The General Term of an Arithmetic Sequence

Insert 2 numbers between 2 and 20 so that the four numbers form an arithmetic sequence.

$2, \_, \_, 20$ (+d, +d, +d)

$t_1 = 2$, $t_4 = 20$

$2 + 3d = 20$

$2 + 6 = 8$

$8 + 6 = 14$

$14 + 6 = 20$

8, 14 are the two numbers

$2 + 3d = 20$

$3d = 18$

$d = 6$

$2 + 3d = 20$

$a + (n-1)\,d$

This leads to an equation for the general term:

$t_n = a + (n-1)d$	where a = the first term n = number of terms d = common difference

Ex. 1) Given the sequence 2, 9, 16, ... (+7, +7)

a) Determine a simplified equation for the general term.

$t_n = 2 + (n-1)\cdot 7$

$= 2 + 7n - 7$

$t_n = 7n - 5$

b) Determine t_{30}.

$t_{30} = 7(30) - 5$

$= 210 - 5 = 205$

c) Determine t_{100}.

$t_{100} = 7(100) - 5$

$= 700 - 5 = 695$

Ex. 2) - 65 is a term in the sequence 19, 13, 7, ... Which term is it? (-6, -6)

$t_n = 19 + (n-1)(-6)$

$= 19 - 6n + 6$

$t_n = 25 - 6n$

$-65 = 25 - 6n$

$-25 \quad -25$

$\frac{-90}{-6} = \frac{-6n}{-6}$

$n = 15$

term 15 is -65

$t_{15} = -65$

Ex. 3) In an arithmetic sequence, the 5th term is 53 and the 12th term is 102.

a) What is the first term?

$t_5 = 53$ $t_{12} = 102$

t_5 53, —, —, —, —, —, —, t_{12} 102

49

$49 \div 7 = 7$ ← common difference

$t_4 = 53 - 7 = 46$

$t_3 = 46 - 7 = 39$

$t_2 = 39 - 7 = 32$

$t_1 = 32 - 7 = 25$

$\therefore t_1 = 25$

OR

$53 = a + (5-1)(7)$

$53 = a + 4(7)$

$53 = a + 28$

$a = 53 - 28$

$a = 25$

b) Write the general term.

$t_n = 25 + (n-1)(7)$

$= 25 + 7n - 7$

$t_n = 7n + 18$

c) How many terms in the sequence are less than 150?

$150 > 7n + 18$

$-18 \quad -18$

$\frac{132}{7} > \frac{7n}{7}$

$18.86 > n$

$\therefore$ There are 18 terms less than 150

Practice Work: Arithmetic Sequences Worksheet