PHYSICS 12 KINEMATICS

SOLVAY CONFERENCE 1927

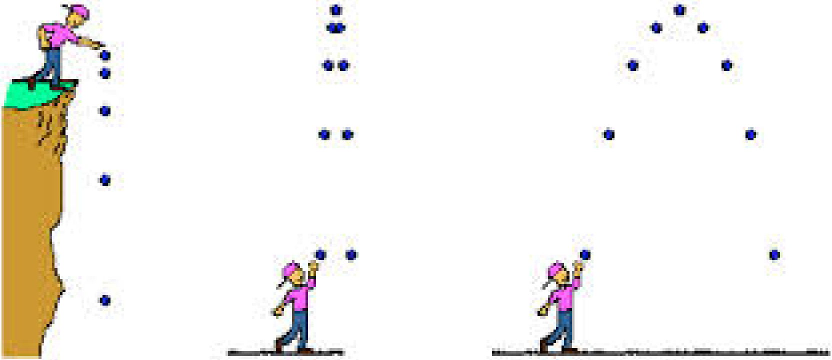
A. PICARD E. HENRIOT P. EHRENFEST Ed. HERSEN Th. DE DONDER E. SCHRÖDINGER E. VERSCHAFFELT W. PAULI W. HEISENBERG R.H FOWLER L. BRILLOUIN

P. DEBYE M. KNUDSEN W.L. BRAGG H.A. KRAMERS P.A.M. DIRAC A.H. COMPTON L. de BROGLIE M. BORN N. BOHR

I. LANGMUIR M. PLANCK Mme CURIE H.A. LORENTZ A. EINSTEIN P. LANGEVIN Ch.E. GUYE C.T.R. WILSON O.W. RICHARDSON

Absents : Sir W.H. BRAGG, H. DESLANDRES et E. VAN AUBEL

Types of Projectiles



PHYSICS 12 – TABLE OF CONSTANTS

Gravitational constant	G	$= 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$
Acceleration due to gravity at the surface of Earth (for the purposes of this examination)	g	$= 9.80\ \mathrm{m/s^2}$
Earth		
radius		$= 6.38 \times 10^{6}\,\mathrm{m}$
radius of orbit about Sun		$= 1.50 \times 10^{11}\,\mathrm{m}$
period of rotation		$= 8.61 \times 10^{4}\,\mathrm{s}$
period of revolution about Sun		$= 3.16 \times 10^{7}\,\mathrm{s}$
mass		$= 5.98 \times 10^{24}\,\mathrm{kg}$
Moon		
radius		$= 1.74 \times 10^{6}\,\mathrm{m}$
radius of orbit about Earth		$= 3.84 \times 10^{8}\,\mathrm{m}$
period of rotation		$= 2.36 \times 10^{6}\,\mathrm{s}$
period of revolution about Earth		$= 2.36 \times 10^{6}\,\mathrm{s}$
mass		$= 7.35 \times 10^{22}\,\mathrm{kg}$
Sun		
mass		$= 1.98 \times 10^{30}\,\mathrm{kg}$
Constant in Coulomb's Law	k	$= 9.00 \times 10^{9}\,\mathrm{N \cdot m^2/C^2}$
Elementary charge	e	$= 1.60 \times 10^{-19}\,\mathrm{C}$
Mass of electron	m_e	$= 9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of proton	m_p	$= 1.67 \times 10^{-27}\,\mathrm{kg}$
Mass of neutron	m_n	$= 1.68 \times 10^{-27}\,\mathrm{kg}$
Permeability of free space	μ_o	$= 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$
Speed of light	c	$= 3.00 \times 10^{8}\,\mathrm{m/s}$

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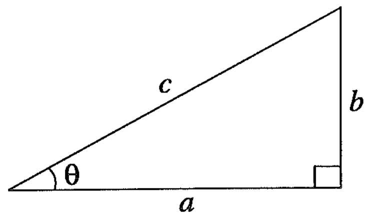
Exercise care when tearing along perforations.

Mathematical Equations

For Right-angled Triangles:

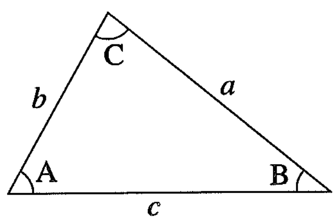


$$a^2 + b^2 = c^2$$

$$\sin\theta = \frac{b}{c} \qquad \cos\theta = \frac{a}{c} \qquad \tan\theta = \frac{b}{a}$$

$$\text{area} = \frac{1}{2}\,ab$$

For All Triangles:



$$\text{area} = \frac{1}{2}\ \text{base} \times \text{height}$$

$$\sin 2\mathrm{A} = 2\sin\mathrm{A}\cos\mathrm{A}$$

Sine Law: $\dfrac{\sin\mathrm{A}}{a} = \dfrac{\sin\mathrm{B}}{b} = \dfrac{\sin\mathrm{C}}{c}$

Cosine Law: $c^2 = a^2 + b^2 - 2ab\ \cos\mathrm{C}$

Circle:

$$\text{Circumference} = 2\pi r$$

$$\text{Area} = \pi r^2$$

Sphere:

$$\text{Surface area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

Quadratic Equation:

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vector Kinematics in Two Dimensions:

$$v = v_0 + at \qquad \bar{v} = \frac{v + v_0}{2}$$

$$v^2 = v_0^2 + 2ad \qquad d = v_0 t + \frac{1}{2}at^2$$

Vector Dynamics:

$$F_{\text{net}} = ma \qquad F_{\text{g}} = mg$$

$$F_{\text{fr}} = \mu F_{\text{N}}$$

Work, Energy, and Power:

$$W = Fd \qquad E_{\text{p}} = mgh$$

$$E_{\text{k}} = \frac{1}{2}mv^2 \qquad P = \frac{W}{t}$$

Momentum:

$$p = mv \qquad \Delta p = F\Delta t$$

Equilibrium:

$$\tau = Fd$$

Circular Motion:

$$a_{\text{c}} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

Gravitation:

$$F = G\frac{m_1 m_2}{r^2} \qquad E_{\text{p}} = -G\frac{m_1 m_2}{r}$$

You may detach this page for convenient reference.
Exercise care when tearing along perforations.

Physics 12 – Lab Write-Up Instructions

Purpose: Clearly state the purpose of the lab.

Equipment: List necessary equipment

Procedure: Reference the procedure from manual or handout. Clearly state any changes made to the procedure. Include any diagrams that clarify your procedure.

Data: Include all quantitative (numbers) and qualitative (observations) measurements. *Neatness and clarity* are of the utmost importance. Clearly label all data and use data tables where appropriate.

Calculations: Show all calculations. For repetitive calculations you only need to show one sample calculation.

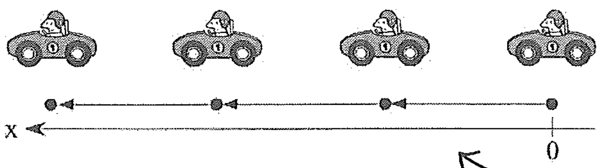
Discussion: No measurement can be perfect. Measurements always have some uncertainty. Due to the presence of measurement uncertainty, measured values will never be equal to predicted values. So the question is not: "are the values equal to each other" but instead "do the values agree with each other within acceptable uncertainty (±5.00%). The values should agree within this margin, i.e. the percent difference should be less than the percent uncertainty. If the values are in agreement, we will conclude that the data has supported the predictions of the theory. No data can ever prove a theory, only support or disprove.

If the values do not agree then there are several options:
i) calculation errors were made (REDO LAB!)
ii) there were errors in experimental technique (didn't follow procedure correctly) (REDO LAB!)
iii) the equipment is malfunctioning (INFORM INSTRUCTOR)
iv) there are flaws in the design of the experiment (procedure does not work) (INFORM INSTRUCTOR)
v) the hypothesis/theory is flawed and must be revised. (WIN NOBEL PRIZE!)

Conclusion:
i) restate the purpose: what were you trying to measure? what is the hypothesis?
ii) state the measured (and % uncertainty) and predicted values
iii) state the percent difference between these values
iv) state whether the values agree (is the percent difference less than the percent uncertainty)
v) state whether the theory is supported by your data
vi) discuss the largest source of uncertainty - the main reason that the values are different from each other.

Physics 12 – Kinematics 1 Lesson 1

Kinematics is the study of motion. Motion is observed, described and quantified. The simplest way to do this is through pictures.



The positive direction of motion can be chosen to the right or to the left. In this case is chosen to be to the left.

UNIFORM MOTION

The **speed** of an object is defined as the distance the object travels in a certain amount of time. For example, if you are driving your car on the highway your *speedometer* (speed meter) may say 100 km/hr. That means that you will travel 100 km if you drive for one hour. If you drive for 2 hours, you will go twice as far and therefore 2 x 100 km is 200 km.

Speed is a **scalar** and is NEVER negative. If you put your car in reverse and drive, your speedometer still will just tell you a number. The formula for average speed is:

1. A car travels 200 m in 4.0 seconds. What is its average speed? ______________

2. A car travels 2000m in 2.0 minutes. What is its average speed? ______________

3. A toy car travels 1.0 kilometre in 0.223 hours. What the average speed in km/hr? What is the average speed in m/s?

______________ ______________

4. You start out on a road trip travelling at 60 km/h. You travel 100 km in 1.67 hours.
Did the speed of your car have to be exactly 60 km/hr for the entire trip? Why or why not?

In Physics, saying your *speed* is – 10 m/s is incorrect! What do you *really* mean by saying **negative** 10 m/s? (recall from Physics 11)

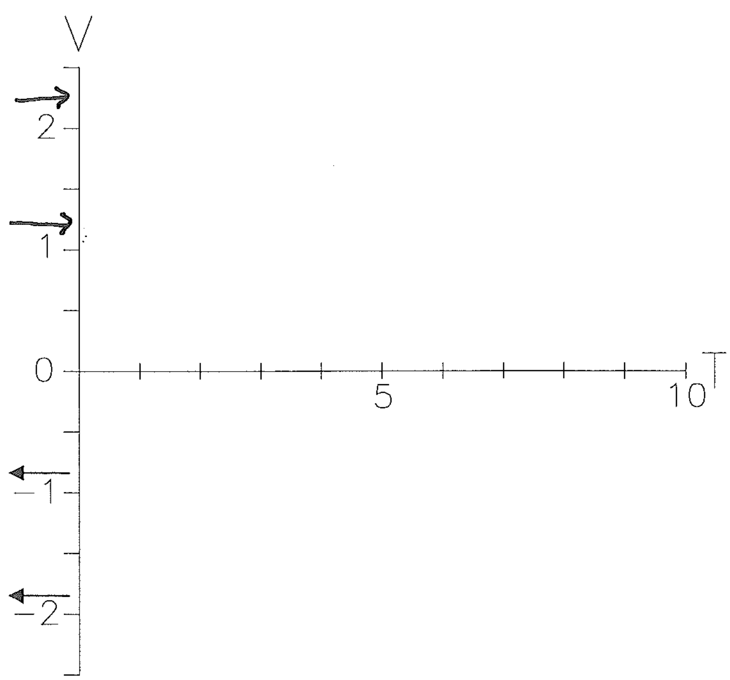
The addition of the direction information turns the *scalar* number into a **VECTOR**. We have a special word for a *speed* with a *direction*. This word is called **VELOCITY**. The equation for velocity is very similar to the equation for speed:

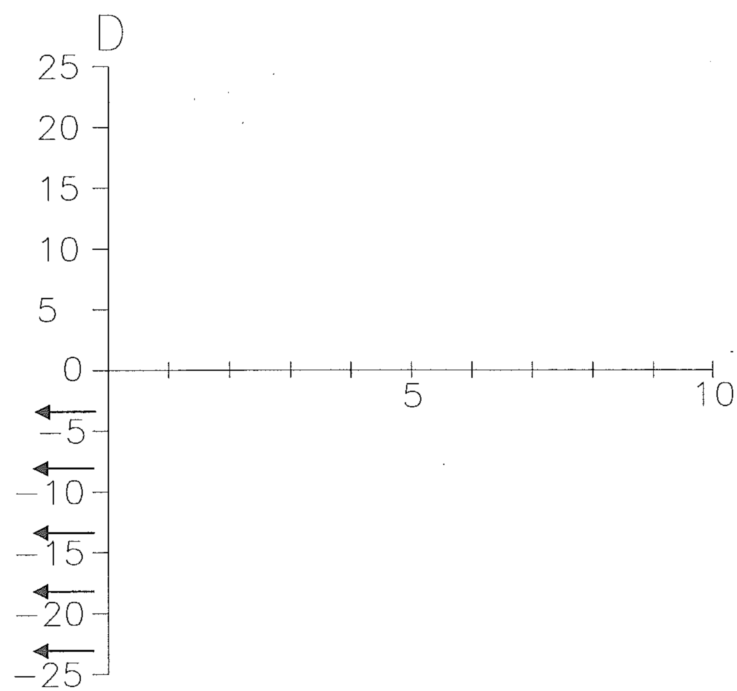
$$\boxed{}$$

REMEMBER - DISPLACEMENT IS A **VECTOR** AND IT HAS **DIRECTION**.

Graphing Motion:

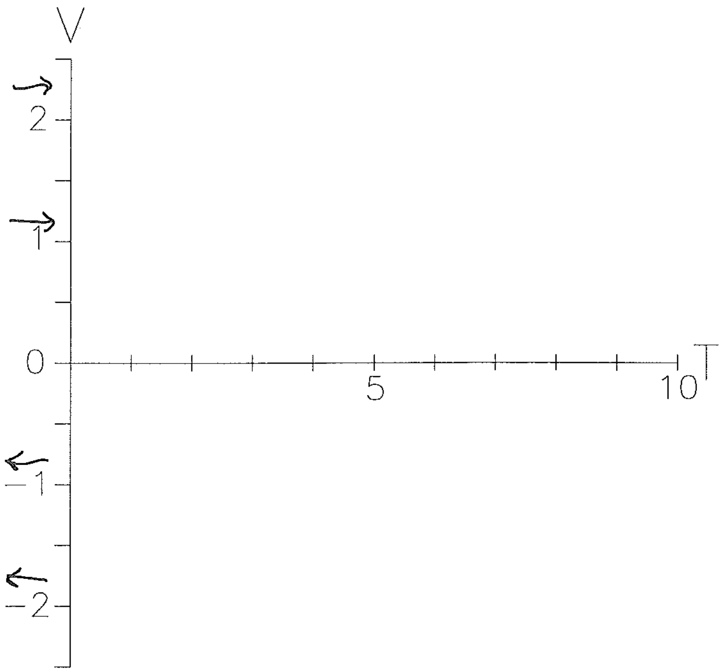
1) A toy travels FORWARD 20 m in 10 s, what is the car's average velocity?
2) Fill in the following graphs for the cars *velocity vs. time* and the car's *displacement vs. time.*

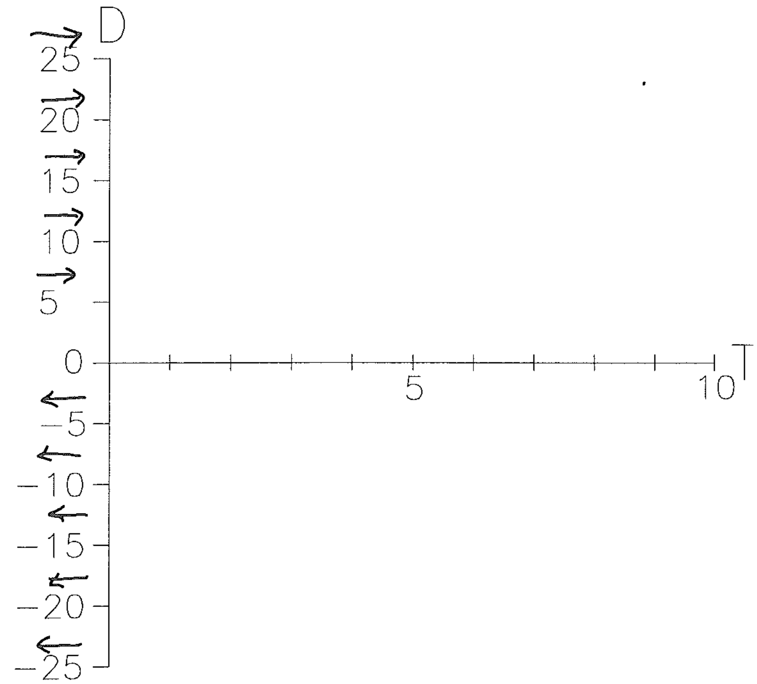




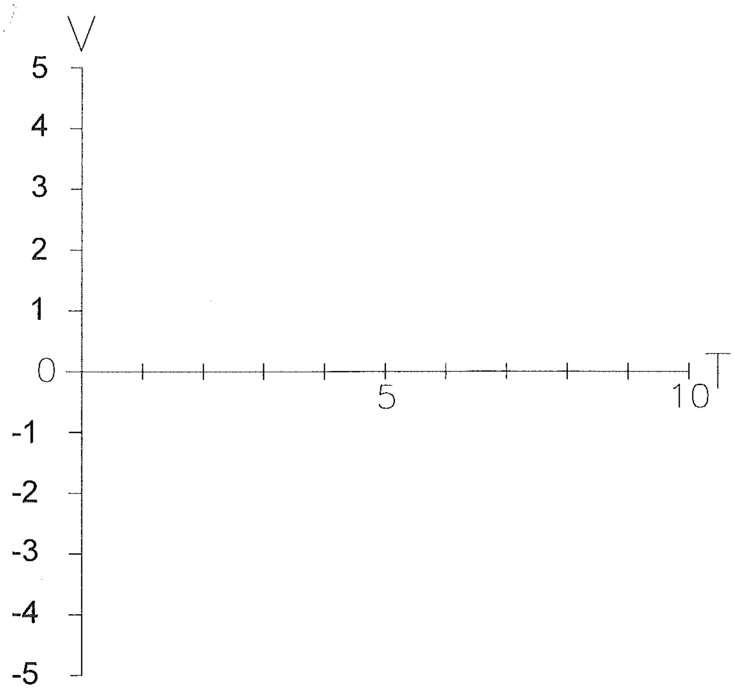
3) A toy car travels BACKWARDS 20 m in 10 s, what is the car's average velocity?

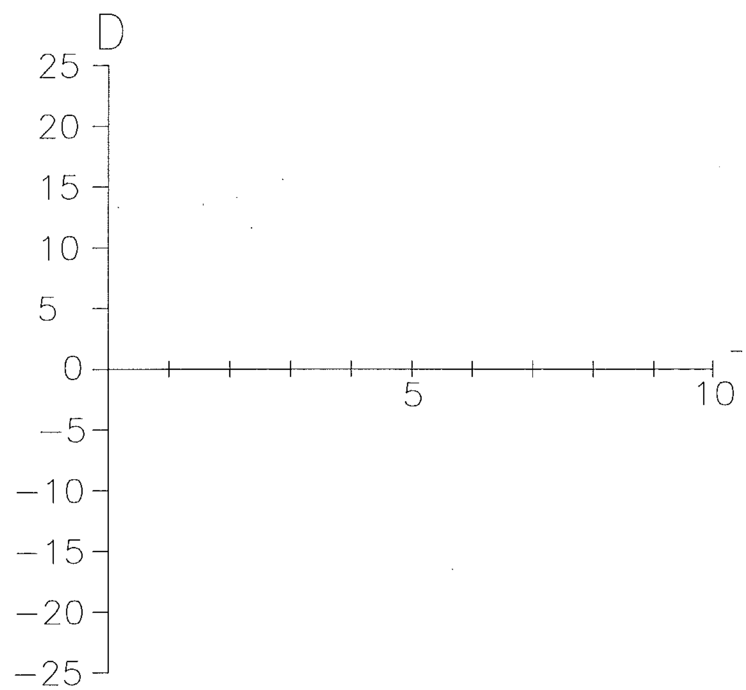
4) Fill in the following graphs for the cars *velocity vs. time* and the car's *displacement vs. time*.

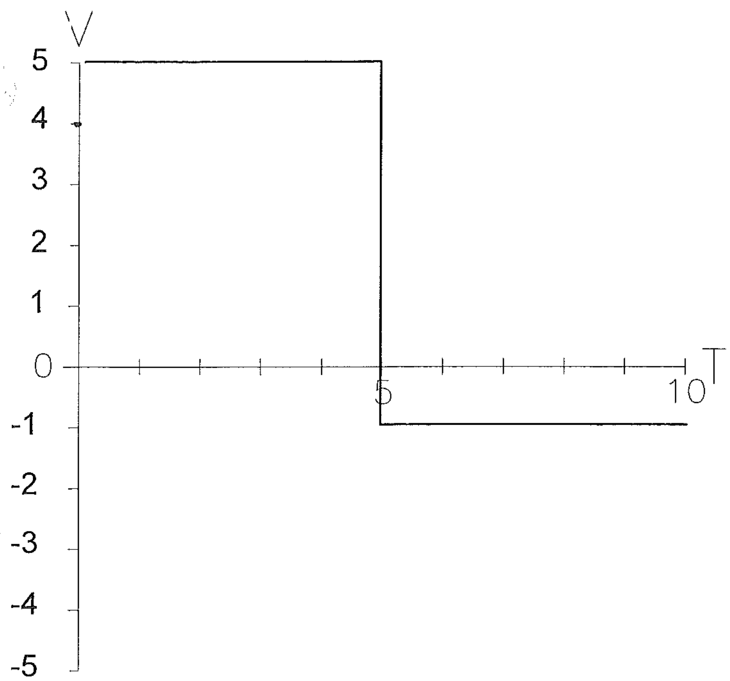


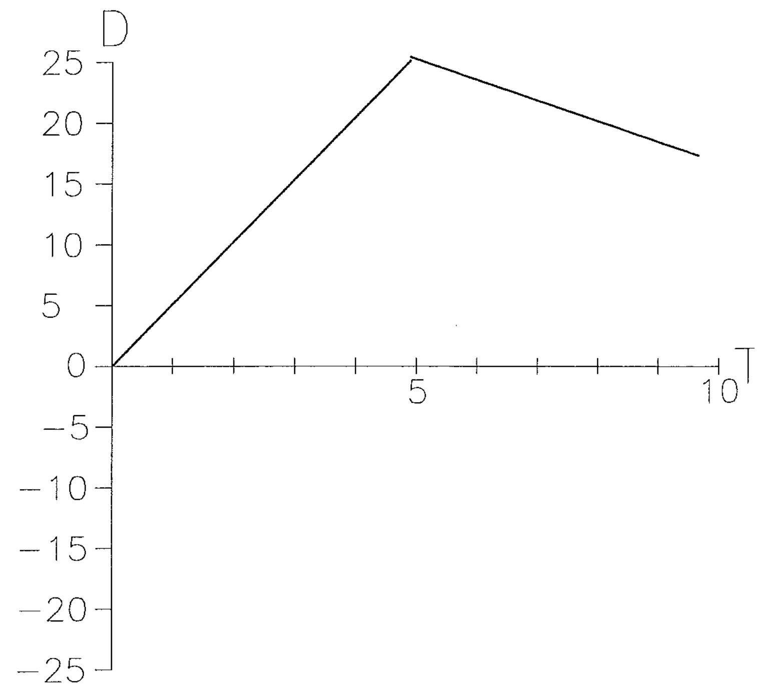


5) A toy car travels forward 25m in 5 s and backwards 5 m in 5 seconds.









i. What is the car's average velocity for the first 5 seconds? ($T_0 \rightarrow T_5$)

ii. What is the car's average velocity for the last 5 seconds? ($T_5 \rightarrow T_{10}$)

iii. On the graph above, fill in the *velocity vs. Time* graph and the *displacement vs. Time* graph.

iv. What is the **total** displacement?

v. What is the **total** time?

vi. What is the **average** velocity over the entire 10 seconds?

vii. Fill in the graphs for the cars ***average velocity*** *vs. time* and the car's *displacement vs. Time* ***assuming the car travelled the average velocity for the entire 10 seconds.*** Use a red pen.

viii. What is the area under to v/t graph from t = 0 to t = 5 seconds?

ix. Is the area is *positive* or *negative?* (Above or Below the *'X'* axis)

x. What is the displacement of the car between 0 and 5 seconds?

xi. What is the area under to v/t graph from t = 5 to t = 10 seconds?

xii. Is the area is *forward* or *backwards?* (Above or Below the *'X'* axis)

xiii. What is the displacement of the car between 5 seconds and 10 seconds?

xiv. Looking at the original question, how far is the car from the origin after 10 seconds (displacement)?

xv. What is the sum of the areas from part **x.** and part **xiii.**?

6) What conclusions about how the area under to v/t graph relates to the displacement of the car?
7) What is the formula for finding the area of a rectangle?
8) When you calculated the area of the rectangles from the velocity time graph, what did the height of the rectangle represent? (velocity or time)
9) When you calculated the area of the rectangles in part **viii.** and part **xiii,** what did the length of the rectangle represent? (velocity or time)
10) What is the formula for finding the displacement of any moving object?

Physics 12 – **Kinematics 2** - <u>**Motion Graphs**</u> Lesson 1 continued

Describing the motion of an object can be assisted through the use of graphs. As you become more proficient at creating and reading motion graphs, you should find the motion easier to picture and understand.

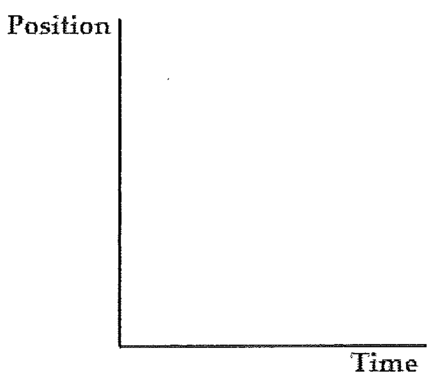
Remember:

- **Motion** is a change in position measured by distance and time.
- **Speed** tells us the rate at which an object moves.
- **Velocity** tells the speed and direction of a moving object.
- **Acceleration** tells us the rate speed or direction changes.

POSITION-TIME GRAPHS:

This analysis also applies to **distance-time graphs** and **displacement-time graphs**.

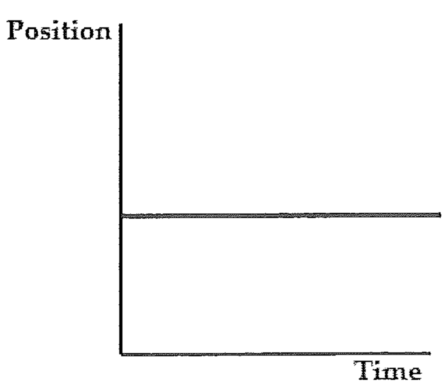
Plotting position against time can tell you a lot about motion. Let's look at what information is available on each axis.



Time is always plotted on the **X-axis** (bottom of the graph). The further to the right on the axis, the more time passes from the start.

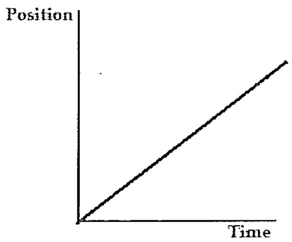
Distance is plotted on the **Y-axis** (side of the graph). The higher up the graph, the further the object has travelling from the reference point.

If an object is not moving, a horizontal line is shown on a position-time graph.



Time is increasing to the right, but its distance does not change. It is not moving. We say that it is **at rest.**

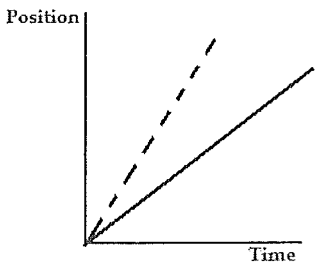
If an object is moving at a constant velocity (or speed), it means that it has the same increase in distance in a given time.



Time is increasing to the right, and distance is increasing constantly with time. The object moves at a **constant velocity.**

Constant velocity is shown by straight lines on the graph.

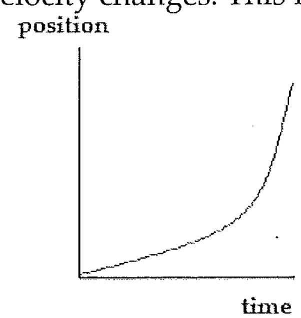
The graph below shows two moving objects. Both of the lines in the graph show that each object moved the same distance, but the steeper dashed line got there before the other one (in less time).



A steeper line indicates a larger distance moved in a given time. In other words, it has a **higher velocity.**

Bo, so both lines are **straight**, so both speeds are **constant.**

When there is a change in velocity (acceleration), the graph now indicates a changing slope as the velocity changes. This results in a curved line.



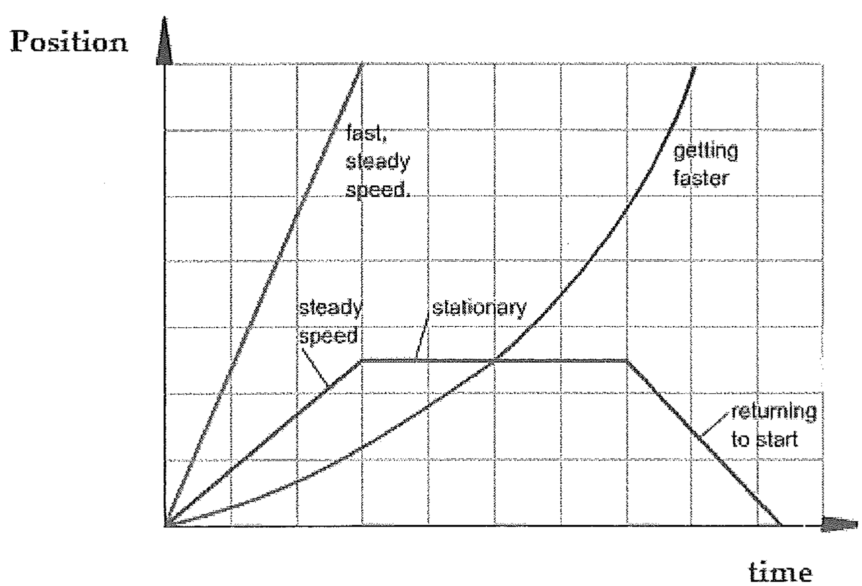
The line on this graph is curving upwards. This shows an **increase in speed** since the line is getting **steeper.**

In other words, in a given time, the distance the object moves is changing (getting larger) in each equal time interval. It is **accelerating.**

Summary:

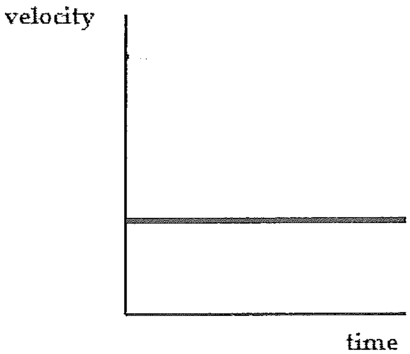
A position-time graph (distance-time, displacement-time) shows us how far an object has moved with time.

- The steeper the graph, the faster the motion.
- A horizontal line means the object is not changing its position - it is not moving, it is at rest.
- A downward sloping line means the object is returning to the start.

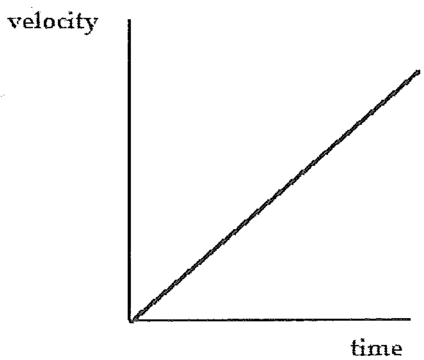


VELOCITY-TIME GRAPHS

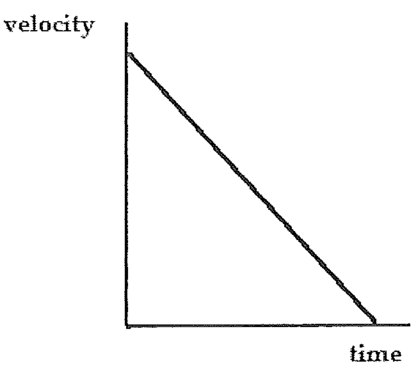
This analysis also applies to **speed-time** graphs.



Velocity-Time graphs look much like Position-Time graphs. Be sure to read the labels!!
Time is plotted on the X-axis. Velocity or speed is plotted on the Y-axis.
A straight horizontal line on a velocity-time graph means that the velocity is constant. It is not changing over time.
A straight line does not mean that the object is not moving!



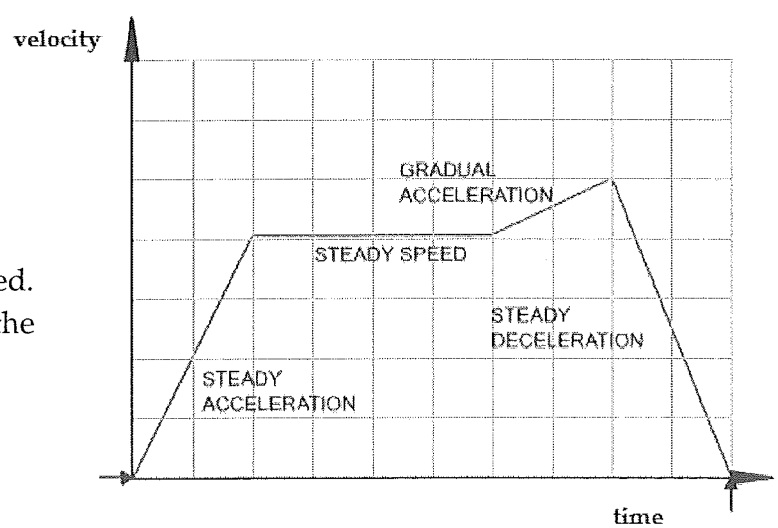
This graph shows **increasing velocity** in the **positive direction.** (speeding up)
Time is increasing to the right, and velocity is increasing constantly with time.
Constant acceleration is shown by straight lines on the graph.



This graph shows **decreasing velocity** in the **positive direction.** (slowing down)
Time is increasing to the right, and velocity is decreasing constantly with time.
Constant acceleration (deceleration in this case) is shown by straight lines on the graph.

Summary:

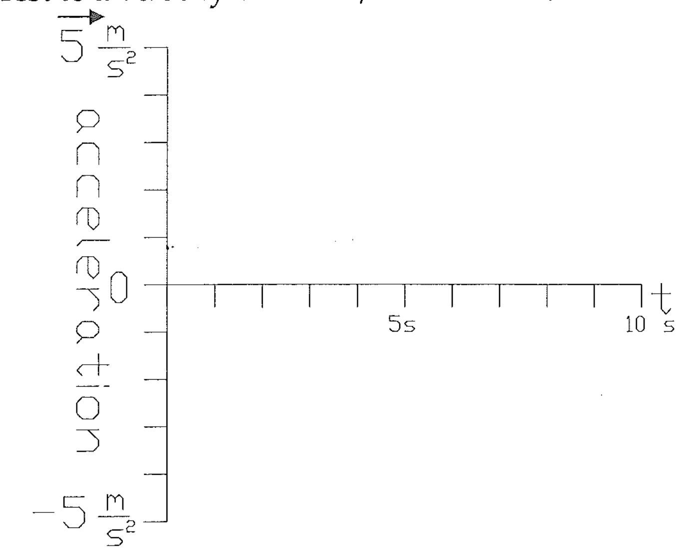
- The steeper the graph, the greater the acceleration.
- A horizontal line means the object is moving at a constant speed.
- A downward sloping line means the object is slowing down.



Examples:

1. A runner racing in a 100 m dash accelerates from rest to a velocity of 10.0 m/s in 5 seconds.

   i. What was his average acceleration during these 5 seconds?



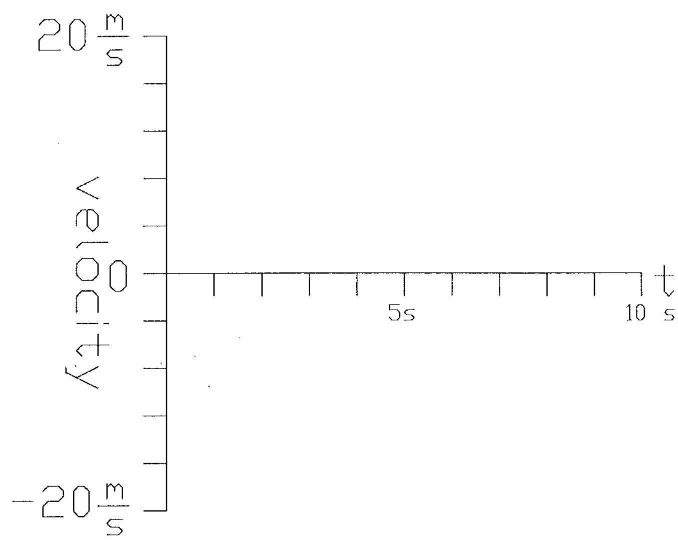
   ii. Fill in the acceleration graph.

   iii. Calculate the area under the acceleration/time graph up to time = 5 seconds.

   iv. What are the 'units' of the area?

   v. Fill in the velocity time graph.

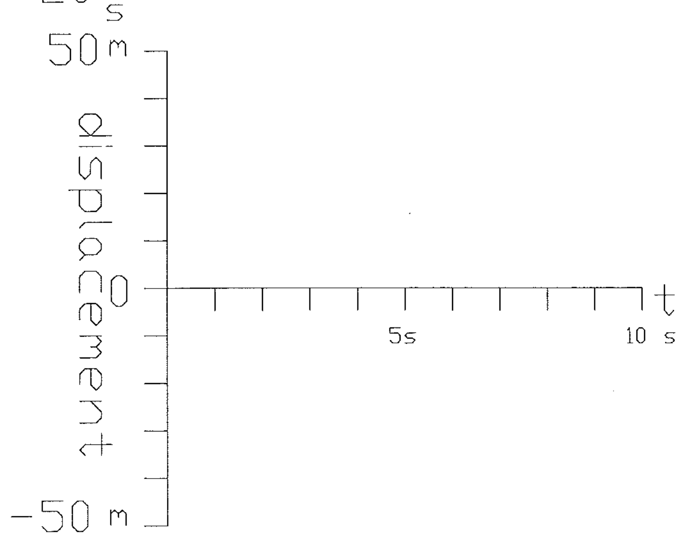
   vi. What is the formula for the area of a triangle?



   vii. How far did the runner travel during the first second? (0 to 1s)

   viii. How far did the runner travel from (0 to 2s)

   ix. How far did the runner travel from (0 to 3s)



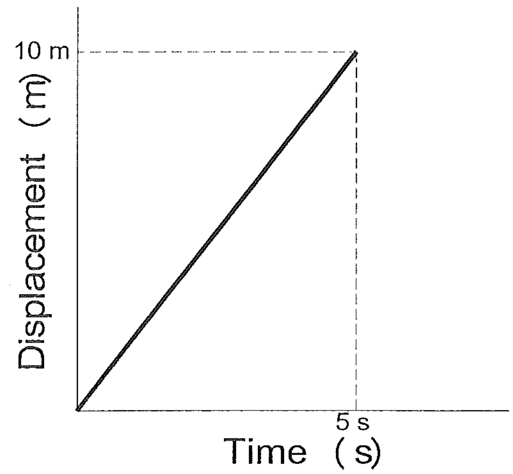
   x. How far did the runner travel from (0 to 4s)

   xi. How far did the runner travel from (0 to 5s)

Fill in the displacement-time graph.

Creating Motion Graphs - **What does each part actually mean?**

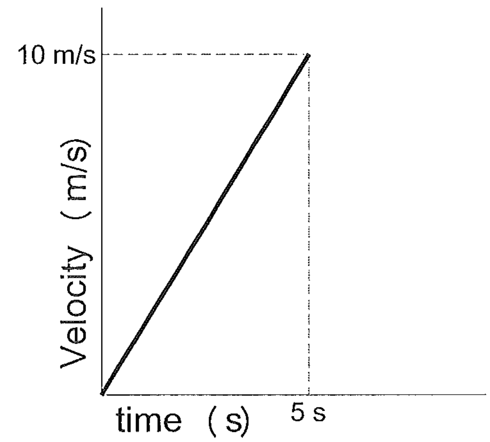
When we find the slope of a line, we simply use: **rise/run**



Displacement/time graph-
Calculate the slope INCLUDING UNITS!!!

What does this tell you about the slope of a ***displacement/time*** graph?

- Is the slope of the graph *changing?* **Yes –No**
- Is the velocity of the object *changing?* **Yes – No**
- Is the object accelerating? **Yes – No**

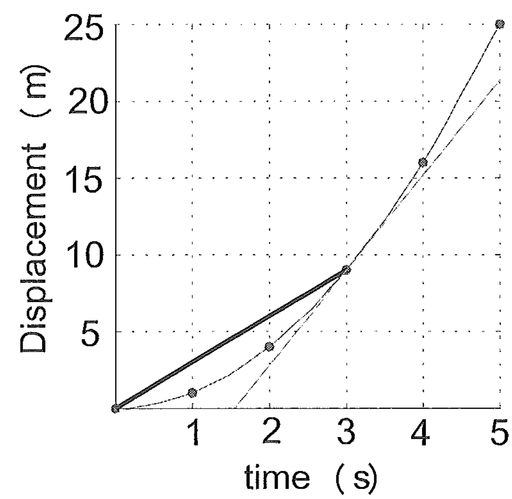


Velocity/time **graph**-
Calculate the slope INCLUDING UNITS!!!

- Is the slope of the graph *changing?* **Yes – No**
- Is the velocity of the object *changing?* **Yes – No**
- Is the object accelerating? **Yes – No**

What does this tell us about the slope of a ***velocity/time*** graph?

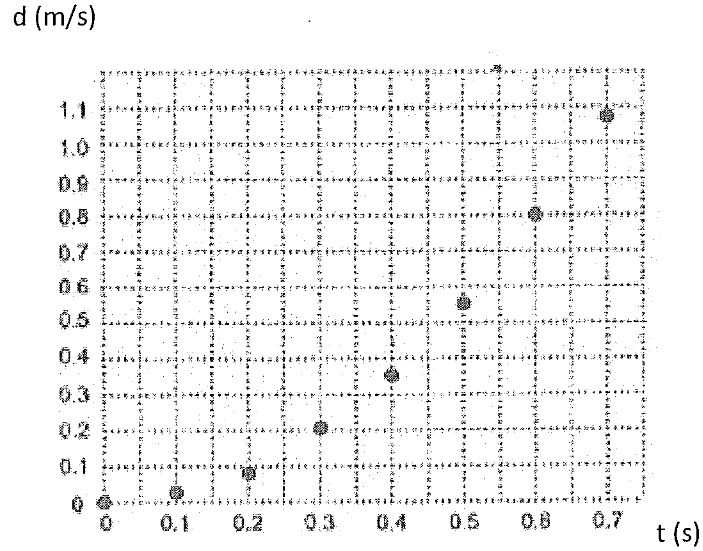
Now back to a **Displacement-Time Graph** -



- The graph to the right is a displacement/time graph with positive acceleration
- The AVERAGE velocity for **the first 3** seconds is the slope of the black line.
- The INSTANTANEOUS velocity **AT** 3 seconds is the slope of the dotted line.

When acceleration is NOT equal to zero, the **displacement graph** is a parabola (curved).

The slope of the TANGENT line of the displacement graph is the INSTANTANEOUS VELOCITY!



Draw the curved line by connecting the data points.

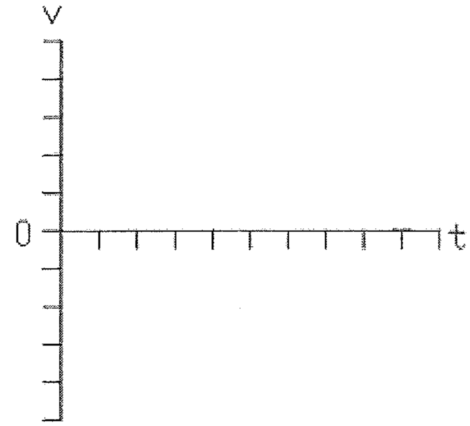
A. Calculate the average velocity between 0.0 and 0.6 s

B. Calculate the instantaneous velocity at 0.4 s

Lesson 1

Assignment Problems:

1. If the initial velocity is +10 m/s and the final velocity is +15 m/s and the time interval is from 0.0 to 5.0s, find the following:



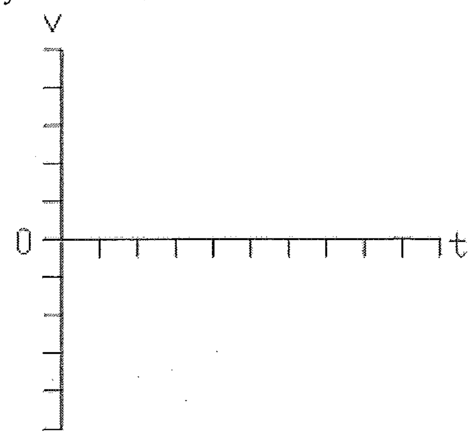
$a = \frac{v_f \quad v_i}{\Delta t}$ a=

d=1/2 (v_o+v_f)t d =

Sketch the velocity/time graph and calculate the area of the triangle + rectangle

Area = Displacement =

2. If the initial velocity is +12 m/s and the final velocity is - 28 m/s and the time interval is from 0.0 to 10s, find the following:



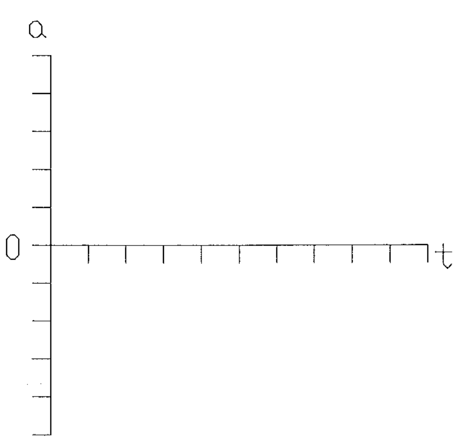
$a = \frac{v_f - v_i}{\Delta t}$ a=

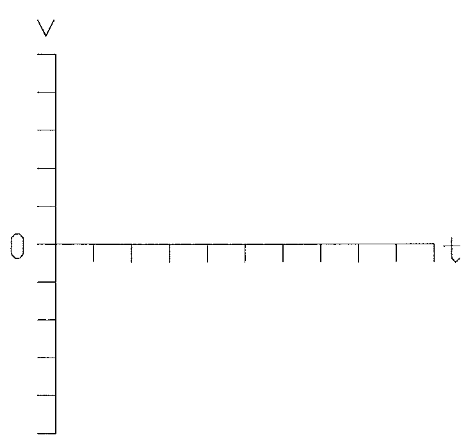
d=1/2 (v_o+v_f)t d =

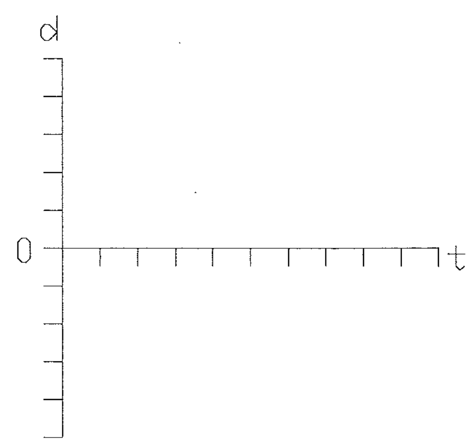
Sketch the velocity/time graph and calculate the area of the triangle + triangle

Area = Displacement =

3. A car accelerates from +0 m/s to +20 m/s in 10 seconds. What is the car's acceleration? How far does the car travel during the ten seconds? Solve the problem using the graphs below.



