

## The General Term of an Arithmetic Sequence

Insert 2 numbers between 2 and 20 so that the four numbers form an arithmetic sequence.

$$\begin{array}{c}
 \xrightarrow{+d} \quad \xrightarrow{+d} \quad \xrightarrow{+d} \\
 t_1 \rightarrow 2, \quad \_, \quad \_, \quad 20 \leftarrow t_4 \\
 2 + 3d = 20 \\
 \uparrow \quad \uparrow \quad \uparrow \\
 a + (n-1)d
 \end{array}$$

$$2 + 3d = 20$$

$$3d = 18$$

$$d = 6$$

$$2 + 6 = 8$$

$$8 + 6 = 14$$

$$14 + 6 = 20$$

8, 14 are  
the two  
numbers

This leads to an equation for the general term:

$$t_n = a + (n-1)d$$

where  $a$  = the first term

$n$  = number of terms

$d$  = common difference

Ex. 1) Given the sequence 2, 9, 16, ...

a) Determine a simplified equation for the general term.

$$\begin{aligned}
 t_n &= 2 + (n-1) \cdot 7 \\
 &= 2 + 7n - 7
 \end{aligned}$$

$$t_n = 7n - 5$$

b) Determine  $t_{30}$ .

$$\begin{aligned}
 t_{30} &= 7(30) - 5 \\
 &= 210 - 5 = \boxed{205}
 \end{aligned}$$

c) Determine  $t_{100}$ .

$$\begin{aligned}
 t_{100} &= 7(100) - 5 \\
 &= 700 - 5 = \boxed{695}
 \end{aligned}$$

Ex. 2) -65 is a term in the sequence 19, 13, 7, .... Which term is it?

$$\begin{aligned}
 t_n &= 19 + (n-1)(-6) \\
 &= 19 - 6n + 6
 \end{aligned}$$

$$t_n = 25 - 6n$$

$$\begin{aligned}
 -65 &= 25 - 6n \\
 -25 &\quad -25
 \end{aligned}$$

$$\begin{aligned}
 -90 &= -6n \\
 -6 &\quad -6 \\
 n &= 15
 \end{aligned}$$

term 15 is -65

$$\hookrightarrow t_{15} = -65$$

Ex. 3) In an arithmetic sequence, the 5<sup>th</sup> term is 53 and the 12<sup>th</sup> term is 102.

a) What is the first term?

$$t_5 = 53 \quad t_{12} = 102$$

Diagram showing an arithmetic sequence with 8 terms between  $t_5$  and  $t_{12}$ . The terms are represented by horizontal lines with arcs above them, indicating a constant difference. The first term is labeled  $t_5$  and the last term is labeled  $t_{12}$ .

$$49 \div 7 = 7 \leftarrow \text{common difference}$$

$$t_4 = 53 - 7 = 46$$

$$t_3 = 46 - 7 = 39$$

$$t_2 = 39 - 7 = 32$$

$$t_1 = 32 - 7 = 25$$

$$\therefore t_1 = 25$$

OR

$$53 = a + (5-1)(7)$$

$$53 = a + 4(7)$$

$$53 = a + 28$$

$$a = 53 - 28$$

$$a = 25$$

b) Write the general term.

$$t_n = 25 + (n-1)(7)$$

$$= 25 + 7n - 7$$

$$t_n = 7n + 18$$

c) How many terms in the sequence are less than 150?

$$150 > 7n + 18$$

$$\begin{array}{r} 150 \\ -18 \\ \hline \end{array} > \begin{array}{r} 7n + 18 \\ -18 \\ \hline \end{array}$$

$$\frac{132}{7} > \frac{7n}{7}$$

$$18.86 > n$$

$\therefore$  There are 18 terms less than 150