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Relations Lesson #3:
x- and y-intercepts and Interpreting Relations

Review

a) A **relation** is a connection between two quantities. A relation can be represented graphically by a set of ordered pairs.

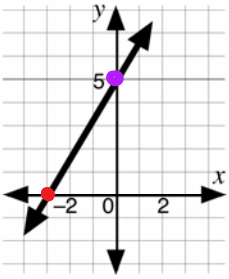
b) The first component of a set of ordered pairs is the x coordinate, also known as the input. Values of the input are values of the independent variable.

c) The second component of a set of ordered pairs is the y coordinate, also known as the output. Values of the output are values of the dependent variable.

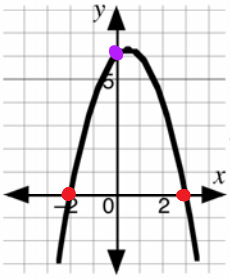
Exploring x- and y-intercepts

Consider the following graphs.

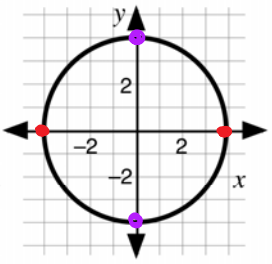
Graph 1



Graph 2



Graph 3



a) List the coordinates of the point(s) where each graph crosses the x-axis.

- Graph 1 crosses the *x*-axis at (-3, 0).
- Graph 2 crosses the *x*-axis at (-2, 0) and (3, 0).
- Graph 3 crosses the *x*-axis at (-4, 0) and (4, 0).

b) What do all the points in a) have in common?

y-coordinate = 0

c) List the coordinates of the point(s) where each graph crosses the y-axis.

- Graph 1 crosses the *y*-axis at (0, 5).
- Graph 2 crosses the *y*-axis at (0, 6).
- Graph 3 crosses the *y*-axis at (0, 4) and (0, -4).

d) What do all the points in c) have in common?

x-coordinate = 0

x- and y- intercepts of a Graph

(x,0)

The *x*-intercept of a graph is the *x*-coordinate of the ordered pair where the graph intersects the *x*-axis. An *x*-intercept occurs at a point on the graph where the *y*-coordinate is zero. The *x*-intercept can be given as a value or as an ordered pair.

(0,y)

The *y*-intercept of a graph is the *y*-coordinate of the ordered pair where the graph intersects the *y*-axis. A *y*-intercept occurs at a point on the graph where the *x*-coordinate is zero. The *y*-intercept can be given as a value or as an ordered pair.

1. Given the equation of the graph of a relation:
   - to determine the *x*-intercept, set $y = 0$ and solve for x.
   - to determine the *y*-intercept, set $x = 0$ and solve for y.
2. The equation of a graph can be written in different forms, all of which are equivalent. The equation of Graph 1 on the previous page is $y = \frac{5}{3}x + 5$, which can be written as $3y = 5x + 15$ or $5x - 3y + 15 = 0$. Equivalent forms of an equation will be studied in detail in a later unit. For the time being, use the instruction in note 1 to find the *x*- and *y*-intercepts of the graph of an equation given in any form.

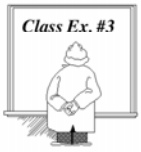
The equation of Graph 1 on the previous page is $3y = 5x + 15$.
Algebraically determine the values of the *x*-intercept and the *y*-intercept of Graph 1.

x-intercept $y=0$	*y*-intercept $x=0$
$3(0) = 5x + 15$	$3y = 5(0) + 15$
$0 = 5x + 15$	$3y = 15$
$-5x = 15$	$y = 5$
$x = -3$	*y*-int = 5
x-int = −3	

The equation of Graph 3 on the previous page is $x^2 + y^2 = 16$.
Calculate the *x*-intercept and the *y*-intercept of the graph of $x^2 + y^2 = 16$. Give the answers as ordered pairs.

x-intercept $y=0$	*y*-intercept $x=0$
$x^2 + 0^2 = 16$	$0^2 + y^2 = 16$
$\sqrt{x^2} = \sqrt{16}$	$y^2 = 16$
$x = \pm 4$	$y = \pm 4$
ordered pairs → (4,0) and (−4,0)	(0,4) and (0,−4)