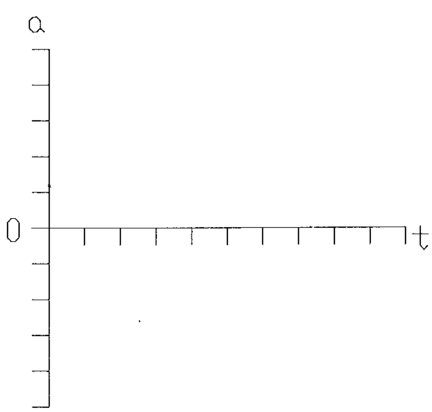


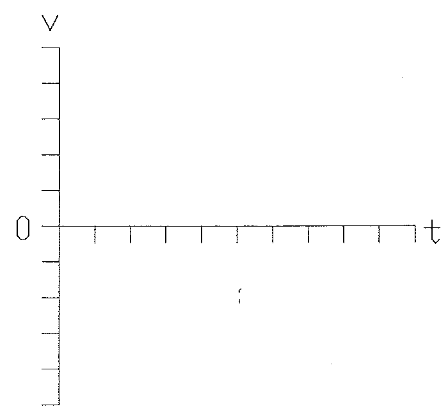


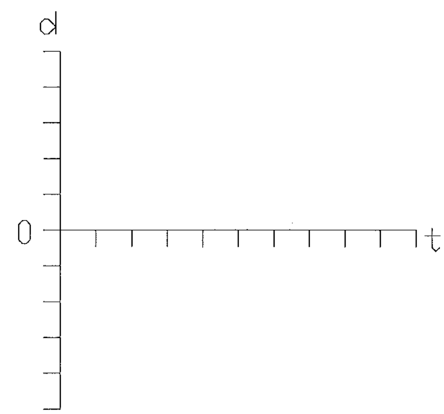
HW

4. How far does the car from question 3 travel in the first 5 seconds? How far does it travel in the last 5 seconds?

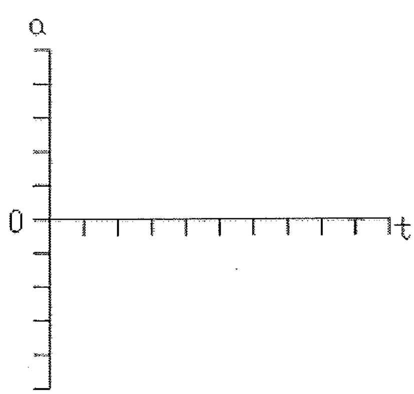
5. A car accelerates from +20 m/s to +40 m/s in 10 seconds. What is the car's acceleration? How far does the car travel during the ten seconds?

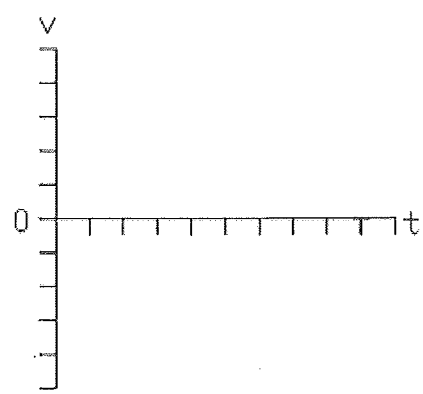


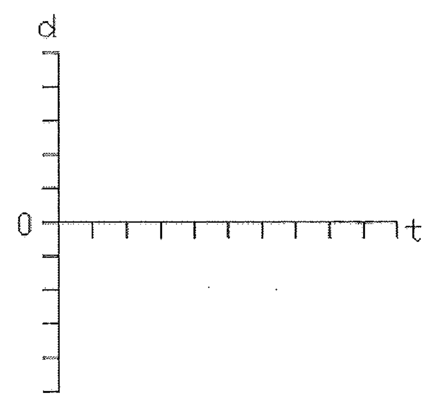




6. A car travelling +50 m/s brakes hard to avoid hitting a deer on the road, slowing down to +10 m/s in 5 seconds. What is the acceleration? What does the negative sign on acceleration mean?

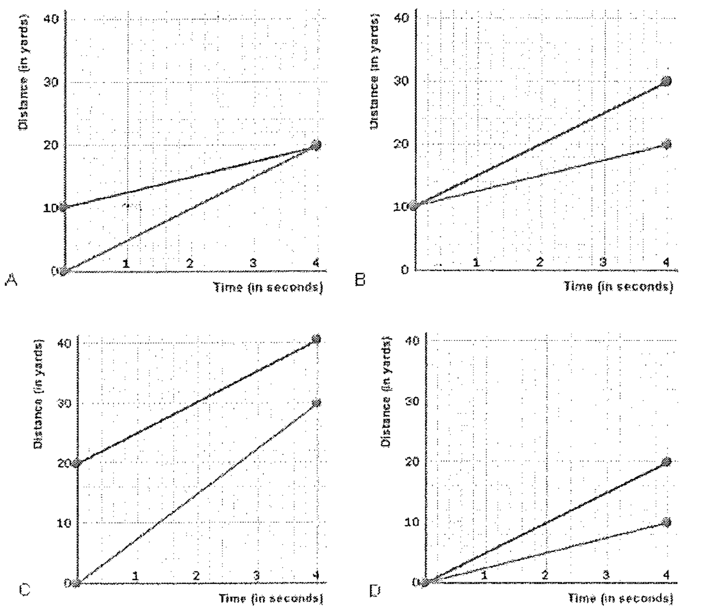






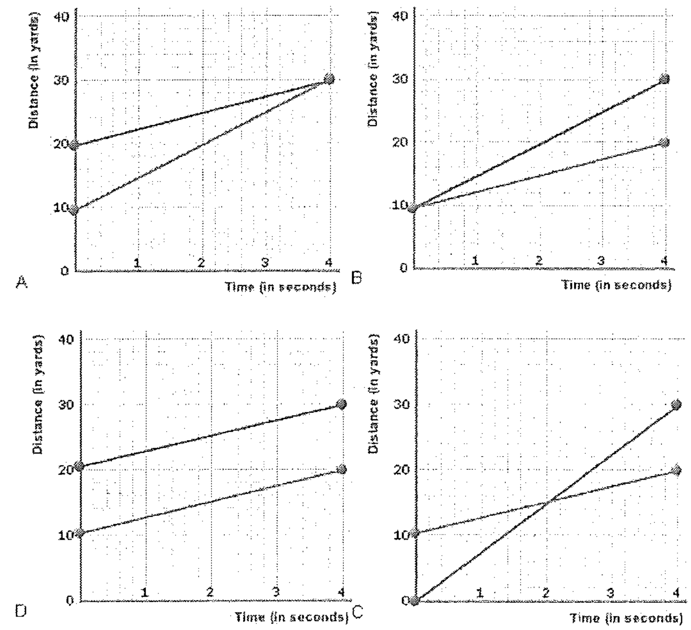
Part 2:

Examine the graphs below.



Which of the graphs shows that one of runners started 10 yards further ahead of the other? Explain your answer.

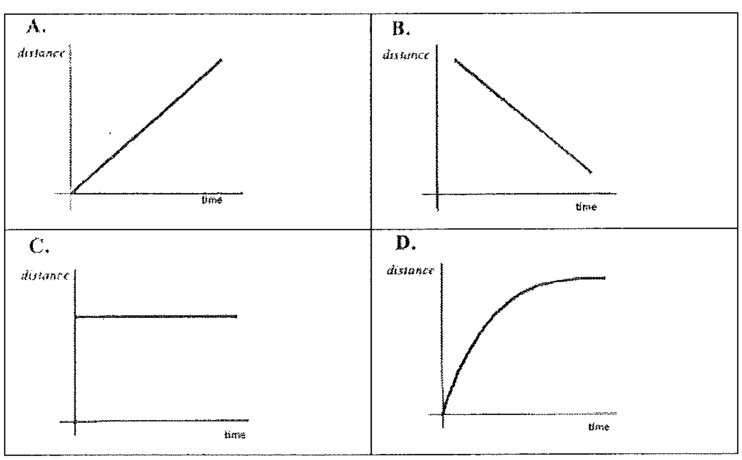
In which of the following graphs below are both runners moving at the same speed? Explain your answer.



The distance-time graphs below represent the motion of a car. Match the descriptions with the graphs. **Explain your answers.**

Descriptions:

1. The car is stopped.
2. The car is traveling at a constant speed.
3. The speed of the car is decreasing.
4. The car is coming back.



Graph A matches description _____ because ______________________________.

Graph B matches description _____ because ______________________________.

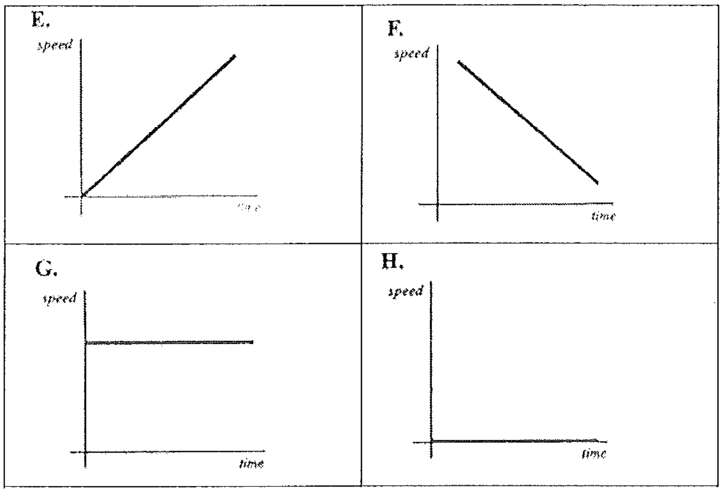
Graph C matches description _____ because ______________________________.

Graph D matches description _____ because ______________________________.

The speed-time graphs below represent the motion of a car. Match the descriptions with the graphs. **Explain your answers.**

Descriptions:

5. The car is stopped.
6. The car is traveling at a constant speed.
7. The car is accelerating.
8. The car is slowing down.



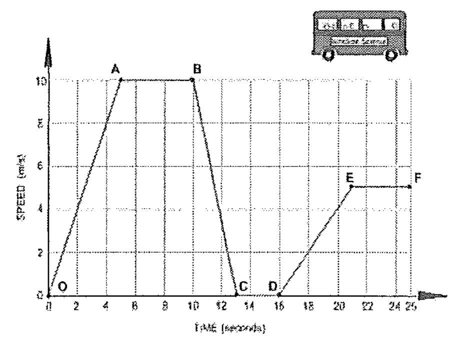
Graph E matches description _____ because ________________________________.

Graph F matches description _____ because ________________________________.

Graph G matches description _____ because ________________________________.

Graph H matches description _____ because ________________________________.

The graph below shows how the speed of a bus changes during part of a journey



Choose the correct words from the following list to describe the motion during each segment of the journey to fill in the blanks.

- accelerating
- decelerating
- constant speed
- at rest

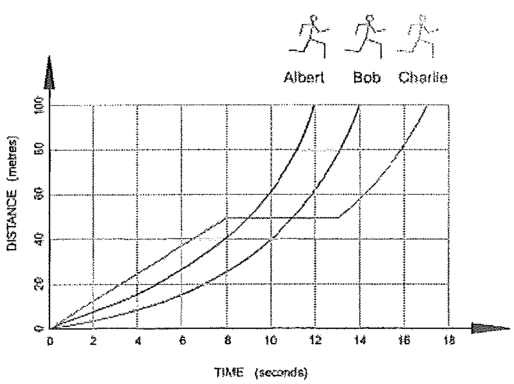
Segment 0-A The bus is ______________________________. Its speed changes from 0 to 10 m/s in 5 seconds.

Segment A-B The bus is moving at a ______________________________of 1 m/s for 5 seconds.

Segment B-C The bus is ______________________________. It is slowing down from 10 m/s to rest in 3 seconds.

Segment C-D The bus is ______________________________. It h stopped.

Segment D-E The bus is ______________________________. It is gradually increasing in speed.

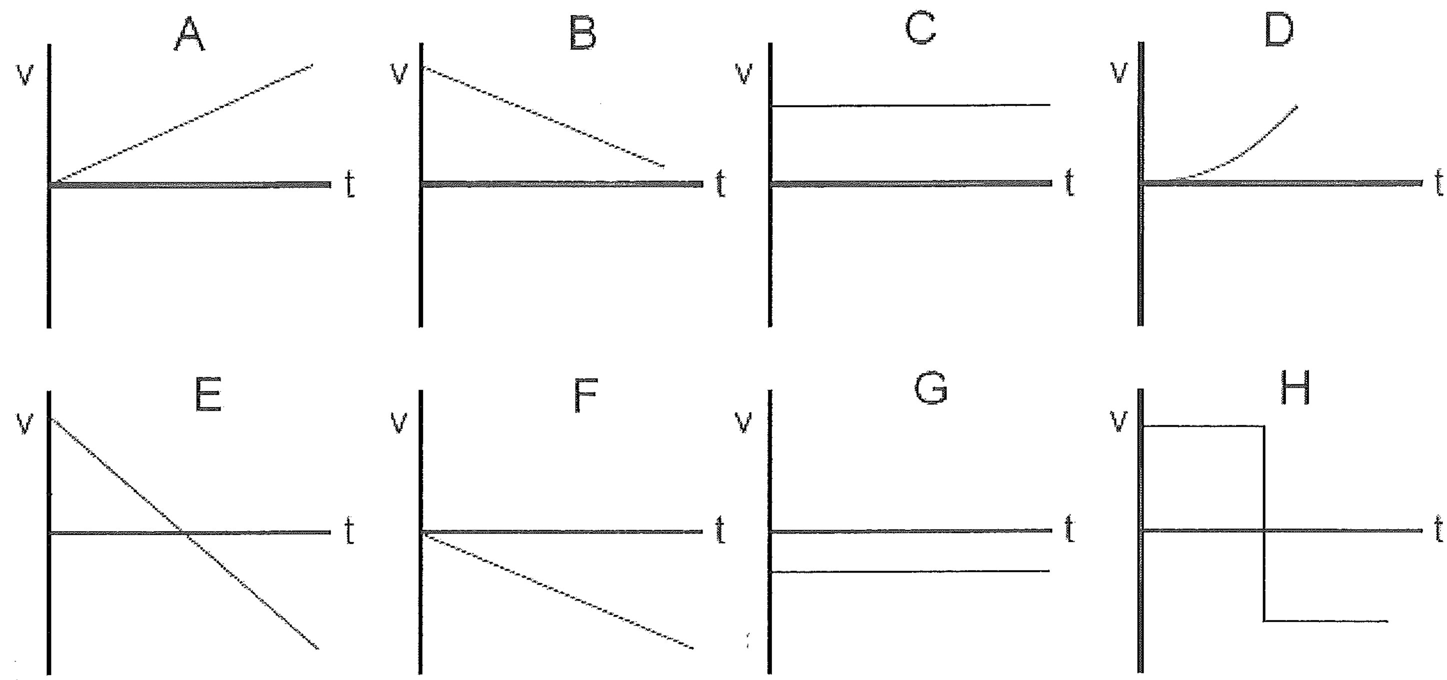


Look at the graph above. It shows how three runners ran a 100-meter race.

Which runner won the race? Explain your answer.

Which runner stopped for a rest? Explain your answer.

Part 3: Select from the following graphs to answer the following questions. Select all graphs that apply (ie. there may be more than one correct answer!)



1. A marble rolls at a constant speed along a horizontal surface away from the origin.

2. A driver accelerates away from his house with *increasing acceleration*.

3. A driver rolls toward his house at constant speed. (origin is house)

4. A marble is rolled from the top of an inclined plane. Assume that 'down' the ramp is '-'.

5. A block is dropped from one meter above the floor and it falls to the ground. Assume 'down' is '+'

6. A ball rolls along a horizontal surface at a constant speed. The ball strikes a wall and rebounds toward the origin at about the same speed as before.

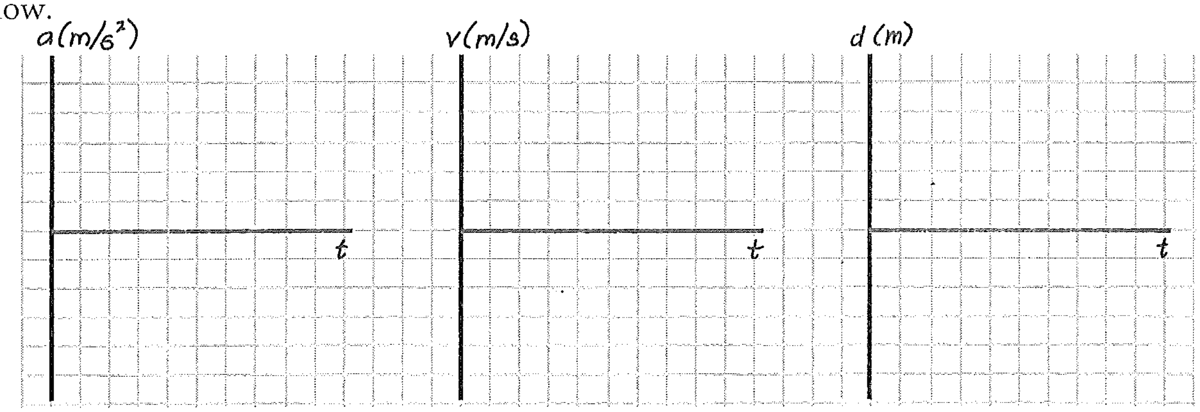
7. A ball is tossed up into the air and is caught at the same height it was released at.

8. A car driver slams on his brakes to avoid hitting a deer.

Physics 12 – **Motion Graphing** Names:

In partners, discuss and then **sketch** the displacement-time, velocity-time and acceleration-time graphs for each of the following scenarios. This will be handed in and assessed.

You must label the axis and **use a ruler**. Use the graph paper provided and set up as shown below.



Scenarios:

A. An elevator moves from the ground floor to the 36th floor, stops and then moves to the 27th floor, stops and returns to the lobby.

B. A basketball is dropped on the court and is allowed to bounce up and down several times.

C. A car on a test track performing a zero-to-sixty m/s acceleration test (straight track)

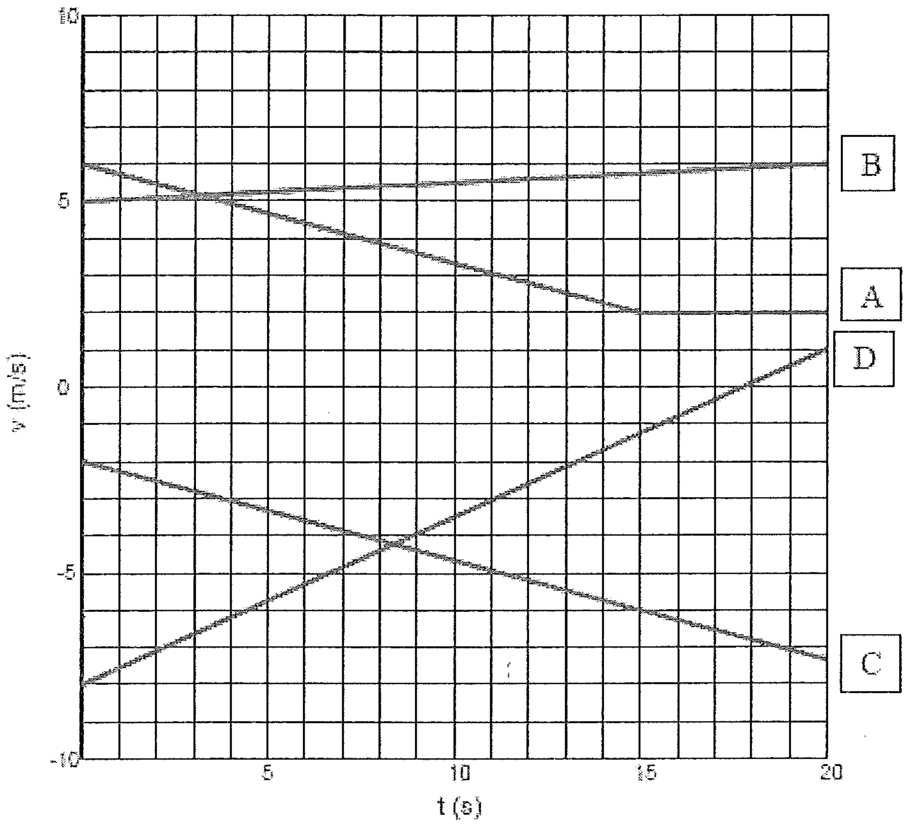
D. A race between a tortoise and a hare that happens just like the story of the same name. (An acceleration-time graph is not needed for this scenario).

E. Two cars are adjacent to each other on a four-lane highway. The first car accelerates uniformly from rest the moment the light changes to green. The second car approaches the intersection already moving and is beside the car the instant the light changes. It then continues to drive with a constant velocity.

F. Traffic lights on some streets are timed to facilitate traffic flow at a certain speed. Car A and Car B are stopped at a red light on this kind of street. When the light changes Car A accelerates at the maximum that the car can handle and exceeds the speed limit. He arrives at the first light which is still red and stops. Car B accelerates at a reasonable rate and never exceeds the speed limit. The second light turns green at just the right instant so that Car B never needs to brake at an intersection. Car A and Car B continue driving this way for three lights.

Complete the following questions and hand this paper in with your graph paper.

Use the graph of velocity vs. time below to answer the following questions.



1. What is the average acceleration over the interval 0 – 5 seconds for object A?

2. What is the average acceleration over the interval 5 – 10 seconds for object A?

3. What is A's average acceleration over the entire 20 seconds of its motion?

4. Is your answer to 1 the same as your answer to 2? Are either of these answers the same as your answer to questions 3? Explain.

5. What is the acceleration of object B over the interval from 5 to 10 seconds?

6. What is the average acceleration of object C over the interval from 10 to 15 seconds?

7. Do any two objects ever have the same accelerations?

8. Which object has the largest (magnitude) acceleration at 5 seconds?

9. Do object A, C, and D have a higher velocity at 5 seconds or a higher velocity at 20 seconds?

10. Do object A, C, and D have a higher speed at 5 seconds or a higher speed at 20 seconds?

METRIC

Accelerated Motion: Lesson 2

Up to this point, the velocity has been constant. However, when velocity is changing, we have **acceleration**.

Recall from Physics 11:

If the change in velocity (acceleration) is in the same direction as the velocity = speeds up

If the change in velocity (acceleration) is in the opposite direction to the velocity = slows down.

Acceleration has more to it than just a change in velocity. Acceleration is the "rate" of change in velocity which means we are also concerned with time.

Accelerated Motion Formulas:

$$\vec{a} = \frac{\vec{v}_F - \vec{v}_0}{t} \qquad \vec{d} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$\vec{d} = \left(\frac{\vec{v}_0 + \vec{v}_F}{2}\right)t \qquad \vec{v}_F^{\,2} = \vec{v}_0^{\,2} + 2\vec{a}\vec{d}$$

Example 1 - An object that is initially travelling at a velocity of 7.0 m/s east accelerates uniformly to a velocity of 22.0 m/s east in a time of 1.7 s. Calculate the acceleration of the object.

Free-Falling Objects

Recall that when air friction is minimal or non-existent (in a vacuum = no air present), acceleration is constant due to the pull of the Earth's gravity on an object close to the Earth's surface.

$g = 9.80\ m/s^2$ (g = acceleration due to gravity)

Example 2 - A cement block falls from the roof of a building. If the time of fall was 5.60s, what is the height of the building?

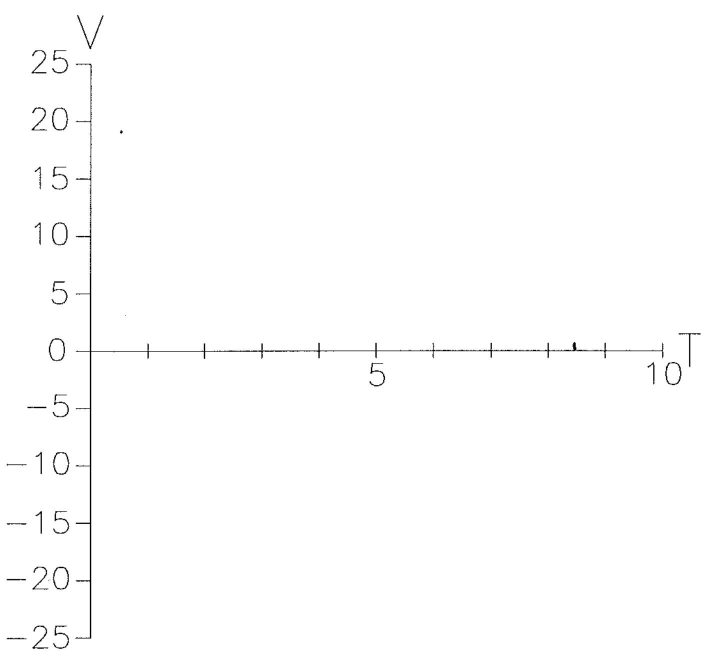
Example 3 - A ball is rolled up a constant slope with an initial velocity of 12.0 m/s. If the ball's displacement is 0.500 m up the slope after 3.60s, what is the velocity of the ball at this time?

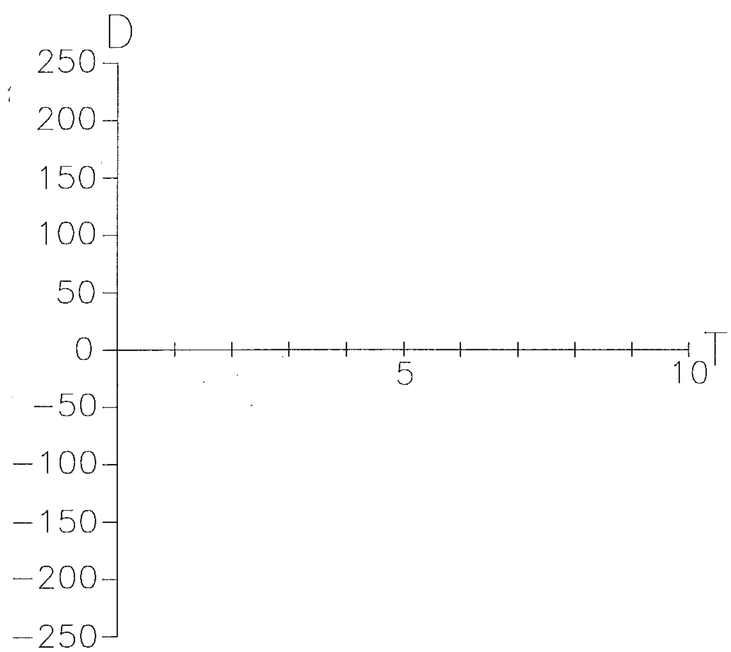
Accelerated Motion Problems: Lesson 2

1. An object uniformly accelerates from a velocity of 8.9 m/s west to 29.0 m/s west. What is the **average velocity** of the object?

2. An object is displaced 55.0 m north while accelerating uniformly. If a velocity of 18.0 m/s north is reached in 4.5 s, what was the initial velocity?

3. A car travels at a constant velocity of 20 m/s for 8 seconds. How far has the car traveled? Sketch the motion on the following graphs.

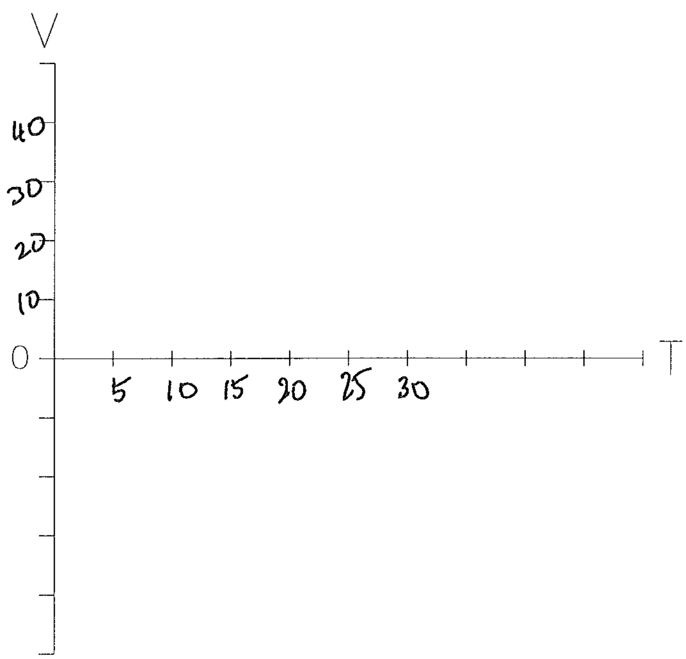


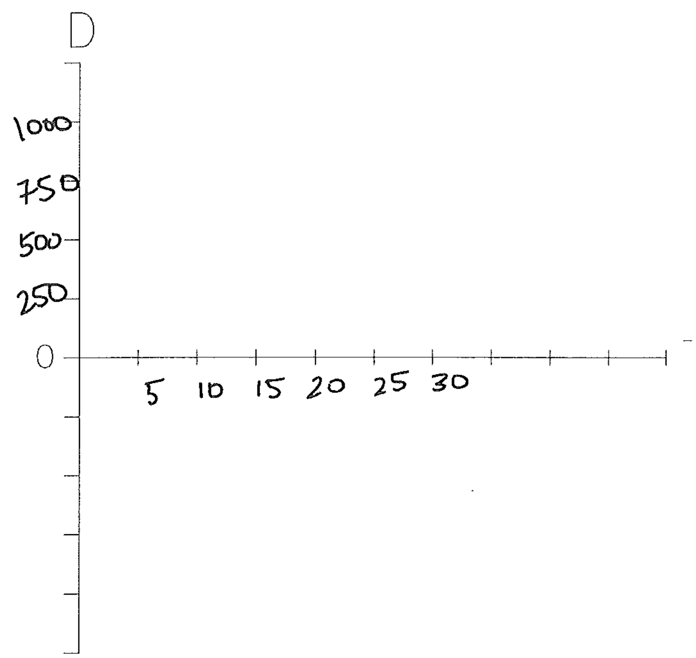


4. A book falls from a cabinet that is 2.45m above the floor. How long will it take the book to reach the floor?

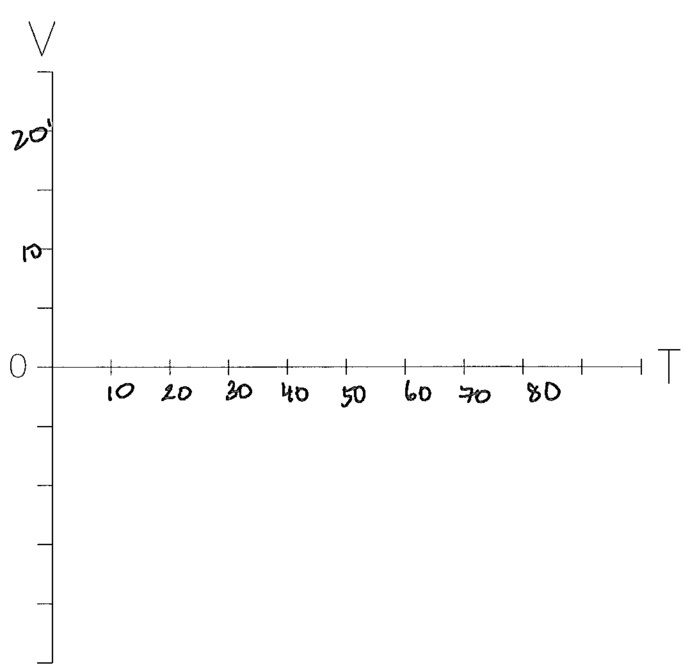
HW

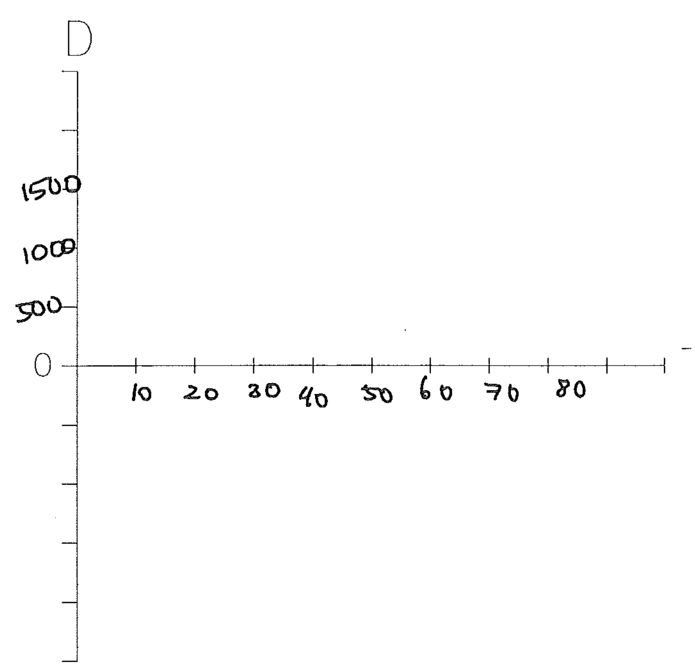
5. A car travels 1000 meters in 30 seconds. What is the cars velocity?
Sketch the motion on the following graphs.





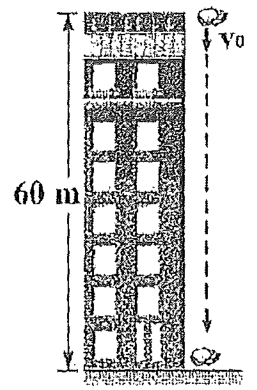
6. How long does it take for a car traveling at 20 m/s to travel 1500 m?
Sketch the motion on the following graphs.





7. An object accelerates uniformly from rest for 112s. What was the velocity of the object in this time if the displacement was 75.0 m west?

8. A rock was thrown downward from an overpass onto the highway below. If the rock was released when it was 12.0m above the highway and it took 1.30s for the rock to reach the road, what was the velocity of the rock when it was released?



9. A stone is dropped from the top of a 60 m high building. Ignore air resistance.

a) What is the velocity and position of the stone after 3.5s?

b) How far does the stone fall during the second and third seconds?

10. An object is thrown vertically upward from a helicopter that is hovering 44.0m above the ground. The initial velocity of the object was 10.0 m/s.

a) Calculate the velocity with which the object hits the ground.

b) Calculate the time it took to reach the ground.

11. While riding on an amusement park ride, you drop an object. The vehicle was rising vertically at a velocity of 8.40 m/s and was 7.00 m above the ground when the object was dropped. How long does it take the object to reach the ground?

Answers:

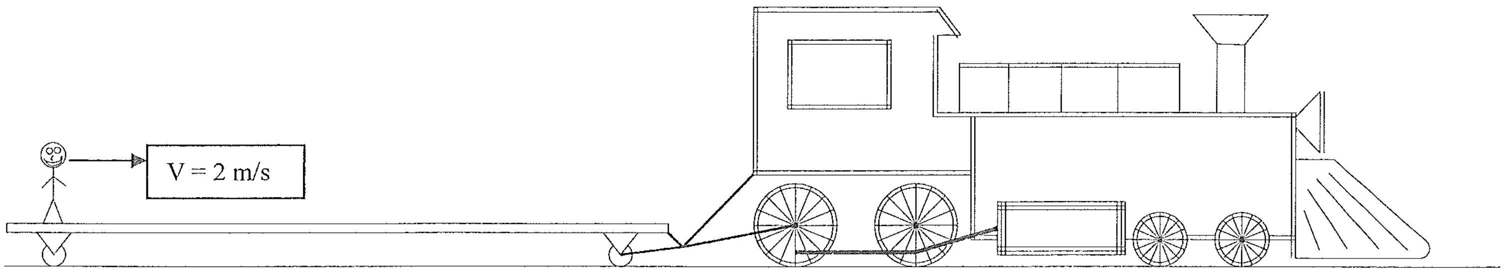
1) 19.0 m/s [W]	2) 6.44 m/s [N]	3) +160 m	4) 0.707s
5) +33.3 m/s	6) 75 s	7) 1.34 m/s [W]	8) -2.86 m/s
9) 34.3 m/s [down], 15m, 24m	10) -31.0 m/s, 4.18 s		11) 2.3 s

HW 7

Lesson 3

Physics 12 – **Kinematics 3 - Accelerated Motion Continued**

1) A train is at rest on the track while you walk at a constant velocity of 2 m/s forward (relative to the train) for 10 seconds;

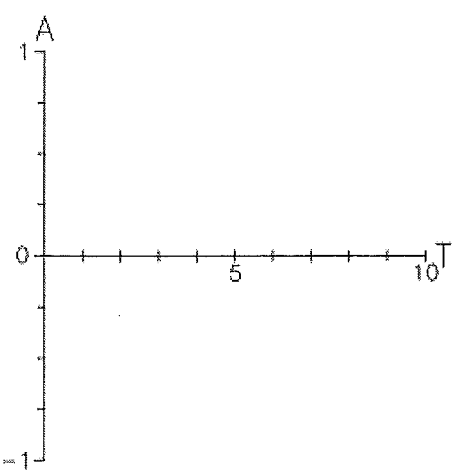


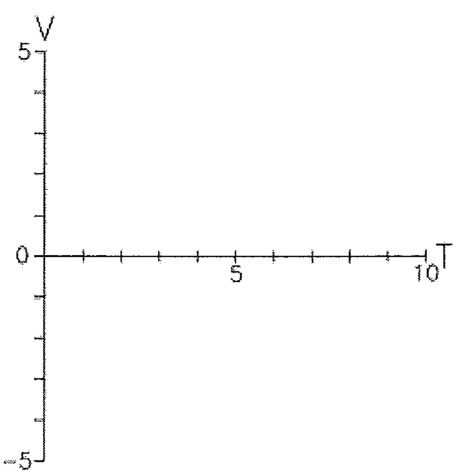
a) Complete the graphs below for the described motion.

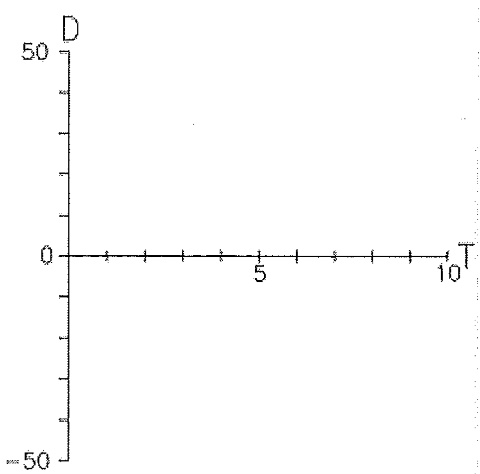
b) How far have you walked (relative to the train) after ten seconds?

c) Draw a vector representing velocity (relative to the train) for you at t = 10 seconds. Use a scale of 1 cm = 1 m/s

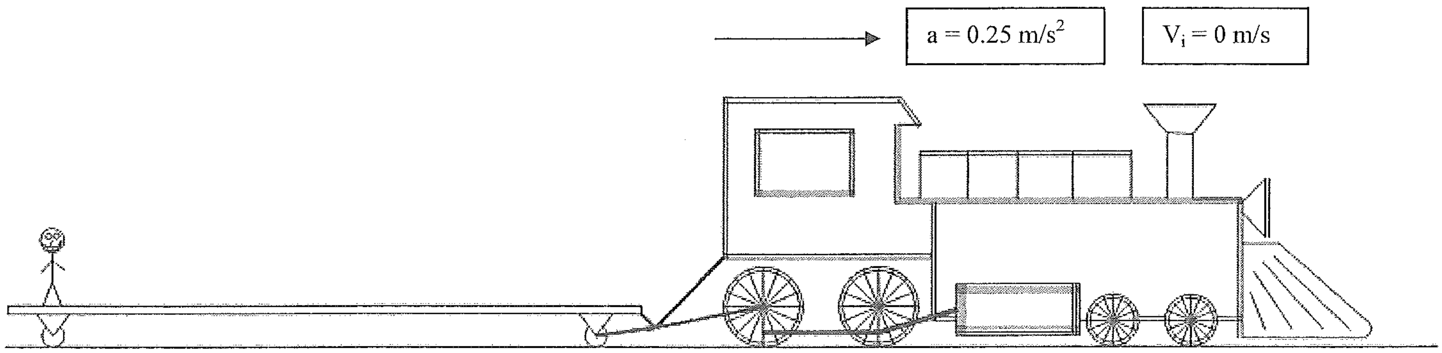
d) Draw a vector representing your displacement (relative to the train) at t = 10 seconds. Use a scale of 1 cm = 10 m







2) You are standing still on a train while the train is accelerating to the right at .25 m/s² from rest.

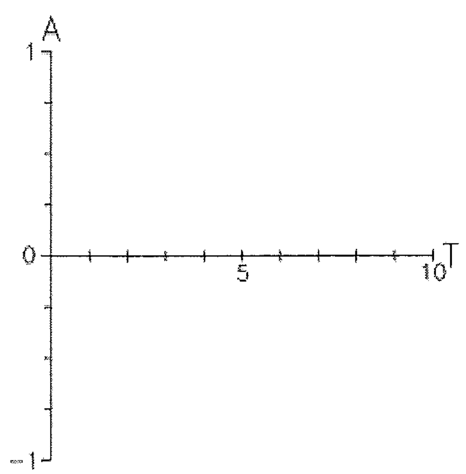


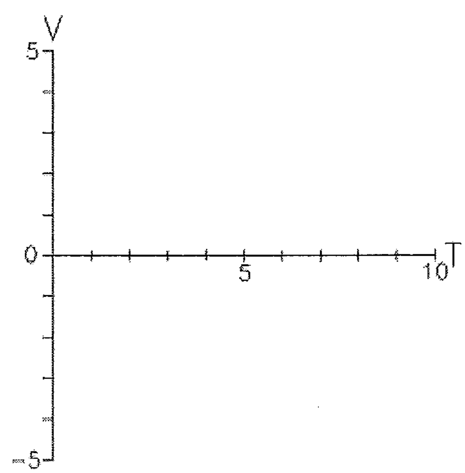
a) Complete the graphs below for your motion. (for 10 seconds)

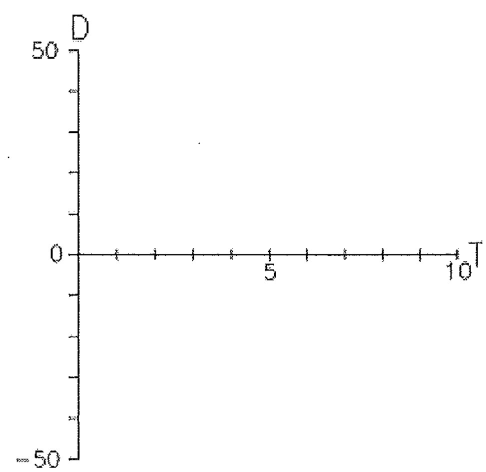
b) What is your displacement (relative to the ground) after ten seconds?

c) Draw a velocity vector representing your velocity (relative to the ground) at t = 10 seconds. Use a scale of 1 cm = 1 m/s

d) Draw a displacement vector representing your displacement (relative to the ground) at t = 10 seconds. Use a scale of 1 cm = 10 m

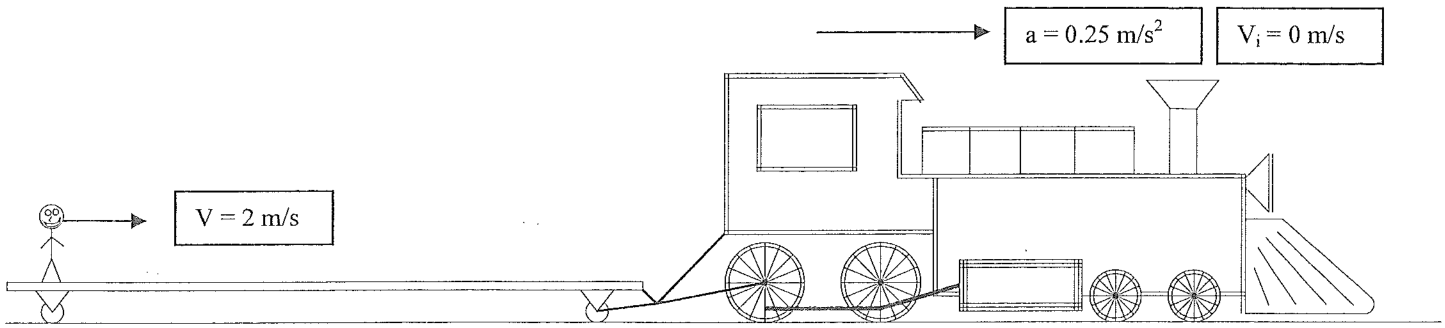






KL3

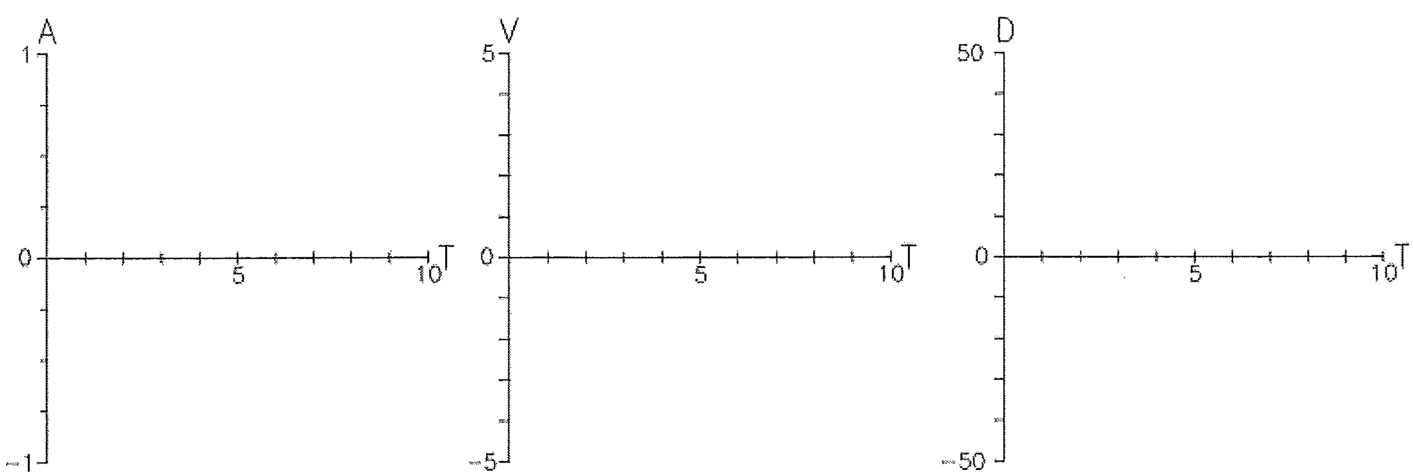
3) You are walking at a constant velocity of 2 m/s relative to the train while the train is accelerating to the right at 0.25 m/s² from rest relative to the ground.



