Prime Factorization and Exponents Lesson #3:
Powers with Whole Number Exponents *(pg.19)*

In this lesson we review numbers written as powers, and the exponent laws applied to powers with numerical bases and whole number exponents.
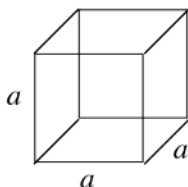We extend the work to consider bases which are variable.

Exponents

In mathematics, exponents are used as a ==short way to write repeated multiplication==.
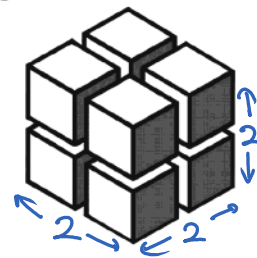
expanded form

The number of small cubes in the diagram can be calculated by the ==repeated multiplication $2 \times 2 \times 2$==.
This can be written in **exponential form** as 2^3.

Exponents can also be used with variables.
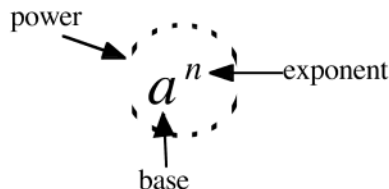
The volume of the cube to the left can be determined by repeated multiplication $a \times a \times a$ or in exponential form a^3.

Powers

A **power** is a ==number written in exponential form==.

It consists of a **base** and an **exponent**.

power → a^n ←exponent
base

Class Ex. #1 State the base and the exponent in each of the following powers.

a) 4^5 base 4
         exponent 5

b) $(-3)^6$ base -3
           exponent 6

c) x^y base x
        exponent y

Note A number that multiplies a variable is called a ==coefficient==.
In the expression $7p^3$ the coefficient is 7.

Class Ex. #2 State the coefficient in each of the of the following.

a) $8x^2$ 8

b) $-3z^9$ -3

c) $\dfrac{1a^8}{7} = \dfrac{1}{7}a^8$ $\dfrac{1}{7}$

Note

Note that written as a repeated multiplication $7p^3 = 7 \times p \times p \times p$, whereas $(7p)^3 = 7p \times 7p \times 7p = 7 \times p \times 7 \times p \times 7 \times p = 7 \times 7 \times 7 \times p \times p \times p = 343p^3$.

* Coefficient is not repeated

7 is part of the base

Class Ex. #3

Write each of the following as a repeated multiplication.

a) $3a^4b$ *just a is the base*

$3 \times a \times a \times a \times a \times b$

b) $3ab^4$

$3 \times a \times b \times b \times b \times b$

c) $3(ab)^4$

$= 3 \times ab \times ab \times ab \times ab$
$= 3 \times a \times a \times a \times a \times b \times b \times b \times b$

d) $(3ab)^4$

$= 3ab \times 3ab \times 3ab \times 3ab$
$= 3 \times 3 \times 3 \times 3 \times a \times a \times a \times a \times b \times b \times b \times b$

Evaluating Powers

$10^3 = 10 \times 10 \times 10 = 1000$

$(-6)^2 = (-6) \times (-6) = 36$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$-6^2 = -(6 \times 6) = -36$

not part of the base

The Zero Exponent

Complete the patterns below by adding one more row.

$10^4 = 10000$ $\quad) \div 10$
$10^3 = 1000$ $\quad) \div 10$
$10^2 = 100$ $\quad) \div 10$
$10^1 = 10$ $\quad) \div 10$
$10^0 = 1$

$3^4 = 81$ $\quad) \div 3$
$3^3 = 27$ $\quad) \div 3$
$3^2 = 9$ $\quad) \div 3$
$3^1 = 3$ $\quad) \div 3$
$3^0 = 1$

The results above are examples of a general rule when a base is raised to the exponent zero.
Complete: $a^0 = \underline{1}$.

Class Ex. #4

Evaluate the following.   a) 6^0 b) $(-9)^0$ c) $-9^0 = -1$ d) $2(6^2)^0 = 2(1)$

1 1 *not part of the base* $= 2$

Complete Assignment Questions #1 - #7

The Exponent Laws

The exponent laws with whole number exponents and numerical bases were covered in previous math courses.

The chart below extends the exponent laws to bases which are variables.

Complete the table as a review of the exponent laws.