Complete Assignment Questions #1 - #3



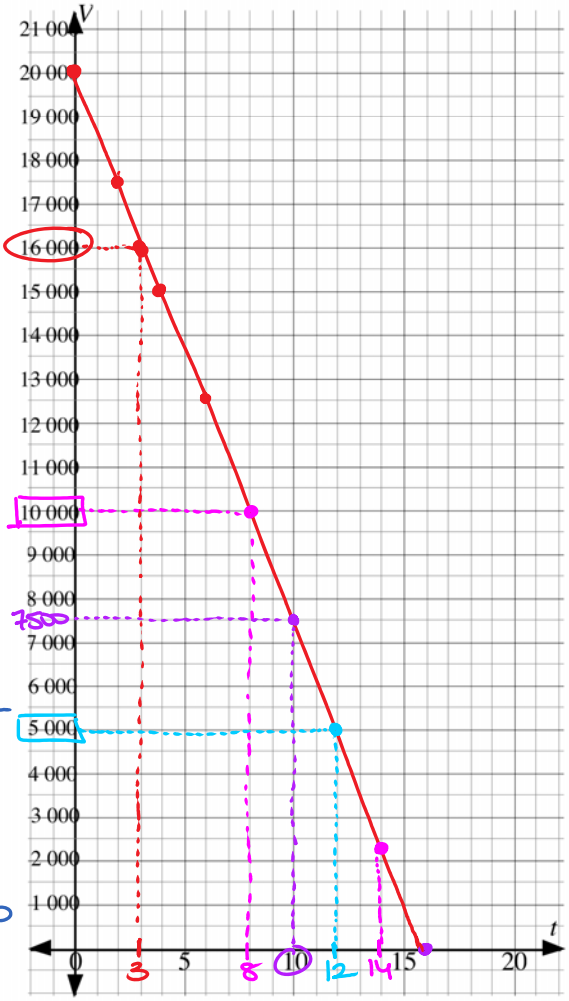
Lisa purchases a new car for $20 000. The value of the car can be represented by the formula $V = 20\,000 - 1250t$, where V is the value of the car in dollars, and t is the age of the car in years.

a) Complete the table of values and plot the ordered pairs on the grid.

$V = 20000 - 1250(0)$
$V = 20000 - 1250(2)$
$V = 20000 - 1250(4)$
$= 20000 - 5000$
$V = 20000 - 1250(6)$
$= 20000 - 7500$

Input (t)	Output (V)	Ordered pair (t, V)
0	20000	(0, 20000)
2	17500	(2, 17500)
4	15000	(4, 15000)
6	12500	(6, 12500)

Connect the points with a straight line, and extend the line.



b) What does the ordered pair (0, 20 000) represent?

The value of the new car (value at $t = 0$)

c) Use the graph to estimate the t-intercept. What does the t-intercept represent?

t-int = 16
The car has no value after 16 years

d) Use the graph to estimate the value of the car after

interpolation extrapolation extrapolation

i) 3 years **ii)** 10 years **iii)** 14 years.

$16000 $7500 $2500

e) Use the formula to verify **d) ii)**. $t = 10$

$V = 20000 - 1250(10)$
$V = 20000 - 12500$
$V = \$7500$

f) Use the graph to estimate when the car will be worth

i) $5 000 12 years **ii)** half of the purchase price: $10000 ✓ 8 years

g) Use the formula to verify **f) ii)**.

$10000 = 20000 - 1250t$
$-10000 = -1250t$
$t = 8$ years

h) Complete the following statement to describe the relation:

The original value of the car is $20000. It depreciates in value by $1250 per year and has no value after 16 years.

In this lesson, using algebra determines the exact values for intercepts, etc. whereas using graphs gives an **estimate** for intercepts, etc. In lesson 5 we use the features of a graphing calculator to determine more accurate results from a graph.

In part **d)i)** we were asked to use the graph to find values lying between given points. This process is called **interpolation**. Extending the graph to predict values outside the plotted points is called **extrapolation**. Examples of extrapolation are **d)ii)** and **d)iii)**.

Complete Assignment Questions #4 - #9

Assignment

1. Determine the value of the *y*-intercept of the graph of each equation.

a) $y = x - 5$ **b)** $y = 3x - 15$ **c)** $2y + 3x - 12 = 0$

d) $0.5x - 2.4y + 0.8 = 0$ **e)** $2y = x^2 - 60$ **f)** $y = 0.001x^2 - 0.001x + 12.44$

2. Determine the value of the *x*-intercept(s) of the graph of each equation.

a) $y = x - 2$ **b)** $y = 2x - 8$ **c)** $3y + 2x - 12 = 0$

d) $0.6x - 2y + 0.5 = 0$ **e)** $y = x^2 - 9$ **f)** $y = 12 - 3x$