#4

a) Complete the graphs below for your motion.
b) Draw a vector representing **your** velocity **<u>relative to the ground</u>** at t = 10 seconds. (hint, there are TWO components!!)

c) Draw a vector representing **your** displacement **<u>relative to the ground</u>** at t = 10 seconds. (hint, there are TWO components!!)



Combining all of your knowledge of uniform motion, motion graphing and accelerated motion-

A car begins a trip by accelerating from rest with an acceleration of $+0.75\ m/s^2$ over 12 seconds. Once it reaches this velocity, it cruises at a constant velocity for 2.0 minutes before stepping on the brake causing an acceleration of $-2.3\ m/s^2$ until it reaches a stop. How far did the car travel over its entire motion?

A dog runs forward from his doghouse at a constant velocity of 2.45 m/s for 62 s before slowing down to 0.50 m/s to investigate an odor for 30 s before turning around and running back to his doghouse. What is the total distance that the dog ran throughout his entire motion?

Lesson 3

Accelerated Motion Problem Assignment – Complete on separate sheet of paper.

1. The McLaren F1 is a great car! Costing a million dollars, it is the end result of a lifetime fascination for racing by a very successful formula one car designer. In its road-legal configuration, the F1 can accelerate from zero to 160 Km/h in about six seconds, beating a Porsche 911 Turbo by about 4.0 seconds. If you round off the numbers, this works out to an acceleration from rest to 50 m/s in 6.0 s. How far does the car travel in these six seconds? What is the rate of acceleration? Sketch a velocity time graph, as well as the kinematics equations to solve this problem.(+8.3 m/s^2,+149m)

2. A jogger runs at a constant velocity of 4.0 m/s for a time of 10 minutes. He then slows to a trot of 2.0 m/s in the same direction for a time of 10 more minutes. He then jogs back toward his starting point, where his car is parked, at a rate of 4 m/s without stopping. How far has the man jogged, and how long does it take him to return to his car? Draw vector diagrams with your solutions. (7200 m, 15 minutes)

3. A subway car accelerates uniformly from rest at a rate of 3.0 m/s^2 for a time of 10 seconds, and then travels at a constant speed for 30 seconds. It then slows down at a rate for -2.0 m/s^2 until it is stopped. Determine the distance traveled by the train for each of the three sections of its motion. Draw a vector diagram. (+150m,+900m,+225m)

4. A ball is thrown up in the air at a speed of 30 m/s. How high does the ball go? How high is the ball after two seconds? How high is the ball after 4.0 seconds? (46 m, 40m,42m)

5. Based on the information from questions three and four, what do you think it means to have a positive velocity and a positive acceleration? How about a negative velocity and a negative acceleration? Finally, how about a positive velocity and a negative acceleration? *(see posted solutions)*

6. A car accelerates uniformly from rest to a speed of 30 m/s in a time of 10 seconds. It then stops in a time of one half of a second. Find its acceleration, and the distance traveled by the car during its speeding up and slowing down periods. What do you suppose happened to create such a deceleration? (+3.0 m/s^2, +150m, -60 m/s^2, +7.5m)

7. A man riding upward in a hot air balloon at a constant rate of 10 m/s drops a sandbag out of his balloon to lighten his craft. If the sandbag falls freely for 10 seconds, what will be its velocity at this time? After ten seconds, how far below the point of release will the bag be? After ten seconds, will this be the same as the distance that the bag is below the balloon? (-88 m/s, 390 m below, no, the balloon is still rising up at +10 m/s)

8. The driver of a Porsche 944 is tooling down a one lane country road at 27 m/s when he crests a hill land sees a cement truck parked in the road 40 m ahead of him. If the maximum deceleration which can be supplied by his brakes and tires is 8.5 m/s^2, will he avoid a crash or not? (+43 m, no he will need 43 m to stop completely and the cement truck is 40 m away)

9. A sling-shot can speed up a 30 gram ball bearing from zero to 100 m/s in 0.30 seconds. What is the acceleration of the metal ball? (+333 m/s^2)

10. If a ball is launched upward at 20 m/s, after 1.5 seconds, what is its velocity? How high above the point of release will the ball be? (+5.3 m/s, 19 m high)

11. If a ball is thrown upward at 20 m/s, after 2.0 seconds what will its velocity be? What will its instantaneous acceleration be? Is this the same as its constant acceleration? (0.40 m/s, -9.8 m/s^2, yes it is the same as the only acceleration acting on the object is acceleration due to gravity.)

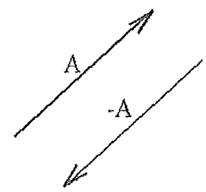
12. A toboggan full of little kids accelerates from rest down a hill with a constant acceleration of 2 m/s^2. How long will they have to keep this up before the exceed 100 km/h? (14 s)

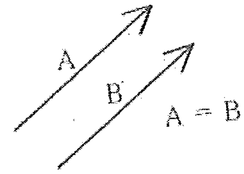
13. A box slides down a ramp and accelerates from 2.0 m/s to 4.0 m/s in a period of ten seconds. How far has the box gone in this time? (30 m)

14. Jimmy backs his car out of its parking space and smacks into a shopping cart which has been left in the parking lot, sending it at 6.0 m/s toward another row of cars 15 meters away. If the cart loses 2 m/s from its velocity every second that passes, how far will the cart go before it stops? Will it hit the other row of cars? (9.0 m, no)

15. A hockey player is checked into the boards, and in 0.50 seconds, changes his speed from 10 m/s to -5 m/s. What acceleration does he experience? If the acceleration of gravity (called 'g') is 9.8 m/s, how many g's does the player experience from the hit? (-30 m/s^2, 3.1 'g's)

Physics 12 – Vectors Lesson 4





Scalars

Scalar quantities require only **<u>magnitude</u>** to specify them.
Examples:

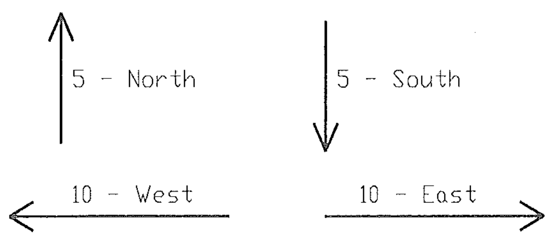
- distance (NOT displacement)
- mass (NOT weight),
- speed (NOT velocity),
- volume, area, density, time and temperature.

Vectors

Vector quantities require both **<u>magnitude</u>** and **<u>direction</u>** to specify them.
Examples: displacement, weight, velocity, acceleration, force and momentum.

Representing Vectors Graphically

Vectors can be represented graphically by drawing an arrow. The arrow you draw will have a length, and a direction. The **length** of the arrow corresponds to the **magnitude** of the vector, and the **direction** that the arrow is pointing corresponds to the **direction** of the vector.



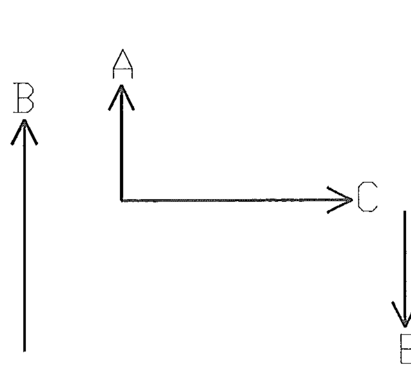
Vector Equality

For two vectors to be equal, they must have the same direction and the same magnitude.

B ≠ A (Same direction but different magnitudes)

A ≠ E (Same magnitude but different directions)

D = A (Same magnitude AND same direction)



Adding Vectors

When you add the vectors together, the result is also a vector. We call this the ***resultant.***

The rule we use to add vectors is called the 'tip to tail' rule. If you want to add two vectors, ***translate*** (move) the tail of one vector to the tip of another vector.
The ***resultant*** is drawn from the tail of the first vector (WHERE YOU STARTED) to the tip of the second vector (WHERE YOU ENDED). In text books resultants are usually shown with dashed lines.
We are going to use the analytical method in which we will draw a reasonable representation of the vector problem (as opposed to the graphical method where a diagram is drawn to scale using a ruler and protractor).

If the vectors are perpendicular to each other, you can use the Pythagorean Theorem to determine the magnitude of the resultant.

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

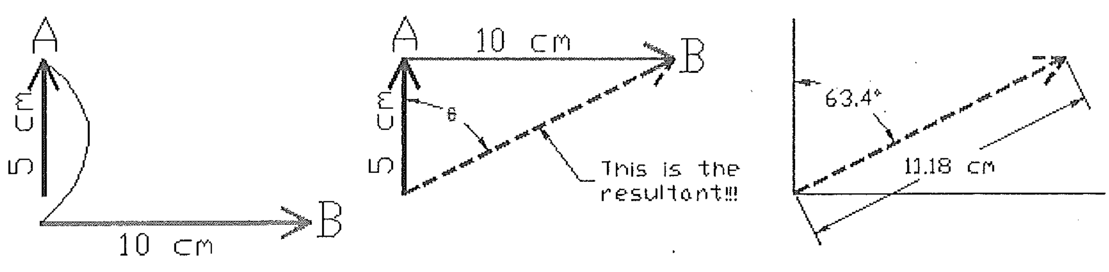
In the following example we will show how to add two vectors. The two vectors are: ***Vector A = 5 up,*** added to ***Vector B = 10 right.*** We can then see how the addition of the two vectors is NOT 15.

To add vector A to vector B:

1) translate the tail of B to the tip of A

2) draw a vector from the TAIL of A to the TIP of B.

3) now use trigonometry to solve for the length and direction

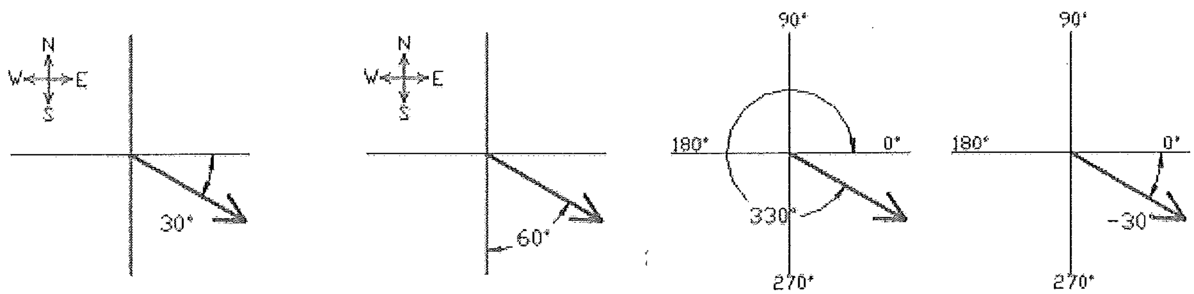


The reason that we "translate" the vectors is so that the resultant reflects the individual vectors. Vector **A** was pointing **up.** Vector **B** was pointing **right**. Therefore the resultant should be pointing **up** and **right.**

Direction Specification

We also need a method to describe the *direction* that vectors point in. There is more than one way to specify the direction of a vector. Depending on the situation we may specify the same direction in different ways, but **all are correct.**

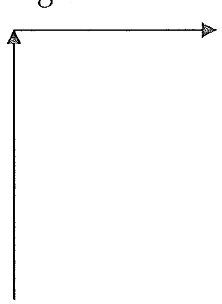
In the first diagram on the left the vector is **not** pointing straight East, but is it pointing at an angle *30° towards South* of **East**. Looking at the second vector, the direction specification is now is **not** pointing straight South, but is it pointing at an angle *60° towards East* of **South.** The third diagram is showing the vector at **+330°** which can also be described as **–30°**. Note how there are four directions that sound different but when you sketch out the direction is can be seen that all four are the same. They are all correct directions for the vector.



30° S of E 60° E of S 330° –30°

An object moves 73 m north and 62 m east. What is the resultant?

Diagram: Magnitude of *R*



Direction of *R*

An object moves 5.0 m north, 10.0 m west and 9.0 m south

Vector Components

When we split vectors up into pieces we call the pieces *components*. Normally, we want to split up vectors into their 'X' and 'Y' components. Another way to think of this is the ***amount*** that the vector points in the **X** and **Y** directions.

1) List the ***components*** of the following vectors *A – F* in the spaces provided.

A_x= ______
A_y= ______

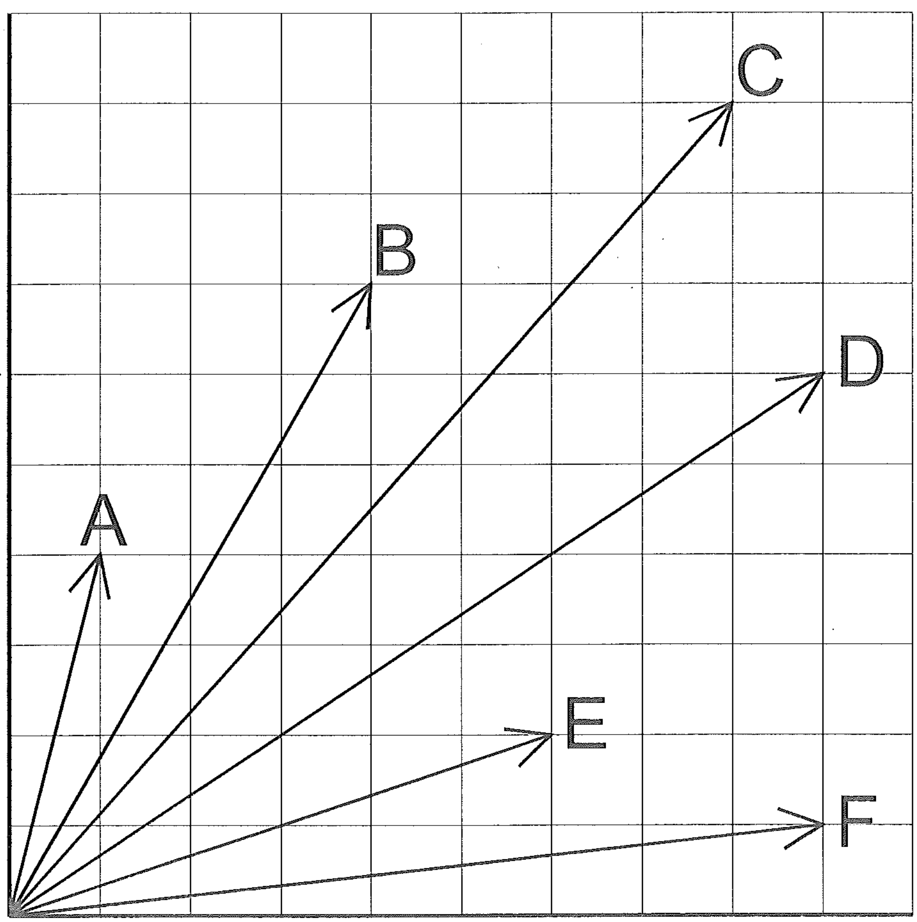
B_x= ______
B_y= ______

C_x= ______
C_y= ______

D_x= ______
D_y= ______

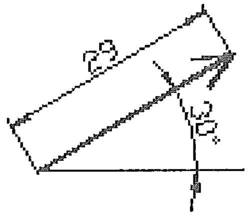
E_x= ______
E_y= ______

F_x= ______
F_y= ______



When resolving vectors into their components we use trigonometry to determine the components.

A.



B. 15.0 m 34.0° S of E

Vectors Part One Assignment: Lesson 4

1. Solve the following displacement vectors by finding the net displacement and direction.

A) 3.0 m south and 4.0 m south

B) 3.0 m south and 4.0 m north

C) 3.0 m south and 4.0 m east

D) 8.0 m west and 5.0 m north

E) 7.0 m south, 6.0 m east and 8.0 m north

F) 15 m west, 12 m north and 20 m east

2. Determine the ***x* and *y* components** of the following displacements:

A) 16.0 m north

Rx = ____________ Ry = ____________

B) 16.0 m 27.0° E of N

Rx = ____________ Ry = ____________

C) 20.0 m 52.0° W of N

Rx = ____________ Ry = ____________

D) 10.0 m 327°

Rx = ____________ Ry = ____________

HW

3. Resolve the following problems into their ***x* and *y* components**.

A) A person walks 200 meters at 27° degrees North of East.

Rx = ______________

Ry = ______________

B) A magnet attracts a steel ball with a force of 220 N at 25° North of West.

Rx = ______________

Ry = ______________

C) A rocket accelerates at 45 m/s^2 at 65 degrees South of East.

Rx = ______________

Ry = ______________

D) A cannonball is launched with a speed of 170 m/s at 40° above the horizontal.

Rx = ______________

Ry = ______________

HW 4

Answers:

1. A) 7.0 m south B) 1.0 m north C) 5.0 m 53° E of S D) 9.4 m 32° N of W

   E) 6.1 m 9.5° N of E F) 13 m 23° E of N

2. A) Rx = 0 m Ry = 16.0 m north B) Rx = 7.26 m east Ry = 14.2 m north

   C) Rx = 15.8 m west Ry = 12.3 m north D) Rx = 8.39 m east Ry = 5.45 m south

3. A) Rx = 178 m east Ry = 91 m north B) Rx = 199 N west Ry = 93.0 n north

   C) Rx = 19 m/s² east Ry = 41 m/s² south D) Rx = 130 m/s forward Ry = 109 m/s up

HV.11

Physics 12 – **Vectors Part Two - Adding Vectors** Lesson 5

Last class, we began adding vectors together.

12 m/s east + 24 m/s north →

Non-90⁰ Vector Addition:

Adding vectors that are completely in the 'X' or 'Y' directions is easy as they form nice right-angle triangles and the basic trig laws and Pythagorean theorem work.

However, often the vectors are not all in the 'X' and 'Y' direction. How do we solve these problems?

Step One: We take the vector at a *weird* angle (ie, not N, E, S, or W) and *resolve* (break apart) the vector to its 'X' and 'Y' components.

Step Two: We add up all the 'X' components, add up all the 'Y' components, and create a right triangle to use basic trig and Pythagorean theorem to calculate the magnitude and direction of the resultant!!!!

Examples - Add the following vectors:

10 m @ 37° N of W + 50 m North

62 N @ 30° + 50 N 53°

5.0 m/s² @ 57° N of W + 2.0 m/s² @ 22° S of W

Review of Velocity Vectors

Velocity vectors are added together in the same way that we added displacement vectors together.

1. Use tip-to-tail to find the resultant.
2. Find the magnitude of R through Pythagorean Theorem $R = \sqrt{(R_x)^2 + (R_y)^2}$
3. Find the direction of the vector using: $\tan\theta = \frac{opp}{adj} = \frac{R_y}{R_x}$

River Problems (2-D motion)

A boat whose speed in still water is 4.5m/s travels north across a river. The river current is 2.0 m/s east. What is the velocity relative to the shore?

We can also put vectors together with kinematics formulas such as v=d/t. This is illustrated in the problem of a boat crossing a river.

A boat whose speed in still water is 3.0 m/s is headed east across a river. The river current is 1.3 m/s south.

a) What is the velocity of the boat relative to the shore?

b) If the river is 2000m wide, how long does it take to cross the river?

c) How far downstream is the boat when it reaches the other side?

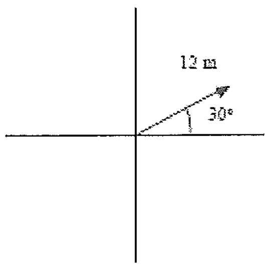
The other use of velocity vectors is to determine the initial direction needed to result in desired destination when more than one velocity is acting on the object.

A pilot wants to fly south. If the plane has an airspeed of 75 m/s and there is a 15 m/s wind blowing east.

Vectors Part Two Assignment:

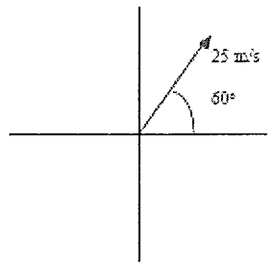
Part I:

Find the x and y components of each of the following vectors.



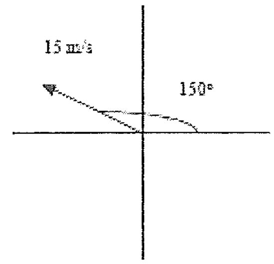
x = ______________

y = ______________



x = ______________

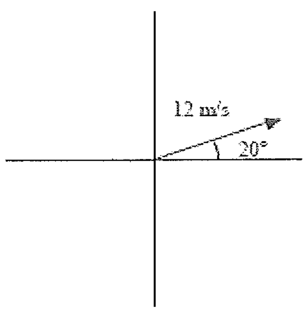
y = ______________



x = ______________

y = ______________

Add the following vectors.



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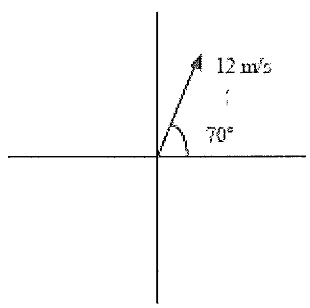

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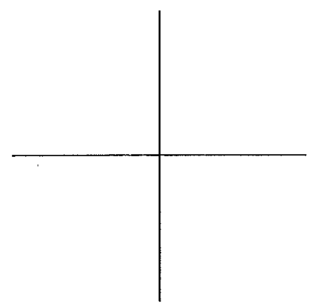
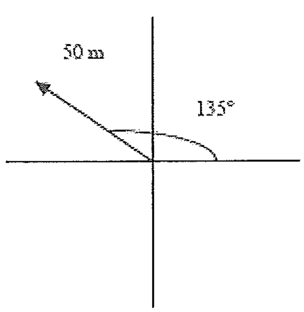


x_1 = ______________ + x_2 = ______________ = x_{tot} = ______________

y_1 = ______________ + y_2 = ______________ = y_{tot} = ______________



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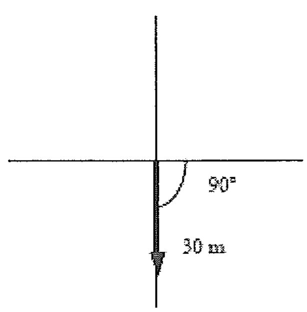

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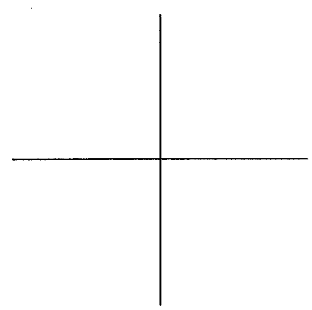


x_1 = ______________ + x_2 = ______________ = x_{tot} = ______________

y_1 = ______________ + y_2 = ______________ = y_{tot} = ______________

2. Add the following displacement vectors:

a) 8.0 m east and 6.0 m 35^0 N of E

b) 12 m south and 15 m 55^0 E of N

c) 5.0 m 26^0 S of E and 7.0 m 58^0 W of N

d) 9.0 m 35^0 N of E and 7.0m 25^0 S of E

3. A *GT SnowRacer*® had a momentum of 100 kg•m/s [20°N of W] as it slid on a perfectly smooth hill. It received an impulse of 50.0 N•s [10°N of E] from a barrier placed on one edge of the hill. What was the resulting momentum of the *GT SnowRacer*®?

4. Two forces act on an object. One has a magnitude of 25.0 N at an angle of 75°, the other has a magnitude of 40.0 N at an angle of 170°. What is the resultant force acting on the object?

5. A boat that can travel 3.6 m/s on still water heads directly north across a river that is 600 m wide. The river current is 1.2 m/s east. Draw the vector diagram.

a) What is the velocity of the boat with respect to the shore?

b) How long does it take the boat to reach the opposite side?

c) How far downstream is the boat when it reaches the opposite shore?

6. A passenger in a train travelling at 6.8 m/s [N] walks across the train car at 0.90 m/s [E] to the snack bar. If the car is 3.8 m wide, how long does it take the passenger to reach the other side? What is his velocity relative to the ground?

<u>**Answers:**</u>

2. a) 13 m 15° N of E b) 13 m 75° E of S c) 2.1 m 47° N of W d) 14 m 9.1° N of E

3) 62.0 kg•m/s 44° N of W

4) 45.0 N 43° N of W

5) a) 3.79 m/s 20° E of N b) 167 s c) 200 m [E]

6) 6.9 m/s 7.5° E of N

Velocity Vector Problems:

Draw a vector diagram for each question and then solve for the resultant with direction.

1. A truck is travelling in a straight line with uniform motion. The east component of this motion is 14.0 m/s, and the south component of the motion is 21.0 m/s. What is the velocity of the car?

2. A pilot heads her plane north with a velocity of 140 km/h. If there is a strong wind of 75 km/h blowing east, what is the velocity of the plane with reference to the ground?

3. An airplane is headed due north at an airspeed of 55 m/s. A sudden wind of 32 m/s arises from the west (blowing east). What is the velocity of the plane relative to the ground while the wind is blowing?

4. A boat whose speed in still water is 4.5 m/s is in a river whose current velocity is 2.0 m/s east. What is the velocity of the boat relative to the shore when the boat is heading:

a) east?

b) west?

c) north?

5. A boat that can travel on still water at a speed of 4.0 m/s wants to travel north perpendicular to the river current. If the river current is 2.2 m/s east, in what direction must the boat be held? (Note: Boat's resultant is to be perpendicular to the river current)

6. A pilot wants to fly west (the resultant will be west). If the plane has an air speed of 105 m/s, and there is a 33 m/s wind blowing north, in what direction must she head?

7. A boat whose speed in still water is 7.4 m/s is headed east across a river. The river current is 1.5 m/s south.

a) What is the velocity of the boat relative to the shore?

b) If the river is 6000 m wide, how long does it take the boat to cross the river?

c) How far downstream is the boat when it reaches the other side of the river?

8. A boat that can travel 3.6 m/s on still water heads directly north across a river that is 600 m wide. The river current is 1.2 m/s east.

a) What is the velocity of the boat with respect to the shore?

b) How long does it take the boat to reach the opposite side?

c) How far downstream is the boat when it reaches the opposite shore?

9. A passenger in a train travelling at 6.8 m/s [N] walks across the train car at 0.90 m/s [E] to the snack bar. If the car is 3.8 m wide, how long does it take the passenger to reach the other side? What is his velocity relative to the ground?

Physics 12 Lab - Investigating Vectors

Name:

PART ONE-

Problem: How does a boat travel on a river?

Materials: Meter stick, constant speed vehicle, strip of paper ("river")

Procedure:

1. The car will serve as the boat. Determine the "boat's" speed and show your calculations.

2. Your boat will start with all wheels on the paper river. Measure the width of the river and predict how much time is needed for your boat to go directly across the river. Show your data and calculations.

3. Determine the time needed to cross the river when your boat is placed on the edge of the river (all wheels on the paper). Complete three trials and record the times in your data table.

4. Do you think it will take more or less time to cross when the river is flowing? Explain your prediction.

5. Have one group member pull the "river" at a slow, constant speed along the floor. Measure the time it takes for the boat to cross the flowing river. Complete three trials and record the times in your data table. Compare your results with your prediction.

6. Devise a method to measure the speed of the river. Have a group member pull the river a slow, constant speed (as in last step) and collect the necessary data.

Data and Observations:

Table 1:

Still River		Moving River	
Trial #	Time (s)	Trial #	Time (s)
1		1	
2		2	
3		3	

Table 2:

Data and Calculations for Measuring the River's Speed

Analyze and Conclude:

1. Does the boat move in the direction that it is pointing when the river is moving? Draw a vector diagram to indicate the resultant direction of motion.

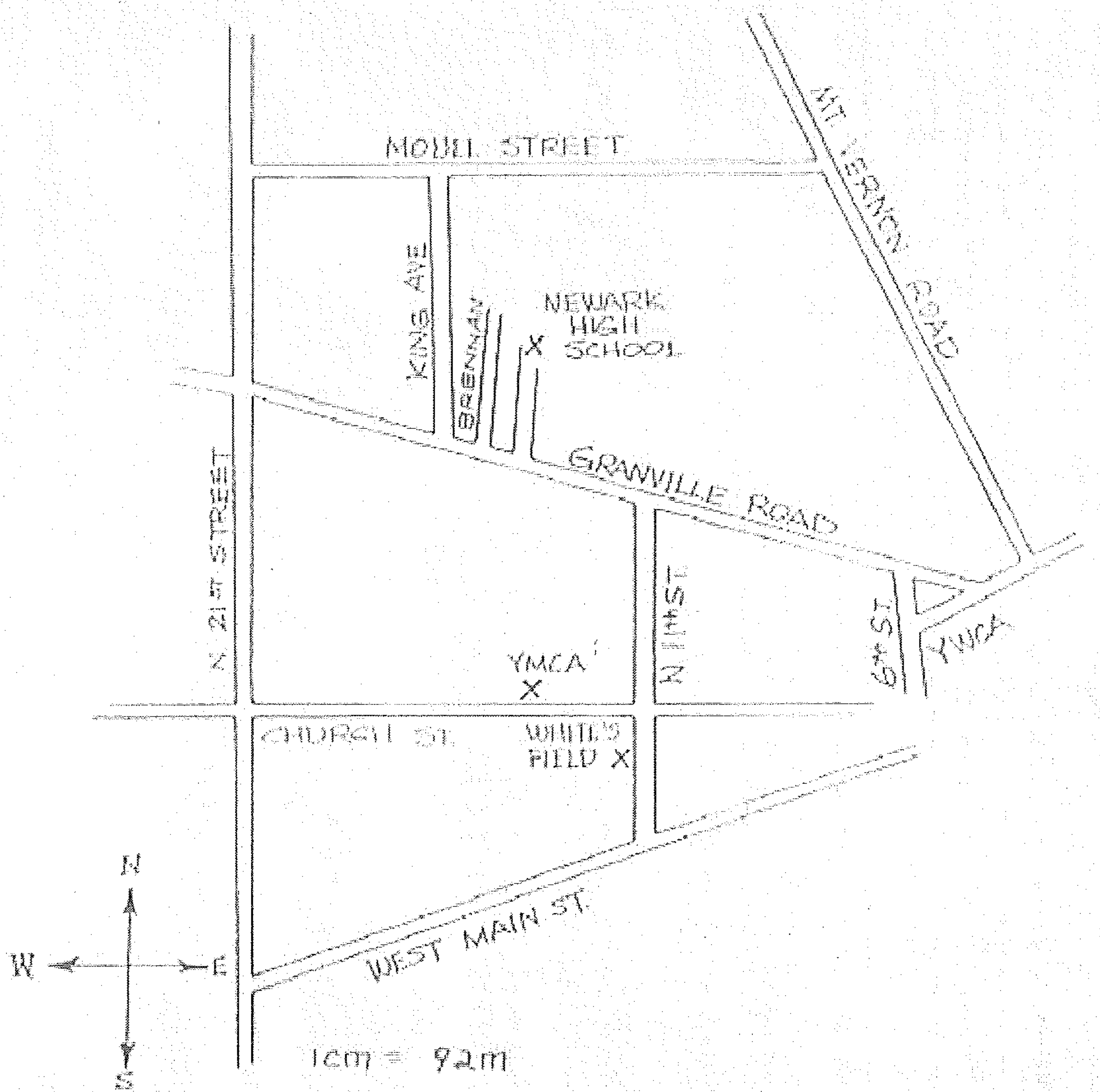
2. Did the motion of the water affect the time needed when the boat was point straight across?

3. Which had the greater speed, the boat or the river? Explain and use a diagram to help.

4. What was the calculated speed of the "river" current?

5. Using your results for the speed of the boat and the speed of the river, calculate the speed as seen relative to the "shore". Draw a vector diagram and include the angle in your final answer.

PART TWO: Scale Map



PART TWO – Using Scale Drawings to Determine Distance and Vector Addition to Determine Displacement **<u>(the map is on the back of the final page so that you can remove it while answering these questions)</u>**

<u>A: Using the Map Scale</u>

1. Use the scale on the map (1cm=92m) to **determine the distance in meters** for

   a) From Granville Road along N 11th Street to Church Street and down Church Street to the YMCA____

   b) Along Moull Street from N 21st Street to Mount Vernon Road.____

<u>B: Adding Two Vectors</u>

1. Take a walk from the corner of N 21st Street and Granville Road down N 21st Street to Church Street and E down Church Street to the YMCA.

Using the map scale (1cm = 92m), find the **magnitude in meters** for the distance covered.

____

2. Using the map scale (1cm = 92m), draw a vector diagram representing your trip and calculate the resultant displacement (in meters).

3. Now determine the **resultant displacement (in meters)** for the two trips in part A.

a) From Granville Road along N 11th Street to Church Street and down Church Street to the YMCA

b) Along Moull Street from N 21st Street to Mount Vernon Road.

Questions: Draw a vector diagram for each question and then solve for the resultant with direction.

1. A pilot heads her plane north with a velocity of 140 km/h. If there is a strong wind of 75 km/h blowing east, what is the velocity of the plane with reference to the ground?

2. An airplane is headed due north at an airspeed of 55 m/s. A sudden wind of 32 m/s arises from the west (blowing east). What is the velocity of the plane relative to the ground while the wind is blowing?

3. A boat whose speed in still water is 4.5 m/s is in a river whose current velocity is 2.0 m/s east. What is the velocity of the boat relative to the shore when the boat is heading:

a) east?

b) west?

c) north?

4. A pilot wants to fly west (the resultant will be west). If the plane has an air speed of 105 m/s, and there is a 33 m/s wind blowing north, in what direction must she head?

5. A boat whose speed in still water is 7.4 m/s is headed east across a river. The river current is 1.5 m/s south.

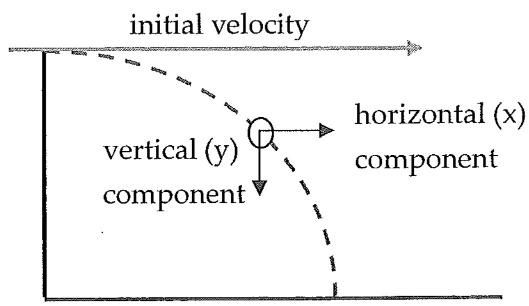
a) What is the velocity of the boat relative to the shore?

b) If the river is 6000 m wide, how long does it take the boat to cross the river?

c) How far downstream is the boat when it reaches the other side of the river?

Physics 12 – **Projectile Motion 1 (Horizontal Launch)** Lesson 6

When an object is thrown into the air, it is a projectile. Any object that **curves** downward in response to gravity is called a **projectile**. The motion of a projectile under the influence of gravity is called **projectile motion**.

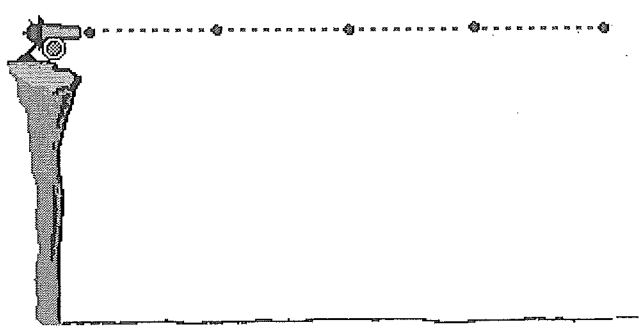


HORIZONTAL COMPONENT:

Why do we describe this horizontal component as uniform motion?

Imagine a cannonball shot horizontally from a very high cliff at a high speed. And suppose for a moment that the *gravity switch* could be *turned off.*

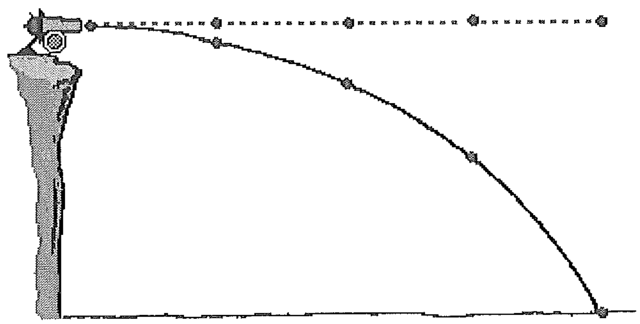
According to Newton's first law of motion, such a cannonball would continue in motion in a straight line at constant speed (in the absence of an unbalanced force)



However, is it an object's path under the influence of gravity that is considered **projectile motion**.

VERTICAL COMPONENT:

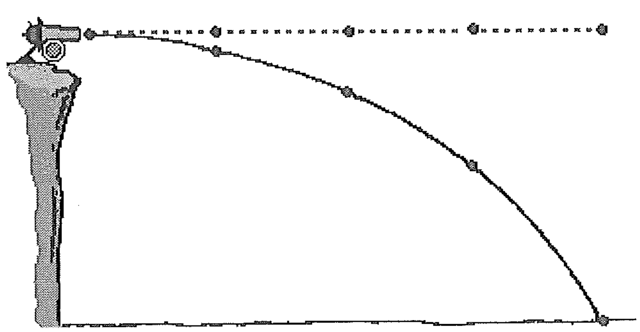
When we have gravity, it will act downwards upon the cannonball to affect its vertical motion. Gravity causes a vertical acceleration ($-9.8 m/s^2$). The ball will drop vertically below its otherwise straight-line, inertial path. As gravity is a downward force, it will affect the projectile's vertical motion and cause the parabolic trajectory that we see in projectile motion.



TIME:

Assuming no air resistance, **<u>the time it takes to fall when dropped straight down is the same as the time it takes to complete the projectile path.</u>**

This happens for the same reason as it did when we were calculating velocity vectors with a boat crossing the river. Even though the displacement is higher (path is greater), the velocity is also higher (due to an initial horizontal velocity).



Examples: An object is thrown horizontally at a velocity of 20.0 m/s from the top of a building 50.0m tall. What is the range of this object?

An object is thrown horizontally at a velocity of 32.0 m/s from the top of a building. If the range of the object is 55.0m, how high is the building?

During a police chase, a car drives off the edge of a cliff that is 47.5 m high. When the police look over the edge to check on the suspect, they find that the car landed in the lake that is 200 m from the base of the cliff. How fast was the car travelling when it went over the edge of the cliff?

KLC

Projectile Motion-1 Assignment: Lesson 6

1. An object is thrown horizontally from the top of a cliff at a velocity of 20.0 m/s.

   a) If the object takes 4.20s to reach the ground, what is the range of this object? (84.0m)

   b) What is the velocity of the object just before it hits the ground? (Remember, this will be a resultant velocity) (45.8 m/s)

2. A bullet is fired from a rifle that is held 1.60 m above the ground in a horizontal position. The initial speed of the bullet is 1100 m/s. Find (a) the time it takes for the bullet to strike the ground and (b) the horizontal distance travelled by the bullet. (0.571 s, 629 m)

3. A car drives straight off the edge of a cliff that is 54.0 m high. The police at the scene of the accident note that the car landed on a tree that was growing 130 m from the base of the cliff. How fast was the car travelling when it went over the edge of the cliff? (39.2 m/s)

HW 5

4. A tennis ball is struck such that it leaves the racket horizontally with a speed of 28.0 m/s. The ball hits the court at a horizontal distance of 19.6 m from the racket. What is the height of the tennis ball when it leaves the racket? (2.40 m)

5. A diver pushes off horizontally with a speed of 2.00 m/s from a platform edge 10.0 m above the surface of the water.

a) At what horizontal distance from the edge is the diver 0.800s after pushing off? (1.60m)

b) At what vertical distance above the surface of the water is the diver at that point? (from part a) (6.87 m above surface)

c) At what horizontal distance does the diver strike the water? (2.86 m)

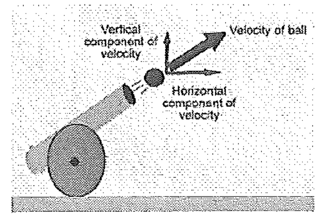
6. A horizontal rifle is fired at a bull's eye. The muzzle speed of the bullet is 670 m/s. The bullet strikes the target 0.025 m below the center of the bull's-eye. What is the horizontal distance between the end of the rifle and the target? (48 m)

Hus

Lesson 7

Projectile Motion (Objects Thrown at an Angle)

Objects that follow a path characteristic with projectile motion can also be thrown or launched into the air at an angle.

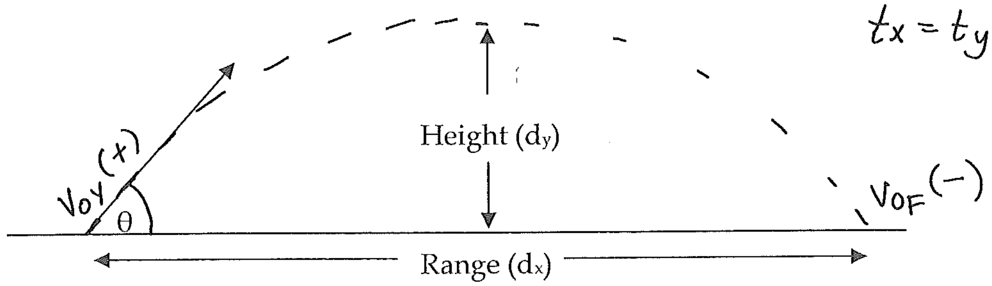


These types of problems also have a vertical and horizontal component in which the vertical motion is uniform and the horizontal component is accelerated by the force of gravity (as seen in the last lesson). However, in these problems the initial vertical velocity (v_{0y}) is no longer 0 m/s as it has some positive initial value.

The path taken by a projectile launched at an angle to the horizontal can be described as follows:

A projectile is launched at an angle to the horizontal and rises upwards to a peak while moving horizontally. Upon reaching the peak, the projectile falls with a motion that is symmetrical to its path upwards to the peak. Predictable unknowns include the time of flight, the horizontal range, and the height of the projectile when it is at its peak.

