

A) the 1.5 kg mass? ( $+1.4 \text{ m/s}^2$ )

$$+1.4 \text{ m/s}^2$$

B) the 2.0 kg mass? ( $-1.4 \text{ m/s}^2$ )

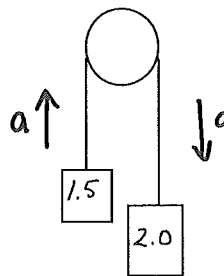
$$-1.4 \text{ m/s}^2$$

$$\Sigma F = F_{g2.0} + F_{g1.5}$$

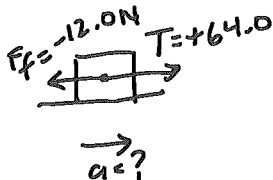
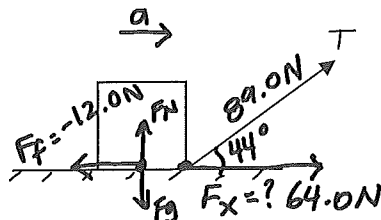
$$= (2.0)(9.8) + (-1.5)(9.8)$$

$$= +4.9 \text{ N}$$

$$\vec{a} = \frac{4.9}{3.5} = \pm 1.4 \text{ m/s}^2$$



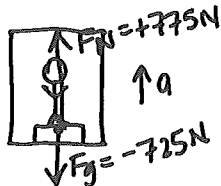
3. A 125N box is pulled east along a horizontal surface with a force of 89.0 N acting at an angle of  $44.0^\circ$  as shown in the diagram. If the force of friction on the box is 12.0 N, what is the acceleration produced? ( $+4.06 \text{ m/s}^2$ )



$$(12.8)(a) = 64.0 + (-12.0)$$

$$\vec{a} = 4.06 \text{ m/s}^2 [E]$$

4. A 725 N student stands on a bathroom scale while riding in an elevator. The student observes that the scale reads 775 N as the elevator begins to move. Find the acceleration and direction of acceleration of the elevator. ( $+0.676 \text{ m/s}^2$ )

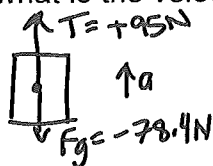


$$m = 74.0 \text{ Kg}$$

$$(74.0)(a) = 775 + (-725)$$

$$\vec{a} = +0.676 \text{ m/s}^2$$

5. An 8.0 kg object is pulled vertically upward by a rope. If the tension in the rope is constant at 95 N, what is the velocity of the object after 1.1s if it is starting at rest? (+2.3 m/s)



$$(8.0)a = 95 + (-78.4) \quad \vec{a} = +2.08 \text{ m/s}^2$$

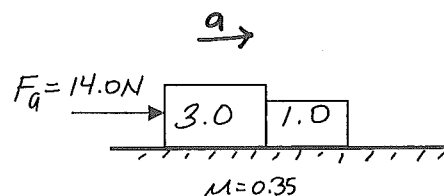
$$2.08 = \frac{v_f - 0}{1.1} \quad \vec{v}_f = +2.3 \text{ m/s}$$

6. A force of 14.0 N is applied to the block as shown in the diagram. If the coefficient of friction between the blocks and horizontal surface is 0.35,

A) What is the acceleration of the two blocks? (+0.070 m/s<sup>2</sup>)

$$(4.0)a = 14.0 + (-0.35)(4.0)(9.8)$$

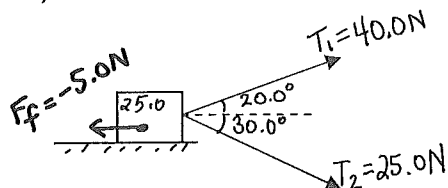
$$\vec{a} = 0.070 \text{ m/s}^2$$



B) What is the net force that the 3.0 kg block exerts on the 1.0 kg block? (hint: use the acceleration acting on the object to determine this.) (+0.21 N)

$$\Sigma F = ma = (3.0)(0.070) = +0.21 \text{ N}$$

7. Two students are dragging a 25.0 kg object along the hall as shown in the diagram below. If the force of friction acting on the object is 5.0 N, what is the acceleration of the object? (+2.2 m/s<sup>2</sup>)



$$\cos 20(40) = R_x = +37.6 \text{ N}$$

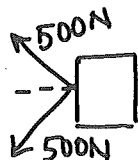
$$\cos 30(25) = R_x = +21.7 \text{ N}$$

$$(25.0)a = 59.3 + (-5.0)$$

$$\vec{a} = +2.2 \text{ m/s}^2$$

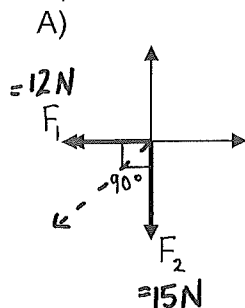
$$\Sigma: 37.6 + 21.7 = +59.3 \text{ N}$$

8. A 500 N force acts in the northwestern direction. A second 500 N force must be exerted in what direction so that the resultant of the two vectors points westward? (SW)



SW

9. The two forces  $F_1$  and  $F_2$  shown in the diagram act on a 10 kg object. If  $F_1 = 12 \text{ N}$  and  $F_2 = 15 \text{ N}$ , find the acceleration produced. (Assume no friction.)



