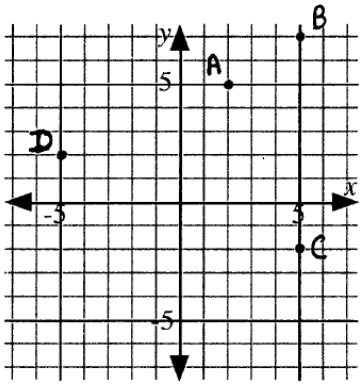
# Characteristics of Linear Relations Lesson #5:
## Parallel and Perpendicular Lines

**Warm-Up #1** *Review: Transformations*

In earlier mathematics courses we studied transformations: translations, reflections, and rotations. In order to investigate parallel and perpendicular line segments, we will review translations and rotations.

On the grid, show the image of the point $A(2, 5)$ after the following transformations. In each case write the coordinates of the image.




**a)** A translation 3 units right and 2 units up.

$A(2, 5) \rightarrow$ $B(5, 7)$

**b)** A 90° clockwise rotation about the origin.

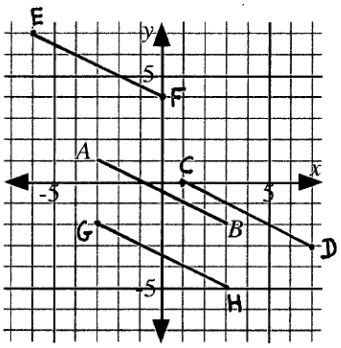
$A(2, 5) \rightarrow$ $C(5, -2)$

**c)** A 90° counterclockwise rotation about the origin.

$A(2, 5) \rightarrow$ $D(-5, 2)$

**Warm-Up #2** *Investigating Parallel Line Segments*

**a)** On the grid show the image of line segment $AB$ after the following transformations.




**i)** A translation 4 units right and 1 unit down to form line segment $CD$.

**ii)** A translation 3 units left and 6 units up to form line segment $EF$.

**iii)** A translation 3 units down to form line segment $GH$.

**b)** Calculate the slope of each of the line segments.

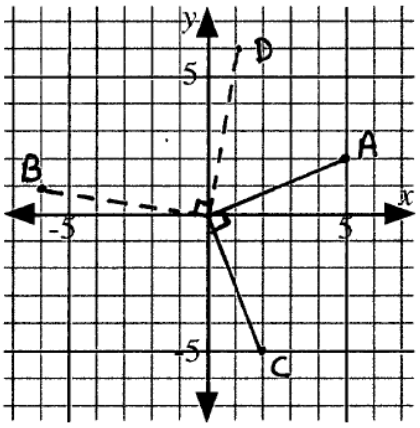
$m_{AB} = -\frac{3}{6} = -\frac{1}{2}$ $m_{CD} = -\frac{3}{6} = -\frac{1}{2}$

$m_{EF} = -\frac{3}{6} = -\frac{1}{2}$ $m_{GH} = -\frac{3}{6} = -\frac{1}{2}$

**c)** The four line segments are parallel. Make a conjecture about the slopes of parallel line segments.

Parallel line segments have the same slope.

## *Warm-Up #3* *Investigating Perpendicular Line Segments*




**a) i)** On the grid plot the point $A(5, 2)$ and draw the line joining the point to the origin, $O$.

**ii)** Rotate the line through an angle of 90° clockwise about $O$ and show the image on the grid.

**iii)** Find the slopes of the two perpendicular lines and multiply them together.

$$m_{OA} = \frac{2-0}{5-0} = \frac{2}{5}$$

$$m_{OC} = \frac{-5-0}{2-0} = -\frac{5}{2}$$

$$m_{OA} \times m_{OC} = \left(\frac{2}{5}\right)\left(-\frac{5}{2}\right) = -1$$

**b)** Repeat part a) for the point $B(-6, 1)$.

$$m_{OB} = \frac{1-0}{-6-0} = -\frac{1}{6}$$

$$m_{OD} = \frac{6-0}{1-0} = 6$$

$$m_{OB} \times m_{OD} = \left(-\frac{1}{6}\right)(6) = -1$$

**c)** Make a conjecture about the slopes of perpendicular line segments.

The slopes of perpendicular line segments have a product equal to -1.

## *Parallel Lines and Perpendicular Lines*

Recall that the slope of any line segment within a line represents the slope of the line.

Consider then two lines with slopes $m_1$ and $m_2$.

- The lines are **parallel** if they have the same slope, i.e. $m_1 = m_2$.
- The lines are **perpendicular** if the product of the slopes is $-1$, i.e. $m_1 \times m_2 = -1$ or $m_1 m_2 = -1$ or $m_1 = -\frac{1}{m_2}$
- For perpendicular lines, each slope is the negative reciprocal of the other, provided neither slope is equal to zero.

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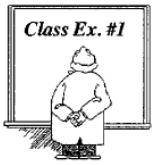


Consider line segment $AC$ with a slope of $\frac{3}{4}$.

**a)** Write the slope of line segment $GH$ which is parallel to $AC$.

$m_{GH} = \frac{3}{4}$

**b)** Write the slope of line segment $BF$ which is perpendicular to $AC$.

$m_{BF} = -\frac{4}{3}$

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The slopes of two lines are given. Determine if the lines are parallel, perpendicular, or neither.

**a)** $m_1 = \frac{1}{4}$, $m_2 = \frac{3}{12} = \frac{1}{4}$

parallel

**b)** $m_1 = \frac{5}{7}$, $m_2 = \frac{14}{10} = \frac{7}{5}$

neither

**Class Ex. #3**

If $P$ is the point $(4, 7)$ and $Q$ is the point $(6, -2)$, find the slope of a line segment

**a)** parallel to line segment $PQ$ **b)** perpendicular to line segment $PQ$

$m_{PQ} = \frac{-2-7}{6-4} = \frac{-9}{2}$ a) $-\frac{9}{2}$ b) $\frac{2}{9}$

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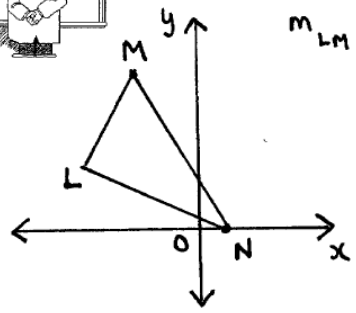
**Class Ex. #4**

$\triangle LMN$ has coordinates $L(-4, 2)$, $M(-2, 7)$, and $N(1, 0)$. Use slopes to show that the triangle is right-angled at $L$.




$m_{LM} = \frac{7-2}{-2+4} = \frac{5}{2}$ $m_{LN} = \frac{0-2}{1+4} = -\frac{2}{5}$

$m_{LM} \times m_{LN} = \left(\frac{5}{2}\right)\left(-\frac{2}{5}\right) = -1$

since the product of the slopes $= -1$

the lines LM and LN are perpendicular.

the triangle is right-angled at L.


Two lines have slopes of $-\frac{3}{4}$ and $\frac{k}{5}$ respectively. Find the value of $k$ if the lines are

**a)** parallel

$-\frac{3}{4} = \frac{k}{5}$

$-15 = 4k$

$k = \frac{-15}{4}$

**b)** perpendicular

$\left(-\frac{3}{4}\right)\left(\frac{k}{5}\right) = -1$

$\frac{-3k}{20} = -1$

$-3k = -20$

$k = \frac{20}{3}$