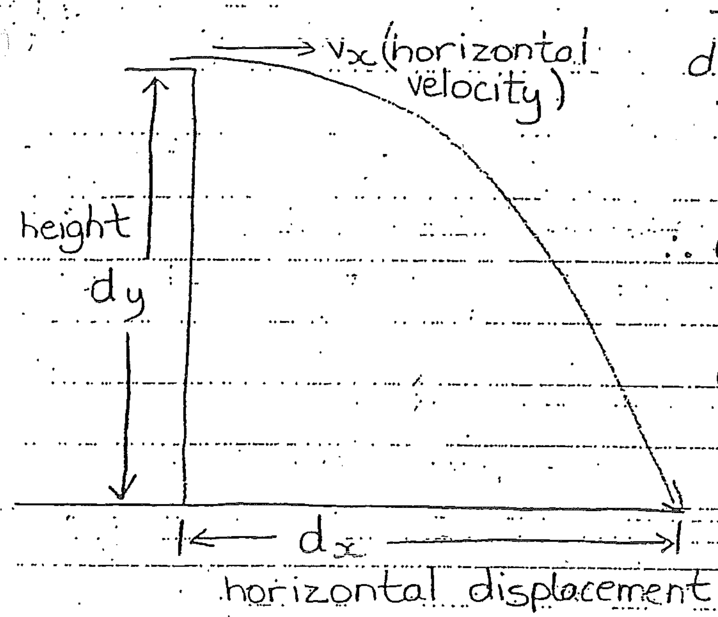
$t_x = t_y$



Using the launch velocity we need to determine the x and y-components:

KL6

Motion in 2-D - Physics 11 review



vertical	horizontal
$d_y = v_y t + \frac{1}{2} a_g t^2$	$d_x = v_x t$
$v_y = 0$ (dropping)	
$\therefore d_y = \frac{1}{2} a_g t^2$	

1ST Find time from:

$t = \sqrt{\frac{2d_y}{a_g}}$ OR $t = \frac{d_x}{v_x}$

and Use time to find

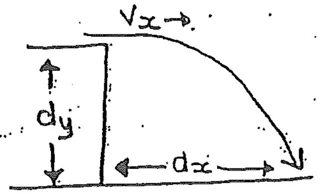
$d_y = \frac{1}{2} a_g t^2$ OR $d_x = v_x t$

$v_x = \frac{d_x}{t}$

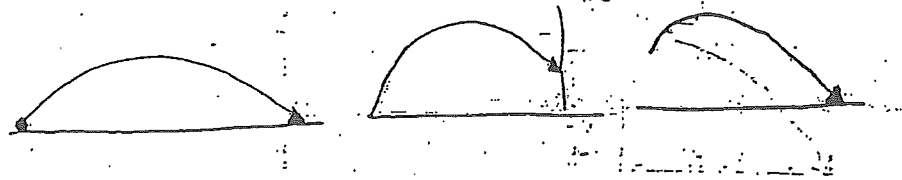
VECTOR ANALYSIS FOR MOTION IN 2 DIMENSIONS

There are two main types of motion in two dimension questions:

a) just down:

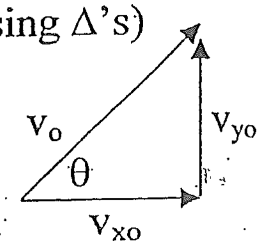
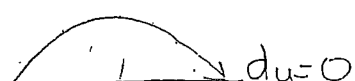


b)) up and down:



Motion in 2-D has movement in both the "x"and "y"directions so you must keep track of these separately as they are independent of each other

b)up and down

	Horizontal (x)	**Vertical (y)**
1. **Make a diagram** (using Δ's) eg.  if we know v_i & θ:		
2. **ALWAYS find v_{xo} and v_{yo}.**	$v_{xo} = \cos\theta\ v_o$	$v_{yo} = \sin\theta\ v_o$
3. Find **total time in flight** from: 	$d_y = v_{yo}t + \frac{1}{2}at^2$ $d_y = 0$ since vertical displacement is 0. so $0 = v_{yo}t + \frac{1}{2}at^2$ $-v_y = \frac{1}{2}at$ $t = \frac{-2v_y}{a}$ or $t = \frac{-v_y}{\frac{1}{2}a}$	

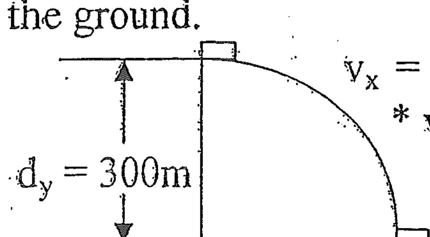
4. Find **MAX height** using: Or **if** d_y=0m for total flight, max height occurs half way through the flight: $t_{½} = \frac{t_{flight}}{2}$		$v_y^2 = v_{yo}^2 + 2ad_y$ (at max height: v_y=0.0m/s) ½ t_{flight} is used to **find d_y maximum.** $t_{½flight}$ = ? $d_y = v_y t_{½} + ½ a t_{½}^2$
5. Find **HORIZONTAL distance** using <u>total flight time</u> and $d_x = v_x t_{flight}$	$v = \frac{d}{t}$ $d_x = v_x t_{flight}$ **uniform motion (horizontally)**	

•**Horizontal and Vertical velocities are independent of each other!!**

<u>**Examples**</u>

1. As an experiment in motion in 2 dimensions, an unwise Physics student decides to throw his/her Physics book off a 300m tall building with a velocity of 25.0 m/s.

Find how far it flew horizontally (the range) and how long it took to hit the ground.

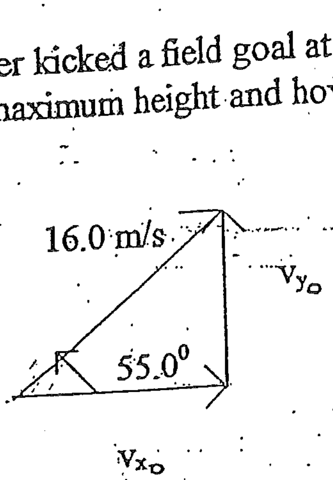


v_x = 25.0 m/s

* v_{oy} = 0.0 m/s (dropping)

Examples

1. A football player kicked a field goal at an angle of 55.0° and with a velocity of 16.0 m/s. Find the balls maximum height and how far it travelled.



$v_{x_o} = \cos\theta\, v_i = (\cos 55.0°)(16.0\text{ m/s})$

$v_{x_o} = 9.18$ m/s

$v_{y_o} = \sin\theta\, v_i = (\sin 55.0°)(16.0\text{ m/s})$

$v_{y_o} = 13.1$ m/s

$d_y = 0$ at maximum d_x

maximum horizontal distance ; $d_y = v_{y_o}t + \frac{1}{2}at^2 \;\therefore\; 0 = v_{y_o}\cancel{t} + \frac{1}{2}at^{\cancel{2}}$

$t_{flight} = \dfrac{-2\,v_y}{a} = \dfrac{-2(13.1)\text{m/s})}{-9.80\text{ m/s}^2}$ $\quad t_{flight} = 2.67$ s

$d_x = v_{x_o} t_{flight}$ $\quad d_x = (9.18\text{m/s})(2.67\text{s}) = d_x = 24.5$ m

maximum height by either:

a) $t_{½} = \dfrac{t_{flight}}{2}$ therefore $t_{½} = 1.34$ s

$d_y = v_y\, t_{½} + ½\, a\, t_{½}^2$

$d_y = (13.1\text{ m/s})(1.34\text{ s}) + ½(-9.80\text{ m/s}^2)(1.34)^2$

$d_y = 17.55$ m + -8.798

$d_y = 8.76$ m

or b) $v_y^2 = v_{oy}^2 + 2ad_y$

max height $v_y = 0.0$ m/s

$0 = (13.1\text{m/s})^2 + 2(-9.80\frac{\text{m}}{\text{s}^2})d_y$

$19.6\,d_y = 171.61$

$d_y = 8.76$m

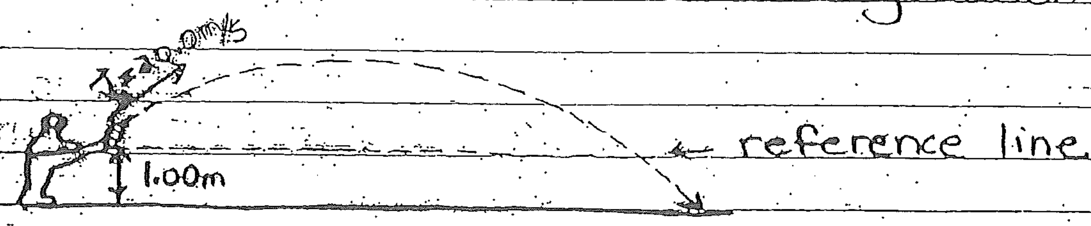
Additional Info: $\theta = 45°$ produces largest possible d_x (range)

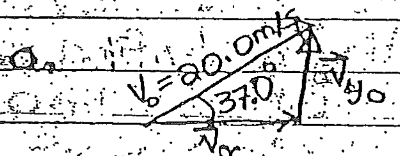
at angles other than 45°, each d_x can result from 2 angles that are complimentary & mirrored on either side of 45°.

More Types of Motion in 2-D Problems

- For vertical displacement the reference point is the initial placement of the object therefore displacement ABOVE the vertical reference point or line is POSITIVE (d_y is positive) AND displacement BELOW the reference point results in a NEGATIVE d_y.

eg 1. A football was kicked at an angle of 37.0°, 1.00 m above the ground with an initial velocity of 20.0 m/s. Calculate
 a) the time the football remains in the air
 b) how far it travels horizontally (← range) before it hits the ground.





1st draw a diagram + find $\vec{v}_{x_0}$ + $\vec{v}_{y_0}$:

$\vec{v}_{y_0} = \sin 37.0° \times 20.0\,m/s = 12.0\,m/s$

$\vec{v}_{x_0} = \cos 37.0° \times 20.0\,m/s = 16.0\,m/s$

2nd use $d_y = v_{y_0}t + \frac{1}{2}at^2$ to find total time:

Note: d_y is BELOW reference line ∴ $d_y = -1.00$

$$-1.00\,m = 12.0\,m/s(t) + \tfrac{1}{2}(-9.8\,m/s^2)t^2$$

rearrange: $(4.90\,m/s^2)t^2 + (-12.0\,m/s)(t) + (-1.00\,m) = 0$

To solve a quadratic use:

$$ax^2 + bx + c = 0 \quad \text{so} \quad x = \frac{-b \pm \sqrt{b^2 - 4a}}{2a}$$

in this case

$$4.90\,\frac{m}{s^2}\,t^2 + \left(-12.0\,\frac{m}{s}\right)t + (-1.00\,m) = 0$$

in this

$$\therefore\ t = \frac{-\left(-12.0\,\frac{m}{s}\right) \pm \sqrt{\left(-12.0\,\frac{m}{s}\right)^2 - (4)\left(4.90\,\frac{m}{s^2}\right)(-1.00\,m)}}{2(4.90\,m/s^2)}$$

$t = +2.53\,s$ or $-0.081\,s$

↳ negative time c not apply

∴ the time the ball remains in the air

$t = 2.53\,s$.

b. $d_x = v_{x_0} t = 16.0\,\frac{m}{s} \times 2.53\,s = 40.5\,m$

A long jumper leaves the ground with an initial velocity of 12 m/s at an angle of 28-degrees above the horizontal. Determine the time of flight, the horizontal distance, and the peak height of the long-jumper.

An object is thrown through the air at a velocity of 20.0 m/s at an angle of 30.0^{0} *with the horizontal. What is the range of the object?*

Another method of determining the range of a projectile launched at an angle is by using:

Compare with the previous example:

An object is thrown through the air at a velocity of 20.0 m/s at an angle of 30.0⁰ with the horizontal. What is the range of the object?

Determining velocity:

A water ski jumper has a range of 84.0 m. The ramp has an angle of 14.0⁰ to the horizontal. Neglecting air resistance, determine her take-off speed.

Lesson 7

Projectile Motion-2 Assignment:

1. An object is thrown from the ground into the air at an angle of 40.0^0 from the horizontal at a velocity of 18.0 m/s. What is the range of this object? (32.7 m)

2. An object is thrown from the ground into the air with a velocity of 20.0 m/s at an angle of 27.0^0 to the horizontal. What is the maximum height reached by the object? (4.21 m)

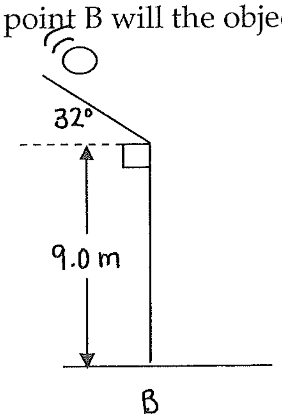
3. An object is thrown from the ground into the air at an angle of 30.0^0 to the horizontal. If this object reaches a maximum height of 5.75m, at what velocity was it thrown? (21.2 m/s)

4. An object is projected from the ground into the air at an angle of 35.0^0 to the horizontal. If this object is in the air for 9.26s, at what velocity was it thrown? (79.1 m/s)

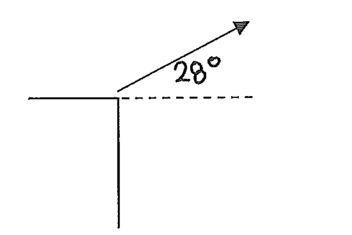
HW 6

5. An object is thrown from the ground into the air at a velocity of 15.7 m/s at an unknown angle to the horizontal. If this object has a range of 25.0 m and was in the air for 2.15 s, at what angle was this object thrown? (42.0°)

6. A ball rolls off an incline, as shown in the diagram, at a velocity of 22 m/s. How far from point B will the object hit the floor? (11 m)



7. An object is projected from the top of a building at an angle of 28.0°, as shown in the diagram, at a velocity of 15.0 m/s. If the object hits the ground 32.0 m from the base of the building, how high is the building? (11.8 m)



HW6

8. The punter on a football team tries to kick a football so that it stays in the air for a long "hang time". If the ball is kicked with an initial velocity of 25.0 m/s at an angle of 60.0° above the ground, with is the 'hang time'? (4.43 s)

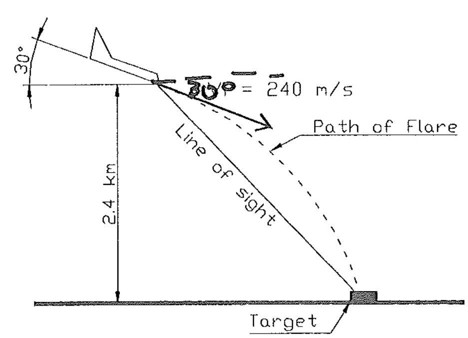
9. With a particular club, the maximum speed that a golfer can impart to a ball is 30.3 m/s. (a) How much time does the ball spend in the air? (b) What is the longest hole in one that the golfer can make, if the ball does not roll when it hits the green? (maximum displacement will come from an angle of 45.0° above the horizontal). (4.37 s, 93.5 m)

10. During a fireworks display a rocket is launched with an initial velocity of 35 m/s at an angle of 75° above the ground. The rocket explodes 3.7 s later. What is the height of the rocket when it explodes? (58 m)

HW6

11. From the edge of a 60.0 m cliff, a small rocket is fired upward with an initial velocity of 23.0 m/s at an angle of 50.0° with respect to the horizontal. At what point above the ground does the rocket strike the wall of a vertical cliff located 20.0 m away? (74.8 m)

12. * An airplane is flying with a speed of 240 m/s at an angle of 30.0° with the horizontal, as the drawing shows. When the altitude of the plane is 2.40 km, a flare is released from the plane. The flare hits the target on the ground. What is the angle θ? (42.0°)



13. *A diver springs upward from a board that is three meters above the water. At the instant she contacts the water her speed is 8.90 m/s and her body makes an angle of 75.0° with respect to the surface of the water. Determine her initial velocity, both magnitude and direction. (59.3° above horizontal)

14. *A golf ball is driven from a level fairway. At a time of 5.10 s later, the ball is travelling downward with a velocity of 48.6 m/s at an angle of 22.2° below the horizontal. Calculate the initial velocity (magnitude and direction) of the ball. (55.0 m/s 35.1° above horizontal)

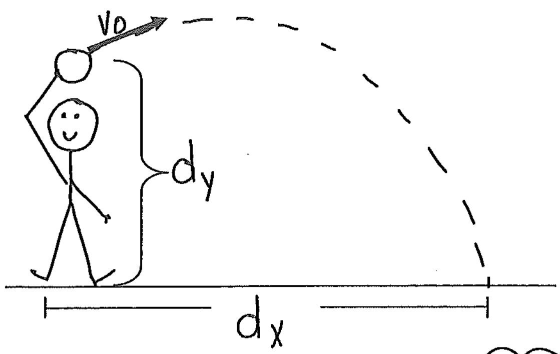
BONUS A garden hose, pointed at an angle of 25° above the horizontal, splashes water on a sunbather lying on the ground 4.4 m away in the horizontal direction. If the hose is held 1.4 m above the ground, at what speed does the water leave the nozzle? (5.8 m/s)

Lesson 8

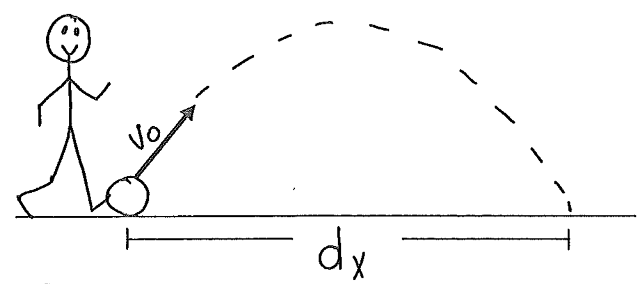
Physics 12 – **<u>Lab Activity: How Far and How Fast?</u>** Name:

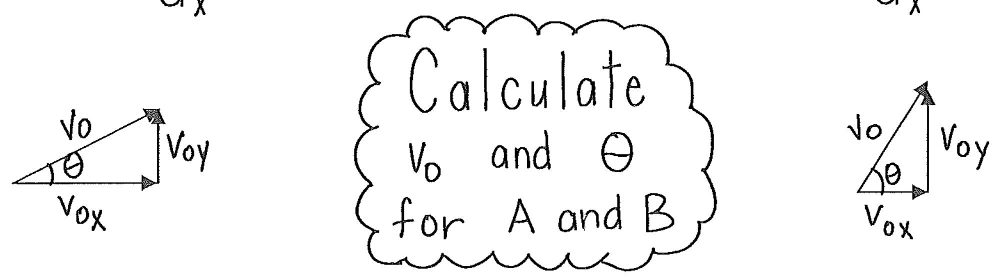
<u>Purpose:</u> You will use your knowledge of projectile motion (range and time) to find out how fast you can throw a softball and kick a soccer ball.

<u>Part A:</u>



<u>Part B:</u>





Materials: softball, soccer ball, stopwatch, meter stick or tape measure, paper, pencil and calculator

<u>Procedure A (with the softball):</u>

1. Split into groups of fours. You will each, in turn, be a thrower, a timer and a distance marker.

2. Take all materials (one per group) out to the field.

3. Record the data for each person in your group **BUT <u>do the calculations only with your own data.</u>**

Name	Height of Hand (d_y)	Time of Flight (t)	Range (d_x)

Procedure B (with the soccer ball):

Repeat steps as indicated in procedure A.

Name	Time of Flight (t)	Range (d_x)

Calculations for both parts A and B:

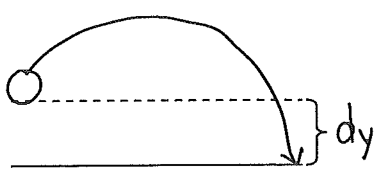
1. Calculating horizontal component: $v_x = \dfrac{d_x}{t}$

Part A:

Part B:

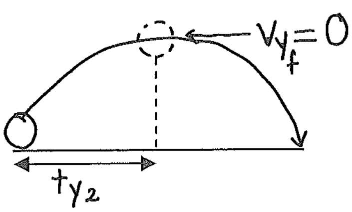
2. Calculating vertical component v_y:

Part A:



$$d_y = \overset{?}{v_{y0}}\, t + \frac{1}{2} a t^2$$

Part B:



$$a = \frac{v_{yf} - \overset{?}{v_{y0}}}{t_{y2}}$$

3. Using v_x and v_y to calculate the resultant velocity: (Draw diagrams)

Part A: **Part B:**

Observations and Data Analysis: (check with your group members)

1. Did all of the people in your group throw at about the same range? If not, what was the variation?

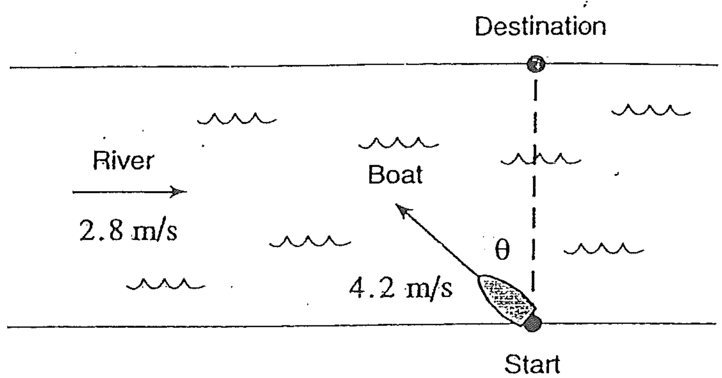
2. Did all of the people in your group throw at around the same speed? If not, what was the variation?

3. In order to achieve the largest range, should the thrower try to throw with a larger v_x or a larger v_y? Explain.

Lesson 9 Physics 12
Kinematics 1-D and 2-D Provincial Exam Questions

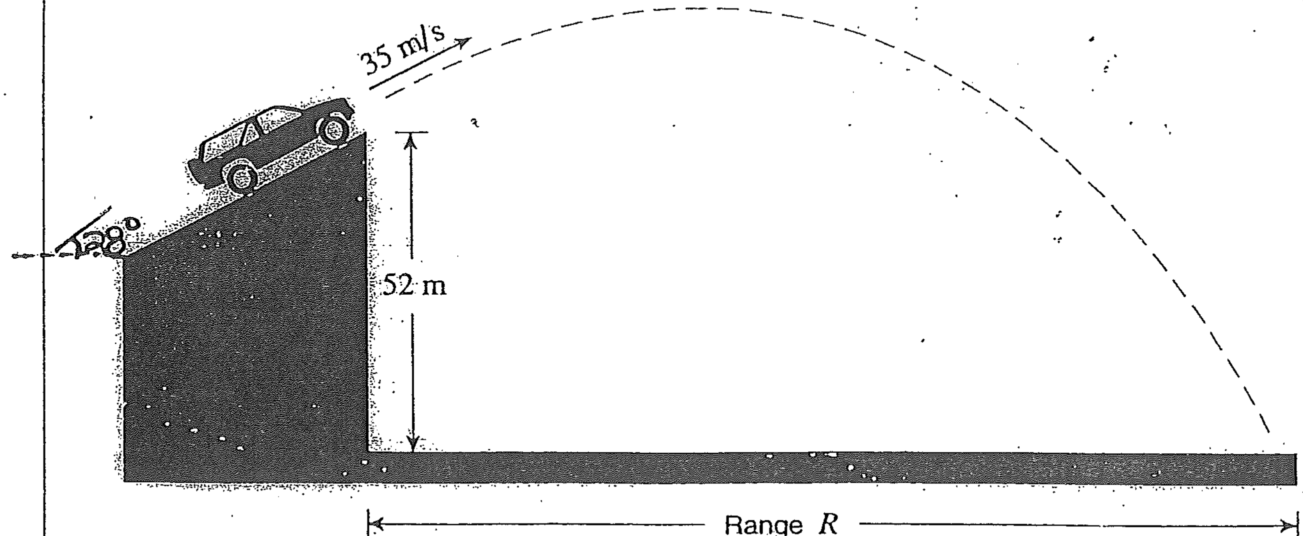
1. An object is launched over level ground at 35° above the horizontal with an initial speed $52\ \text{m/s}$. What is the time of flight?

2. A boat shown below travels at $4.2\ \text{m/s}$ relative to the water, in a river flowing at $2.8\ \text{m/s}$.



At what angle θ must the boat head to reach the destination directly across the river?

3. A stunt vehicle leaves an incline with a speed of $35\ \text{m/s}$ at a height of $52\ \text{m}$ above level ground. Air resistance is negligible.

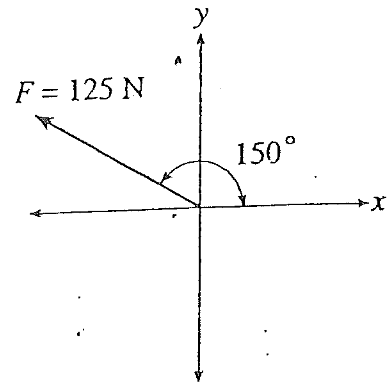


a) What are the vehicle's vertical and horizontal velocity components as it leaves the incline? **(1 mark)**

b) What is the vehicle's time of flight? **(4 marks)**

c) What is the vehicle's range, R? **(2 marks)**

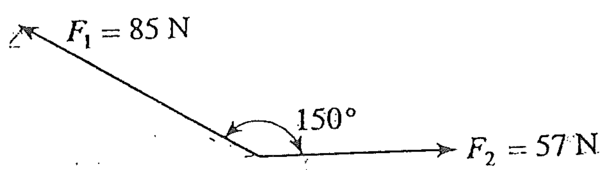
4. Consider the diagram below.



What are the components of the 125 N force?

5. A projectile is launched at 35.0° above the horizontal with an initial velocity of 120 m/s. What is the projectile's speed 3.00 s later?

6. What is the magnitude of the sum of the two forces shown in the diagram below?



7. The data table shows the velocity of a car during a 5.0 s interval.

t (s)	0.0	1.0	2.0	3.0	4.0	5.0
v (m/s)	12	15	15	18	20	21

a) Plot the data and draw a best-fit straight line. (2 marks)

8. An aircraft heads due south with a speed relative to the air of 44 m/s. Its resultant speed over the ground is 47 m/s. The wind blows from the west.

a) What is the speed of the wind? (4 marks)

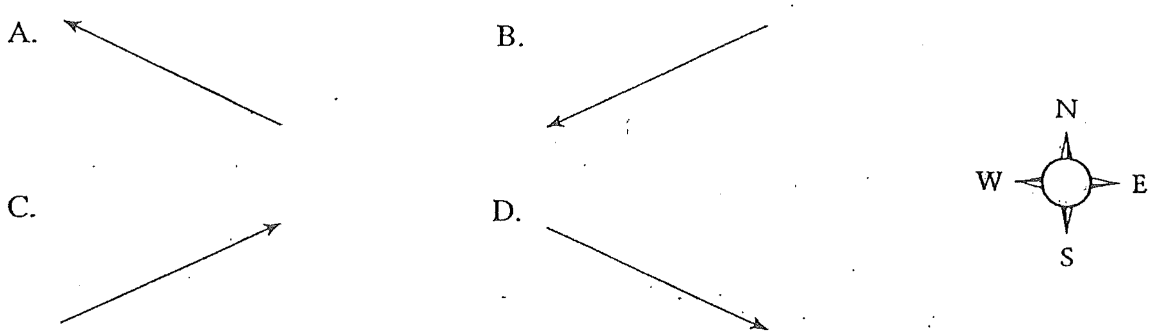
b) What is the direction of the aircraft's path over the ground? (3 marks)

9. Which of the following is true for projectile motion? (Ignore friction.)

	HORIZONTAL COMPONENT	VERTICAL COMPONENT
A.	constant velocity	constant velocity
B.	constant velocity	changing velocity
C.	changing velocity	constant velocity
D.	changing velocity	changing velocity

10. A ball is thrown vertically upward at 20 m/s from a height of 30 m above the ground. W is its speed on impact with the ground below?

11. A car travelling north at 20 m/s is later travelling west at 30 m/s. a) What is the direction of the change in velocity? b) Which one represents the sum or resultant?



12. Which of the following is constant for all projectiles?

A. vertical velocity
B. horizontal velocity
C. vertical displacement
D. horizontal displacement

13. A projectile is launched at 30 m/s over level ground at an angle of 37° to the horizontal. What maximum height does this projectile reach?

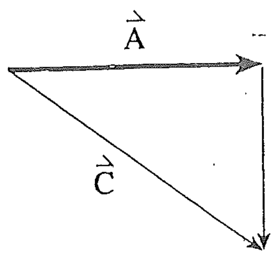
A. 3.1 m
B. 17 m
C. 29 m
D. 46 m

14. Which of the following contains vector quantities only?

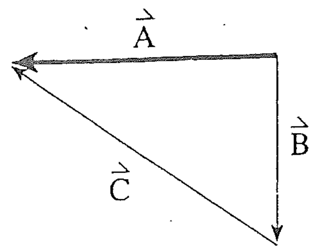
A. mass, speed
B. energy, velocity
C. displacement, energy
D. displacement, velocity

15. Which of the following vector diagrams shows: a) $\vec{A}$ as the sum of $\vec{B}$ and $\vec{C}$ (i.e. $\vec{A} = \vec{B} + \vec{C}$)?

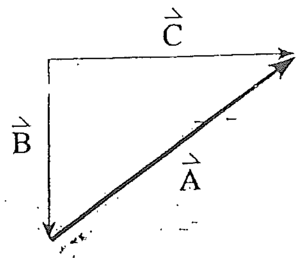
A.



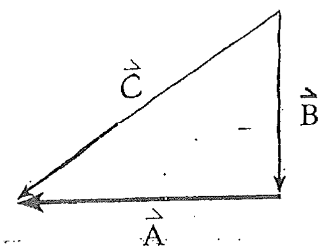
B.



C.



D.



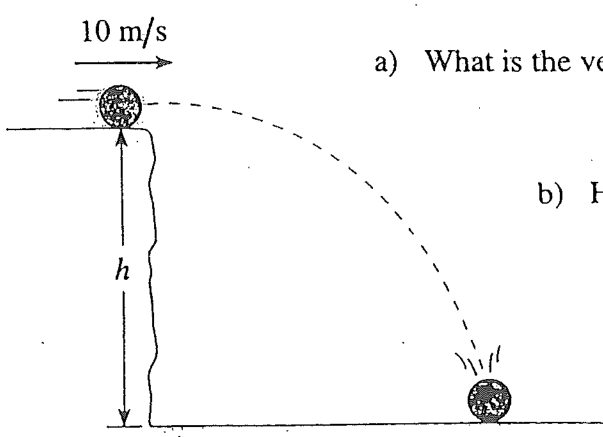
16. Which of the following statements is always correct about an object in motion?

A. It has a tendency to accelerate.
B. A net force must be acting on it.
C. It has a tendency to keep moving.
D. The net force acting on it must be zero.

17. A projectile is launched with a velocity of 35 m/s at 55° above the horizontal. What is the maximum height reached by the projectile? Ignore friction.

5.3 m
B. 42 m
C. 54
D. 63 m

18. A blue ball rolls off the cliff shown below at 10 m/s and hits the ground with a speed of 30 m/s.



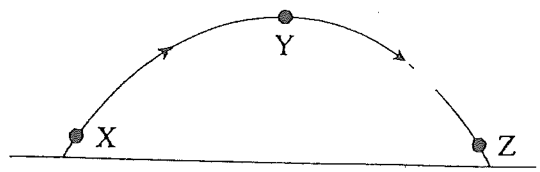
a) What is the vertical component of the ball's impact velocity?

b) How high (h) is the cliff?

19. A few minutes after takeoff a jet is heading due east with an air speed of 300 km/h. If the wind is blowing at 60 km/h, towards 40° S of E, what is the jet's ground speed?

A. 260 km/h
B. 340 km/h
C. 350 km/h
D. 360 km/h

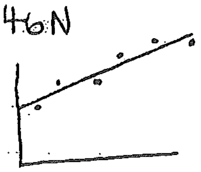
20. Consider three points in the path of a certain projectile as shown in the diagram below.



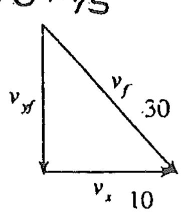
What is the acceleration of the projectile at each of these points?

	ACCELERATION (m/s^2)		
	At X	At Y	At Z
A.	+9.8	0	−9.8
B.	+9.8	0	+9.8
C.	−9.8	0	−9.8
D.	−9.8	−9.8	−9.8

Answers:

1. 6.1s
2. 42°
3. a) V_{oy} = 16m/s
   V_{ox} = 31m/s
   b) 5.3s
   c) 1.6×10^2 m
4. F_x = −108 N
   F_y = 62.5 N
5. 106 m/s
6. 46N
7. 
8. a) 17m/s
   b) 69° S of E
   OR 21° E of S
9. B
10. 31 m/s
11. a) B
    b) A
12. B
13. B
14. D
15. B
16. C
17. B
18. see below
19. C
20. D

18. a)



← 1 mark

$v_{yf}^2 = v_f^2 - v_x^2$ ← 2 marks

$v_{yf}^2 = 30^2 - 10^2$

$v_{yf} = -28.3\ m/s$ ← $\frac{1}{2}$ mark for magnitude, $\frac{1}{2}$ mark for direction

b) $v_{yf}^2 = v_{yi}^2 + 2(a_g)d_y$ ← 2 marks

$-28.3^2 = 0^2 + 2(-9.8)d_y$

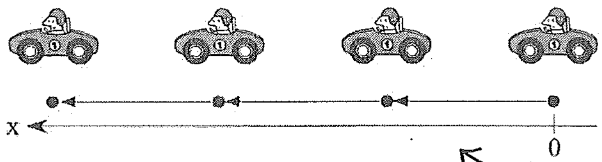
$d_y = -40.9$ m ← 1 mark

$\therefore h = 41$ m

Kinematics
Key

Physics 12 – Kinematics 1

Kinematics is the study of motion. Motion is observed, described and quantified. The simplest way to do this is through pictures.



The positive direction of motion can be chosen to the right or to the left. In this case is chosen to be to the left.

UNIFORM MOTION

The **speed** of an object is defined as the distance the object travels in a certain amount of time. For example, if you are driving your car on the highway your *speedometer* (speed meter) may say 100 km/hr. That means that you will travel 100 km if you drive for one hour. If you drive for 2 hours, you will go twice as far and therefore 2 x 100 km is 200 km.

Speed is a **scalar** and is NEVER negative. If you put your car in reverse and drive, your speedometer still will just tell you a number. The formula for average speed is:

$$\boxed{\text{speed} = \frac{\text{distance}}{\text{time}}}$$

1. A car travels 200 m in 4.0 seconds. What is its average speed? 50 m/s

2. A car travels 2000m in 2.0 minutes. What is its average speed? 17 m/s

3. A toy car travels 1.0 kilometre in 0.223 hours. What the average speed in km/hr? What is the average speed in m/s?

4.5 km/h → 1.2 m/s

$$\square \frac{\cancel{km}}{\cancel{h}} \times \frac{1\cancel{h}}{3600 s} \times \frac{1000 m}{1 \cancel{km}} = \square \frac{m}{s}$$

4. You start out on a road trip travelling at 60 km/h. You travel 100 km in 1.67 hours.
Did the speed of your car have to be exactly 60 km/hr for the entire trip? Why or why not?

Could have gone slower & then faster than 60 km/h and still have an average of 60 km/h for the trip.

In Physics, saying your <u>*speed*</u> is – 10 m/s is <u>incorrect</u>! What do you *really* mean by saying **negative** 10 m/s? (recall from Physics 11)

the object is moving in the opposite direction to original motion

The addition of the direction information turns the *scalar* number into a **VECTOR**. We have a special word for a *speed* with a *direction*. This word is called **<u>VELOCITY</u>**. The equation for velocity is very similar to the equation for speed:

$$\boxed{\text{velocity} = \frac{\text{displacement}}{\text{time}}}$$

REMEMBER - DISPLACEMENT IS A **<u>VECTOR</u>** AND IT HAS **<u>DIRECTION</u>**.

Graphing Motion:

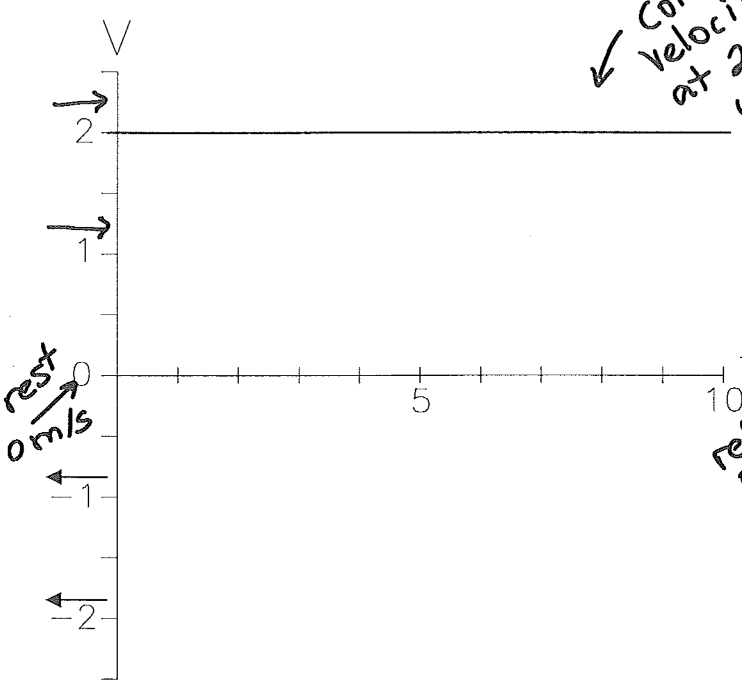
1) A toy travels <u>FORWARD</u> 20 m in 10 s, what is the car's average velocity?
2) Fill in the following graphs for the cars *velocity vs. time* and the car's *displacement vs. time*.

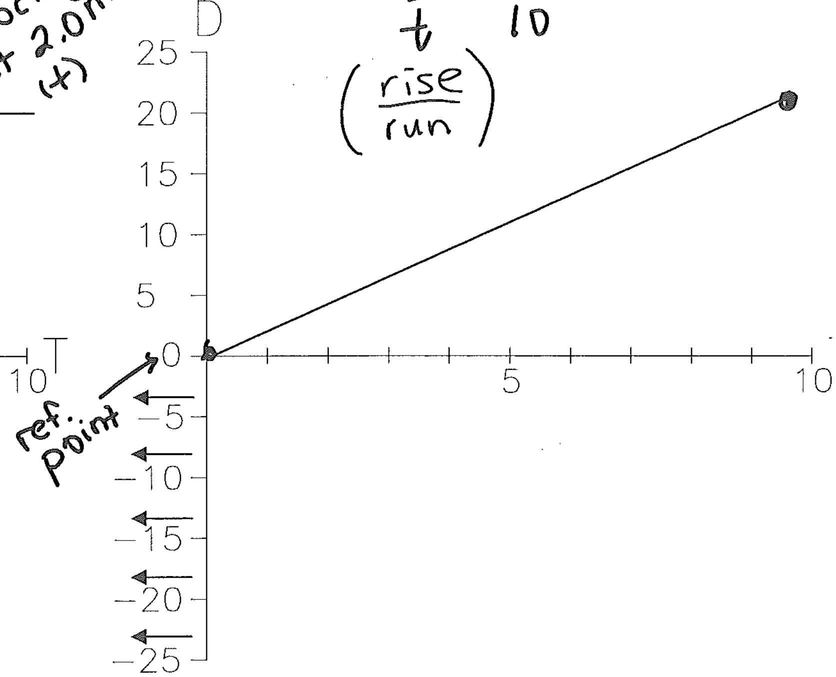
remember – on v-t graph, pos. direction is above x-axis, below neg direction

constant velocity at 2.0 m/s (+)

$\vec{v} = \frac{\vec{d}}{t} = \frac{20}{10} = +2.0\ m/s$

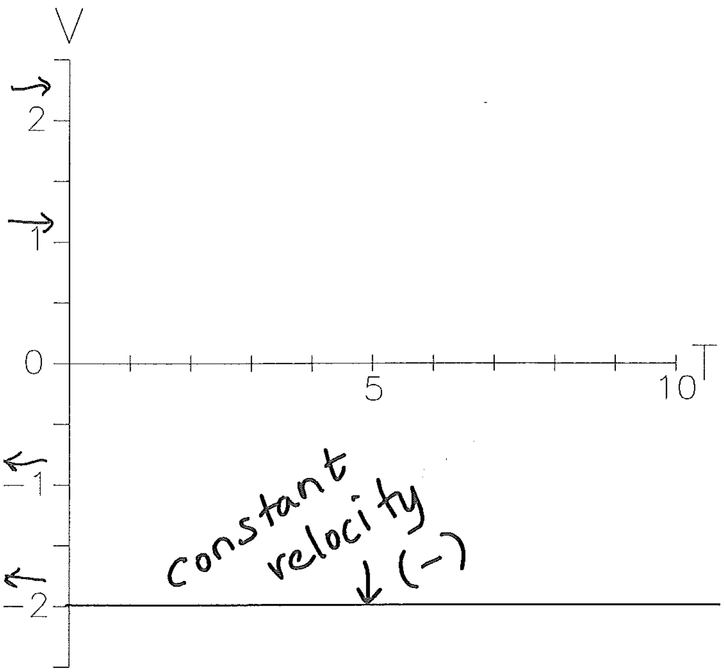
$\left(\frac{rise}{run}\right)$

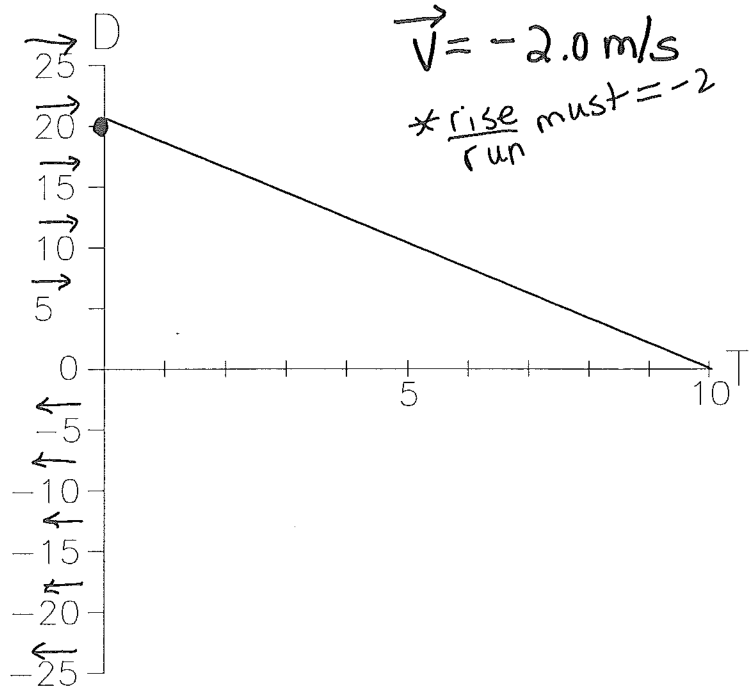




3) A toy car travels BACKWARDS 20 m in 10 s, what is the car's average velocity?

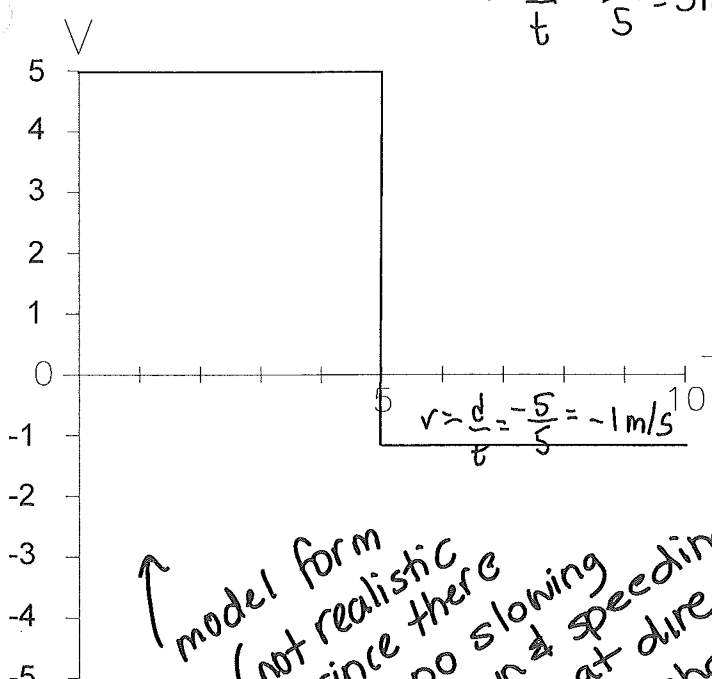
4) Fill in the following graphs for the cars *velocity vs. time* and the car's *displacement vs. time.*

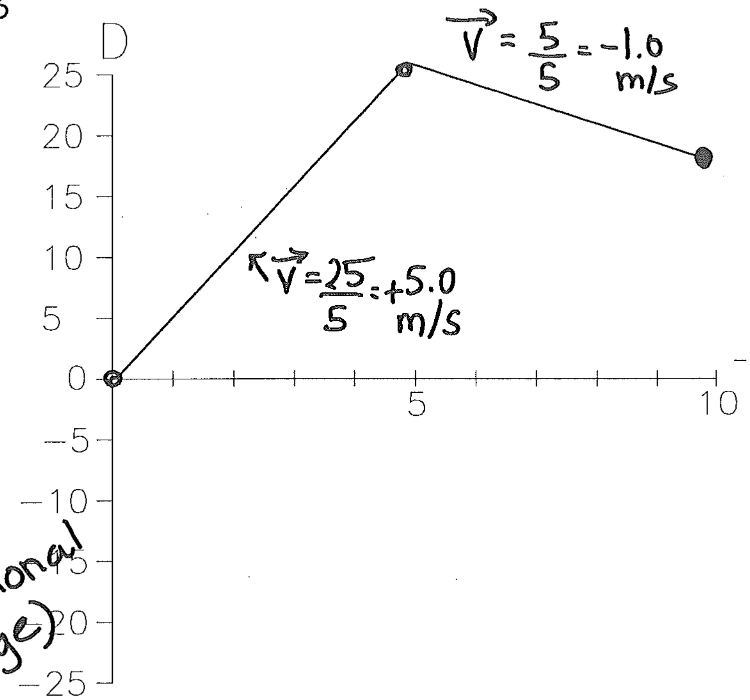




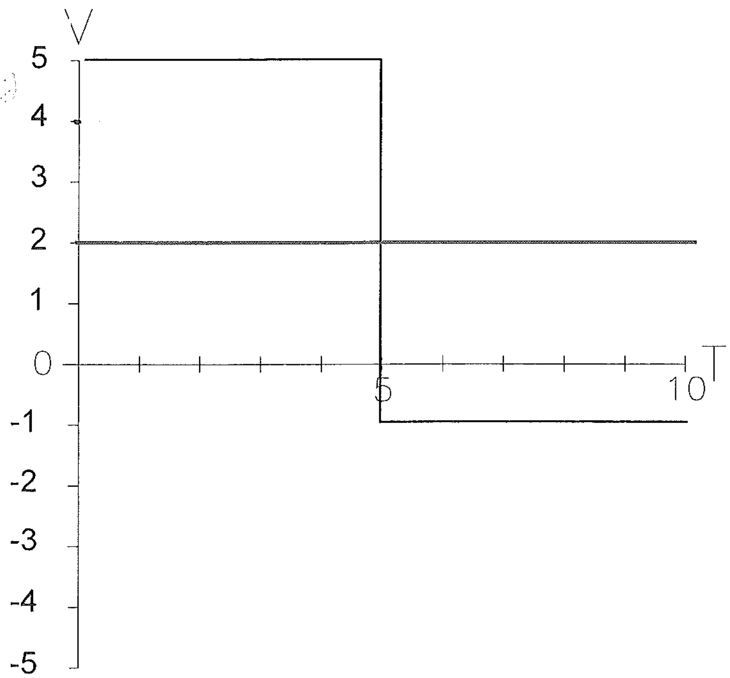
5) A toy car travels forward 25m in 5 s and backwards 5 m in 5 seconds.