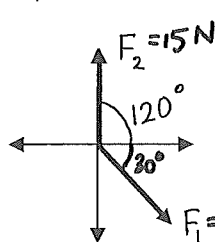
$$R = \sqrt{12^2 + 15^2} = 19 \text{ N}$$

$$\tan^{-1}\left(\frac{12}{15}\right) = 39^\circ \text{ W of S}$$

$$(10) a = 19 + 0$$

$$\vec{a} = 1.9 \text{ m/s}^2 \text{ } 39^\circ \text{ W of S}$$

(1.9 m/s<sup>2</sup> @ 39° W of S)

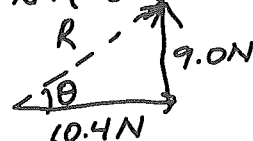


$$\cos 30 (12) = R_x = 10.4 \text{ N [E]}$$

$$\sin 30 (12) = R_y = 6.0 \text{ N [S]}$$

$$x: 10.4 \text{ N} + 0 \text{ N} = 10.4 \text{ N [E]}$$

$$y: 15 \text{ N} + (-6.0) = 9.0 \text{ N [N]}$$



$$R = \sqrt{10.4^2 + 9.0^2} = 13.8 \text{ N}$$

$$\tan^{-1}\left(\frac{9.0}{10.4}\right) = 41^\circ \text{ N of E}$$

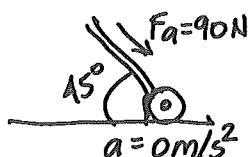
$$(10) a = 13.8 + 0$$

$$\vec{a} = 1.4 \text{ m/s}^2 \text{ } 41^\circ \text{ N of E}$$

(1.4 m/s<sup>2</sup> @ 41° N of E)

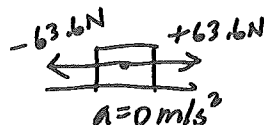
10. A man pushes a 16 kg lawn mower at a constant speed with a force of 90 N directed along the handle which is at an angle of 45° to the horizontal. Calculate

A) The horizontal frictional force acting on the mower. (-63.9 N)



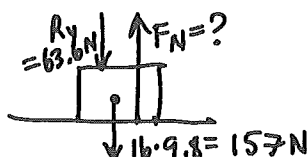
$$\cos 45 (90) = R_x = 63.6 \text{ N}$$

$$F_f = -63.6 \text{ N}$$



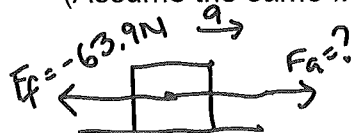
B) The normal force exerted vertically upward on the mower. (+221 N)

$$\sin 45 (90) = R_y = 63.6 \text{ N}$$



$$F_N = 63.6 + 157 = +221 \text{ N}$$

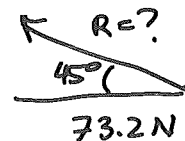
C) The force the man must exert on the mower to accelerate is from rest to 1.5 m/s in 2.5 s (Assume the same frictional force as calculated in part A). (+104 N)



$$a = \frac{1.5}{2.5} = +0.60 \text{ m/s}^2$$

$$(16)(0.60) = F_a + (-63.9)$$

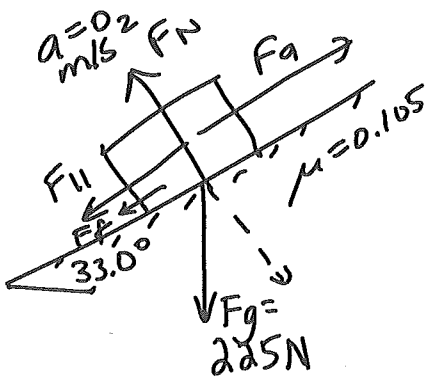
$$F_a = 73.2 \text{ N}$$



$$\cos 45 = \frac{73.2}{R} = +104 \text{ N}$$

$$R = +1.0 \times 10^2 \text{ N}$$

11. A student pulls a 225 N object up a  $33.0^\circ$  inclined plane. If the coefficient of friction between the object and the incline is 0.105, what force must the student apply to pull the object up the incline at a constant velocity? (+143 N)



$$F_{||} = \sin 33(225) = 123 \text{ N}$$

$$F_{\perp} = \cos 33(225) = 189 \text{ N}$$

$$F_f = (0.105)(189) = 19.8 \text{ N}$$

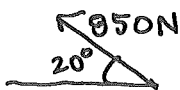
$$ma = F_a + F_{||} + F_f$$

$$(23)(0) = F_a + (-123) + (-19.8)$$

$$F_a = \underline{143 \text{ N}}$$

12. At the instant a race begins, a 60 kg sprinter was found to exert a force of 850 N on the starting block at a  $20^\circ$  angle with respect to the horizontal. (assume no air friction)