Numerical Bases	Variable Bases	Exponent Laws
$8^3 \times 8^2 = (8 \cdot 8 \cdot 8)(8 \cdot 8)$ $= 8^5$ or 8^{3+2}	$a^3 \times a^2 = (a \cdot a \cdot a)(a \cdot a)$ $= a^5$ or a^{3+2}	**Product Law** $(a^m)(a^n) = a^{m+n}$
$8^3 \div 8^2 = \dfrac{\cancel{8} \cdot \cancel{8} \cdot 8}{\cancel{8} \cdot \cancel{8}}$ $= 8^1$ or 8^{3-2}	$a^3 \div a^2 = \dfrac{\cancel{a} \cdot \cancel{a} \cdot a}{\cancel{a} \cdot \cancel{a}}$ $= a^1$ or a^{3-2}	**Quotient Law** $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$
$(8 \cdot 7)^3 = (8 \cdot 7)(8 \cdot 7)(8 \cdot 7)$ $= (8 \cdot 8 \cdot 8)(7 \cdot 7 \cdot 7)$ $= 8^3 \cdot 7^3$	$(a \cdot b)^3 = (a \cdot b)(a \cdot b)(a \cdot b)$ $= (a \cdot a \cdot a)(b \cdot b \cdot b)$ $= a^3 b^3$	**Power of a Product Law** $(ab)^m = a^m b^m$
$\left(\dfrac{8}{7}\right)^3 = \left(\dfrac{8}{7}\right)\left(\dfrac{8}{7}\right)\left(\dfrac{8}{7}\right)$ $= \dfrac{8^3}{7^3}$	$\left(\dfrac{a}{b}\right)^3 = \left(\dfrac{a}{b}\right)\left(\dfrac{a}{b}\right)\left(\dfrac{a}{b}\right)$ $= \dfrac{a^3}{b^3}$	**Power of a Quotient Law** $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ $(b \neq 0)$
$(8^3)^2 = (8^3)(8^3)$ $= (8 \cdot 8 \cdot 8)(8 \cdot 8 \cdot 8)$ $= 8^6$ or $8^{3 \times 2}$	$(a^3)^2 = (a^3)(a^3)$ $= (a \cdot a \cdot a)(a \cdot a \cdot a)$ $= a^6$ or $a^{3 \times 2}$	**Power of a Power Law** $(a^m)^n = a^{mn}$

handwritten annotation: bases must be the same

Class Ex. #5   Use the exponent laws to simplify and then evaluate.

$3 \boxed{x^y} 6$

$\boxed{\wedge}$

Class Ex. #5   Use the exponent laws to simplify and then evaluate.

a) $3^4 \cdot 3^2 = 3^{4+2}$
$= 3^6$
$= 729$

b) $\dfrac{(-2)^5}{(-2)^3} = (-2)^{5-3}$
$= (-2)^2$
$= 4$

c) $(5^2)^3 = 5^{2 \times 3}$
$= 5^6$
$= 15625$

Class Ex. #6   Use the exponent laws to simplify.

a) $(a)^4(a)^3$ **b)** $x^6 x$ **c)** $b^7 \times b^0 \times b^4$ **d)** $\dfrac{x^8}{x^4} = x^{8-4}$ **e)** $\dfrac{y^{10}}{y^2} = y^{10-2}$ **f)** $(a^4)^3$

$= a^7$ $= x^7$ $= b^{11}$ $= a^{12}$

g) $(y^5)^5$ **h)** $\left(\dfrac{x}{y}\right)^9$ **i)** $\left(\dfrac{c}{4}\right)^2 = \dfrac{c^2}{4^2}$ **j)** $(st)^6$ **k)** $(2a)^5$ **l)** $(-3pq)^4$

$= y^{5 \times 5}$ $= \dfrac{x^9}{y^9}$ $= \dfrac{c^2}{16}$ $= s^6 t^6$ $= 2^5 a^5$ $= (-3)^4 p^4 q^4$

$= y^{25}$ $= 32a^5$ $= 81 p^4 q^4$

Complete Assignment Questions #8 - #22

Assignment

1. State the base and the exponent in each of the following powers.

     a) 8^3 **b)** k^{15} **c)** 2^x **d)** $(-x)^4$ **e)** $\left(\dfrac{3}{4}\right)^6$

2. State the coefficient in each of the following.

     a) $5x^7$ **b)** $-6z^2$ **c)** a^3 **d)** $\dfrac{y^3}{4}$ **e)** $\dfrac{5y^9}{8}$

3. Write each of the following as a repeated multiplication.

     a) c^4 **b)** $5x^3$ **c)** $(ab)^2$ **d)** $(-5)^3$ **e)** $s^2 t$

     f) $2\left(\dfrac{5}{4}\right)^3$ **g)** $(4a)^3$ **h)** $3cd^2$ **i)** $3(cd)^2$ **j)** $(3cd)^2$