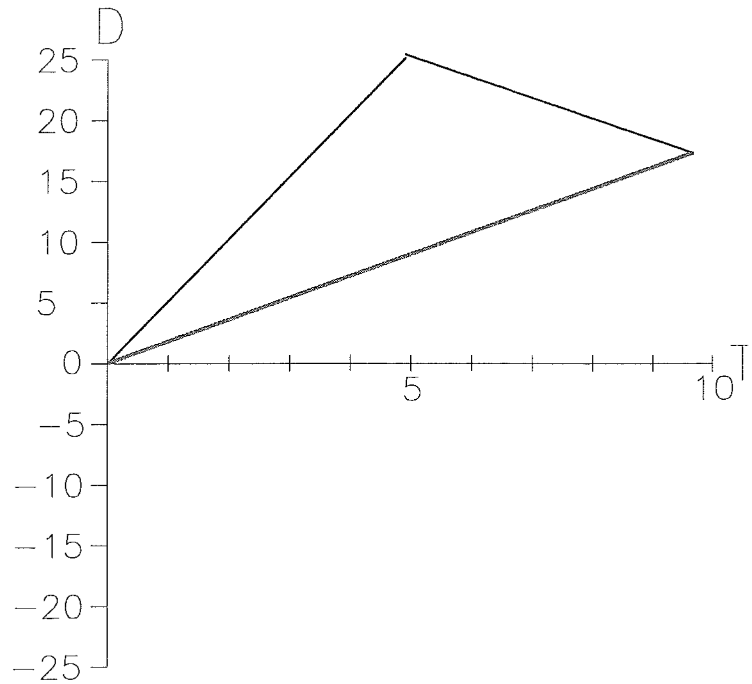
$V = \frac{d}{t} = \frac{25}{5} = 5 m/s$





↑ model form (not realistic since there is no slowing down & speeding up at directional change)





i. What is the car's average velocity for the first 5 seconds? ($T_0 \rightarrow T_5$) +5.0 m/s
ii. What is the car's average velocity for the last 5 seconds? ($T_5 \rightarrow T_{10}$) − 1.0 m/s
iii. On the graph above, fill in the *velocity vs. Time* graph and the *displacement vs. Time* graph.
iv. What is the **total** displacement? +20 m
v. What is the **total** time? 10 s
vi. What is the **average** velocity over the entire 10 seconds? $\frac{+20}{10} = +2.0 m/s$
vii. Fill in the graphs for the cars ***average velocity*** *vs. time* and the car's *displacement vs. Time* ***assuming the car travelled the <u>average velocity</u> for the entire 10 seconds.*** Use a red pen.
viii. What is the area under to v/t graph from t = 0 to t = 5 seconds? (5x5)=25.0
ix. Is the area is *positive* or *negative?* (Above or Below the 'X' axis) (+)
x. What is the displacement of the car between 0 and 5 seconds? +25.0 m
xi. What is the area under to v/t graph from t = 5 to t = 10 seconds? (5.0x1) = 5.0
xii. Is the area is *forward* or *backwards?* (Above or Below the 'X' axis) (−)
xiii. What is the displacement of the car between 5 seconds and 10 seconds? −5.0 m
xiv. Looking at the original question, how far is the car from the origin after 10 seconds (displacement)? +20 m
xv. What is the sum of the areas from part **x.** and part **xiii.**? 25.0 + (−5.0) = +20.0 m

6) What conclusions about how the area under to v/t graph relates to the displacement of the car? area = displacement

7) What is the formula for finding the area of a rectangle? $L \cdot W$

8) When you calculated the area of the rectangles from the velocity time graph, what did the height of the rectangle represent? (velocity or time) $\vec{v}$

9) When you calculated the area of the rectangles in part **viii.** and part **xiii,** what did the length of the rectangle represent? (velocity or time) t

10) What is the formula for finding the displacement of any moving object? $\vec{d} = \vec{v} \cdot t$

displacement:

graph	formula
↓ area between v-t line and x-axis	$\vec{d} = \vec{v} \cdot t$

Physics 12 – **Kinematics 2** - <u>Motion Graphs</u>

Describing the motion of an object can be assisted through the use of graphs. As you become more proficient at creating and reading motion graphs, you should find the motion easier to picture and understand.

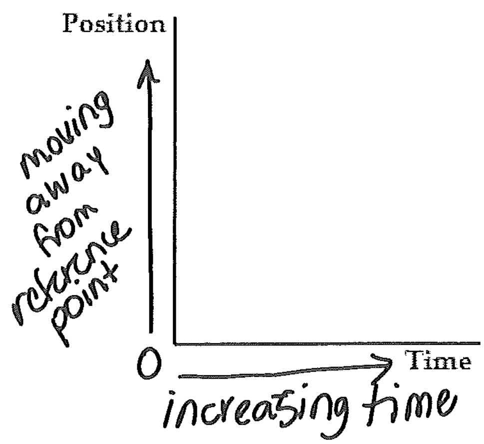
Remember:

- **Motion** is a change in position measured by distance and time.
- **Speed** tells us the rate at which an object moves.
- **Velocity** tells the speed and direction of a moving object.
- **Acceleration** tells us the rate speed or direction changes.

<u>POSITION-TIME GRAPHS:</u>

This analysis also applies to **distance-time graphs** and **displacement-time graphs**.

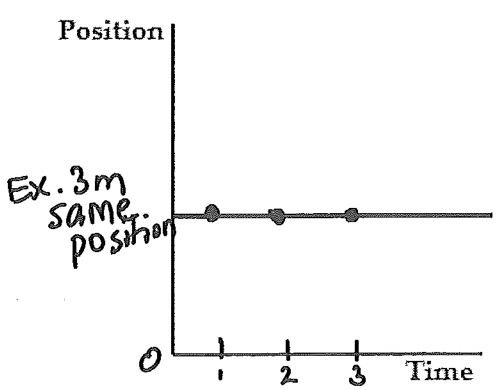
Plotting position against time can tell you a lot about motion. Let's look at what information is available on each axis.



Time is always plotted on the **X-axis** (bottom of the graph). The further to the right on the axis, the more time passes from the start.

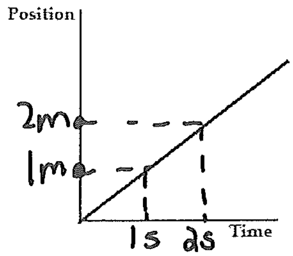
Distance is plotted on the **Y-axis** (side of the graph). The higher up the graph, the further the object has travelling from the reference point.

If an object is not moving, a horizontal line is shown on a position-time graph.



Time is increasing to the right, but its distance does not change. It is not moving. We say that it is **at rest.**

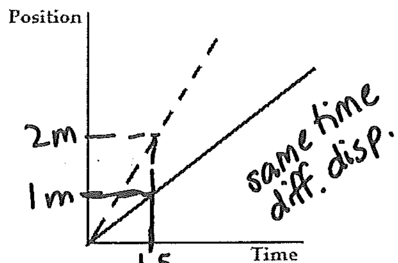
If an object is moving at a constant velocity (or speed), it means that it has the same increase in distance in a given time.



Time is increasing to the right, and distance is increasing constantly with time. The object moves at a **constant velocity.**

Constant velocity is shown by straight lines on the graph.

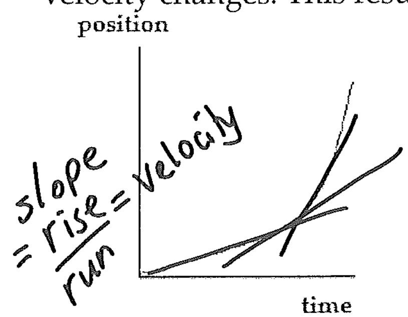
The graph below shows two moving objects. Both of the lines in the graph show that each object moved the same distance, but the steeper dashed line got there before the other one (in less time).



A steeper line indicates a larger distance moved in a given time. In other words, it has a **higher velocity.**

Bo, so both lines are **straight**, so both speeds are **constant.**

When there is a change in velocity (acceleration), the graph now indicates a changing slope as the velocity changes. This results in a curved line.



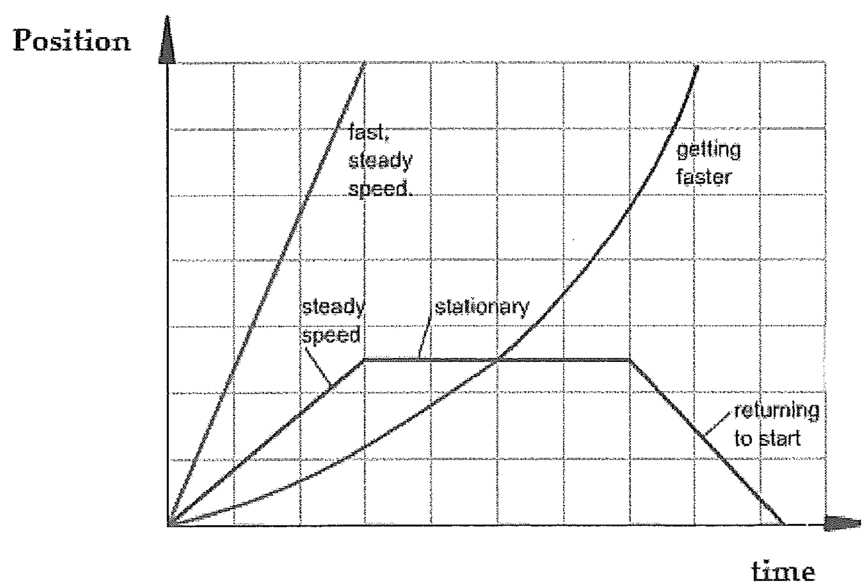
The line on this graph is curving upwards. This shows an **increase in speed** since the line is getting **steeper.**

In other words, in a given time, the distance the object moves is changing (getting larger) in each equal time interval. It is **accelerating.**

Summary:

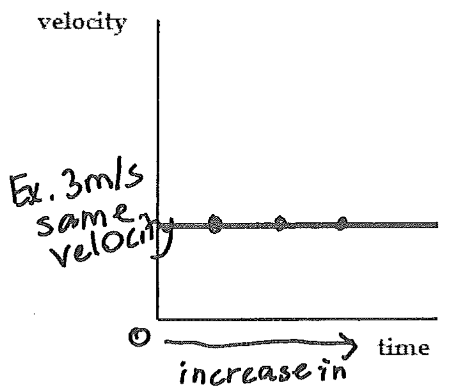
A position-time graph (distance-time, displacement-time) shows us how far an object has moved with time.

- The steeper the graph, the faster the motion.
- A horizontal line means the object is not changing its position - it is not moving, it is at rest.
- A downward sloping line means the object is returning to the start.

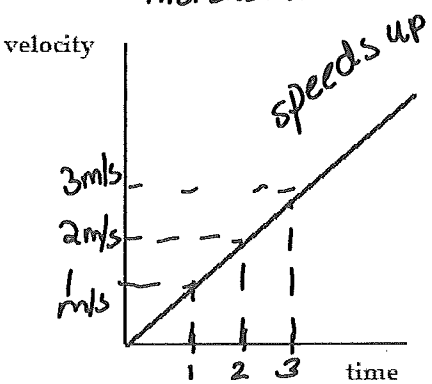


VELOCITY-TIME GRAPHS

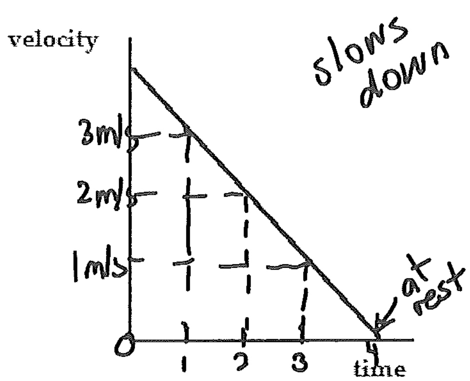
This analysis also applies to **speed-time** graphs.



Velocity-Time graphs look much like Position-Time graphs. Be sure to read the labels!!
Time is plotted on the X-axis. Velocity or speed is plotted on the Y-axis.
A straight horizontal line on a velocity-time graph means that the velocity is constant. It is not changing over time.
A straight line does not mean that the object is not moving!



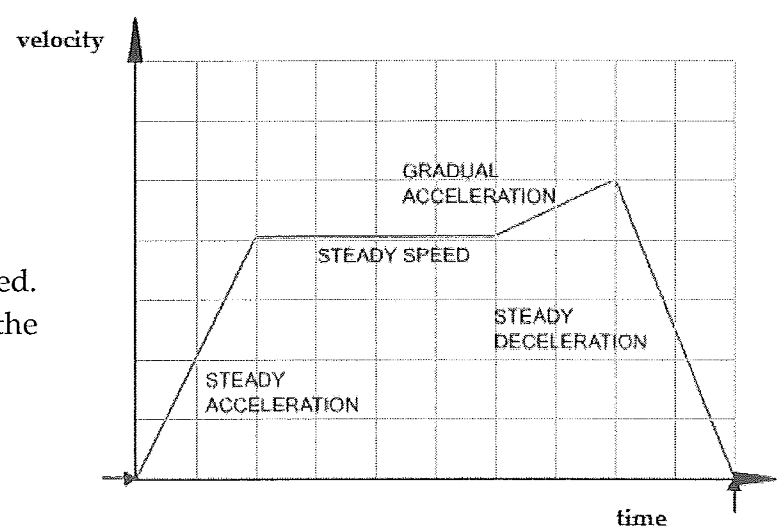
This graph shows **increasing velocity** in the **positive direction.** (speeding up)
Time is increasing to the right, and velocity is increasing constantly with time.
Constant acceleration is shown by straight lines on the graph.



This graph shows **decreasing velocity** in the **positive direction.** (slowing down)
Time is increasing to the right, and velocity is decreasing constantly with time.
Constant acceleration (deceleration in this case) is shown by straight lines on the graph.

Summary:

- The steeper the graph, the greater the acceleration.
- A horizontal line means the object is moving at a constant speed.
- A downward sloping line means the object is slowing down.



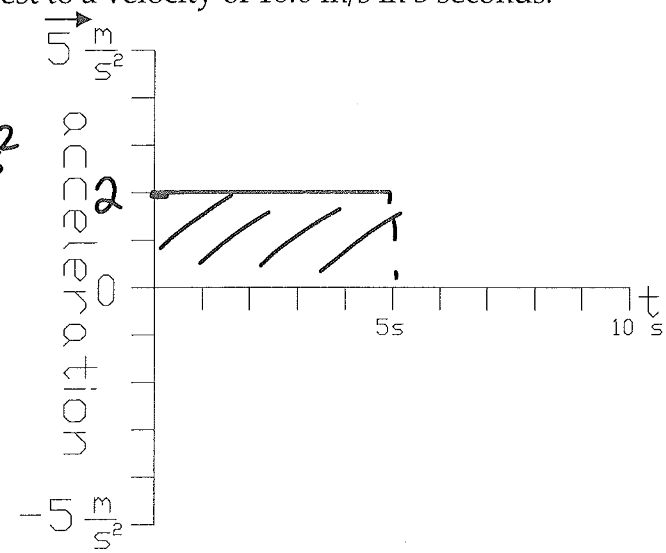
Examples:

1. A runner racing in a 100 m dash accelerates from rest to a velocity of 10.0 m/s in 5 seconds.

   i. What was his average acceleration during these 5 seconds? $a = \frac{\Delta v}{t} = \frac{10}{5} = 2.0\,m/s^2$

   ii. Fill in the acceleration graph.

   iii. Calculate the area under the acceleration/time graph up to time = 5 seconds. $L \cdot W = 2 \cdot 5 = 10\,m/s$

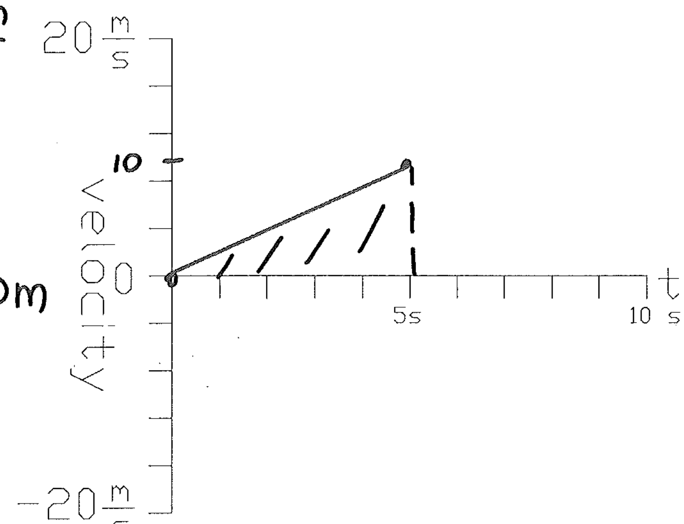


   iv. What are the 'units' of the area?

   m/s

   v. Fill in the velocity time graph.

   vi. What is the formula for the area of a triangle? $\frac{b \cdot h}{2} = \frac{5^{m/s} \cdot 10^{s}}{2} = \frac{50\,m}{2} = 25\,m$



   vii. How far did the runner travel during the first second? (0 to 1s)

   $d = \cancel{v_0 t} + \frac{1}{2}at^2 = \frac{(2.0)(1)^2}{2} = 1.0\,m$

   viii. How far did the runner travel from (0 to 2s) $d = \frac{(2.0)(2.0)^2}{2} = 4.0\,m$

   ix. How far did the runner travel from (0 to 3s)

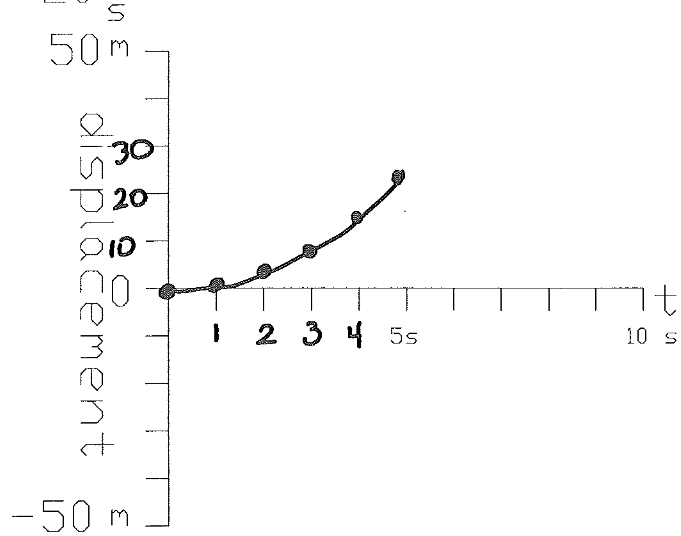
   $d = \frac{(2.0)(3.0)^2}{2} = 9.0\,m$

   x. How far did the runner travel from (0 to 4s)

   $d = \frac{(2.0)(4.0)^2}{2} = 16\,m$

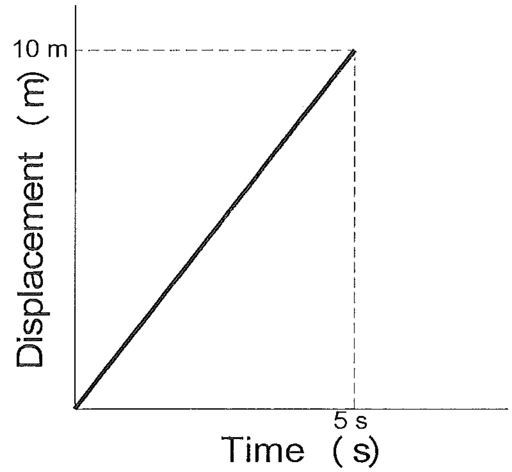
   xi. How far did the runner travel from (0 to 5s) $d = \frac{(2.0)(5.0)^2}{2} = 25\,m$

Fill in the displacement-time graph.



Creating Motion Graphs - What does each part actually mean?

When we find the slope of a line, we simply use: **rise/run**



Displacement/time graph-

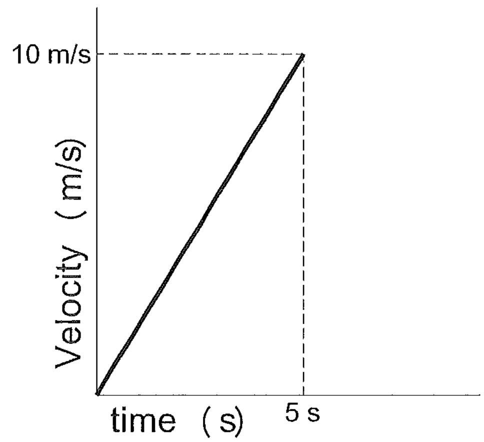
Calculate the slope INCLUDING UNITS!!!

$\frac{rise}{run} = \frac{10m}{5s} = 2\,m/s$

What does this tell you about the slope of a *displacement/time* graph?

= velocity / speed

- Is the slope of the graph *changing?* Yes – No
- Is the velocity of the object *changing?* Yes – No
- Is the object accelerating? Yes – No



Velocity/time graph-

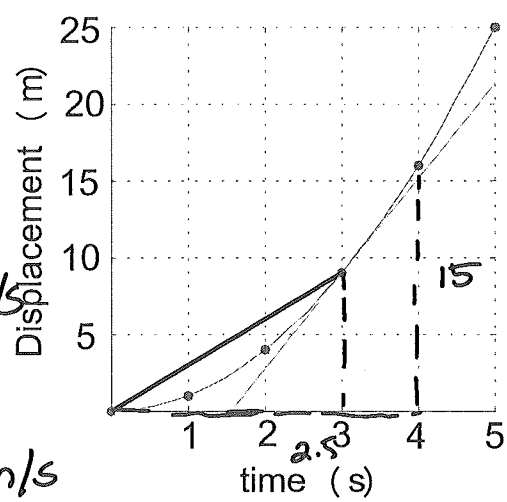
Calculate the slope INCLUDING UNITS!!!

$\frac{rise}{run} = \frac{10m/s}{5s} = 2.0\,m/s^2$

- Is the slope of the graph *changing?* Yes – No
- Is the velocity of the object *changing?* Yes – No
- Is the object accelerating? Yes – No

What does this tell us about the slope of a *velocity/time* graph? = acceleration

Now back to a **Displacement-Time Graph** -

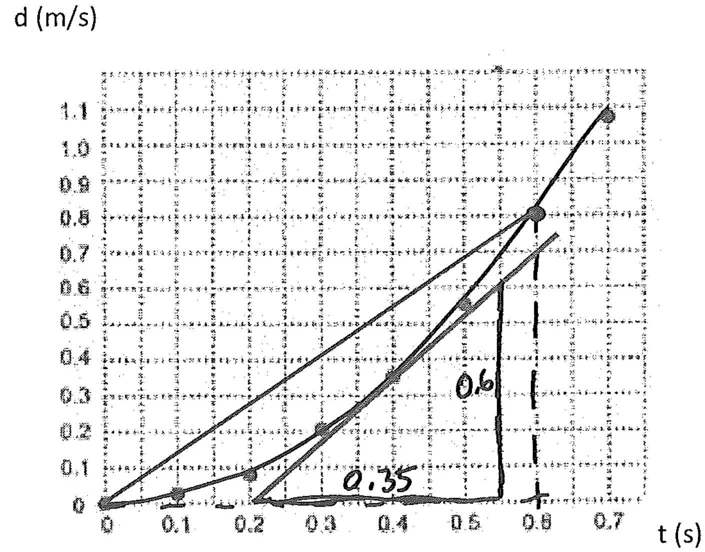


- The graph to the right is a displacement/time graph with positive acceleration
- The AVERAGE velocity for **the first 3** seconds is the slope of the black line. $\frac{rise}{run} = \frac{9}{3} = 3\,m/s$
- The INSTANTANEOUS velocity **AT** 3 seconds is the slope of the dotted line.

$\frac{rise}{run} = \frac{15}{2.5} = 7\,m/s$

When acceleration is NOT equal to zero, the **displacement graph** is a parabola (curved).

The slope of the TANGENT line of the displacement graph is the INSTANTANEOUS VELOCITY!



Draw the curved line by connecting the data points.

A. Calculate the average velocity between 0.0 and 0.6 s

$$\frac{rise}{run} = \frac{0.8}{0.6} = 1.3 m/s$$

B. Calculate the instantaneous velocity at 0.4 s

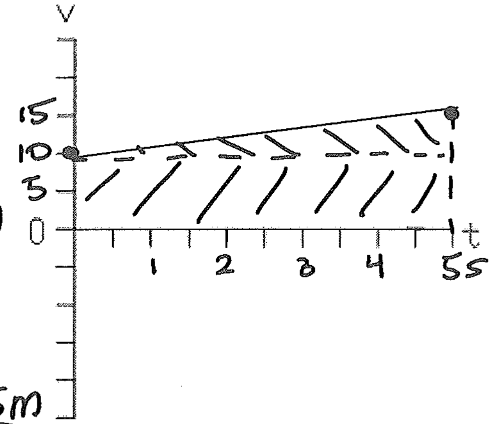
$$\frac{rise}{run} = \frac{0.6}{0.35} = 1.7 m/s$$

Assignment Problems:

1. If the initial velocity is +10 m/s and the final velocity is +15 m/s and the time interval is from 0.0 to 5.0s, find the following:

$a = \frac{V_f \quad V_i}{\Delta t}$ a= $\frac{15-10}{5.0} = 1.0\ m/s^2$

d=1/2 (v_o+v_f)t d = $\left(\frac{10+15}{2}\right)5 = 62.5m$



Sketch the velocity/time graph and calculate the area of the triangle + rectangle

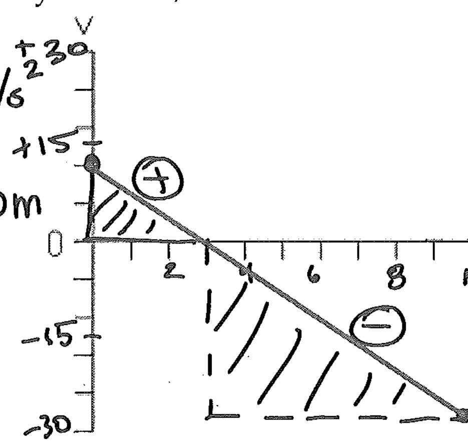
Area = $\frac{b \cdot h}{2} + l \cdot w$ Displacement = 62.5m

$\frac{5 \cdot 5}{2} + 5 \cdot 10 = 62.5$

2. If the initial velocity is +12 m/s and the final velocity is - 28 m/s and the time interval is from 0.0 to 10s, find the following:

$a = \frac{V_f - V_i}{\Delta t}$ a= $\frac{-28-12}{10} = -4.0\ m/s^2$

d=1/2 (v_o+v_f)t d = $\left(\frac{-28+12}{2}\right)10 = -80m$

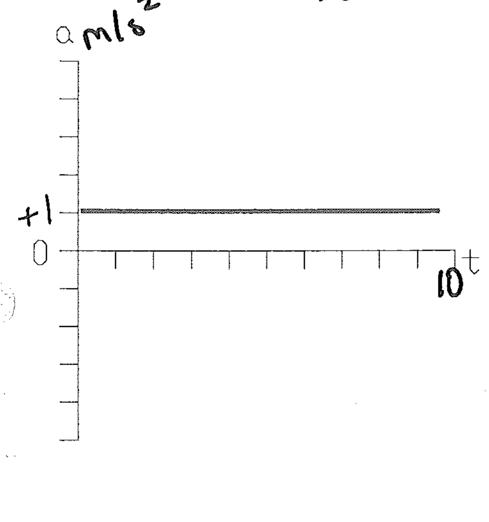


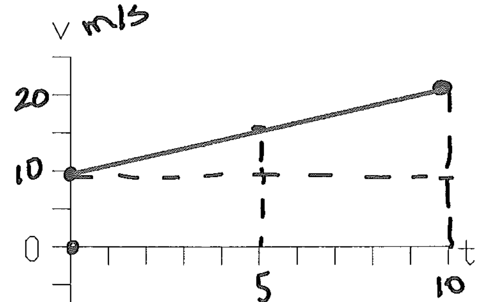
Sketch the velocity/time graph and calculate the area of the triangle + triangle

Area = $\frac{b \cdot h}{2} + \left(\frac{b \cdot h}{2}\right)$ Displacement =

$\frac{3 \cdot 12}{2} + \left(\frac{7 \cdot (-28)}{2}\right)$ $18 + (-98) = -80m$

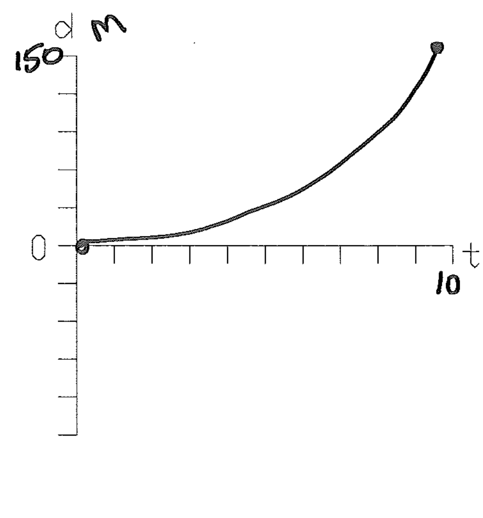
3. A car accelerates from +10 m/s to +20 m/s in 10 seconds. What is the car's acceleration? How far does the car travel during the ten seconds? Solve the problem using the graphs below. $a = \frac{20-10}{10} = +1.0\ m/s^2$





$\frac{10 \cdot 10}{2} + 10 \cdot 10$

$= 150m$



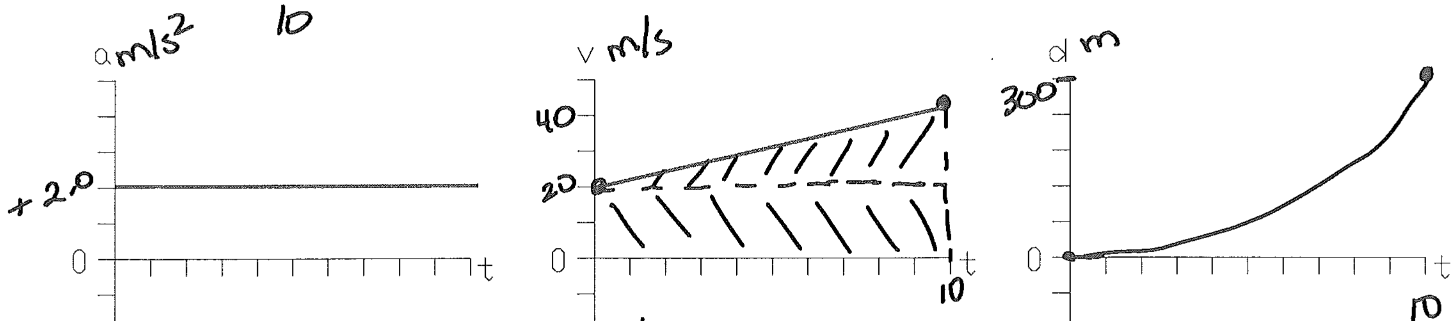
4. How far does the car from question 3 travel in the first 5 seconds? How far does it travel in the last 5 seconds?

$$d = \left(\frac{V_0 + V_2}{2}\right)t = \left(\frac{10+15}{2}\right)5 = 63\,m \text{ (first 5s)}$$

$$d = \left(\frac{V_0 + V_2}{2}\right)t = \left(\frac{15+20}{2}\right)5 = 88\,m \text{ (last 5s)}$$

5. A car accelerates from +20 m/s to +40 m/s in 10 seconds. What is the car's acceleration? How far does the car travel during the ten seconds?

$$a = \frac{40-20}{10} = 2.0\,m/s^2$$

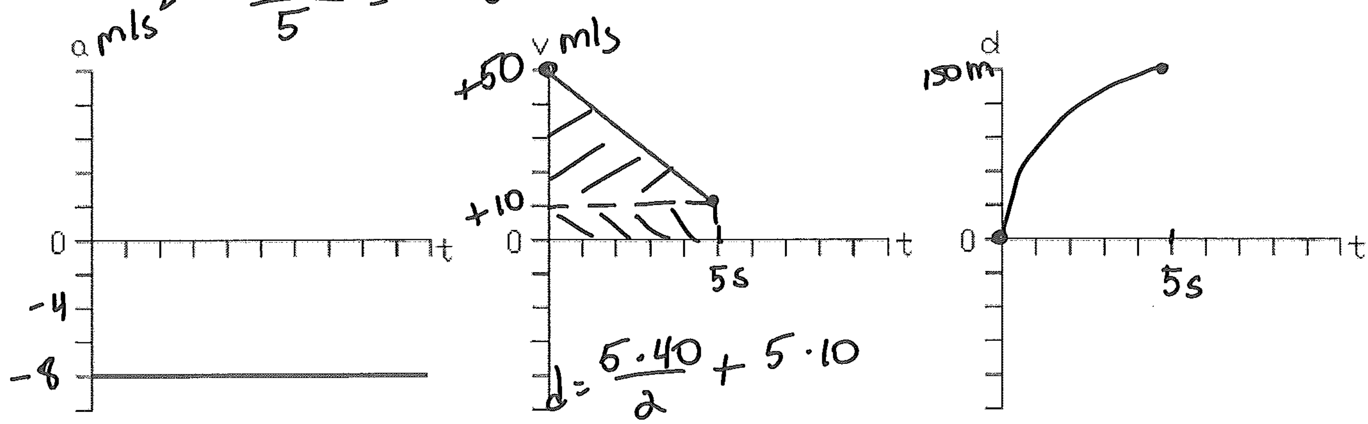


$$d = \frac{10 \cdot 20}{2} + 10 \cdot 20$$

$$= 300\,m$$

6. A car travelling +50 m/s brakes hard to avoid hitting a deer on the road, slowing down to +10 m/s in 5 seconds. What is the acceleration? What does the negative sign on acceleration mean?

$$\frac{10-50}{5} = -8.0\,m/s^2$$

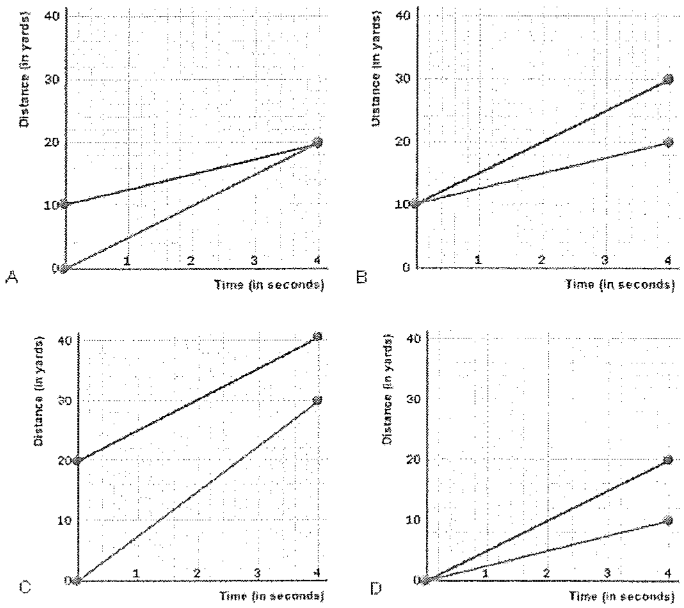


$$d = \frac{5 \cdot 40}{2} + 5 \cdot 10$$

$$= 150\,m$$

Part 2:

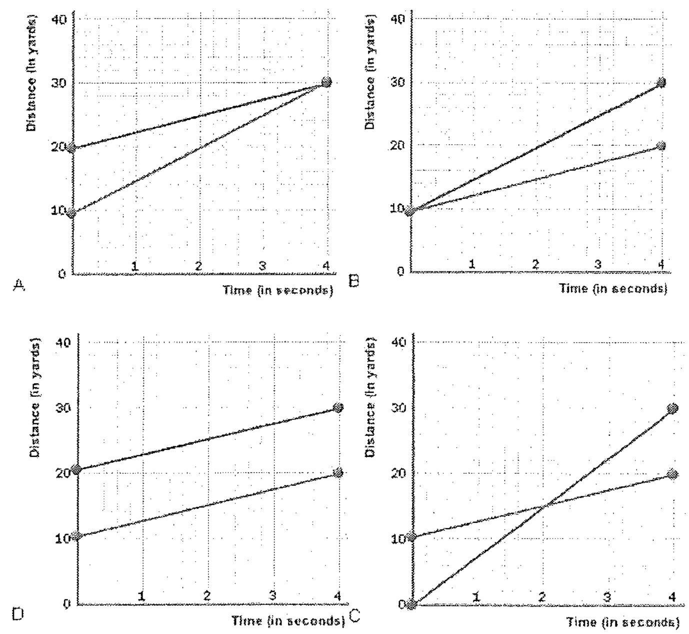
Examine the graphs below.



Which of the graphs shows that one of runners started 10 yards further ahead of the other? Explain your answer.

A → one starts at 0m (reference point)
→ one starts at 10m

In which of the following graphs below are both runners moving at the same speed? Explain your answer.

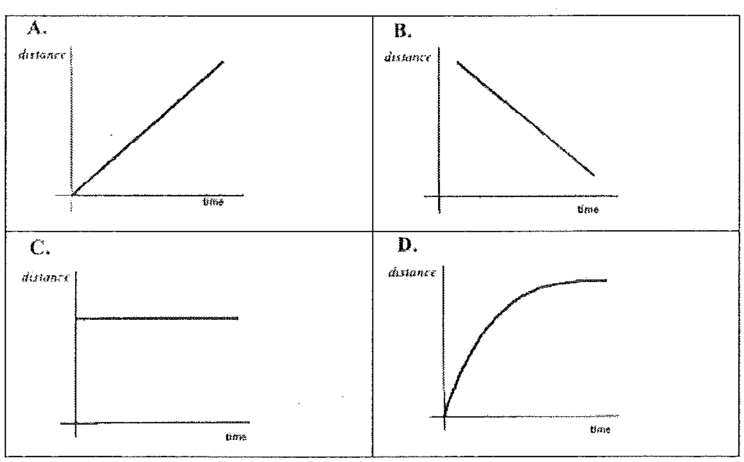


D → same slope = same speed

The distance-time graphs below represent the motion of a car. Match the descriptions with the graphs. **Explain your answers.**

Descriptions:

1. The car is stopped.
2. The car is traveling at a constant speed.
3. The speed of the car is decreasing.
4. The car is coming back.

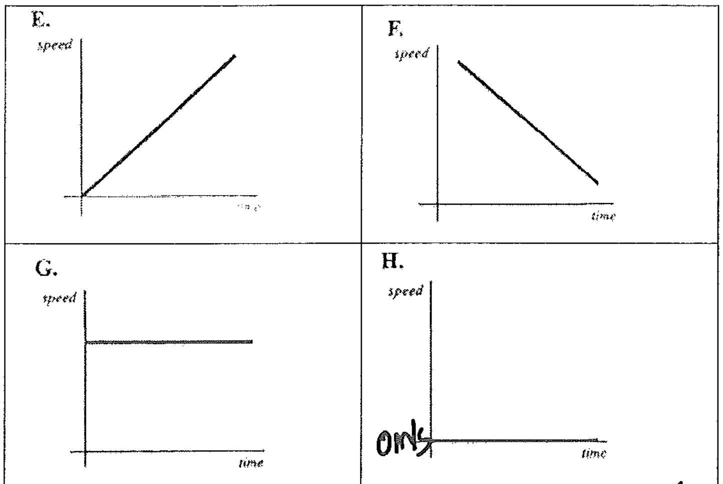


1. ~~Graph A~~ matches description C because no change in position
2. ~~Graph B~~ matches description A or B because constant slope
3. ~~Graph C~~ matches description D because change in slope is decreasing
4. ~~Graph D~~ matches description B because moving closer to 0m over time

The speed-time graphs below represent the motion of a car. Match the descriptions with the graphs. **Explain your answers.**

Descriptions:

5. The car is stopped.
6. The car is traveling at a constant speed.
7. The car is accelerating.
8. The car is slowing down.



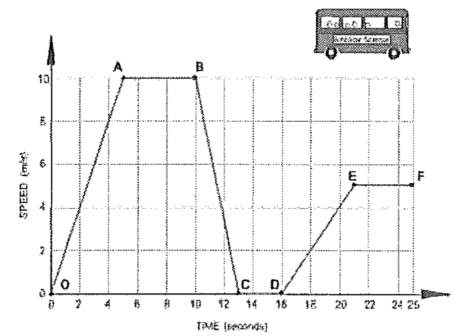
5. ~~Graph E~~ matches description H because only one at 0m/s (x-axis).

6. ~~Graph F~~ matches description G because speed remains the same.

7. ~~Graph G~~ matches description E because increase in speed.

8. ~~Graph H~~ matches description F because decrease in speed as time passes.

The graph below shows how the speed of a bus changes during part of a journey



Choose the correct words from the following list to describe the motion during each segment of the journey to fill in the blanks.

- accelerating
- decelerating
- constant speed
- at rest

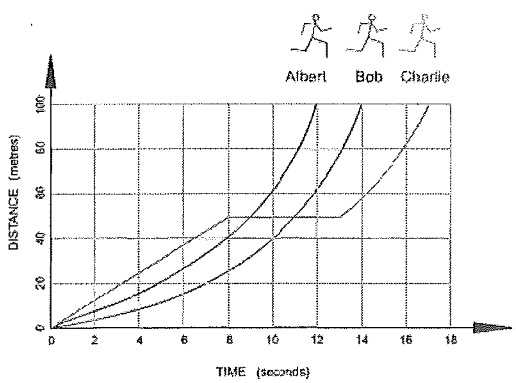
Segment 0-A The bus is accelerating. Its speed changes from 0 to 10 m/s in 5 seconds.

Segment A-B The bus is moving at a constant speed of 1 m/s for 5 seconds.

Segment B-C The bus is decelerating. It is slowing down from 10 m/s to rest in 3 seconds.

Segment C-D The bus is at rest. It h stopped.

Segment D-E The bus is accelerating. It is gradually increasing in speed.



Look at the graph above. It shows how three runners ran a 100-meter race.

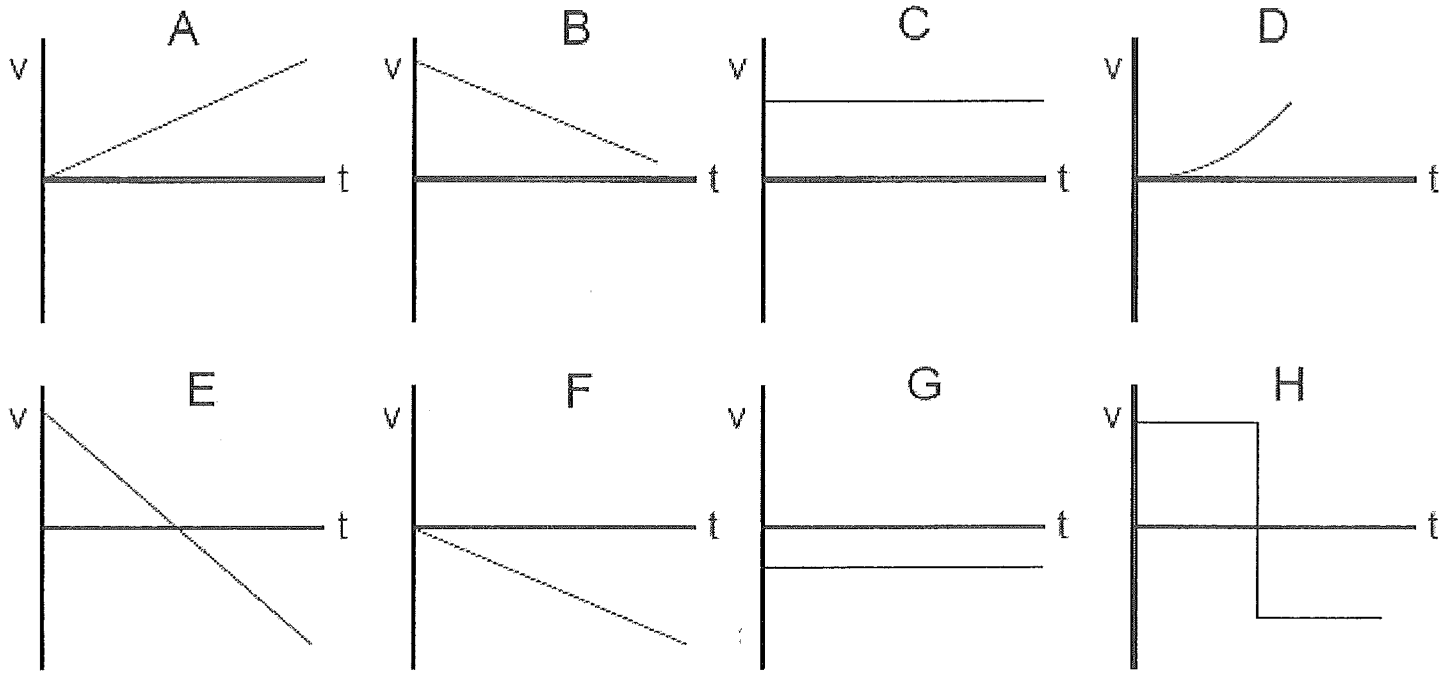
Which runner won the race? Explain your answer.