A) What was the horizontal acceleration of the sprinter? (+13 m/s<sup>2</sup>)



$$\cos 20(850) = 799 \text{ N}$$

$$\Sigma F = ma \quad 799 = (60)(a) \quad \underline{\vec{a} = 13 \text{ m/s}^2}$$

B) If the force was exerted for 0.38 s, with what speed did the sprinter leave the starting block? (+4.9 m/s)

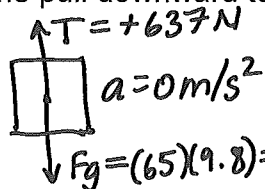
$$13 = \frac{v_f - 0}{0.38}$$

$$v_f = 4.9 \text{ m/s}$$

13. A window washer pulls herself upward using the bucket-pulley apparatus shown in the diagram. The mass of the bucket plus the person is 65 kg.

A) How hard must she pull downward to raise herself at a slow constant speed? (+637 N)

$$\Sigma F = 0 \text{ N}$$

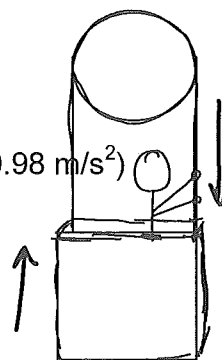


$$= 6.4 \times 10^2 \text{ N}$$

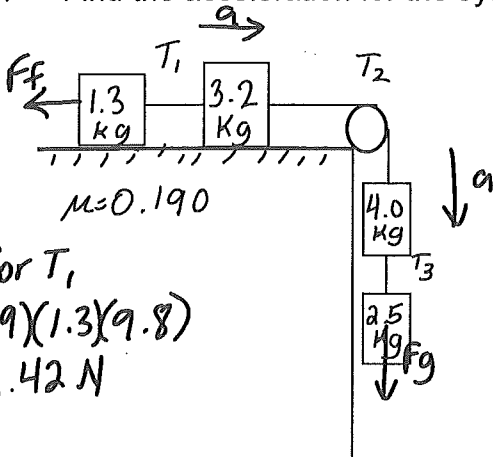
B) If she increases this force by 10%, what will her acceleration be? (+0.98 m/s<sup>2</sup>)

$$637 \times 1.10 = 701 \text{ N}$$

$$(65)a = 701 + (-637) \quad \underline{\vec{a} = +0.98 \text{ m/s}^2}$$



14. Find the acceleration for the system and the tension force for each segment of cable.



$$F_f \text{ for } T_1 \\ = (0.19)(1.3)(9.8) \\ = -2.42 \text{ N}$$

$$\Sigma F = F_g + F_f = (6.5)(9.8) + (-0.19)(4.5)(9.8) \\ (+) (-) = 55.3 \text{ N}$$

$$\vec{a} = \frac{55.3}{11} = \pm 5.03 \text{ m/s}^2$$

$$T_1 \rightarrow \begin{array}{c} \text{1.3 kg} \\ \text{1.3 kg} \end{array} \quad \begin{array}{l} T_1 \\ F_f \end{array} \quad \begin{array}{l} ma = T_1 + F_f \\ (1.3)(5.03) = T_1 + (-2.42) \\ T_1 = +9.0 \text{ N} \end{array}$$

$$T_2 \rightarrow \begin{array}{c} \text{6.5 kg} \\ \text{6.5 kg} \end{array} \quad \begin{array}{l} T_2 \\ F_g \end{array} \quad \begin{array}{l} ma = T_2 + F_g \\ (6.5)(-5.03) = T_2 + (-63.7) \\ T_2 = +31 \text{ N} \end{array}$$

$$T_3 \rightarrow \begin{array}{c} \text{2.5 kg} \\ \text{2.5 kg} \end{array} \quad \begin{array}{l} T_3 \\ F_g \end{array} \quad \begin{array}{l} ma = T_3 + F_g \\ (2.5)(-5.03) = T_3 + (-24.5) \\ T_3 = +12 \text{ N} \end{array}$$

( $\pm 5.03 \text{ m/s}^2$ , +9.0 N, +31 N, +12 N)