Exponents Lesson #3:
Integral Exponents

The Negative Exponent

a) Complete the patterns below.

$10^3 = 1000$	$3^3 = 27$	
$10^2 = 100$	$3^2 = 9$	
$10^1 = 10$	$3^1 = 3$	
$10^0 = 1$	$3^0 = 1$	$a^0 = 1$
$10^{-1} = \frac{1}{10} = \frac{1}{10^1}$	$3^{-1} = \frac{1}{3}$	$a^{-1} = \frac{1}{a}$
$10^{-2} = \frac{1}{100} = \frac{1}{10^2}$	$3^{-2} = \frac{1}{9} = \frac{1}{3^2}$	$a^{-2} = \frac{1}{a^2}$
$10^{-3} = \frac{1}{1000} = \frac{1}{10^3}$	$3^{-3} = \frac{1}{27} = \frac{1}{3^3}$	$a^{-3} = \frac{1}{a^3}$

reciprocal reciprocal reciprocal

$\left(\frac{2}{\pi} x^2 y^{3/2} z^{-8}\right)^0 = 1$

b) Write the following with positive exponents.

i) $10^{-7} = \frac{1}{10^7}$ ii) $3^{-5} = \frac{1}{3^5}$ iii) $a^{-n} = \frac{1}{a^n}$

Using the Exponent Laws to Define the Negative Exponent

Consider the expression $5^4 \div 5^7$.

a) Evaluate the expression as an exact value using a calculator.

b) Complete the following to evaluate the expression.

$$5^4 \div 5^7 = \frac{5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5} = \frac{1}{5^{3}} = 5^{-3}$$

c) Use the quotient law to complete the following.

$5^4 \div 5^7 = 5^{4-7} = 5^{-3}$

d) The results in a) to c) are examples of a general rule when a base is raised to a negative exponent. Complete: $a^{-p} = \frac{1}{a^p}$

e) Write the following with positive exponents and evaluate.

i) 2^{-1} ii) 3^{-2} iii) 4^{-3}

5 ÷ 2 = 10 5 ÷ 2 = 5 · 1 = 5

1/2 3/4 5/6

The Negative Exponent in the Denominator

$4^{-3} = \frac{1}{4^3}$

Use the rule for division of fractions to show that $\frac{1}{4^{-3}} = 4^3$. Use a calculator to confirm.

$$\frac{1}{4^{-3}} = \frac{1}{\frac{1}{4^3}} = 1 \cdot \frac{4^3}{1} = 4^3$$

Negative Exponent Law

A base (not including zero) raised to a negative exponent has the following properties:

* $a^{-n} = \frac{1}{a^n}, \ a \neq 0$ and $\frac{1}{a^{-n}} = a^n, \ a \neq 0$

$3^{-4} = \frac{1}{3^4}$ and $\frac{1}{3^{-4}} = 3^4$

Class Ex. #1

Simplify, express with positive exponents, and evaluate ~~without~~ using a calculator.

a) $4^5 \times 4^{-3}$ b) $3^2 \times 3^{-5}$ c) $\frac{1}{2^{-5}}$ d) $\frac{6^{-7}}{6^{-5}}$ e) $(2^3)^{-1}$

$= 4^{5+-3}$ $= 3^{2+-5}$ $= 2^5$ $= 6^{-7--5}$ $= 2^{-3}$

$= 4^2$ $= 3^{-3}$ $= 32$ $= 6^{-7+5}$ $= \frac{1}{2^3}$

$= \frac{1}{3^3}$ or $\frac{1}{27}$ $= 6^{-2}$ $= \frac{1}{8}$

$= \frac{1}{6^2} = \frac{1}{36}$

Class Ex. #2

Identify the following as true or false.

a) $\frac{8^3}{8^{-1}} = 8^4$ b) $\frac{8^3}{4^{-1}} = 2^4$ c) $a^{-3} = \frac{1}{a^3}$ d) $9a^{-3} = \frac{1}{9a^3}$

$= 8^{3--1}$ False bases must be the same true $9\left(\frac{1}{a^3}\right)$

$= 8^{3+1}$ $\frac{9}{a^3} \neq \frac{1}{9a^3}$

$= 8^4$

TRUE $2 - 7 = -5$

Class Ex. #3

Simplify and write the answer with positive exponents.

a) $a^{-4} \times a^{-3}$ b) $6x^2 \div 2x^7$ c) $\frac{1y^6}{2y^{-5}}$

$= a^{-4+-3}$ $3x^{-5}$ $\frac{1}{2}y^{11}$

$= a^{-7}$ $= \frac{3}{x^5}$ OR

$= \frac{1}{\ }$

$= a^{-7}$ $= \frac{1}{a^7}$ $= \frac{3}{x^5}$ OR $\frac{y^{11}}{2}$

d) $(-2x)^{-3}$ $= \frac{1}{(-2x)^3}$ $= \frac{1}{-8x^3}$ OR $\frac{-1}{8x^3}$

e) $\frac{8a^{-5}}{4b^{-3}}$ $2\frac{b^3}{a^5}$

f) $\frac{(5p)^{-2}}{5q^4}$ $= \frac{1}{(5p)^2 5q^4}$ $= \frac{1}{125p^2q^4}$

Simplify and write the answer with positive exponents.

① rewrite as a multiplication

$5x^3y^{-8}z^{-2} \div \frac{15x^8y^3z^{-1}}{x^5y^{-3}z^2}$

$5x^3y^{-8}z^{-2} \cdot \frac{x^5y^{-3}z^2}{15x^8y^3z^{-1}}$

$= \frac{1}{3}\frac{x^8 y^{-11}}{x^8 y^3 z^{-1}}$ $= \frac{1}{3}\frac{y^{-14}}{z^{-1}}$ $= \frac{1}{3}\frac{z}{y^{14}}$ OR $\frac{z}{3y^{14}}$

Explain why $2p^{-3} \neq \frac{1}{2p^3}$.

$2p^{-3} = \frac{2}{p^3} \neq \frac{1}{2p^3}$

Simplifying a Fractional Base with a Negative Exponent

Consider the expression $\left(\frac{2}{3}\right)^{-4}$.

a) Complete the following $\left(\frac{2}{3}\right)^{-4} = \frac{1}{\left(\frac{2}{3}\right)^{\square}} = \frac{1}{\frac{2^4}{3^4}} = 1 \times \frac{3^4}{2^4} = \frac{81}{16}$ ↑4

b) Evaluate $\left(\frac{3}{2}\right)^4$. $\frac{3^4}{2^4} = \frac{81}{16}$

c) Classify the following statement as true or false. $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4$

d) Suggest a quick method for evaluating $\left(\frac{5}{2}\right)^{-3}$ without using a calculator.

↓

d) Suggest a quick method for evaluating $\left(\frac{5}{2}\right)^{-3}$ without using a calculator.

$$\left(\frac{2}{5}\right)^{3}$$

In general, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$ $a, b \neq 0$.