Albert → reached the finish line (100m) in less time (12s) than the other runners

Which runner stopped for a rest? Explain your answer.

Charlie → 8-13s → not moving = same position for that time period.

Part 3: Select from the following graphs to answer the following questions. Select all graphs that apply.(ie, there may be more than one correct answer!)



1. A marble rolls at a constant speed along a horizontal surface away from the origin.

   C

2. A driver accelerates away from his house with *increasing acceleration.*

   D

3. A driver rolls toward his house at constant speed. (origin is house)

   G

4. A marble is rolled from the top of an inclined plane. Assume that 'down' the ramp is '-'.

   F

5. A block is dropped from one meter above the floor and it falls to the ground. Assume 'down' is '+'

   A

6. A ball rolls along a horizontal surface at a constant speed. The ball strikes a wall and rebounds toward the origin at about the same speed as before.

   H

7. A ball is tossed up into the air and is caught at the same height it was released at.

   E

8. A car driver slams on his brakes to avoid hitting a deer. B

Accelerated Motion:

Up to this point, the velocity has been constant. However, when velocity is changing, we have **acceleration**.

Recall from Physics 11:

If the change in velocity (acceleration) is in the same direction as the velocity = speeds up

$\xrightarrow{a}$ / $\xrightarrow{v}$ speeds up forward

$\xleftarrow{a}$ / $\xleftarrow{v}$ speeds up in opposite direction

If the change in velocity (acceleration) is in the opposite direction to the velocity = slows down.

$\xleftarrow{a}$ / $\xrightarrow{v}$ slows down forward

$\xrightarrow{a}$ / $\xleftarrow{v}$ slows down in opposite direction

Acceleration has more to it than just a change in velocity. Acceleration is the "rate" of change in velocity which means we are also concerned with time.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{change in time}} = \frac{V_f - V_o}{t_f - t_o} = \frac{\Delta V}{\Delta t}$$

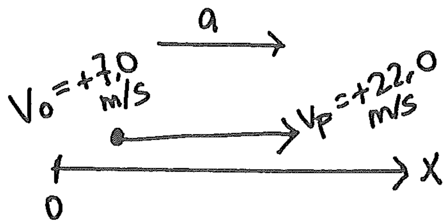
* vector quantity $\quad \frac{m/s}{s} = m/s^2$

Accelerated Motion Formulas:

$$\vec{a} = \frac{\vec{V_F} - \vec{V_o}}{t} \qquad \vec{d} = \vec{V_o}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{d} = \left(\frac{\vec{V_o} + \vec{V_F}}{2}\right)t \qquad \vec{V_F}^2 = \vec{V_o}^2 + 2\vec{a}\vec{d}$$

Example 1 - An object that is initially travelling at a velocity of 7.0 m/s east accelerates uniformly to a velocity of 22.0 m/s east in a time of 1.7 s. Calculate the acceleration of the object.



$V_0 = +7.0$
$V_F = +22.0$
$a = ?$
$d = X$
$t = 1.7s$

$$\vec{a} = \frac{\vec{V}_F - \vec{V}_0}{t}$$

$$\vec{a} = \frac{22.0 - 7.0}{1.7}$$

$$\vec{a} = +8.8 \text{ m/s}^2$$

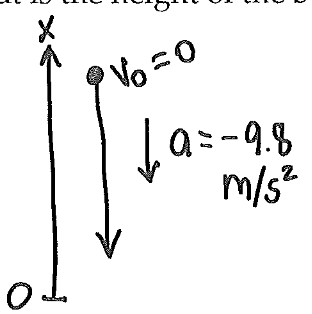
Free-Falling Objects

Recall that when air friction is minimal or non-existent (in a vacuum = no air present), acceleration is constant due to the pull of the Earth's gravity on an object close to the Earth's surface.

for Earth

g= 9.80 m/s² (g = acceleration due to gravity)

Example 2 - A cement block falls from the roof of a building. If the time of fall was 5.60s, what is the height of the building?



$V_0 = 0$
$V_F = X$
$a = -9.8$
$d = ?$
$t = 5.60$

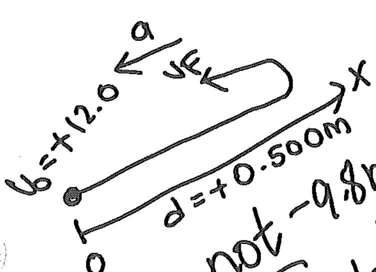
$$\vec{d} = \vec{V}_0 t + \frac{1}{2}\vec{a}t^2$$

$$\vec{d} = 0 + \frac{1}{2}(-9.8)(5.60)^2$$

$$\vec{d} = -154 \text{ m}$$

height = 154 m
(scalar!)

Example 3 - A ball is rolled up a constant slope with an initial velocity of 12.0 m/s. If the ball's displacement is 0.500 m up the slope after 3.60s, what is the velocity of the ball at this time?



not -9.8 m/s²
(not completely vertical)

$V_0 = +12.0$
$V_F = ?$
$a = X$
$d = +0.500$
$t = 3.60s$

$$\vec{d} = \left(\frac{\vec{V}_0 + \vec{V}_F}{2}\right)t$$

$$0.500 = \left(\frac{12.0}{2} + \frac{V_F}{2}\right)3.60$$

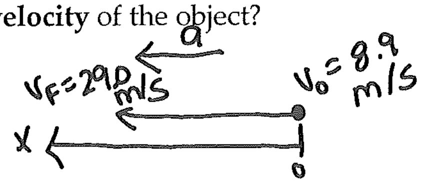
$$0.139 = 6.0 + \frac{V_F}{2}$$

$$-5.86 = \frac{V_F}{2}$$

$V_F = -11.7$ m/s

Accelerated Motion Problems:

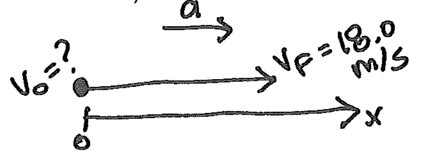
1. An object uniformly accelerates from a velocity of 8.9 m/s west to 29.0 m/s west. What is the **average velocity** of the object?



$$\vec{V}_{av} = \frac{\vec{V}_0 + \vec{V}_F}{2} = \frac{8.9 + 29.0}{2}$$

$$= 19.0 \text{ m/s [W]}$$

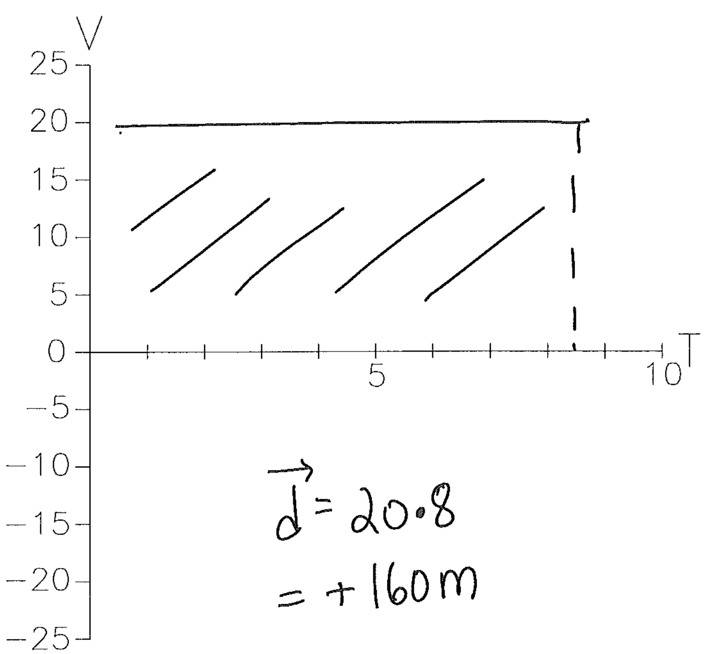
2. An object is displaced 55.0 m north while accelerating uniformly. If a velocity of 18.0 m/s north is reached in 4.5 s, what was the initial velocity?



$$\vec{d} = \left(\frac{\vec{V}_0 + \vec{V}_F}{2}\right)t \qquad 55 = \left(\frac{\vec{V}_0}{2} + \frac{18}{2}\right)4.5$$

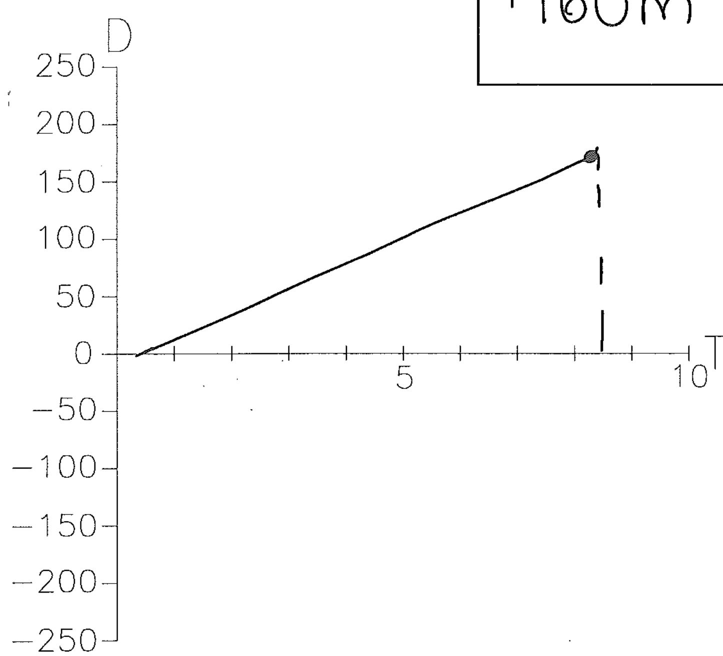
3. A car travels at a constant velocity of 20 m/s for 8 seconds. How far has the car traveled? Sketch the motion on the following graphs.

$\boxed{+160\text{m}}$

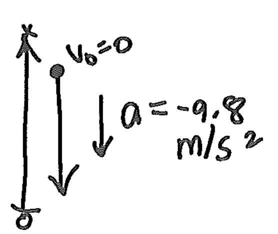


$\vec{d} = 20 \cdot 8$

$= +160\text{m}$



4. A book falls from a cabinet that is 2.45m above the floor. How long will it take the book to reach the floor?

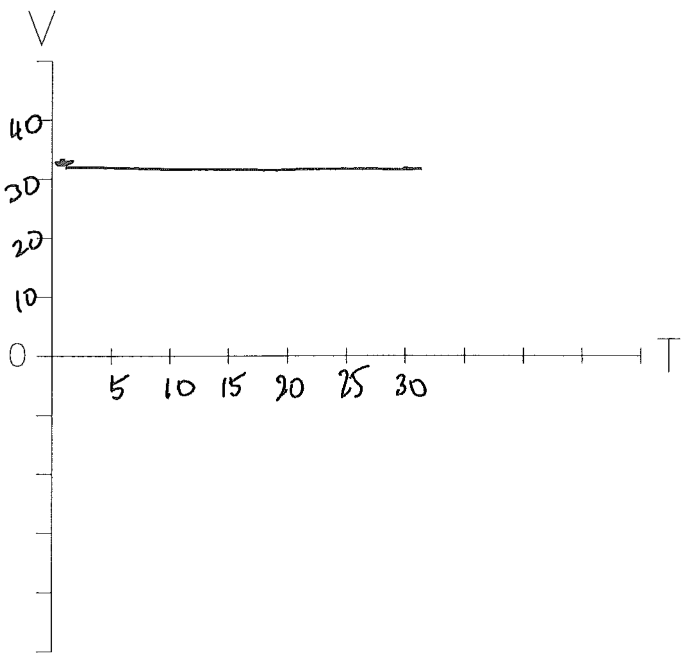


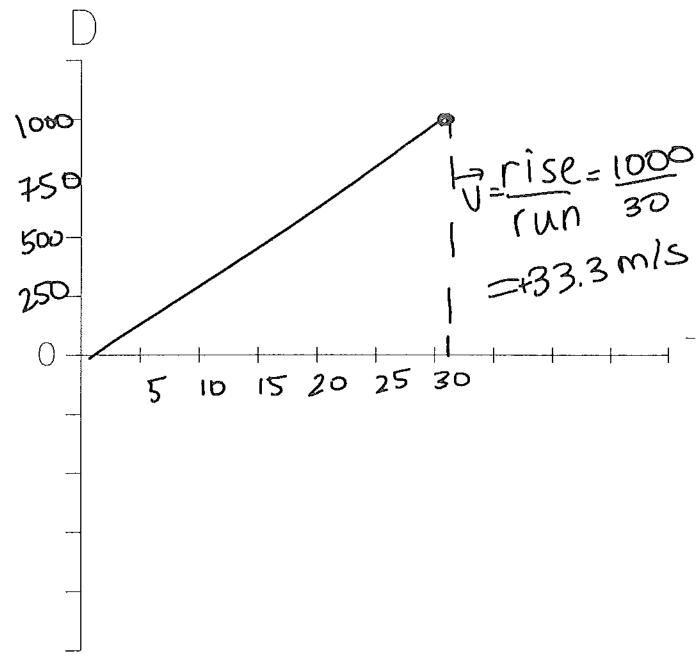
$$\vec{d} = \vec{V}_0 t + \frac{1}{2}at^2 \qquad t^2 = 0.50$$

$$-2.45 = 0 + \frac{1}{2}(-9.8)t^2 \qquad t = 0.707\text{s}$$

5. A car travels 1000 meters in 30 seconds. What is the cars velocity?
Sketch the motion on the following graphs.

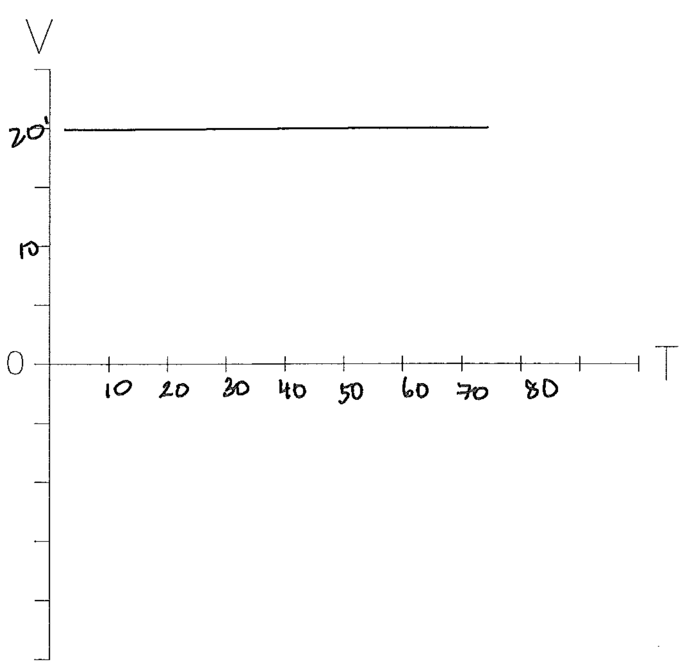
$\boxed{+33.3\ m/s}$

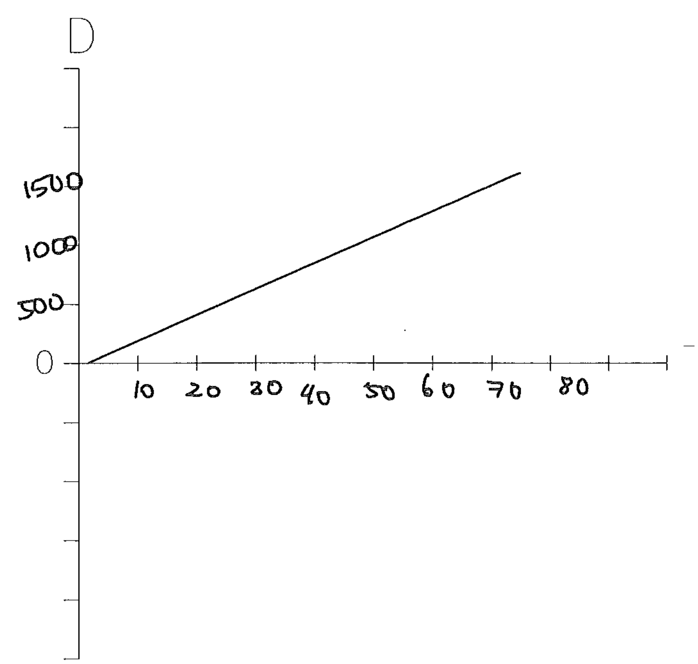




6. How long does it take for a car traveling at 20 m/s to travel 1500 m?
Sketch the motion on the following graphs.

$20 = \frac{1500}{t}$ $t = 75\,s$





7. An object accelerates uniformly from rest for 112s. What was the velocity of the object in this time if the displacement was 75.0 m west?

$v_f = ?$ $\xleftarrow{a}$ $v_0 = 0$

$x \leftarrow$ 0

$$\vec{d} = \left(\frac{\vec{v}_0 + \vec{v}_F}{2}\right)t \qquad 75.0 = \left(\frac{0}{2} + \frac{v_F}{2}\right)112$$

$$0.67 = \frac{v_F}{2} \qquad \vec{v}_F = 1.34\,m/s\ [W]$$

8. A rock was thrown downward from an overpass onto the highway below. If the rock was released when it was 12.0m above the highway and it took 1.30s for the rock to reach the road, what was the velocity of the rock when it was released?

v_0 $\quad a = -9.8\ m/s^2$ $\quad$ 0

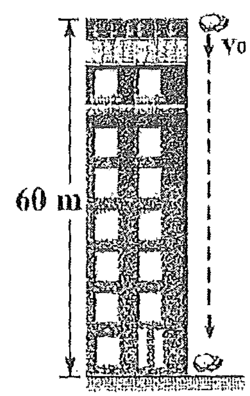
$$\vec{d} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$-12.0 = v_0(1.30) + \frac{1}{2}(-9.8)(1.30)^2$$

$$-3.71 = v_0(1.30)$$

$$v_0 = -2.86\ m/s$$

9. A stone is dropped from the top of a 60 m high building. Ignore air resistance.



a) What is the velocity and position of the stone after 3.5s?

b) How far does the stone fall during the second and third seconds?

a) $\vec{v}_F^2 = 0 + 2(-9.8)(-60)$ $\qquad \vec{d} = 0 + \frac{1}{2}(-9.8)(3.5)^2$

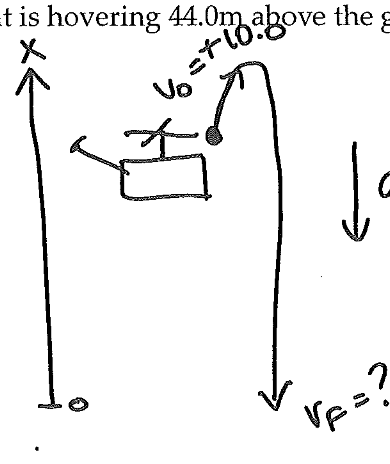
$v_F^2 = 1176 \quad \vec{v}_F = 34.3\ m/s$ [down] $\qquad \vec{d} = -60\,m$

$\vec{d} = 0\,m$

b) $\vec{d} = 0 + \frac{1}{2}(-9.8)(2)^2 = -20\,m$

$\vec{d} = 0 + \frac{1}{2}(-9.8)(3)^2 = -44\,m$ $\qquad \Delta\vec{d} = 24\,m$

10. An object is thrown vertically upward from a helicopter that is hovering 44.0m above the ground. The initial velocity of the object was 10.0 m/s.



$a = -9.8\ m/s^2$

a) Calculate the velocity with which the object hits the ground.

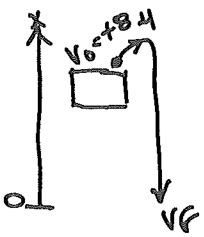
b) Calculate the time it took to reach the ground.

a) $\vec{v}_F^2 = (+10)^2 + 2(-9.8)(-44)$

$\vec{v}_F^2 = 315.6 \quad \vec{v}_F = 31.0\,m/s$ [down]

b) $-9.8 = \dfrac{-31.0 - (+10)}{t} \qquad t = 4.18\,s$

11. While riding on an amusement park ride, you drop an object. The vehicle was rising vertically at a velocity of 8.40 m/s and was 7.00 m above the ground when the object was dropped. How long does it take the object to reach the ground?



$\downarrow a = -9.8\ m/s^2$

① find v_F (because V_o is not zero & looking for t)

$\vec{V_F}^2 = 8.4^2 + 2(-9.8)(-7.00)$ $\quad \vec{V_F} = -14.4$ m/s

② $-9.8 = \frac{-14.4 - (+8.4)}{t}$ $\quad t = 2.3$ s

Answers:

1) 19.0 m/s [W] 2) 6.44 m/s [N] 3) +160 m 4) 0.707s

5) +33.3 m/s 6) 75 s 7) 1.34 m/s [W]

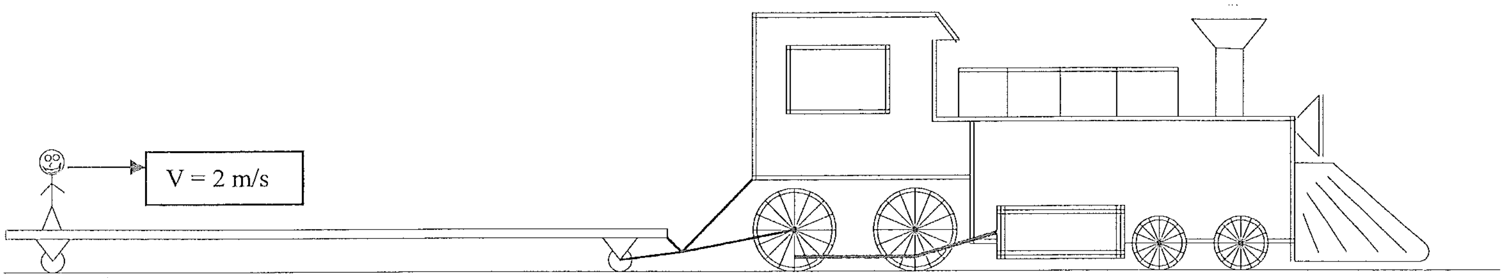
8) -2.86 m/s 9) 34.3 m/s [down], 0m, 24m

10) -31.0 m/s, 4.18 s

11) 2.3 s

Physics 12 – **Kinematics 3 - Accelerated Motion Continued**

1) A train is at rest on the track while you walk at a constant velocity of 2 m/s forward (relative to the train) for 10 seconds;



a) Complete the graphs below for the described motion.

b) How far have you walked (relative to the train) after ten seconds?

20m forward

c) Draw a vector representing velocity (relative to the train) for you at t = 10 seconds. Use a scale of 1 cm = 1 m/s

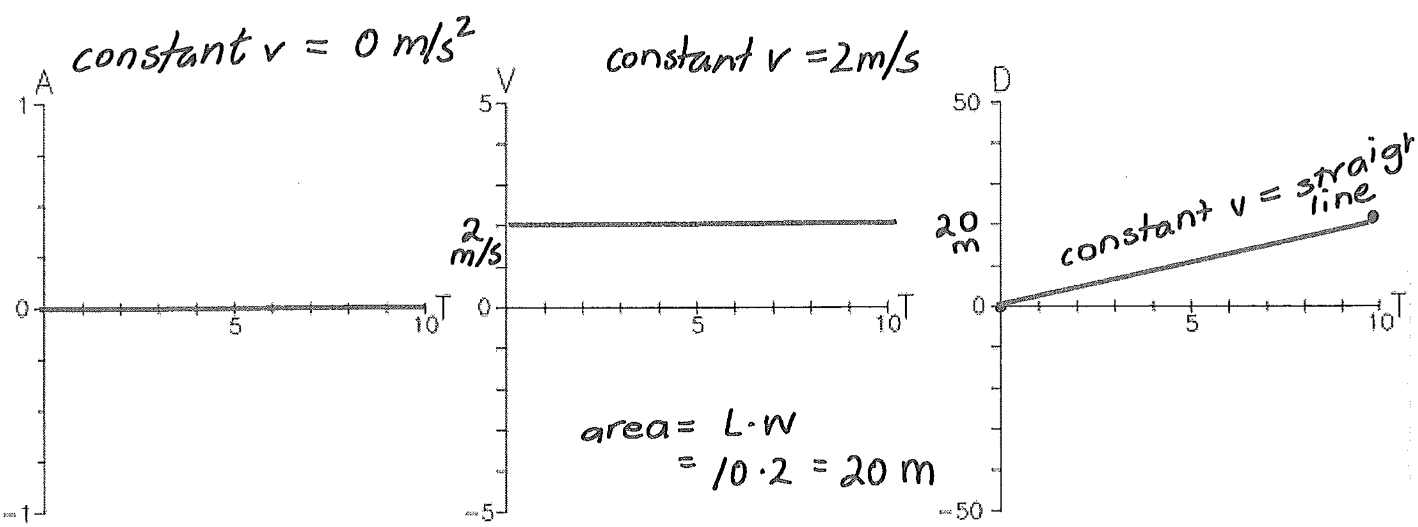
$v = 2.0\,m/s = 2\,cm$ forward

$\xrightarrow{v}$

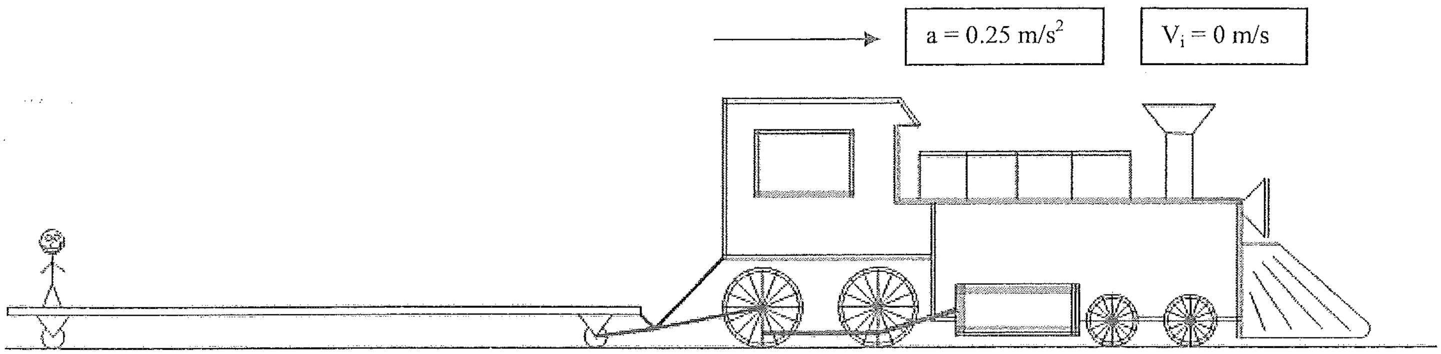
d) Draw a vector representing your displacement (relative to the train) at t = 10 seconds. Use a scale of 1 cm = 10 m

$d = 20\,m = 2\,cm$

$\xrightarrow{d}$



2) You are standing still on a train while the train is accelerating to the right at .25 m/s² from rest.

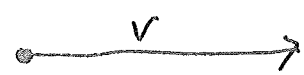


a) Complete the graphs below for your motion. (for 10 seconds)

b) What is your displacement (relative to the ground) after ten seconds?

12.5 m forward

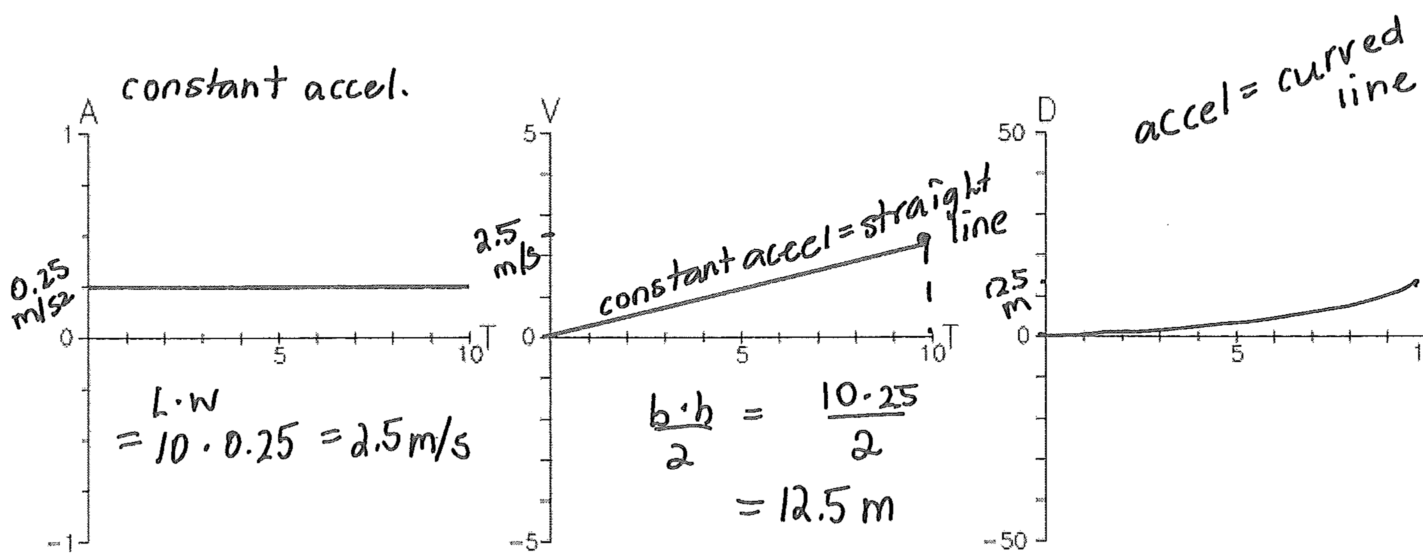
c) Draw a velocity vector representing your velocity (relative to the ground) at t = 10 seconds. Use a scale of 1 cm = 1 m/s



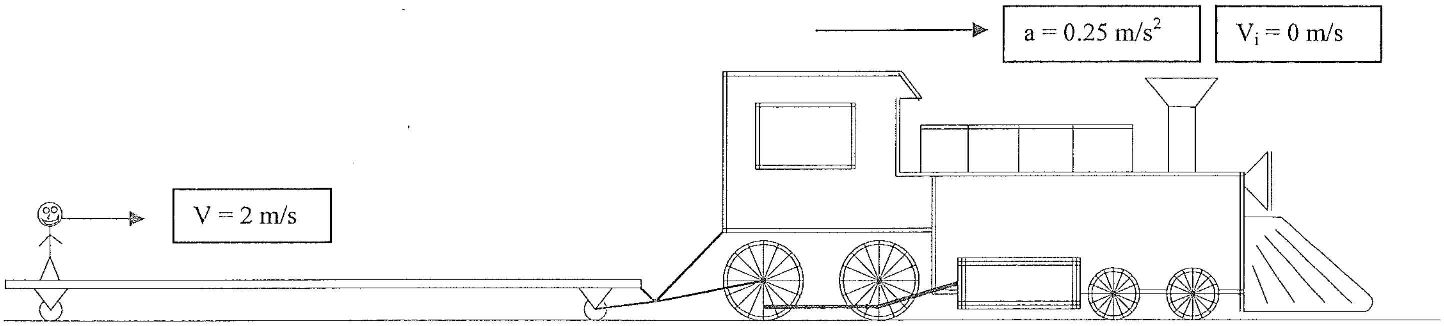
v = 2.5 m/s = 2.5 cm

d) Draw a displacement vector representing your displacement (relative to the ground) at t = 10 seconds. Use a scale of 1 cm = 10 m

d = 12.5 m = 1.25 cm



3) You are walking at a constant velocity of 2 m/s relative to the train while the train is accelerating to the right at 0.25 m/s² from rest relative to the ground.



#4

a) Complete the graphs below for your motion.

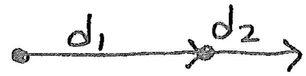
b) Draw a vector representing **your** velocity **relative to the ground** at t = 10 seconds. (hint, there are TWO components!!)

2 m/s + 2.5 m/s
(2 cm) + (2.5 cm)

= 4.5 m/s [f]

c) Draw a vector representing **your** displacement **relative to the ground** at t = 10 seconds. (hint, there are TWO components!!)

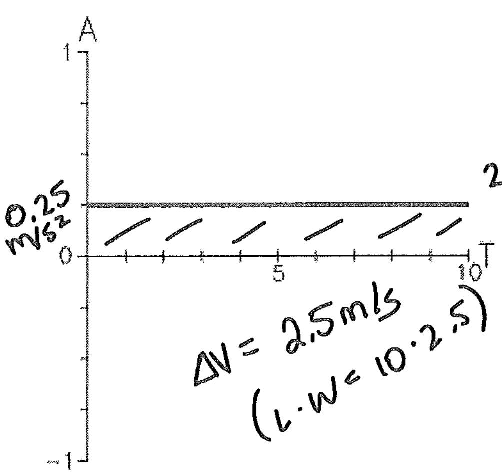
20 m + 12.5 m
(2 cm) + (1.25 cm)

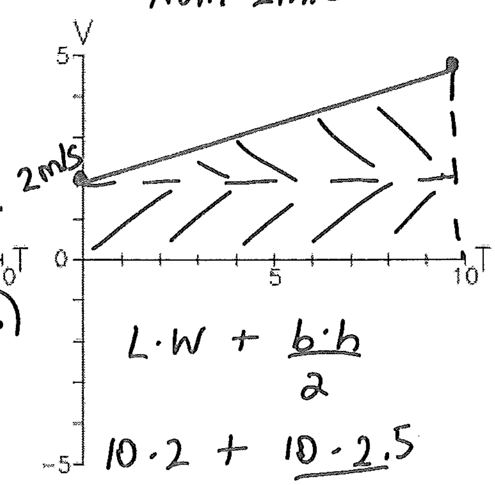


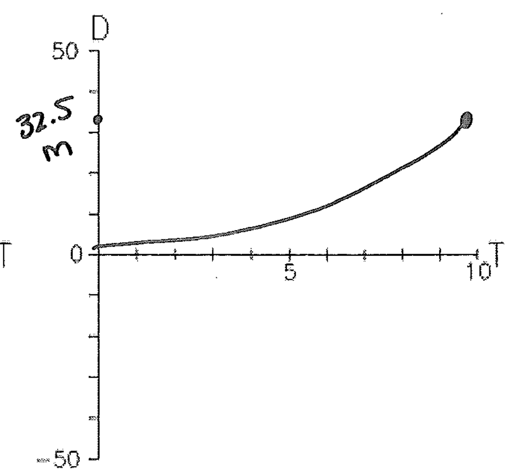
= 32.5 m [f]

$0 m/s^2 + 0.25 m/s^2$
$= 0.25 m/s^2$

start accel
from 2m/s to 4.5m/s







$\Delta V = 2.5 m/s$
$(L \cdot W = 10 \cdot 2.5)$

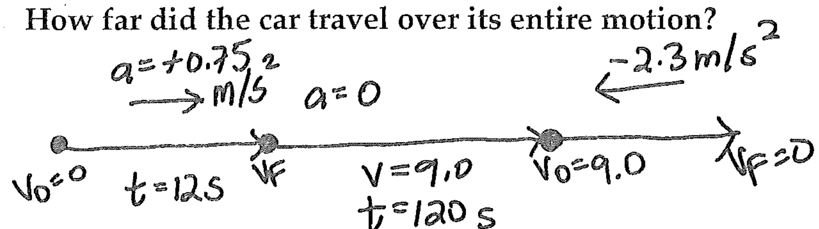
$L \cdot W + \frac{b \cdot h}{2}$

$10 \cdot 2 + \frac{10 \cdot 2.5}{2}$

$= 32.5 m$

Combining all of your knowledge of uniform motion, motion graphing and accelerated motion-

A car begins a trip by accelerating from rest with an acceleration of +0.75 m/s² over 12 seconds. Once it reaches this velocity, it cruises at a constant velocity for 2.0 minutes before stepping on the brake causing an acceleration of -2.3 m/s² until it reaches a stop. How far did the car travel over its entire motion?



$d_1 = 54$ m $d_2 = 1080$m $d_3 = 18$m

total = 54 + 1080 + 18 = 1152 m

$d_1 = v_0 t + \frac{1}{2} a t^2$

$= 0 + \frac{1}{2}(0.75)(12)^2$

$d_1 = 54$ m

$0.75 = \frac{V_F - 0}{12}$ $V_F = 9.0$ m/s

$d_2 = vt = (9.0)(120)$

$= 1080$m

$0^2 = 9.0^2 + 2(-2.3)\, d_3$

$d_3 = 17.6$ m

A dog runs forward from his doghouse at a constant velocity of 2.45 m/s for 62 s before slowing down to 0.50 m/s to investigate an odor for 30 s before turning around and running back to his doghouse. What is the total distance that the dog ran throughout his entire motion?

$V_1 = 2.45$ m/s
$t_1 = 62$ s
$d_1 = ?$

$V_2 = 0.50$ m/s
$t_2 = 30$ s
$d_2 = ?$

$2.45 = \frac{d_1}{62}$ $d_1 = 152$m

$0.50 = \frac{d_2}{30}$ $d_2 = 15$m

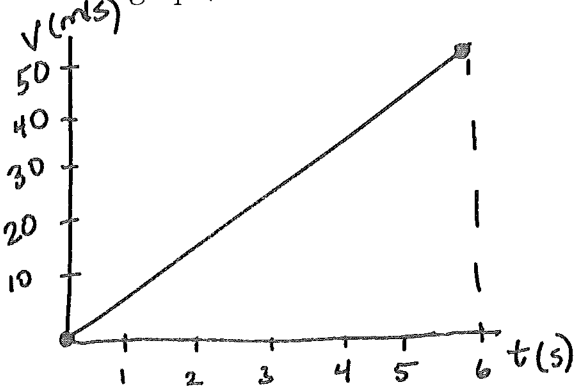
$d_{1+2} = 167$ m [f]

$d_3 = 167$ m [b]

total distance =
2(167) = 334 m

<u>Accelerated Motion Problem Assignment</u> – Complete on separate sheet of paper.

1. The McLaren F1 is a great car! Costing a million dollars, it is the end result of a lifetime fascination for racing by a very successful formula one car designer. In its road-legal configuration, the F1 can accelerate from zero to 160 Km/h in about six seconds, beating a Porsche 911 Turbo by about 4.0 seconds. If you round off the numbers, this works out to an acceleration from rest to 50 m/s in 6.0 s. How far does the car travel in these six seconds? What is the rate of acceleration? Sketch a velocity time graph, as well as the kinematics equations to solve this problem.(+8.3 m/s², +149m)



$a = \frac{rise}{run} = \frac{50}{6} = +8.3\ m/s^2$

$a = \frac{V_F - V_O}{t} = \frac{50-0}{6} = +8.3\ m/s^2$

$d = \frac{b \cdot h}{2} = \frac{6 \cdot 50}{2} = +150m$

$d = V_o t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(8.3)(6)^2 = +149m$

2. A jogger runs at a constant velocity of 4.0 m/s for a time of 10 minutes. He then slows to a trot of 2.0 m/s in the same direction for a time of 10 more minutes. He then jogs back toward his starting point, where his car is parked, at a rate of 4 m/s without stopping. How far has the man jogged, and how long does it take him to return to his car? Draw vector diagrams with your solutions. (7200 m, 15 minutes)

$d_1 = ?$ $d_2 = ?$

$t_1 = (10)(60) = 600s$ $t_2 = 600s$

$v_1 = 4.0 m/s$ $v_2 = 2.0 m/s$

$V_3 = -4 m/s$

$t_3 = ?$

$d_3 = d_1 + d_2 = -3600m$

$d_1 = v_1 t_1 = (4.0)(600) = +2400m$

$d_2 = v_2 t_2 = (2.0)(600) = +1200m$

$t_3 = \frac{d_3}{V_3} = \frac{-3600}{-4} = 900s$ or 15 minutes.

total = 7200m

3. A subway car accelerates uniformly from rest at a rate of 3.0 m/s² for a time of 10 seconds, and then travels at a constant speed for 30 seconds. It then slows down at a rate for -2.0 m/s² until it is stopped. Determine the distance traveled by the train for each of the three sections of its motion. Draw a vector diagram. (+150m, +900m, +225m)

$a_1 = +3.0 m/s^2$ $a_2 = 0$ $a_3 = -2.0 m/s^2$

$t_1 = 10s$ $t_2 = 30s$ $V_o = +30 m/s$

$V_o = 0$ $d_1 = ?$ $V_F = 30 m/s$ $d_2 = ?$ $d_3 = ?$ $V_F = 0$

$V_2 = 30 m/s$

$d_1 = +150m$ $d_2 = +900m$ $d_3 = +225m$

total = 1275m

$d_1 = 0 + \frac{1}{2}(3.0)(10)^2$

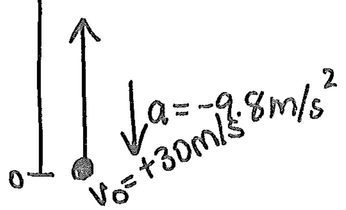
$d_1 = +150m$

$V_F^2 = 0^2 + 2(3.0)(150)$ $V_F = 30\ m/s$

$d_2 = V_2 t_2 = (30)(30) = +900m$

$0^2 = 30^2 + 2(-2.0)d = +225m$

4. A ball is thrown up in the air at a speed of 30 m/s. How high does the ball go? How high is the ball after two seconds? How high is the ball after 4.0 seconds? (46 m, 40m,42m)



$0^2 = 30^2 + 2(-9.8)d \qquad d = \underline{46m}$

$t = 2.0s \rightarrow d = v_0 t + \frac{1}{2}at^2$
$= (30)(2) + \frac{1}{2}(-9.8)(2)^2$
$= \underline{40\ m}$

$t = 3.0s \rightarrow d = (30)(4) + \frac{1}{2}(-9.8)(4)^2$
$= 120 + (-78.4)$
$= \underline{42m}$

5. Based on the information from questions three and four, what do you think it means to have a positive velocity and a positive acceleration? How about a negative velocity and a negative acceleration? Finally, how about a positive velocity and a negative acceleration? (*see posted solutions*)

$\xrightarrow{+v}$ $\xrightarrow{+a}$ increasing speed in positive direction

$\xleftarrow{-v}$ $\xleftarrow{-a}$ increasing speed in negative direction

$\xrightarrow{+v}$ $\xleftarrow{-a}$ decreasing speed in positive direction

6. A car accelerates uniformly from rest to a speed of 30 m/s in a time of 10 seconds. It then stops in a time of one half of a second. Find its acceleration, and the distance traveled by the car during its speeding up and slowing down periods. What do you suppose happened to create such a deceleration? (+3.0 m/s², +150m, -60 m/s², +7.5m)

$a_1 = \frac{30-0}{10} = \underline{+3.0 m/s^2} \qquad d_1 = 0 + \frac{1}{2}(3.0)(10)^2 = \underline{150m}$

$a_2 = \frac{0-30}{0.5} = \underline{-60 m/s^2} \qquad d_2 = (30)(0.5) + \frac{1}{2}(-60)(0.5)^2$
$= \underline{+7.5m}$

→ collided with something with enough mass to change its velocity quickly

7. A man riding upward in a hot air balloon at a constant rate of 10 m/s drops a sandbag out of his balloon to lighten his craft. If the sandbag falls freely for 10 seconds, what will be its velocity at this time? After ten seconds, how far below the point of release will the bag be? After ten seconds, will this be the same as the distance that the bag is below the balloon? (-88 m/s, 390 m below, no, the balloon is still rising up at +10 m/s)

$a = -9.8\ m/s^2$

$V_0 = +10\ m/s$

$-9.8 = \frac{V_F - 10}{10}$ $V_F = -88\ m/s$

$d = \left(\frac{+10 + (-88)}{2}\right)10 = -390\ m$

$= 390\ m$ below

$v = \frac{d}{t}$ $+10 = \frac{d}{10}$ $= +100\ m$ (no because balloon is still rising at $+10\ m/s$)

8. The driver of a Porsche 944 is tooling down a one lane country road at 27 m/s when he crests a hill land sees a cement truck parked in the road 40 m ahead of him. If the maximum deceleration which can be supplied by his brakes and tires is 8.5 m/s², will he avoid a crash or not? (+43 m, no he will need 43 m to stop completely and the cement truck is 40 m away)

$V_F^2 = V_0^2 + 2ad$

$0 = 27^2 + 2(-8.5)d$

$d = +43\ m$

no, he will need 43 m to completely stop.

9. A sling-shot can speed up a 30 gram ball bearing from zero to 100 m/s in 0.30 seconds. What is the acceleration of the metal ball? (+333 m/s²)

$\vec{a} = \frac{100 - 0}{0.30}$ $a = +333\ m/s^2$

10. If a ball is launched upward at 20 m/s, after 1.5 seconds, what is its velocity? How high above the point of release will the ball be? (+5.3 m/s, 19 m high)

x

0

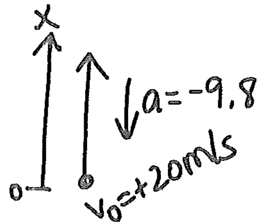
$a = -9.8\ m/s^2$

$V_0 = +20\ m/s$

$-9.8 = \frac{V_F - 20}{1.5}$ $V_F = +5.3\ m/s$

$d = \left(\frac{20 + 5.3}{2}\right)1.5$ $d = 19$ m high

11. If a ball is thrown upward at 20 m/s, after 2.0 seconds what will its velocity be? What will its instantaneous acceleration be? Is this the same as its constant acceleration? (0.40 m/s, -9.8 m/s², yes it is the same as the only acceleration acting on the object is acceleration due to gravity.)



$-9.8 = \frac{v_F - 20}{2.0}$ $\quad v_F = 0.40\ m/s$

inst $\vec{a}$ = $-9.8 m/s$ yes it is the same (gravity)

12. A toboggan full of little kids accelerates from rest down a hill with a constant acceleration of 2 m/s². How long will they have to keep this up before the exceed 100 km/h? (14 s)

100 km/h → 27.8 m/s

$2 = \frac{27.8 - 0}{t}$ $\quad t = 14s$

13. A box slides down a ramp and accelerates from 2.0 m/s to 4.0 m/s in a period of ten seconds. How far has the box gone in this time? (30 m)

$d = \left(\frac{2.0 + 4.0}{2}\right) 10 = \underline{30\,m}$

14. Jimmy backs his car out of its parking space and smacks into a shopping cart which has been left in the parking lot, sending it at 6.0 m/s toward another row of cars 15 meters away. If the cart loses 2 m/s from its velocity every second that passes, how far will the cart go before it stops? Will it hit the other row of cars? (9.0 m, no)

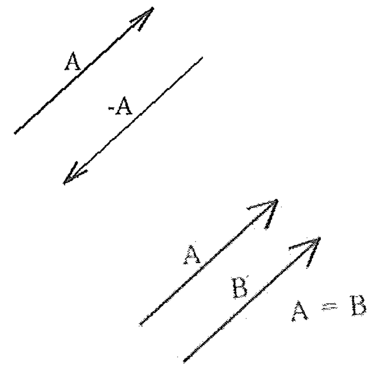
$a = -2 m/s^2$ $\quad 0^2 = 6.0^2 + 2(-2)d$

$d = 9.0\,m$ before stopping; will not hit the other cars.

15. A hockey player is checked into the boards, and in 0.50 seconds, changes his speed from 10 m/s to -5 m/s. What acceleration does he experience? If the acceleration of gravity (called 'g') is 9.8 m/s, how many g's does the player experience from the hit? (-30 m/s², 3.1 'g's)

$a = \frac{-5 - 10}{0.5}$ $\quad a = \underline{-30\,m/s^2}$ $\quad 30 \div 9.8 = \underline{3.1\text{ 'g's}}$

Physics 12 – Vectors



Scalars

Scalar quantities require only **magnitude** to specify them.
Examples:

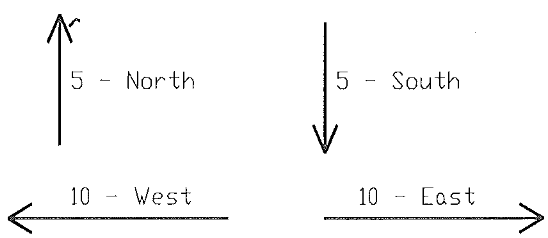
- distance (NOT displacement)
- mass (NOT weight),
- speed (NOT velocity),
- volume, area, density, time and temperature.

Vectors

Vector quantities require both **magnitude** and **direction** to specify them.
Examples: displacement, weight, velocity, acceleration, force and momentum.

Representing Vectors Graphically

Vectors can be represented graphically by drawing an arrow. The arrow you draw will have a length, and a direction. The **length** of the arrow corresponds to the **magnitude** of the vector, and the **direction** that the arrow is pointing corresponds to the **direction** of the vector.



Vector Equality

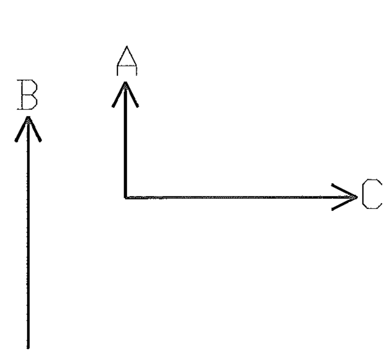
For two vectors to be equal, they must have the same direction and the same magnitude.

B ≠ A (Same direction but different magnitudes)

A ≠ E (Same magnitude but different directions)

D = A (Same magnitude AND same direction)

A and D are the only vectors that are equal



Adding Vectors

When you add the vectors together, the result is also a vector. We call this the *resultant.* = R

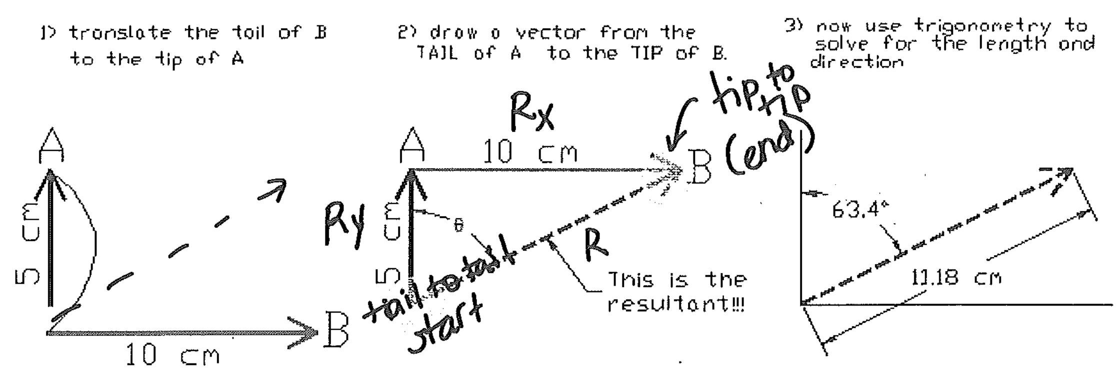
The rule we use to add vectors is called the 'tip to tail' rule. If you want to add two vectors, ***translate*** (move) the tail of one vector to the tip of another vector.
The ***resultant*** is drawn from the tail of the first vector (WHERE YOU STARTED) to the tip of the second vector (WHERE YOU ENDED). In text books resultants are usually shown with dashed lines.
We are going to use the analytical method in which we will draw a reasonable representation of the vector problem (as opposed to the graphical method where a diagram is drawn to scale using a ruler and protractor).

If the vectors are perpendicular to each other, you can use the Pythagorean Theorem to determine the magnitude of the resultant.

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

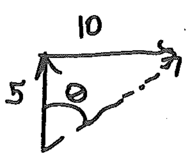
In the following example we will show how to add two vectors. The two vectors are: ***Vector A = 5 up,*** added to ***Vector B = 10 right***. We can then see how the addition of the two vectors is NOT 15.

To add vector A to vector B:



$R = \sqrt{5^2 + 10^2} = 11.2\text{cm}$

* need exact location = direction



$\tan\theta = \frac{10}{5} = 63^\circ \quad (90^\circ - 63^\circ) = 27^\circ$

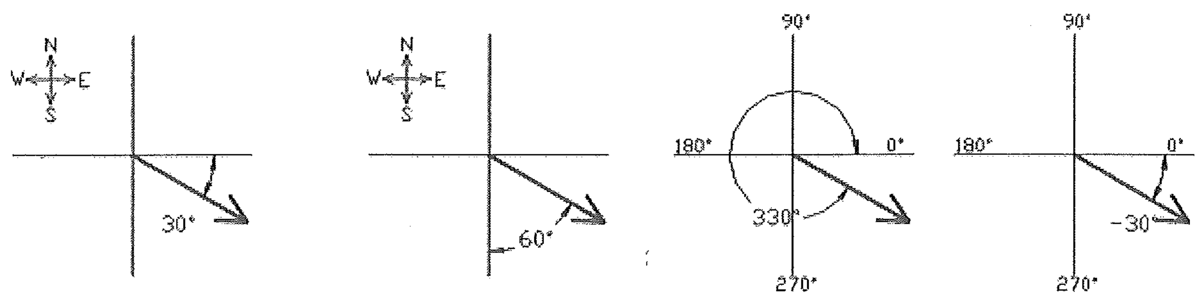
Resultant = 11.2cm 27° from horizontal
or 11.2 cm 63° from vertical

The reason that we "translate" the vectors is so that the resultant reflects the individual vectors. Vector **A** was pointing **up.** Vector **B** was pointing **right**. Therefore the resultant should be pointing **up** and **right.**

Direction Specification

We also need a method to describe the *direction* that vectors point in. There is more than one way to specify the direction of a vector. Depending on the situation we may specify the same direction in different ways, but **all are correct.**

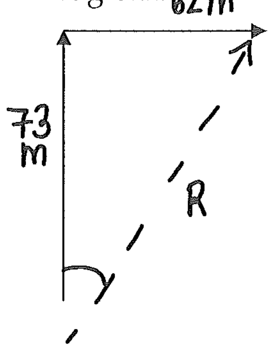
In the first diagram on the left the vector is **not** pointing straight East, but is it pointing at an angle *30° towards South* of **East.** Looking at the second vector, the direction specification is now is **not** pointing straight South, but is it pointing at an angle *60° towards East* of **South.** The third diagram is showing the vector at **+330°** which can also be described as **–30°**. Note how there are four directions that sound different but when you sketch out the direction is can be seen that all four are the same. They are all correct directions for the vector.



30° S of E　　60° E of S　　330°　　-30°

An object moves 73 m north and 62 m east. What is the resultant?

Diagram: 62m



Magnitude of R

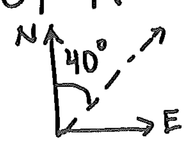
$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

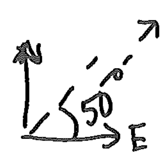
$$= \sqrt{(62)^2 + (73)^2}$$

$$= 96 \text{ m}$$

Direction of R

① $\tan\theta = \frac{62}{73} = 40°$ E of N　　② 50° N of E





choose one:

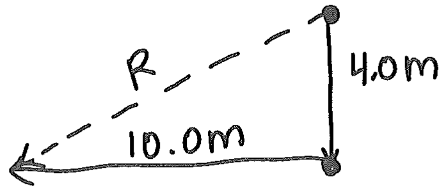
R = 96 m 40° E of N

An object moves 5.0 m north, 10.0 m west and 9.0 m south

step one: combine north & south (on the same plane)

$+5.0\,m + (-9.0\,m) = -4.0\,m = 4.0\,m$ south

step two:



$R = \sqrt{10.0^2 + 4.0^2} = \sqrt{116} = 10.8\,m$

step three: $\tan\theta = \frac{10.0}{4.0} = 68°$ W of S

Solution:
10.8m 68° W of S

Vector Components

When we split vectors up into pieces we call the pieces *components*. Normally, we want to split up vectors into their 'X' and 'Y' components. Another way to think of this is the ***amount*** that the vector points in the **X** and **Y** directions.

1) List the ***components*** of the following vectors ***A – F*** in the spaces provided.

A_x= 1
A_y= 4

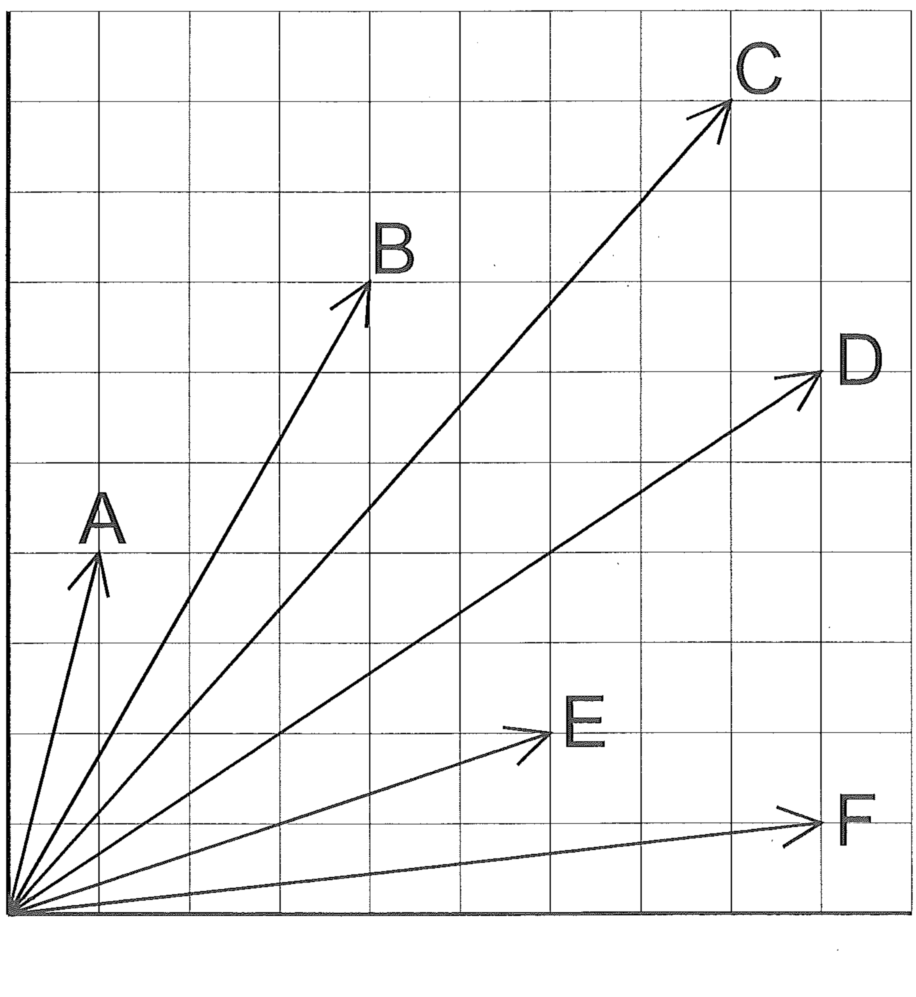
B_x= 4
B_y= 7

C_x= 8
C_y= 9

D_x= 9
D_y= 6

E_x= 6
E_y= 2

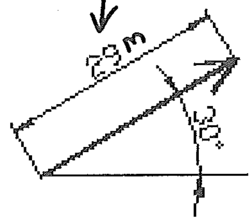
F_x= 9
F_y= 1

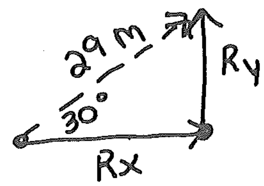


When resolving vectors into their components we use trigonometry to determine the components.

A.

resultant

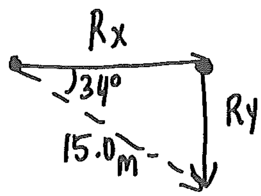




$\sin 30 = \frac{R_y}{29}$ $\cos 30 = \frac{R_x}{29}$

$R_y = 15\text{m [N]}$ $R_x = 25\text{m [E]}$

B. 15.0 m 34.0° S of E



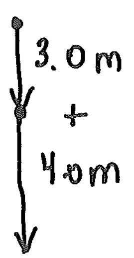
$\sin 34 = \frac{R_y}{15.0}$ $\cos 34 = \frac{R_x}{15.0}$

$R_y = 8.4\text{m [S]}$ $R_x = 12\text{m [E]}$

Vectors Part One Assignment:

1. Solve the following displacement vectors by finding the net displacement and direction.

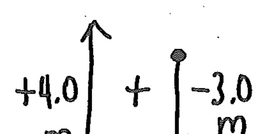
A) 3.0 m south and 4.0 m south



= 7.0m South

B) 3.0 m south and 4.0 m north

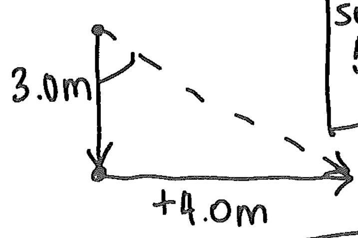
$4.0 + (-3.0) = +1.0\text{m}$



= ↑ +1.0 m

= 1.0m North

C) 3.0 m south and 4.0 m east

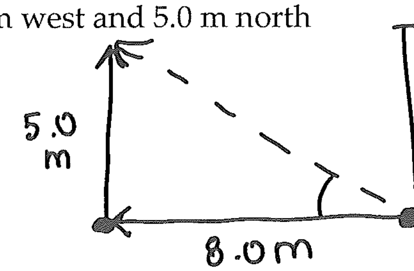


solution:
5.0 m 53° E of S

$R = \sqrt{4.0^2 + (3.0)^2}$ $R = 5.0\text{m}$

$\tan\theta = \frac{4.0}{3.0} = 53°$ E of S

D) 8.0 m west and 5.0 m north



solution:
9.4 m
32° N of W

$R = \sqrt{8.0^2 + 5.0^2}$ $R = 9.4\text{m}$

$\tan\theta = \frac{5.0}{8.0} = 32°$ N of W

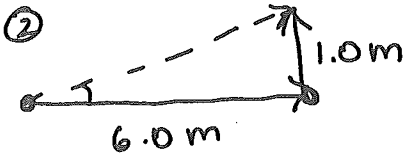
E) 7.0 m south, 6.0 m east and 8.0 m north

F) 15 m west, 12 m north and 20 m east

E)

① $-7.0\,m + 8.0\,m = +1.0\,m$

②



$\boxed{\text{solution: } 6.1\,m\ 9.5^\circ\ N \text{ of } E}$

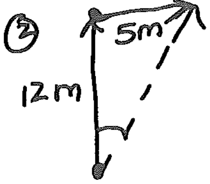
$R = \sqrt{6.0^2 + 1.0^2}$ $R = 6.1\,m$

$\tan\theta = \frac{1.0}{6.0} = 9.5^\circ$ N of E

F)

① $-15\,m + 20\,m = +5\,m$

②



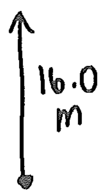
$\boxed{\text{solution: } 13\,m\ 23^\circ\ E \text{ of } N}$

$R = \sqrt{5^2 + 12^2}$, $R = 13\,m$

$\tan\theta = \frac{5}{12} = 23^\circ$ E of N

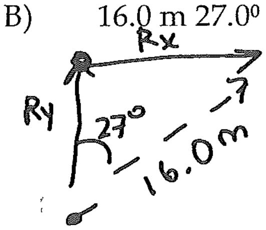
2. Determine the <u>*x* and *y* components</u> of the following displacements:

A) 16.0 m north



Rx = 0 m Ry = 16.0m [N]

B) 16.0 m 27.0° E of N



$Rx = \sin 27 = \frac{Rx}{16.0}$

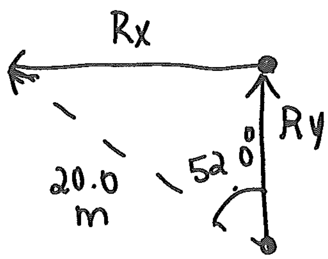
$= 7.26\,m$

$Ry = \cos 27 = \frac{Ry}{16.0}$

$= 14.2\,m$

Rx = 7.26m [E] Ry = 14.2 m [N]

C) 20.0 m 52.0° W of N



$\sin 52 = \frac{Rx}{20.0}$

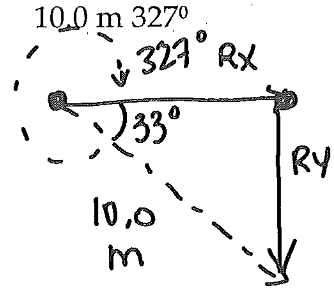
$Rx = 15.8\,m$ W

$\cos 52 = \frac{Ry}{20.0}$

$Ry = 12.3\,m$ N

Rx = 15.8m [W] Ry = 12.3 m [N]

D) 10.0 m 327°



$\sin 33 = \frac{Ry}{10.0}$

$Ry = 5.45\,m$ S

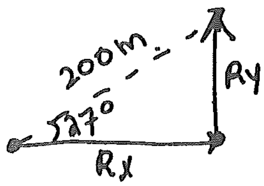
$\cos 33 = \frac{Rx}{10.0}$

$Rx = 8.39\,m$ E

Rx = 8.39m [E] Ry = 5.45m [S]

3. Resolve the following problems into their ***x* and *y* components**.

A) A person walks 200 meters at 27° degrees North of East.



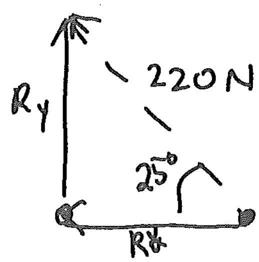
$\sin 27 = \frac{Ry}{200}$ $Ry = 91$ m

$\cos 27 = \frac{Rx}{200}$ $R_x = 178$ m

Rx = 178 m [E]

Ry = 91 m [N]

B) A magnet attracts a steel ball with a force of 220 N at 25° North of West.



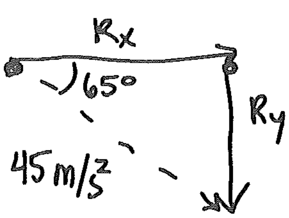
$\sin 25 = \frac{Ry}{220}$ $Ry = 93.0$ N

$\cos 25 = \frac{Rx}{220}$ $Rx = 199$ N

Rx = 199 N [W]

Ry = 93.0 N [N]

C) A rocket accelerates at 45 m/s² at 65 degrees South of East.



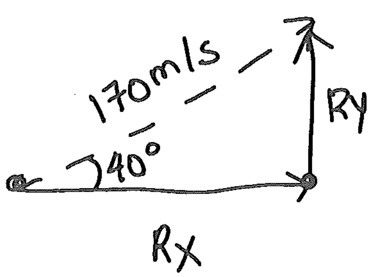
$\sin 65 = \frac{Ry}{45}$ $Ry = 41\ m/s^2$

$\cos 65 = \frac{Rx}{45}$ $Rx = 19\ m/s^2$

Rx = $19\ m/s^2$ [E]

Ry = $41\ m/s^2$ [S]

D) A cannonball is launched with a speed of 170 m/s at 40° above the horizontal.



$\sin 40 = \frac{Ry}{170}$ $Ry = 109$ m/s

$\cos 40 = \frac{Rx}{170}$ $Rx = 130$ m/s

Rx = 130 m/s [f]

Ry = 109 m/s [up]

Answers:

1. A) 7.0 m south B) 1.0 m north C) 5.0 m 53° E of S D) 9.4 m 32° N of W

   E) 6.1 m 9.5° N of E F) 13 m 23° E of N

2. A) Rx = 0 m Ry = 16.0 m north B) Rx = 7.26 m east Ry = 14.2 m north

   C) Rx = 15.8 m west Ry = 12.3 m north D) Rx = 8.39 m east Ry = 5.45 m south

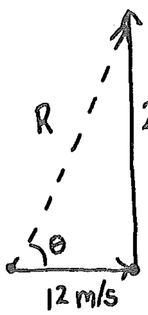
3. A) Rx = 178 m east Ry = 91 m north B) Rx = 199 N west Ry = 93.0 n north

   C) Rx = 19 m/s^2 east Ry = 41 m/s^2 south D) Rx = 130 m/s forward Ry = 109 m/s up

Physics 12 – Vectors Part Two - Adding Vectors

Last class, we began adding vectors together.

12 m/s east + 24 m/s north →



$R = \sqrt{12^2 + 24^2} = 27\,m/s$

$\tan^{-1}\left(\frac{24}{12}\right) = 63^\circ$ N of E

solution: 27 m/s 63° N of E

Non-90⁰ Vector Addition:

Adding vectors that are completely in the 'X' or 'Y' directions is easy as they form nice right-angle triangles and the basic trig laws and Pythagorean theorem work.

However, often the vectors are not all in the 'X' and 'Y' direction. How do we solve these problems?

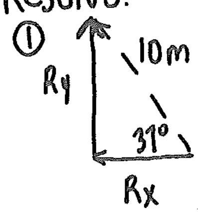
Step One: We take the vector at a *weird* angle (ie, not N, E, S, or W) and *resolve* (break apart) the vector to its 'X' and 'Y' components.

Step Two: We add up all the 'X' components, add up all the 'Y' components, and create a right triangle to use basic trig and Pythagorean theorem to calculate the magnitude and direction of the resultant!!!!

Examples - Add the following vectors:

10 m @ 37° N of W + 50 m North

Resolve:

① 

$\sin 37 = \frac{R_y}{10} = 6.0\,m$ North

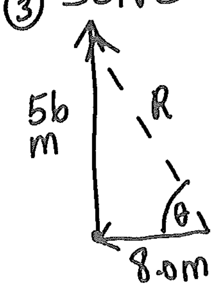
$\cos 37 = \frac{R_x}{10} = 8.0\,m$ West

② determine total X & Y components

Y: 6.0 m [N] + 50 m [N] = 56 m [N]

X: 8.0 m [W] + 0m ← there is no y component to this vector.

③ Solve



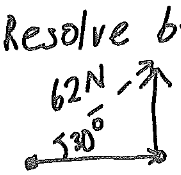
$R = \sqrt{56^2 + 8.0^2} = 57\,m$

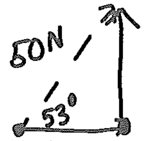
$\tan^{-1}\left(\frac{56}{8}\right) = 82^\circ$ N of W

solution: 57 m 82° N of W

62 N @ 30° + 50 N 53°

① Resolve both into x & y-components:





$\sin 30 = \frac{R_y}{62}$ $\cos 30 = \frac{R_x}{62}$ $\sin 53 = \frac{R_y}{50}$ $\cos 53 = \frac{R_x}{50}$

$R_y = 31\text{ N}$ [N] $R_x = 53\text{ N}$ [E] $R_y = 40\text{ N}$ [N] $R_x = 30\text{ N}$ [E]

② Find total x & y:

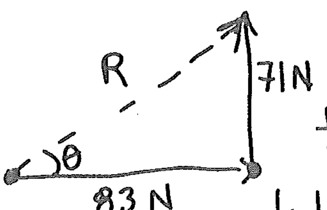
y: $31 + 40 = 71\text{ N}$ [N]

x: $53 + 30 = 83\text{ N}$ [E]

*always pay attention to direction

(+) or (−)

③ Solve:



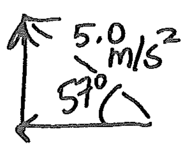
$R = \sqrt{83^2 + 71^2} = 109\text{ N}$

$\tan^{-1}\left(\frac{71}{83}\right) = 41°$ N of E

solution: 109 N 41°

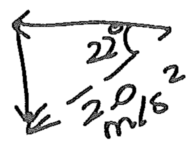
5.0 m/s² @ 57° N of W + 2.0 m/s² @ 22° S of W

① resolve:



$R_x = \cos 57(5.0)$
$= 2.7\text{ m/s}^2$ [W]

$R_y = \sin 57(5.0)$
$= 4.2\text{ m/s}$ [N]



$R_x = \cos 22(2.0)$
$= 1.8\text{ m/s}^2$ [W]

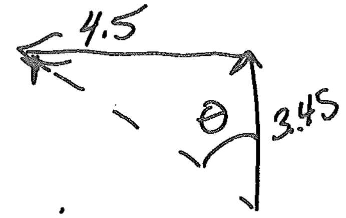
$R_y = \sin 22(2.0)$
$= 0.75\text{ m/s}^2$ [S]

② combine:

x: $-2.7 + (-1.8) = -4.5\text{ m/s}^2$ or 4.5 m/s^2 [W]

y: $4.2 + (-0.75) = 3.45\text{ m/s}^2$ [N]

③ solve



$R = \sqrt{4.5^2 + 3.45^2} = 5.7\text{ m/s}^2$

$\tan^{-1}\left(\frac{4.5}{3.45}\right) = 53°$ W of N

$= 5.7\text{ m/s}^2$ @ 53° W of N

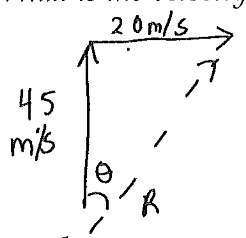
Review of Velocity Vectors (from Physics 11)

Velocity vectors are added together in the same way that we added displacement vectors together.

1. Use tip-to-tail to find the resultant.
2. Find the magnitude of R through Pythagorean Theorem $R = \sqrt{(R_x)^2 + (R_y)^2}$
3. Find the direction of the vector using: $\tan\theta = \frac{opp}{adj} = \frac{R_y}{R_x}$

River Problems (2-D motion)

A boat whose speed in still water is 4.5m/s travels north across a river. The river current is 2.0 m/s east. What is the velocity relative to the shore?



$R = \sqrt{2.0^2 + 4.5^2} = 4.9 \text{ m/s}$

$\tan^{-1}\left(\frac{2.0}{4.5}\right) = 24° \text{ E of N}$

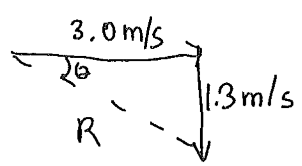
solution:

4.9 m/s 24° E of N

We can also put vectors together with kinematics formulas such as v=d/t. This is illustrated in the problem of a boat crossing a river.

A boat whose speed in still water is 3.0 m/s is headed east across a river. The river current is 1.3 m/s south.

a) What is the velocity of the boat relative to the shore?

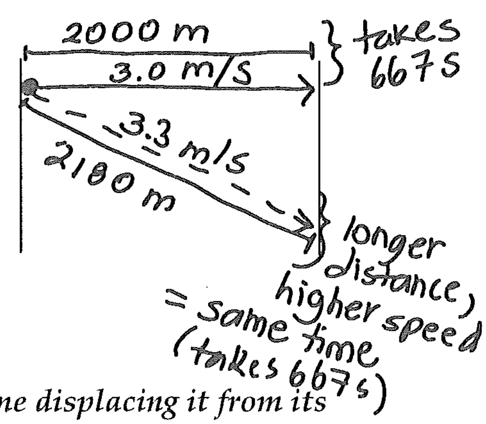


$R = \sqrt{1.3^2 + 3.0^2} = 3.3 \text{ m/s}$

$\tan^{-1}\left(\frac{1.3}{3.0}\right) = 23° \text{ S of E}$

solution:

3.3 m/s 23° S of E



Remember, the time it takes to travel across with no current will be the same as the time it takes to cross with the current.

b) If the river is 2000m wide, how long does it take to cross the river?

$V_x = \frac{d_x}{t} \quad 3.0 = \frac{2000}{t} \quad t = 667 \text{ s}$

This means the southern current acts on the boat for the total time displacing it from its intended destination at 1.3 m/s [S].

c) How far downstream is the boat when it reaches the other side?

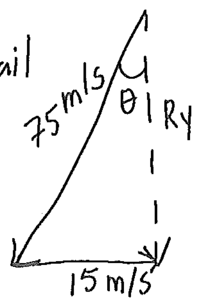
$V_y = \frac{d_y}{t} \quad 1.3 = \frac{d_y}{667} \quad d_y = 867 \text{ m [S]}$

The other use of velocity vectors is to determine the initial direction needed to result in desired destination when more than one velocity is acting on the object.

A pilot wants to fly south. If the plane has an airspeed of 75 m/s, and there is a 15 m/s wind blowing east,

a) What direction must the pilot head the plane in order to travel the desired resultant path?
b) What will the resultant speed of the plane be with the effects of the wind?

use tip-to-tail



$R_y = \sqrt{75^2 - 15^2}$
$= 73\ m/s$

$\tan^{-1}\left(\frac{15}{75}\right) = 11°$

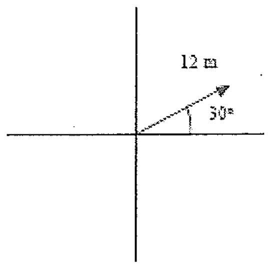
solution:
pilot must head
11° W of S

(resultant speed = 73 m/s [S])

Vectors Part Two Assignment:

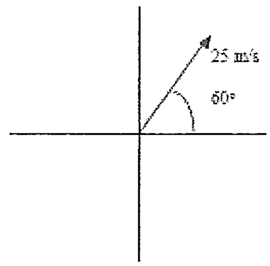
Part I:

Find the x and y components of each of the following vectors.



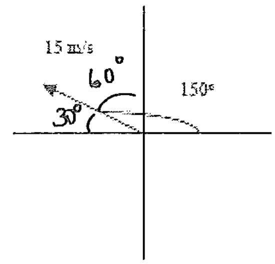
x = 10m E

y = 6.0m N



x = 13m/s E

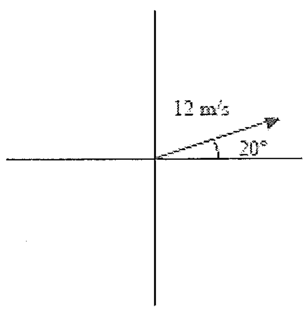
y = 22 m/s N



x = 13 m/s W

y = 7.5m/s N

Add the following vectors.



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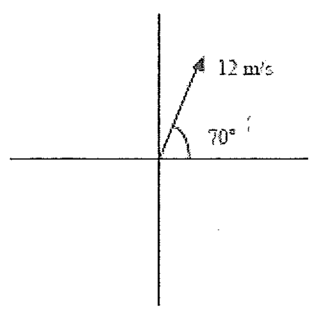

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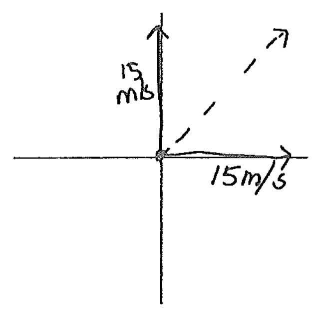
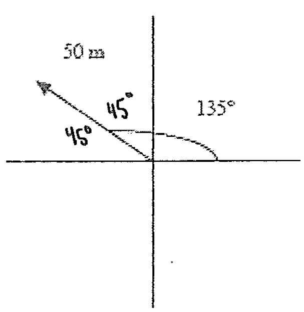


x_1 = 11 m/s E + x_2 = 4.1 m/s E = x_{tot} = 15 m/s E = 21 m/s 45° N of E

y_1 = 4.1 m/s N + y_2 = 11 m/s N = y_{tot} = 15 m/s N



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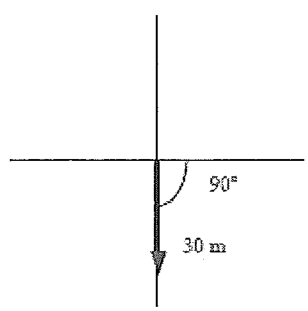

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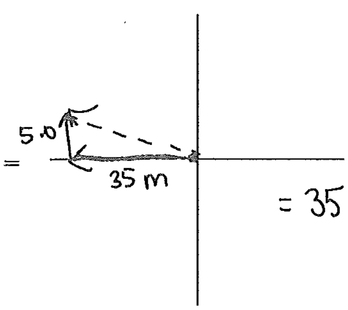


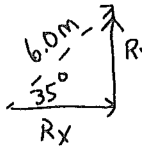
= 35 m 8.0° N of W

x_1 = 35 m W + x_2 = 0 m W = x_{tot} = 35 m W

y_1 = 35 m N + y_2 = 30m S = y_{tot} = 5.0m N

2. Add the following displacement vectors:

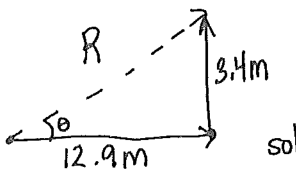
a) 8.0 m east and 6.0 m 35⁰ N of E



$\cos 35 = \frac{R_x}{6.0} = 4.9 \text{ m E}$

$\sin 35 = \frac{R_y}{6.0} = 3.4 \text{ m N}$

8.0m + 4.9m E = 12.9m E and 3.4m N

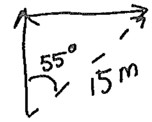


$R = \sqrt{12.9^2 + 3.4^2} = 13.3\text{m}$

$\tan^{-1}\left(\frac{3.4}{12.9}\right) = 15°$

solution:
13m 15° N of E

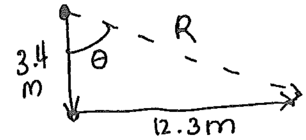
b) 12 m south and 15 m 55⁰ E of N



$R_x = 12.3 \text{ m E}$

$R_y = 8.6 \text{ m N}$

−12m + 8.6m = −3.4m → 3.4 m [S] and 12.3m [E]

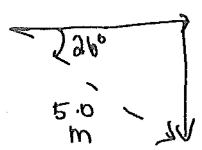


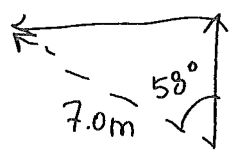
$R = \sqrt{12.3^2 + 3.4^2} = 13 \text{ m}$

$\tan^{-1}\left(\frac{12.3}{34}\right) = 75° \text{ E of S}$

solution:
13m 75° E of S

c) 5.0 m 26⁰ S of E and 7.0 m 58⁰ W of N

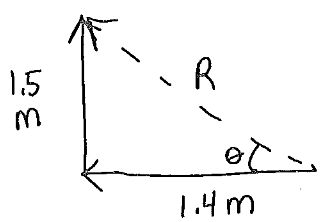




$R_x = 4.5\text{m E}$
$R_y = 2.2\text{m S}$

$R_x = 5.9\text{m W}$
$R_y = 3.7\text{m N}$

X: 4.5 + (−5.9) = 1.4 m W
Y: −2.2 + 3.7 = 1.5m N



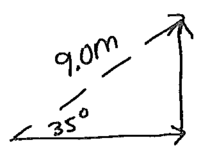
$R = \sqrt{1.4^2 + 1.5^2}$

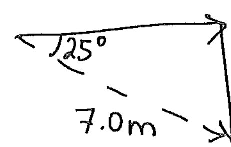
$= 2.1 \text{ m}$

$\tan^{-1}\left(\frac{1.5}{1.4}\right) = 47° \text{ N of W}$

Solution:
2.1m 47° N of W

d) 9.0 m 35⁰ N of E and 7.0m 25⁰ S of E

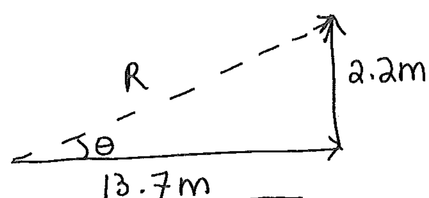




$R_x = 7.4\text{m E}$
$R_y = 5.2\text{m N}$

$R_x = 6.3\text{m E}$
$R_y = 3.0\text{m S}$

X: 7.4 + 6.3 = 13.7 m E
Y: 5.2 + (−3.0) = 2.2 m N

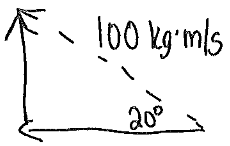


$R = \sqrt{13.7^2 + 2.2^2} = 14 \text{ m}$

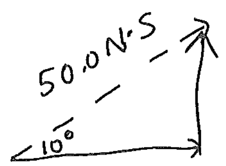
$\tan^{-1}\left(\frac{2.2}{13.7}\right) = 9.1° \text{ N of E}$

solution:
14m 9.1° N of E

3. A *GT SnowRacer®* had a momentum of 100 kg•m/s [20°N of W] as it slid on a perfectly smooth hill. It received an impulse of 50.0 N•s [10°N of E] from a barrier placed on one edge of the hill. What was the resulting momentum of the *GT SnowRacer®*?

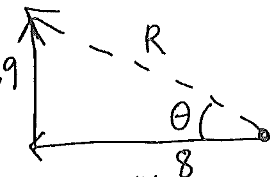


$R_x = 94.0$ kg·m/s W
$R_y = 34.2$ kg·m/s N



$R_x = 49.2$ N·S E
$R_y = 8.7$ N·S N

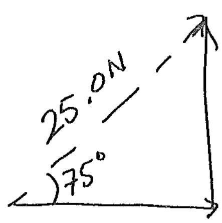
X: $-94.0 + 49.2 = 44.8$ kg·m/s W
Y: $34.2 + 8.7 = 42.9$ kg·m/s N



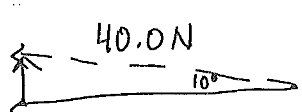
$R = \sqrt{44.8^2 + 42.9^2} = 62.0$ kg·m/s
$\tan^{-1}\left(\frac{42.9}{44.8}\right) = 44°$ N of W

solution:
62.0 kg·m/s 44° N of W

4. Two forces act on an object. One has a magnitude of 25.0 N at an angle of 75°, the other has a magnitude of 40.0 N at an angle of 170°. What is the resultant force acting on the object?

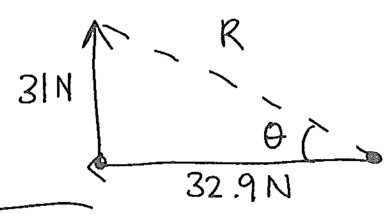


$R_x = 6.47$ N [E]
$R_y = 24.1$ N [N]



$R_x = 39.4$ N [W]
$R_y = 6.9$ N [N]

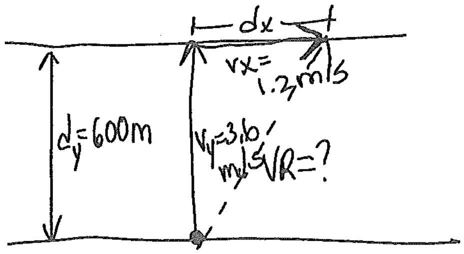
X: $6.47 + (-39.4) = 32.9$ N [W]
y: $24.1 + 6.9 = 31$ N [N]



$R = \sqrt{32.9^2 + 31^2} = 45$ N
$\tan^{-1}\left(\frac{31}{32.9}\right) = 43°$ N of W

Solution:
45.0 N 43° N of W

5. A boat that can travel 3.6 m/s on still water heads directly north across a river that is 600 m wide. The river current is 1.2 m/s east. Draw the vector diagram.



a) What is the velocity of the boat with respect to the shore?

$R = \sqrt{1.2^2 + 3.6^2} = 3.79\,m/s \qquad \tan^{-1}\left(\frac{1.3}{3.6}\right) = 20^\circ$ E of N

$= 3.79\,m/s\ 20^\circ$ E of N

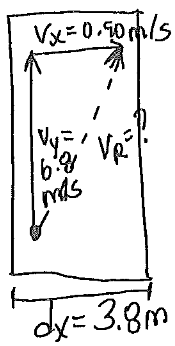
b) How long does it take the boat to reach the opposite side?

$V_y = \frac{d_y}{t} \qquad 3.6 = \frac{600}{t} \qquad t = 167\,s$

c) How far downstream is the boat when it reaches the opposite shore?

$V_x = \frac{d_x}{t} \qquad 1.2 = \frac{d_x}{167} \qquad d_x = 200\,m$ [E]

6. A passenger in a train travelling at 6.8 m/s [N] walks across the train car at 0.90 m/s [E] to the snack bar. If the car is 3.8 m wide, how long does it take the passenger to reach the other side? What is his velocity relative to the ground?



$V_x = \frac{d_x}{t} \qquad 0.90 = \frac{3.8}{t} \qquad t = 4.2\,s$

$V_R = \sqrt{0.9^2 + 6.8^2} \qquad V_R = 6.9\,m/s$

$\tan^{-1}\left(\frac{0.90}{6.8}\right) = 7.5^\circ$ E of N

solution: 6.9 m/s 7.5° E of N

Answers:

1) see posted solutions

2. a) 13 m 15° N of E   b) 13 m 75° E of S   c) 2.1 m 47° N of W   d) 14 m 9.1° N of E

3) 62.0 kg•m/s 44° N of W

4) 45.0 N 43° N of W

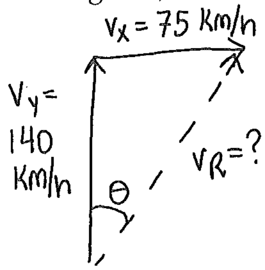
5) a) 3.79 m/s 20° E of N   b) 167 s   c) 200 m [E]

6) 6.9 m/s 7.5° E of N

Lab - Investigating Vectors

Questions: Draw a vector diagram for each question and then solve for the resultant with direction.

1. A pilot heads her plane north with a velocity of 140 km/h. If there is a strong wind of 75 km/h blowing east, what is the velocity of the plane with reference to the ground?



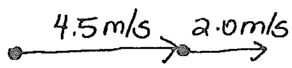
$V_R = \sqrt{140^2 + 75^2} = 159$ Km/h

$\tan^{-1}\left(\frac{75}{140}\right) = 28.2°$ E of N

solution: 159 Km/h 28.2° E of N

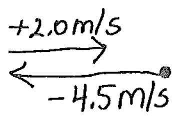
2. A boat whose speed in still water is 4.5 m/s is in a river whose current velocity is 2.0 m/s east. What is the velocity of the boat relative to the shore when the boat is heading:

a) east?



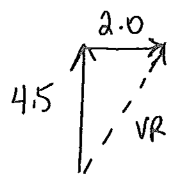
$= 6.5$ m/s [E]

b) west?



$= -2.5$ m/s $= 2.5$ m/s [W]

c) north?

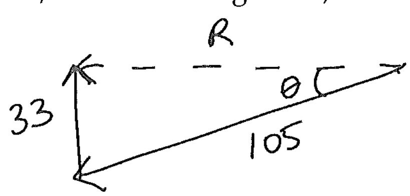


$V_R = \sqrt{4.5^2 + 2.0^2} = 4.9$ m/s

$\tan^{-1}\left(\frac{2.0}{4.5}\right) = 24°$ E of N

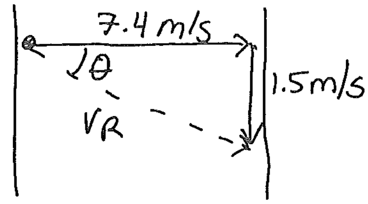
$\boxed{4.9 \text{ m/s } 24° \text{ E of N}}$

3. A pilot wants to fly west (the resultant will be west). If the plane has an air speed of 105 m/s, and there is a 33 m/s wind blowing north, in what direction must she head?



$\sin^{-1}\left(\frac{33}{105}\right) = \boxed{18° \text{ S of W}}$

4. A boat whose speed in still water is 7.4 m/s is headed east across a river. The river current is 1.5 m/s south.



a) What is the velocity of the boat relative to the shore?

$V_R = \sqrt{7.4^2 + 1.5^2} = 7.6\,m/s$

$\tan^{-1}\left(\frac{1.5}{7.4}\right) = 11°$ S of E

7.6 m/s 11° S of E

b) If the river is 6000 m wide, how long does it take the boat to cross the river?

$V_x = \frac{dx}{t}$ $\quad 7.4 = \frac{6000}{t}$ $\quad t = 811\,s$

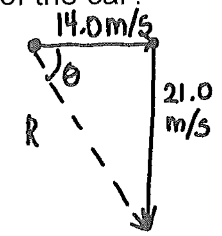
c) How far downstream is the boat when it reaches the other side of the river?

$V_y = \frac{dy}{t}$ $\quad 1.5 = \frac{dy}{811}$ $\quad dy = 1217\,m\,[S]$

Velocity Vector Problems: **Name:**

Draw a vector diagram for each question and then solve for the resultant with direction.

1. A truck is travelling in a straight line with uniform motion. The east component of this motion is 14.0 m/s, and the south component of the motion is 21.0 m/s. What is the velocity of the car?

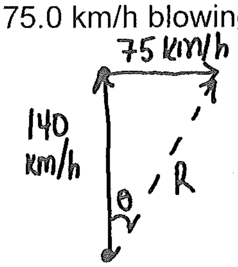


$R = \sqrt{14.0^2 + 21.0^2} = 25.2\ m/s$

$\tan^{-1}\left(\frac{21}{14}\right) = 56.3°$ S of E

solution: 25.2 m/s 56.3° S of E

2. A pilot heads her plane north with a velocity of 140 km/h. If there is a strong wind of 75.0 km/h blowing east, what is the velocity of the plane with reference to the ground?

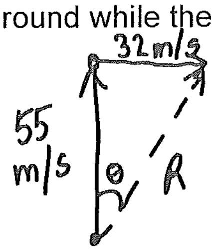


$R = \sqrt{75^2 + 140^2} = 159\ km/h$

$\tan^{-1}\left(\frac{75}{140}\right) = 28.2°$ E of N

solution: 159 km/h 28.2° E of N

3. An airplane is headed due north at an airspeed of 55 m/s. A sudden wind of 32 m/s arises from the west (blowing east). What is the velocity of the plane relative to the ground while the wind is blowing?



$R = \sqrt{32^2 + 55^2} = 64\ m/s$

$\tan^{-1}\left(\frac{32}{55}\right) = 30°$ E of N

solution: 64 m/s 30° E of N

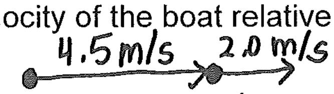
4. A boat whose speed in still water is 4.5 m/s is in a river whose current velocity is 2.0 m/s east. What is the velocity of the boat relative to the shore when the boat is headed:

a) east?

b) west?

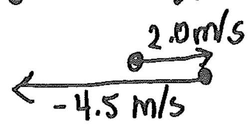
c) north?

a)



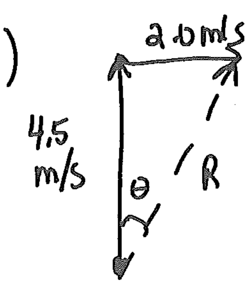
R = 6.5 m/s east

b)



R = 2.5 m/s west

c)

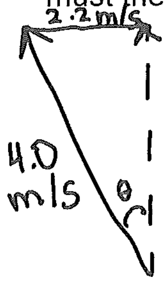


$R = \sqrt{2.0^2 + 4.5^2} = 4.9\ m/s$

$\tan^{-1}\left(\frac{2.0}{4.5}\right) = 24°$ E of N

R = 4.9 m/s 24° E of N

5. A boat that can travel on still water at a speed of 4.0 m/s wants to travel north perpendicular to the river current. If the river current is 2.2 m/s east, in what direction must the boat be held? (Note: Boat's resultant is to be perpendicular to the river current)

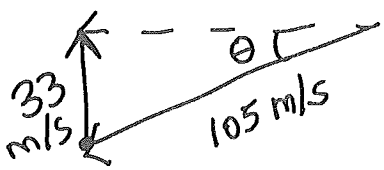


$\sin^{-1}\left(\frac{2.2}{4.0}\right) = 33°$

33° W of N

6. A pilot wants to fly west (the resultant will be west). If the plane has an air speed of 105 m/s, and there is a 33 m/s wind blowing north, in what direction must she head?

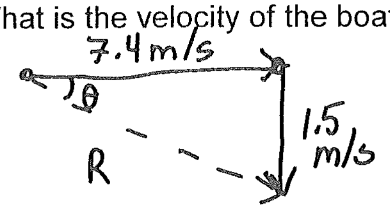
$\sin^{-1}\left(\frac{33}{105}\right) = 18°$



18° S of W

7. A boat whose speed in still water is 7.4 m/s is headed east across a river. The river current is 1.5 m/s south.

a) What is the velocity of the boat relative to the shore?



$R = \sqrt{7.4^2 + 1.5^2} = 7.6\text{ m/s}$

$\tan^{-1}\left(\frac{1.5}{7.4}\right) = 11°$ S of E

solution: 7.6 m/s 11° S of E

b) If the river is 6000 m wide, how long does it take the boat to cross the river?

$V_x = \frac{d_x}{t}$ $\quad 7.4 = \frac{6000}{t}$ $\quad t = 811\text{ s}$

c) How far downstream is the boat when it reaches the other side of the river?

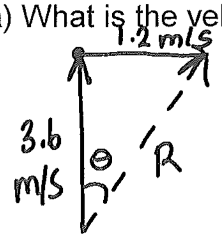
$v_y = \frac{d_y}{t}$ $\quad 1.5 = \frac{d_y}{811}$ $\quad d_y = 1216\text{ m [s]}$

$\downarrow$

$1.2 \times 10^3\text{ m [s]}$

8. A boat that can travel 3.6 m/s on still water heads directly north across a river that is 600 m wide. The river current is 1.2 m/s east.

a) What is the velocity of the boat with respect to the shore?



$R = \sqrt{1.2^2 + 3.6^2} = 3.8\ m/s$

$\tan^{-1}\left(\frac{1.2}{3.6}\right) = 18°$ E of N

solution: 3.8 m/s 18° E of N

b) How long does it take the boat to reach the opposite side?

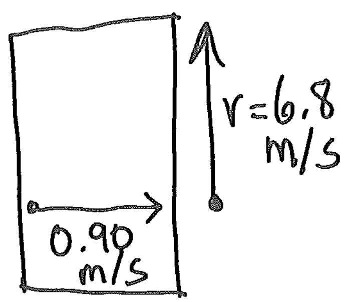
$V_y = \frac{d_y}{t}$ $\quad 3.6 = \frac{600}{t}$ $\quad t = 167\ s$

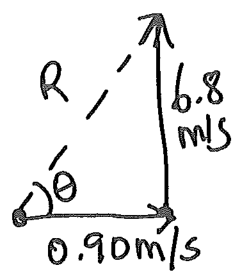
c) How far downstream is the boat when it reaches the opposite shore?

$V_x = \frac{d_x}{t}$ $\quad 1.2 = \frac{d_x}{167}$ $\quad d_x = 200\ m\ [E]$

$(2.0 \times 10^2\ m\ [E])$

9. A passenger in a train travelling at 6.8 m/s [N] walks across the train car at 0.90 m/s [E] to the snack bar. If the car is 3.8 m wide, how long does it take the passenger to reach the other side? What is his velocity relative to the ground?





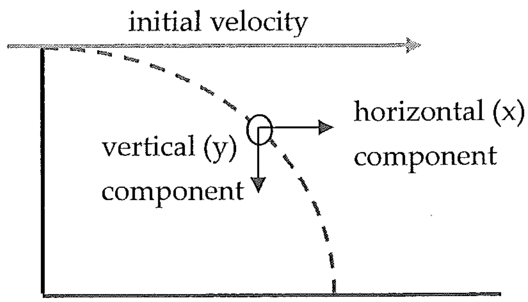
$R = \sqrt{0.90^2 + 6.8^2} = 6.9\ m/s$

$\tan^{-1}\left(\frac{6.8}{0.90}\right) = 82°$ N of E

solution: 6.9 m/s 82° N of E

Physics 12 – **Projectile Motion 1 (Horizontal Launch)**

When an object is thrown into the air, it is a projectile. Any object that **curves** downward in response to gravity is called a **projectile**. The motion of a projectile under the influence of gravity is called **projectile motion**.

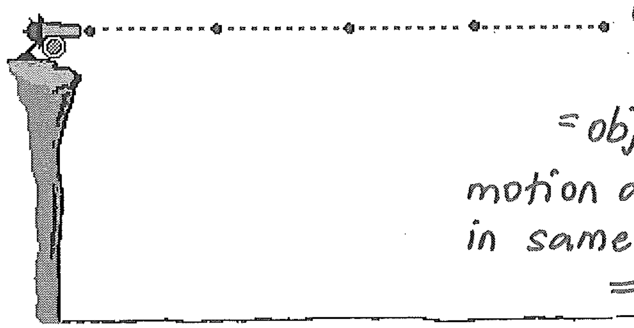


HORIZONTAL COMPONENT:

Why do we describe this horizontal component as uniform motion?

Imagine a cannonball shot horizontally from a very high cliff at a high speed. And suppose for a moment that the *gravity switch* could be *turned off.*

According to Newton's first law of motion, such a cannonball would continue in motion in a straight line at constant speed (in the absence of an unbalanced force)



path taken with no gravity

= object will stay in motion at the same speed in same direction = inertia

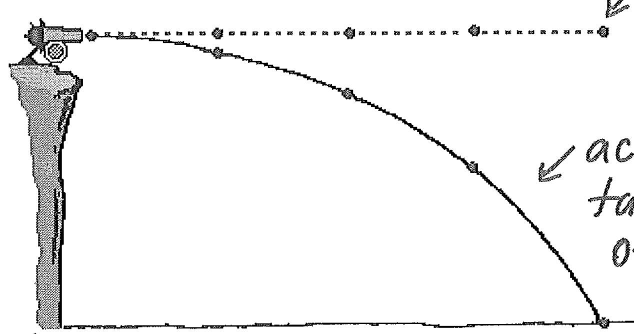
= uniform motion → $V_x = \frac{dx}{t}$

However, is it an object's path under the influence of gravity that is considered **projectile motion**.

VERTICAL COMPONENT:

= accelerated motion (*use kinematics formulas – v_0, v_F, a, d, t)

When we have gravity, it will act downwards upon the cannonball to affect its vertical motion. Gravity causes a vertical acceleration (-9.8m/s²). The ball will drop vertically below its otherwise straight-line, inertial path. As gravity is a downward force, it will affect the projectile's vertical motion and cause the parabolic trajectory that we see in projectile motion.



↙ gravity-free inertial path

↙ actual path take under influence of gravity

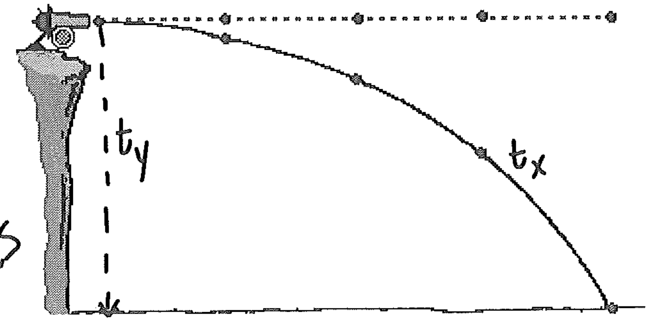
with gravity acting as a downward "unbalanced" force, the projectile will fall below its "inertial" path and follow the parabolic shaped path seen above

TIME:

Assuming no air resistance, **<u>the time it takes to fall when dropped straight down is the same as the time it takes to complete the projectile path.</u>**

This happens for the same reason as it did when we were calculating velocity vectors with a boat crossing the river. Even though the displacement is higher (path is greater), the velocity is also higher (due to an initial horizontal velocity).

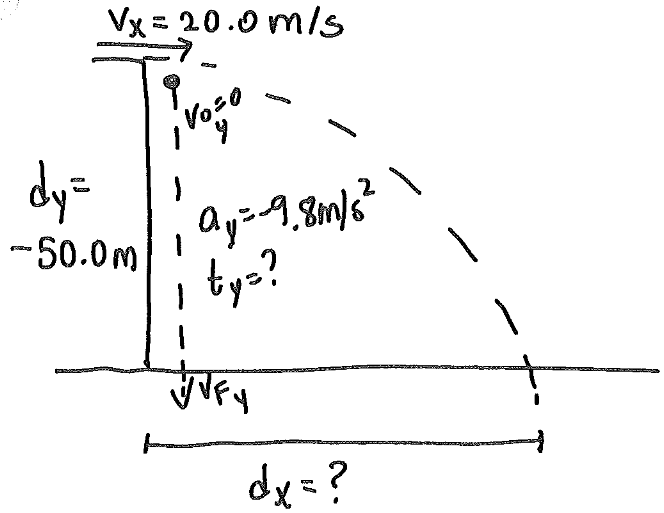
when calculating t_y, it is based on the object being dropped $\Rightarrow V_{0y} = 0$ m/s



$t_x = t_y$

*this allows us to calculate time in one component and use it to solve for the other motion component.

Examples: An object is thrown horizontally at a velocity of 20.0 m/s from the top of a building 50.0m tall. What is the range of this object?



find time through y-component → $V_{oy} = 0$, V_{Fy} , X, $a_y = -9.8$, $d_y = -50$, $t = ?$

$-50 = 0 + \frac{1}{2}(-9.8)t^2$

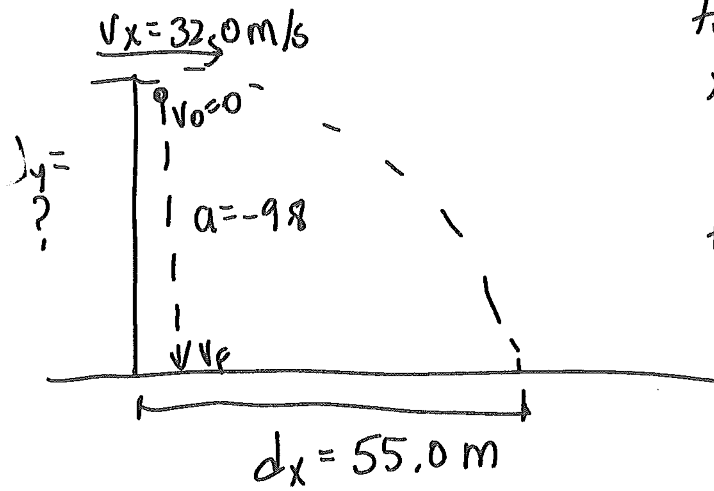
$t = 3.19$s

find d_x for x-component → $V_x = 20.0$, $d_x = ?$, $t = 3.19$

$20 = \frac{d_x}{3.19}$

$\underline{d_x = 63.9\text{ m}}$

An object is thrown horizontally at a velocity of 32.0 m/s from the top of a building. If the range of the object is 55.0m, how high is the building?



find t with x-component data → $V_x = \frac{d_x}{t}$ $32 = \frac{55}{t}$

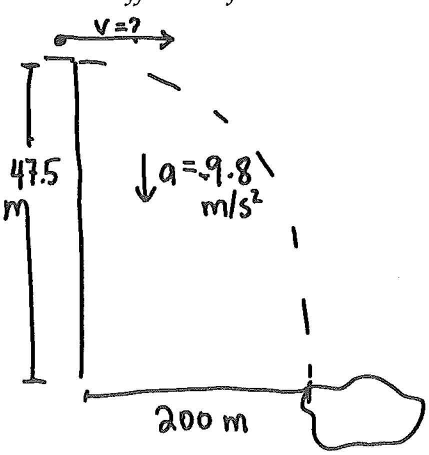
$t = 1.72$s

find dy with y-component data → $V_{oy} = 0$, V_{Fy} ~ X, $a_y = -9.8$, $d_y = ?$, $t = 1.72$

$d_y = 0 + \frac{1}{2}(9.8)(1.72)^2$

$\underline{d_y = 14.5\text{m high}}$

During a police chase, a car drives off the edge of a cliff that is 47.5 m high. When the police look over the edge to check on the suspect, they find that the car landed in the lake that is 200 m from the base of the cliff. How fast was the car travelling when it went over the edge of the cliff?



1. find time it took the car to fall. vertically

$d = v_o t + \frac{1}{2}at^2$

$-47.5 = 0 + \frac{1}{2}(-9.8)t^2$

$t = 3.11$s

2. calculate V_x

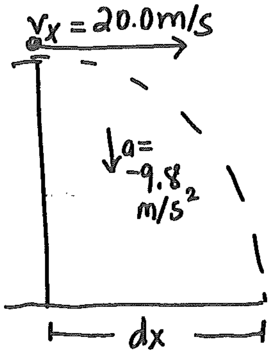
$V_x = \frac{d_x}{t} = \frac{200}{3.11} = \underline{64.3\text{ m/s}}$

Projectile Motion-1 Assignment:

1. An object is thrown horizontally from the top of a cliff at a velocity of 20.0 m/s.

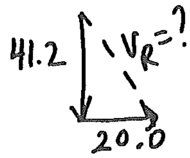
   a) If the object takes 4.20s to reach the ground, what is the range of this object? (84.0m)

   b) What is the velocity of the object just before it hits the ground? (Remember, this will be a resultant velocity) (45.8 m/s)



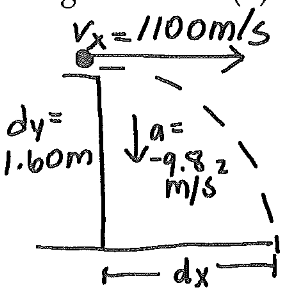
a) $v_x = \frac{d_x}{t}$ $\quad 20.0 = \frac{d_x}{4.20}$ $\quad d_x = \underline{84.0\,m}$

b) $-9.8 = \frac{v_F - 0}{4.20}$ $\quad v_F = -41.2\,m/s$



$v_R = \sqrt{41.2^2 + 20^2}$ $\quad v_R = \underline{45.8\,m/s}$

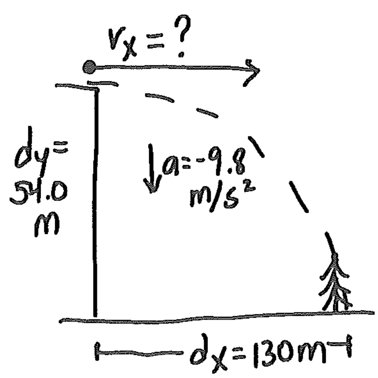
2. A bullet is fired from a rifle that is held 1.60 m above the ground in a horizontal position. The initial speed of the bullet is 1100 m/s. Find (a) the time it takes for the bullet to strike the ground and (b) the horizontal distance travelled by the bullet. (0.571 s, 629 m)



a) $-1.60 = 0 + \frac{1}{2}(-9.8)t^2$ $\quad \underline{t = 0.571\,s}$

b) $1100 = \frac{d_x}{0.571}$ $\quad d_x = \underline{629\,m}$

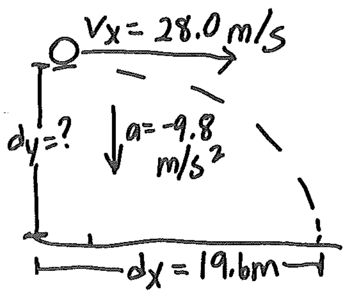
3. A car drives straight off the edge of a cliff that is 54.0 m high. The police at the scene of the accident note that the car landed on a tree that was growing 130 m from the base of the cliff. How fast was the car travelling when it went over the edge of the cliff? (39.2 m/s)



$-54.0 = 0 + \frac{1}{2}(-9.8)t^2$ $\quad t = 3.32\,s$

$v_x = \frac{130}{3.32} = \underline{39.2\,m/s}$

4. A tennis ball is struck such that it leaves the racket horizontally with a speed of 28.0 m/s. The ball hits the court at a horizontal distance of 19.6 m from the racket. What is the height of the tennis ball when it leaves the racket? (2.40 m)



$28.0 = \frac{19.6}{t}$ $t = 0.700\,s$

$d_y = 0 + \frac{1}{2}(-9.8)(0.700)^2$ $d_y = -2.40\,m$

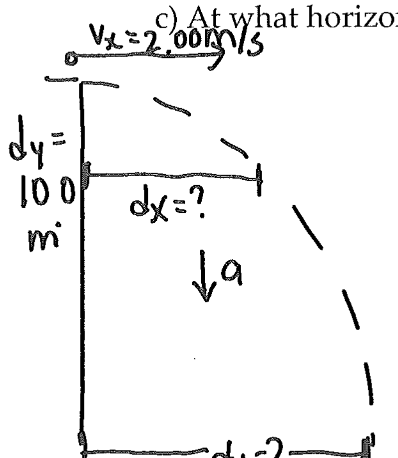
$\underline{h = 2.40\,m}$

5. A diver pushes off horizontally with a speed of 2.00 m/s from a platform edge 10.0 m above the surface of the water.

a) At what horizontal distance from the edge is the diver 0.800s after pushing off? (1.60m)

b) At what vertical distance above the surface of the water is the diver at that point? (from part a) (6.87 m above surface)

c) At what horizontal distance does the diver strike the water? (2.86 m)



a) $2.00 = \frac{d_x}{0.800}$ $d_x = \underline{1.60\,m}$

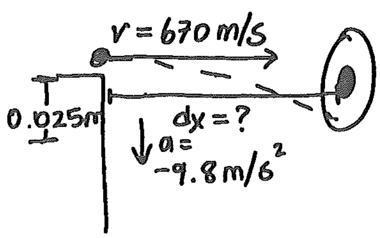
b) $d = 0 + \frac{1}{2}(-9.8)(0.800)^2$ $d_y = -3.13\,m$

$\rightarrow 10.0 - 3.13 = \underline{6.87\,m}$ above surface

c) $-10.0 = 0 + \frac{1}{2}(-9.8)\,t^2$ $t = 1.43\,s$

$2.00 = \frac{d_x}{1.43}$ $\underline{d_x = 2.86\,m}$

6. A horizontal rifle is fired at a bull's eye. The muzzle speed of the bullet is 670 m/s. The bullet strikes the target 0.025 m below the center of the bull's-eye. What is the horizontal distance between the end of the rifle and the target? (48 m)

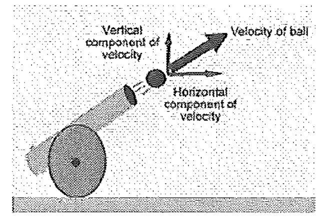


$-0.025 = 0 + \frac{1}{2}(-9.8)t^2$ $t = 0.0714\,s$

$670 = \frac{d_x}{0.0714}$ $d_x = \underline{48\,m}$

Projectile Motion (Objects Thrown at an Angle)

Objects that follow a path characteristic with projectile motion can also be thrown or launched into the air at an angle.

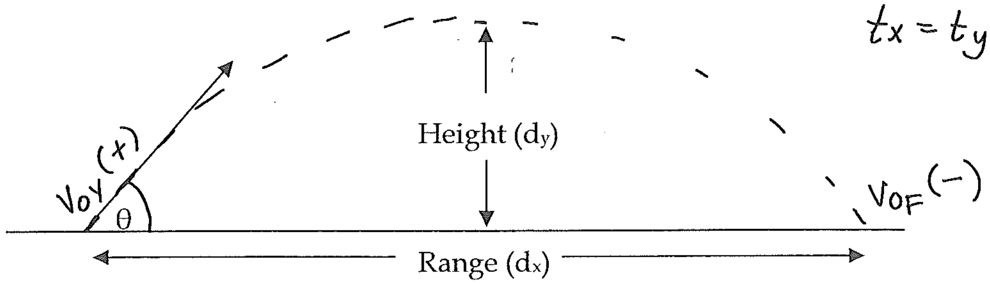


These types of problems also have a vertical and horizontal component in which the vertical motion is uniform and the horizontal component is accelerated by the force of gravity (as seen in the last lesson). However, in these problems the initial vertical velocity (v_{0y}) is no longer 0 m/s as it has some positive initial value.

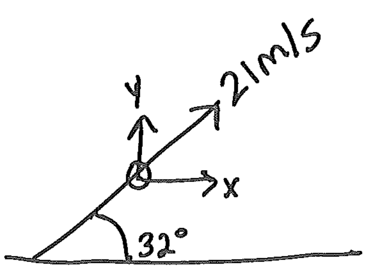
The path taken by a projectile launched at an angle to the horizontal can be described as follows:

A projectile is launched at an angle to the horizontal and rises upwards to a peak while moving horizontally. Upon reaching the peak, the projectile falls with a motion that is symmetrical to its path upwards to the peak. Predictable unknowns include the time of flight, the horizontal range, and the height of the projectile when it is at its peak.

$t_x = t_y$

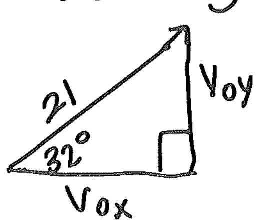


Using the launch velocity we need to determine the x and y-components:



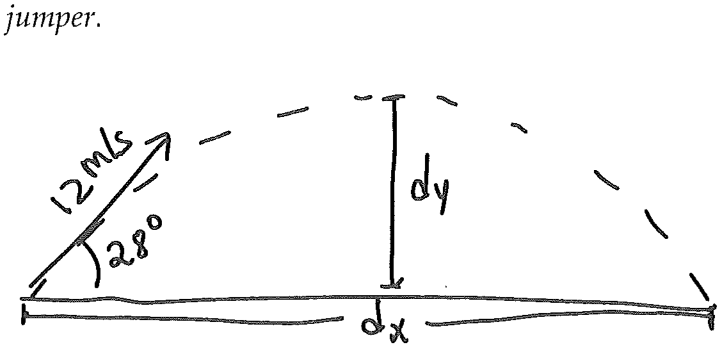
Calculating x & y component of initial velocity (launch velocity)

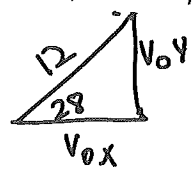
use trig: $\sin 32 = \frac{V_{0y}}{21} = +11\ m/s = V_{0y}$



$\cos 32 = \frac{V_{0x}}{21} = +18\ m/s = V_{0x}$

A long jumper leaves the ground with an initial velocity of 12 m/s at an angle of 28-degrees above the horizontal. Determine the time of flight, the horizontal distance, and the peak height of the long-jumper.





$V_{oy} = \sin 28(12) = 5.6 \text{ m/s}$

$V_{ox} = \cos 28(12) = 10.6 \text{ m/s}$

$X: \; V_x = \frac{d_x}{t}$

$10.6 = \frac{d_x}{1.14}$ $\quad d_x = 12 \text{ m/s}$

$Y: \; V_{oy} = +5.6 \text{ m/s}$

$V_{Fy} = -5.6 \text{ m/s}$

$a_y = -9.8 \text{ m/s}^2$

$d_y = X$

$t = ?$

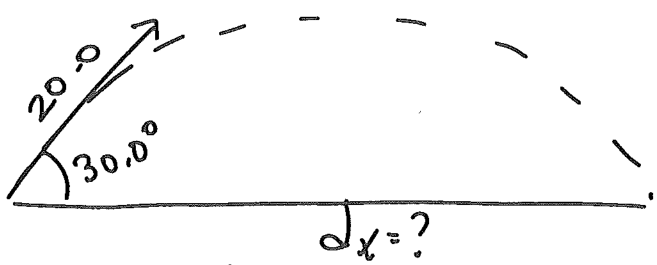
$-9.8 = \frac{-5.6 - 5.6}{t}$

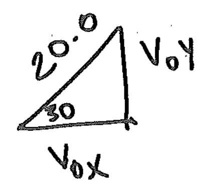
$t = 1.14 \text{ s}$

$d_y = (5.6)(1.14) + \frac{1}{2}(-9.8)(1.14)^2$

$d_y = 0.016 \text{ m}$

An object is thrown through the air at a velocity of 20.0 m/s at an angle of 30.0° with the horizontal. What is the range of the object?





$\sin 30 = \frac{V_{oy}}{20.0} = 10.0 \text{ m/s}$

$\cos 30 = \frac{V_{ox}}{20.0} = 17.3 \text{ m/s}$

$X: \; V_x = \frac{d_x}{t}$

$17.3 = \frac{d_x}{2.04}$

$d_x = 35.3 \text{ m}$

$Y: \; V_{oy} = +10.0$

$V_{Fy} = -10.0$

$a_y = -9.8$

$d_y = X$

$t = ?$

$-9.8 = \frac{-10 - 10}{t}$

$t = 2.04 \text{ s}$

Another method of determining the range of a projectile launched at an angle is by using:

$$\boxed{R = \frac{v^2 \cdot \sin(2 \cdot \theta)}{g}}$$

Compare with the previous example:

An object is thrown through the air at a velocity of 20.0 m/s at an angle of 30.0° with the horizontal. What is the range of the object?

$$R = \frac{20.0^2 \cdot \sin(2 \cdot 30)}{9.8} = \frac{400 \cdot \sin 60}{9.8} = \underline{35.3\text{ m}}$$

*same solution as above

Determining velocity:

A water ski jumper has a range of 84.0 m. The ramp has an angle of 14.0° to the horizontal. Neglecting air resistance, determine her take-off speed.

$$R = \frac{v^2 \cdot \sin(2 \cdot \theta)}{g}$$

$$84.0 = \frac{v^2 \cdot \sin(2 \cdot 14)}{9.8}$$

$$823.2 = v^2(\sin 28)$$

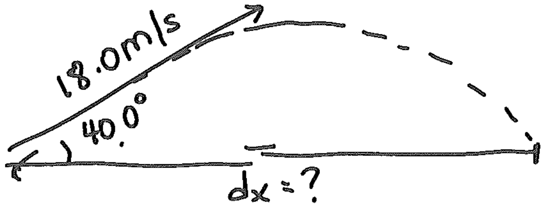
$$v^2 = 1753 \qquad \underline{v = 41.9\text{ m/s}}$$

Projectile Motion-2 Assignment:

1. An object is thrown from the ground into the air at an angle of 40.0° from the horizontal at a velocity of 18.0 m/s. What is the range of this object? (32.7 m)

$\sin 40(18) = V_{oy} = 11.6\ m/s$

$\cos 40(18) = V_x = 13.8\ m/s$



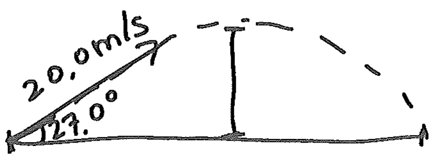
$-9.8 = \frac{-11.6 - 11.6}{t}$ $\quad$ $13.8 = \frac{dx}{2.37}$

$t = 2.37s$ $\quad$ $dx = \underline{32.7m}$

2. An object is thrown from the ground into the air with a velocity of 20.0 m/s at an angle of 27.0° to the horizontal. What is the maximum height reached by the object? (4.21 m)

$\sin 27(20) = V_{oy} = 9.08\ m/s$

$\cos 27(20) = V_x = 17.8\ m/s$



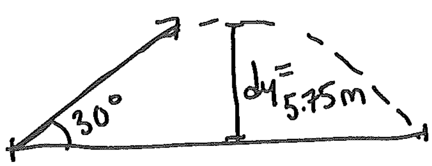
$-9.8 = \frac{0 - 9.08}{t}$ $\quad$ $d = \left(\frac{9.08 + 0}{2}\right)0.927$

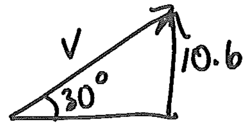
$t = 0.927s$ $\quad$ $d = \underline{4.21m}$

3. An object is thrown from the ground into the air at an angle of 30.0° to the horizontal. If this object reaches a maximum height of 5.75m, at what velocity was it thrown? (21.2 m/s)

$0^2 = V_o^2 + 2(-9.8)(5.75)$

$V_o^2 = 112.7$ $\quad$ $V_o = 10.6\ m/s$



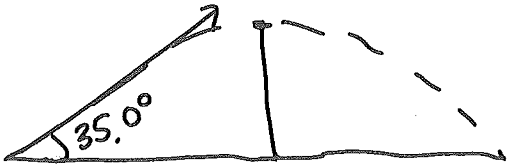


$\sin 30 = \frac{10.6}{V}$

$V = \underline{21.2\ m/s}$

4. An object is projected from the ground into the air at an angle of 35.0° to the horizontal. If this object is in the air for 9.26s, at what velocity was it thrown? (79.1 m/s)

top of motion → $t = \frac{9.26}{2} = 4.63s$



$V_o =$

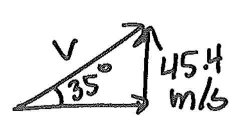
$V_F = 0$

$a = -9.8$

$d = X$

$t = 4.63$

$-9.8 = \frac{0 - V_o}{4.63}$ $\quad$ $V_o = 45.4m/s$



$\sin 35 = \frac{45.4}{V}$

$V = \underline{79.1\ m/s}$

5. An object is thrown from the ground into the air at a velocity of 15.7 m/s at an unknown angle to the horizontal. If this object has a range of 25.0 m and was in the air for 2.15 s, at what angle was this object thrown? (42.0°)

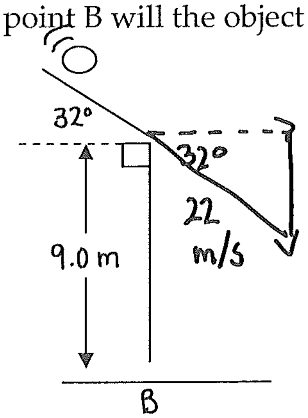
$$R = \frac{v^2 \cdot \sin(2\cdot\theta)}{g} \qquad 25.0 = \frac{15.7^2 \cdot \sin(2\cdot\theta)}{9.8}$$

$$245 = 15.7^2 \cdot \sin(2\cdot\theta)$$

$$0.994 = \sin(2\cdot\theta)$$

$$= \underline{42.0^\circ}$$

6. A ball rolls off an incline, as shown in the diagram, at a velocity of 22 m/s. How far from point B will the object hit the floor? (11 m)



① $\sin 32(22) = V_{oy} = 11.7\ m/s$

$\cos 32(22) = V_x = 18.7\ m/s$

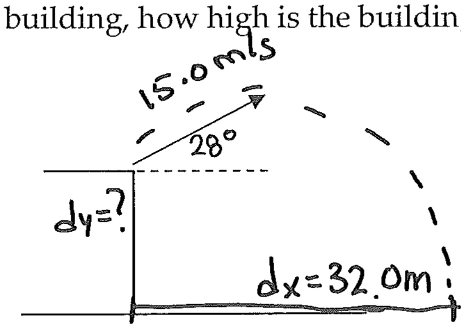
② $V_F^2 = 11.7^2 + 2(-9.8)(9.0)$ $\quad V_F^2 = 313.29$ $\quad V_{F_y} = 17.7\ m/s$

$-9.8 = \frac{-17.7 - (-11.7)}{t}$ $\quad t = 0.612\ s$

③ $18.7 = \frac{d_x}{0.612}$ $\quad d_x = \underline{11 m}$

7. An object is projected from the top of a building at an angle of 28.0°, as shown in the diagram, at a velocity of 15.0 m/s. If the object hits the ground 32.0 m from the base of the building, how high is the building? (11.8 m)

$\sin 28(15) = V_{oy} = 7.04\ m/s$



$\cos 28(15) = V_x = 13.2\ m/s$

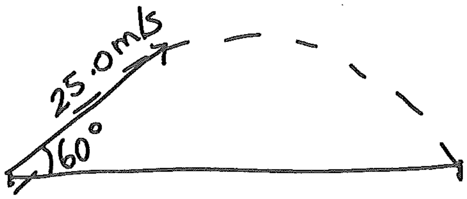
$13.2 = \frac{32.0}{t}$ $\quad t = 2.43\ s$

$d_y = (7.04)(2.43) + \frac{1}{2}(-9.8)(2.43)^2$

$d_y = 17.1 + (-28.9)$

$d_y = -11.8\ m$ $\quad \underline{h = 11.8\ m}$

8. The punter on a football team tries to kick a football so that it stays in the air for a long "hang time". If the ball is kicked with an initial velocity of 25.0 m/s at an angle of 60.0° above the ground, with is the 'hang time'? (4.43 s)



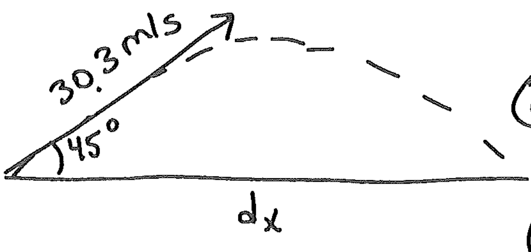
$\sin 60(25) = V_{oy} = 21.7 \text{ m/s}$

$V_{Fy} = -21.7 \text{ m/s}$

$-9.8 = \dfrac{-21.7 - 21.7}{t}$

$t = \underline{4.43 s}$

9. With a particular club, the maximum speed that a golfer can impart to a ball is 30.3 m/s. (a) How much time does the ball spend in the air? (b) What is the longest hole in one that the golfer can make, if the ball does not roll when it hits the green? (maximum displacement will come from an angle of 45.0° above the horizontal). (4.37 s, 93.5 m)



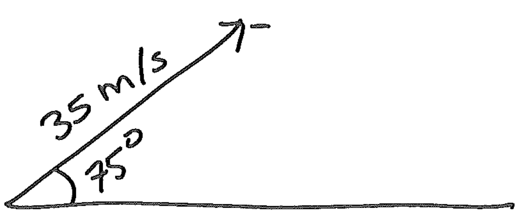
$\sin 45(30.3) = V_{oy} = 21.4 \text{ m/s}$

$\cos 45(30.3) = V_x = 21.4 \text{ m/s}$

(a) $-9.8 = \dfrac{-21.4 - 21.4}{t}$ $\quad t = \underline{4.37 s}$

(b) $21.4 = \dfrac{d_x}{4.37}$ $\quad d_x = \underline{93.5 \text{ m}}$

10. During a fireworks display a rocket is launched with an initial velocity of 35 m/s at an angle of 75° above the ground. The rocket explodes 3.7 s later. What is the height of the rocket when it explodes? (58 m)



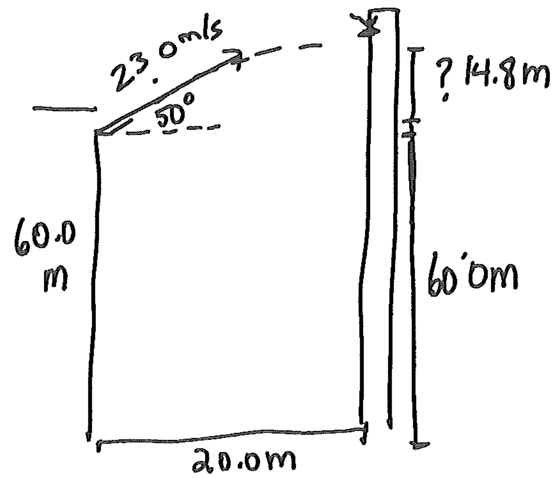
$\sin 75(35) = V_{oy} = 33.8 \text{ m/s}$

$\vec{d} = 33.8(3.7) + \frac{1}{2}(-9.8)(3.7)^2$

$= 125.06 + (-67.08)$

$= \underline{58 \text{ m}}$

11. From the edge of a 60.0 m cliff, a small rocket is fired upward with an initial velocity of 23.0 m/s at an angle of 50.0° with respect to the horizontal. At what point above the ground does the rocket strike the wall of a vertical cliff located 20.0 m away? (74.8 m)



$\sin 50(23) = V_{oy} = 17.6\ m/s$

$\cos 50(23) = V_x = 14.8\ m/s$

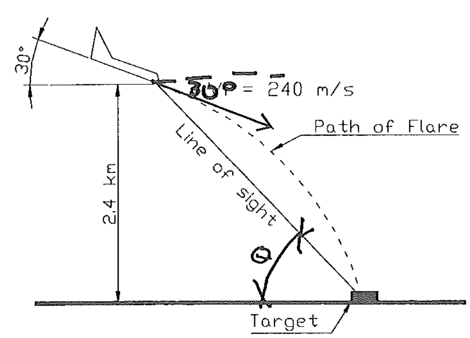
$14.8 = \frac{20.0}{t} \quad t = 1.35\ s$

$d = (17.6)(1.35) + \frac{1}{2}(-9.8)(1.35)^2$

$= +14.8\ m$

$60.0 + 14.8 = \underline{74.8\ m \text{ above ground}}$

12. * An airplane is flying with a speed of 240 m/s at an angle of 30.0° with the horizontal, as the drawing shows. When the altitude of the plane is 2.40 km, a flare is released from the plane. The flare hits the target on the ground. What is the angle θ? (42.0°)



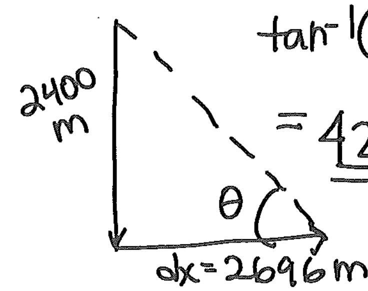
$\sin 30(240) = V_{oy} = 120\ m/s$

$\cos 30(240) = V_x = 208\ m/s$

$V_f^2 = (120)^2 + (2)(-9.8)(-2400) \quad V_f = 247.8\ m/s$

$-9.8 = \frac{-247.8 - (-120)}{t} \quad t = 12.96\ s$

$208 = \frac{d_x}{12.96} \quad d_x = 2696\ m$

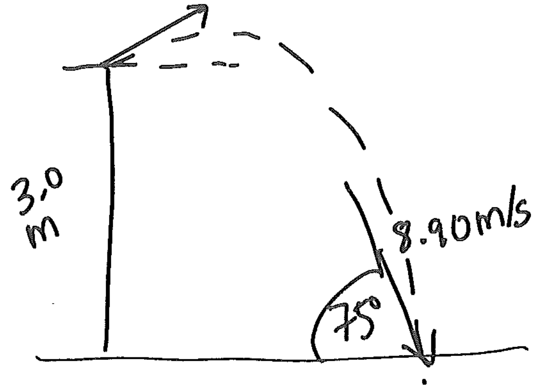


$\tan^{-1}\left(\frac{2400}{2696}\right)$

$= \underline{42.0°}$

13. *A diver springs upward from a board that is three meters above the water. At the instant she contacts the water her speed is 8.90 m/s and her body makes an angle of 75.0° with respect to the surface of the water. Determine her initial velocity, both magnitude and direction. (59.3° above horizontal)

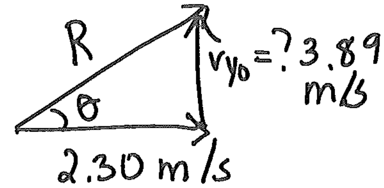
$\sin 75(8.9) = V_{yF} = 8.60\,m/s$



$\cos 75(8.9) = V_x = 2.30\,m/s$

$8.60^2 = V_0^2 + 2(-9.8)(-3.0)$

$V_{oy} = 3.89\,m/s$



$R = \sqrt{3.89^2 + 2.30^2}$

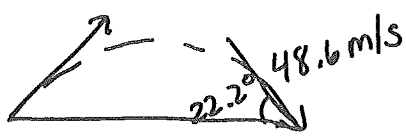
$= 4.52\,m/s$

$\tan^{-1}\left(\frac{3.89}{2.30}\right) = 59.3°$ above horizontal

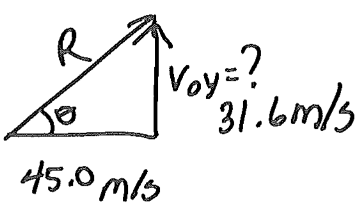
14. *A golf ball is driven from a level fairway. At a time of 5.10 s later, the ball is travelling downward with a velocity of 48.6 m/s at an angle of 22.2° below the horizontal. Calculate the initial velocity (magnitude and direction) of the ball. (55.0 m/s 35.1° above horizontal)

$\sin 22.2(48.6) = V_{Fy} = 18.4\,m/s$

$\cos 22.2(48.6) = V_x = 45.0\,m/s$



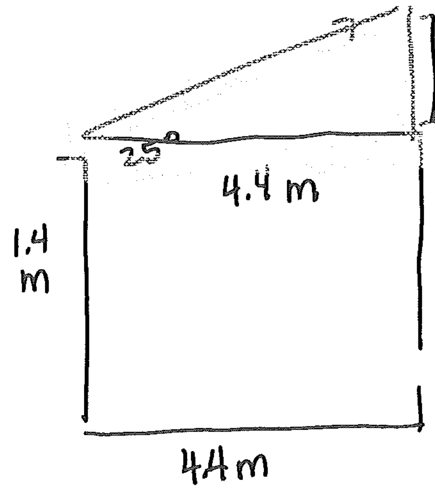
$-9.8 = \frac{-18.4 - V_0}{5.10s}$ $\quad V_{oy} = +31.6\,m/s$



$R = \sqrt{31.6^2 + 45.0^2} = 55.0\,m/s$

$\tan^{-1}\left(\frac{31.6}{45}\right) = 35.1°$ above horizontal

BONUS A garden hose, pointed at an angle of 25° above the horizontal, splashes water on a sunbather lying on the ground 4.4 m away in the horizontal direction. If the hose is held 1.4 m above the ground, at what speed does the water leave the nozzle? (5.8 m/s)



$V_0 = 0$ at top
$d = 4.9t^2$

$d = 0 + \frac{1}{2}(9.8)t^2 = 4.9t^2 - 1.4m = d$

$$\tan 25 = \frac{4.9t^2 - 1.4}{4.4}$$

$t = 0.84s$

$$V_x = \frac{4.4}{0.84} = 5.24 \text{ m/s}$$

$$-1.4 = V_0(0.84) + \frac{1}{2}(-9.8)(0.84)^2$$

$V_{oy} = 2.45$ m/s

$$R = \sqrt{5.24^2 + 2.45^2} = \underline{5.8 \text{ m/s}}$$